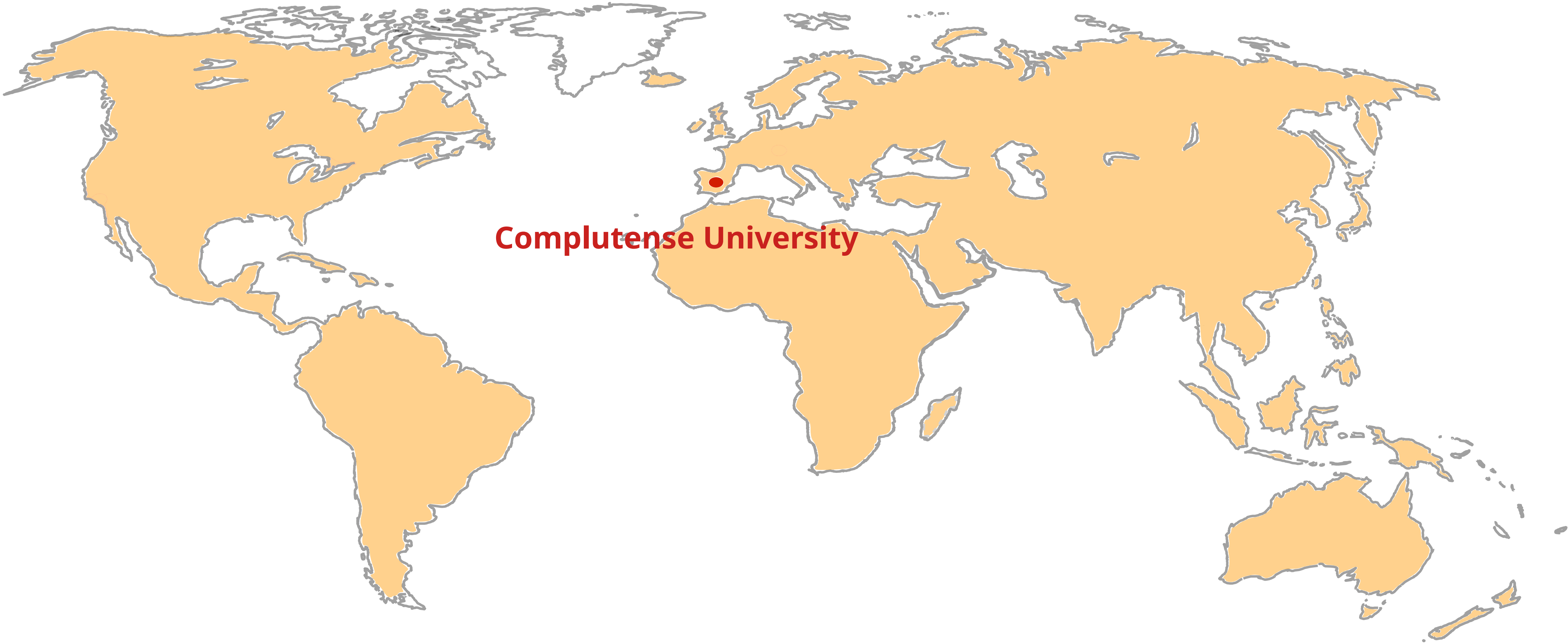
Phase transitions in the early Universe

Carlos Tamarit, Johannes Gutenberg-Universität Mainz

Career and research journey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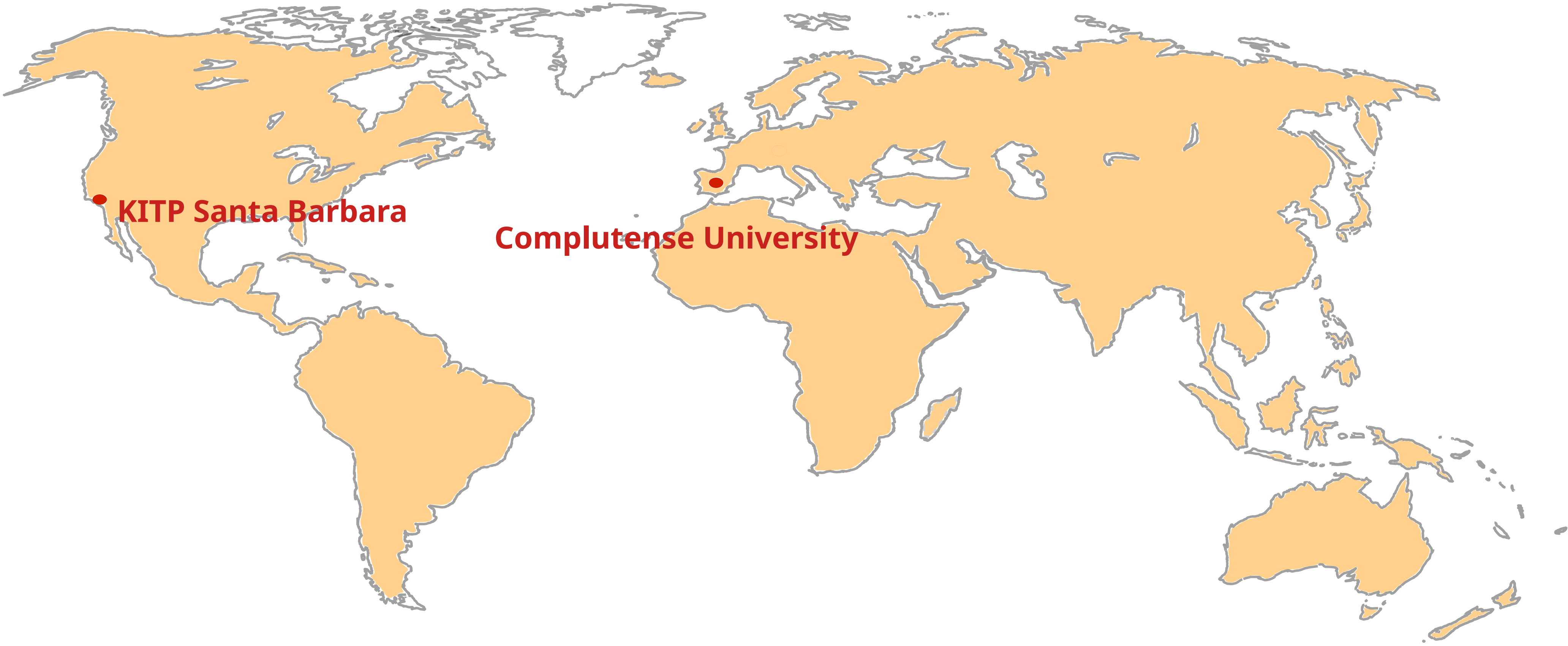
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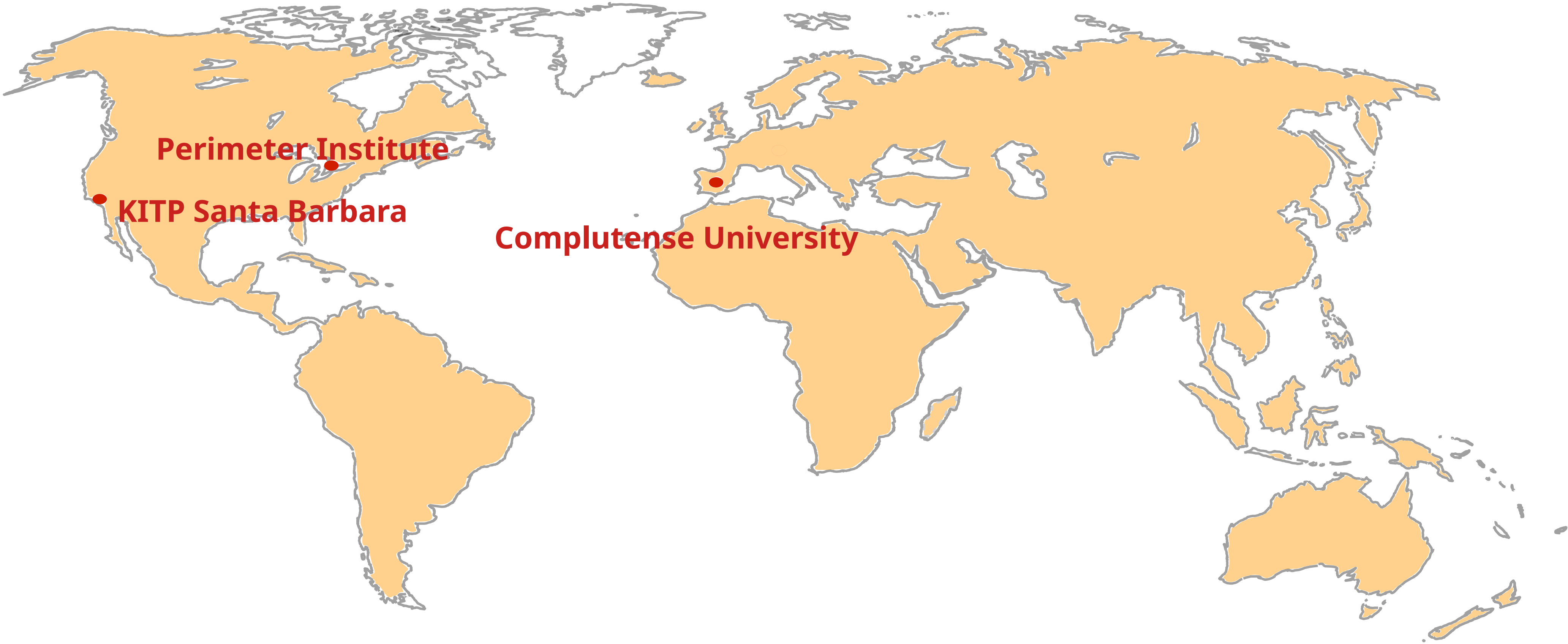
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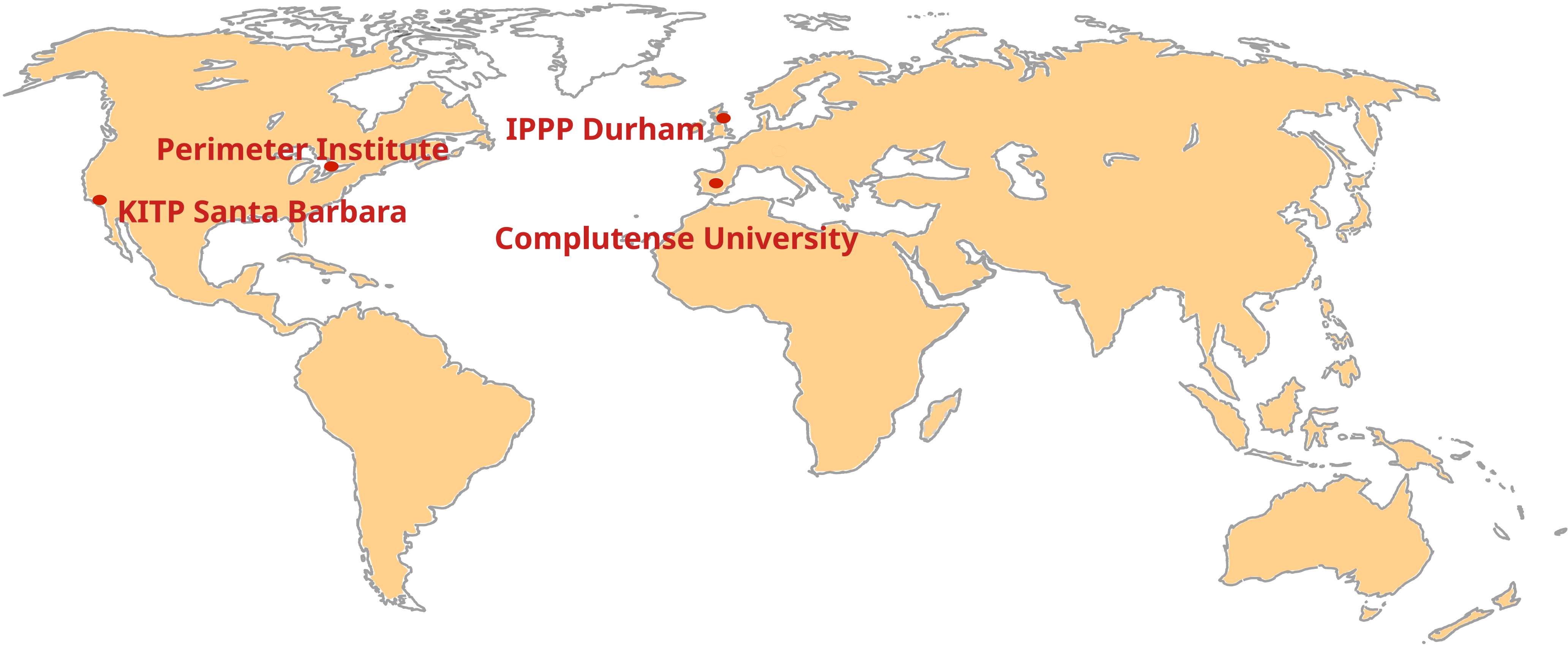

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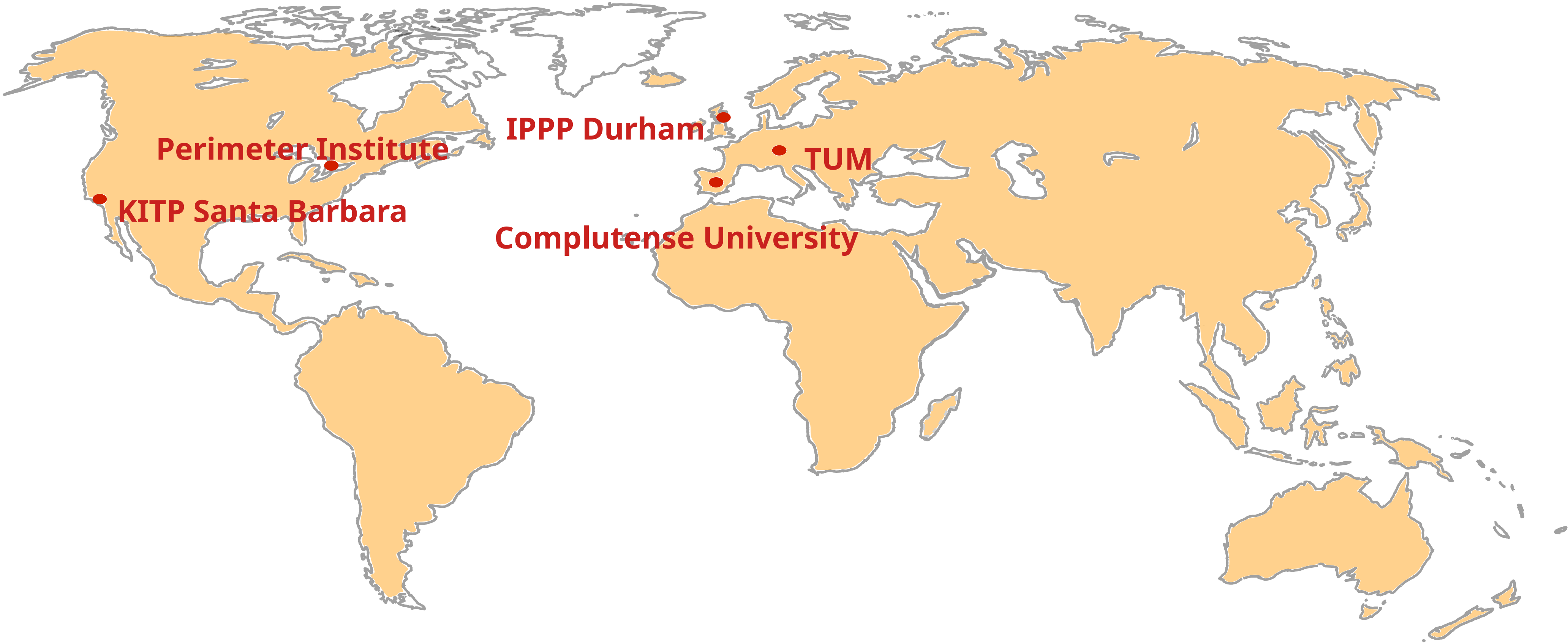

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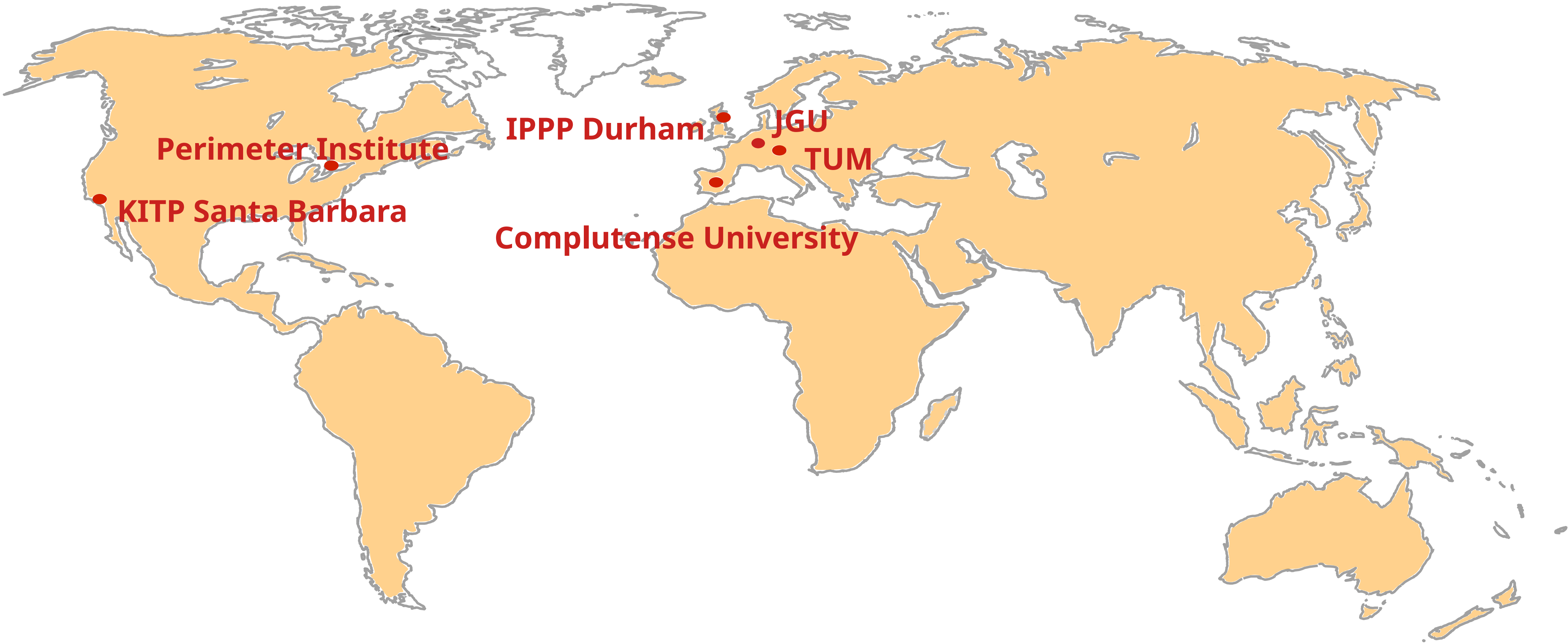

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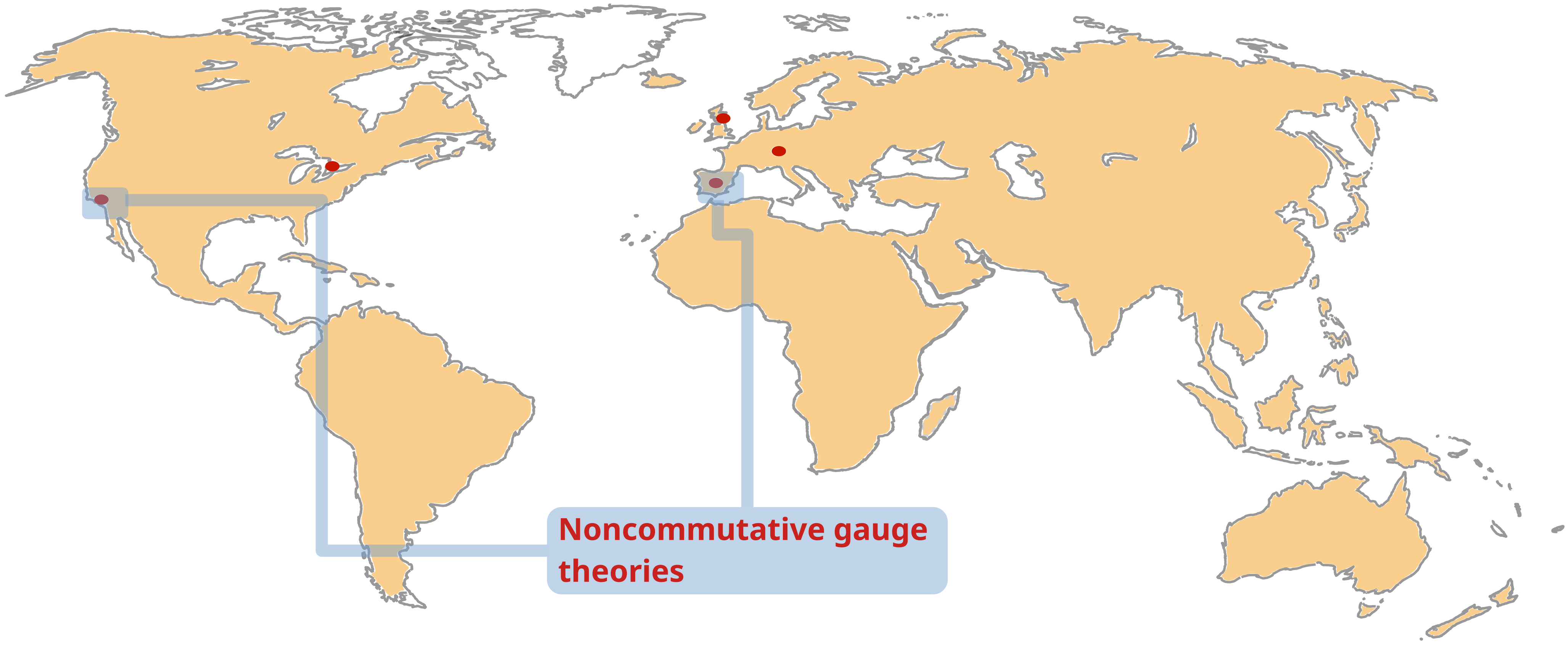
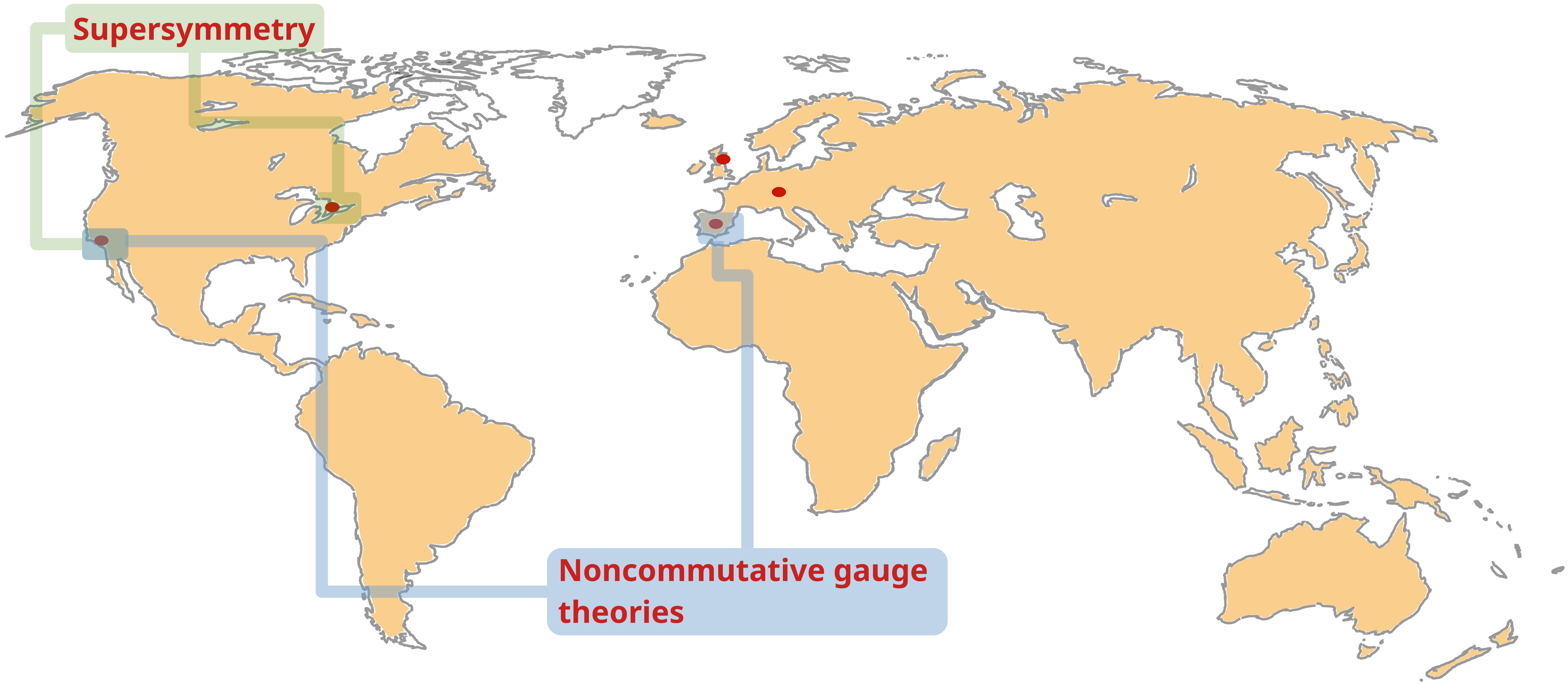
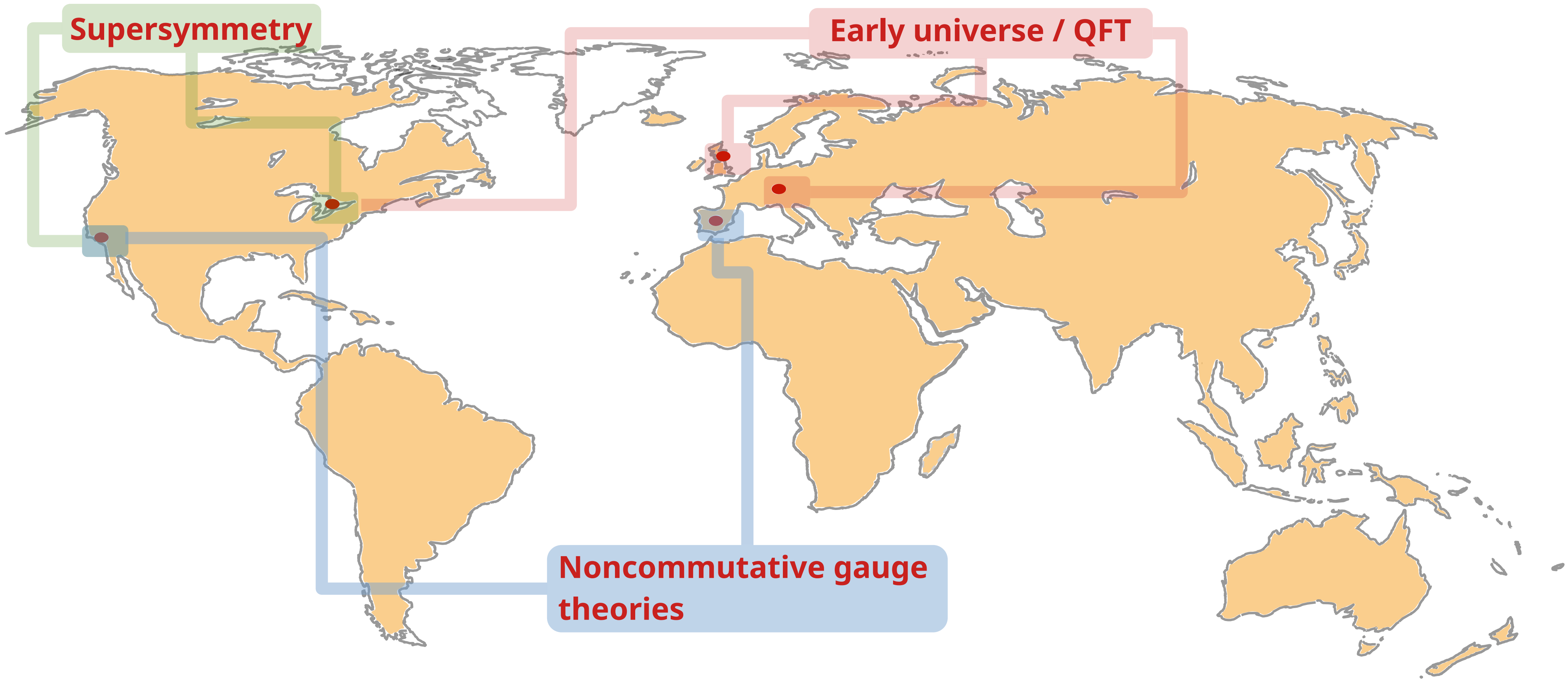

Noncommutative gauge theories


Supersymmetry
Noncommutative gauge theories


Supersymmetry
Early universe / QFT
Noncommutative gauge theories

The aim:

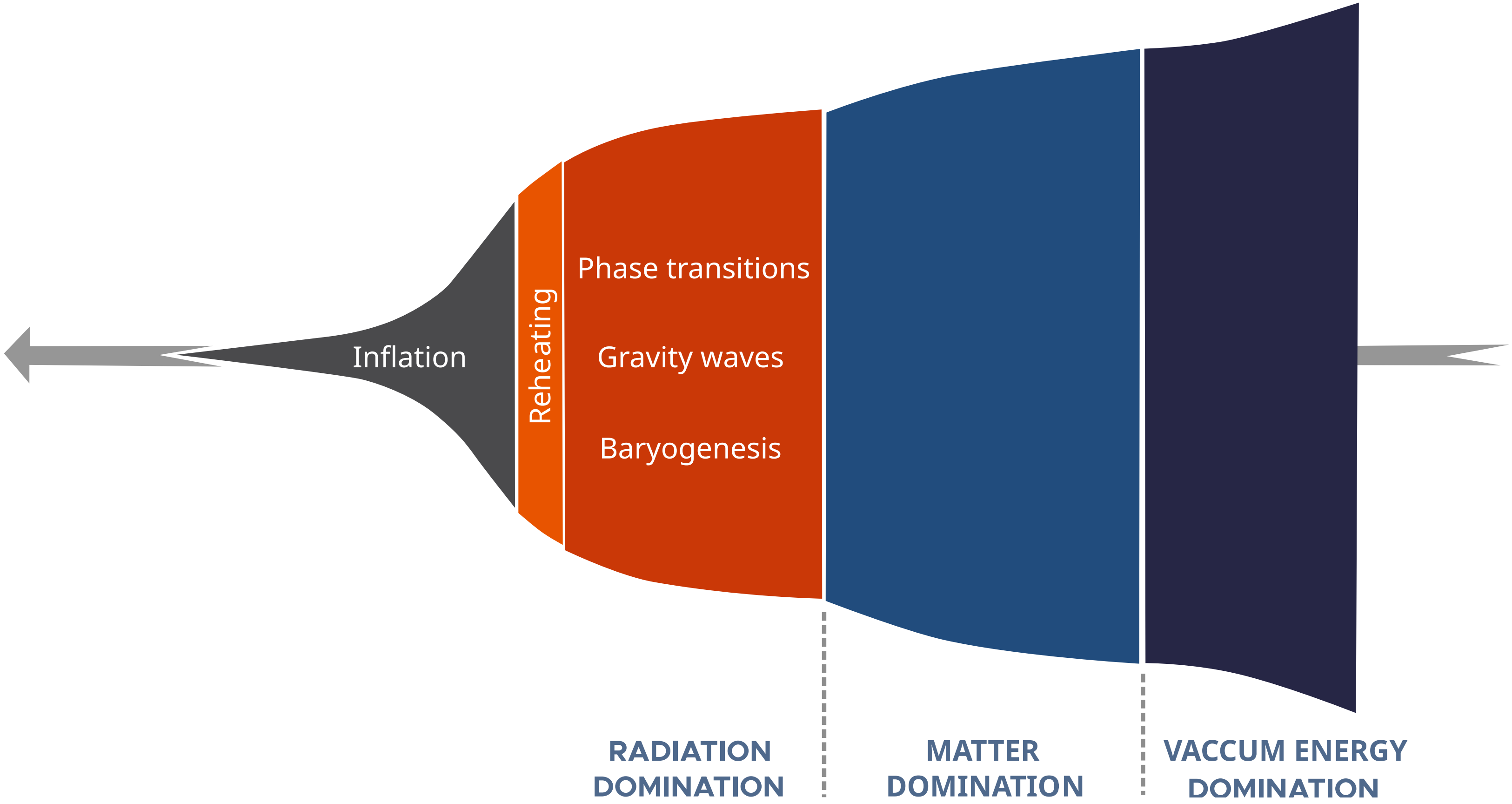
To present an **overview of phase transitions** in the early universe: why they are interesting, what is the theoretical framework used to describe them, what are some of the current **open questions** and areas of **active research**

The plan:

2. What are phase transitions, and why are they interesting?
3. The beauty and the challenges of thermal field theory
5. Bubble velocities
6. CP-violating sources in electroweak baryogenesis

1. What are phase transitions, and why are they interesting?

The contex of the early universe



The primordial plasma

It is believed that the early universe was a **hot plasma** containing

- **Standard Model** d.o.f
- **Extra** d.o.f needed to explain **neutrino masses, dark matter, baryogenesis**

Many of the of the degrees of freedom had sizable interactions, allowing them to reach **thermal equilibrium**

▶ One can characterize the bulk properties of the primordial plasma with **thermodynamical quantities**:

Temperature, pressure, energy density, free energy

A throwback to first-year physics

First principle of thermodynamics:

$$dU = TdS - pdV$$

Helmholtz free energy: energy of a closed system available for useful work:

$$F = U - TS$$

$$dF = -SdT - pdV$$

$$p = -\left.\frac{\partial F}{\partial V}\right|_T$$

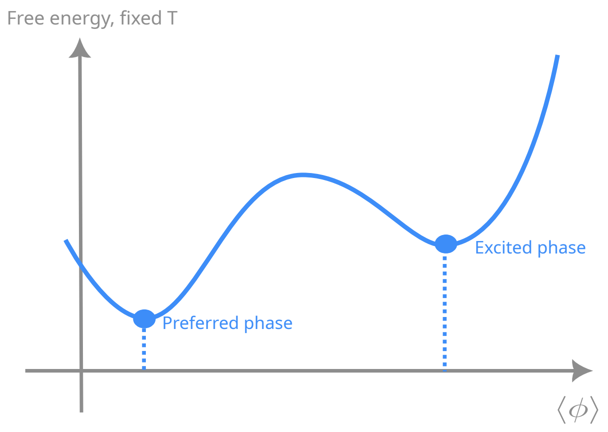
From this we identify the **pressure** as minus the **free energy density**

Phases = Minima of free energy

The plasma will tend to relax to the **minimum free energy density (max. p)**

Assuming Lorentz invariance, F will be a function of temperature T and the expectation values of scalar fields $\langle\phi\rangle$ or fermion condensates $\langle\bar{\psi}\psi\rangle$

Different **phases** ⟺ different **local minima** of free energy



Phase transitions

A **phase transition** happens when the **favoured phase** (i.e. with **lowest** F) **changes** as a function of **temperature**

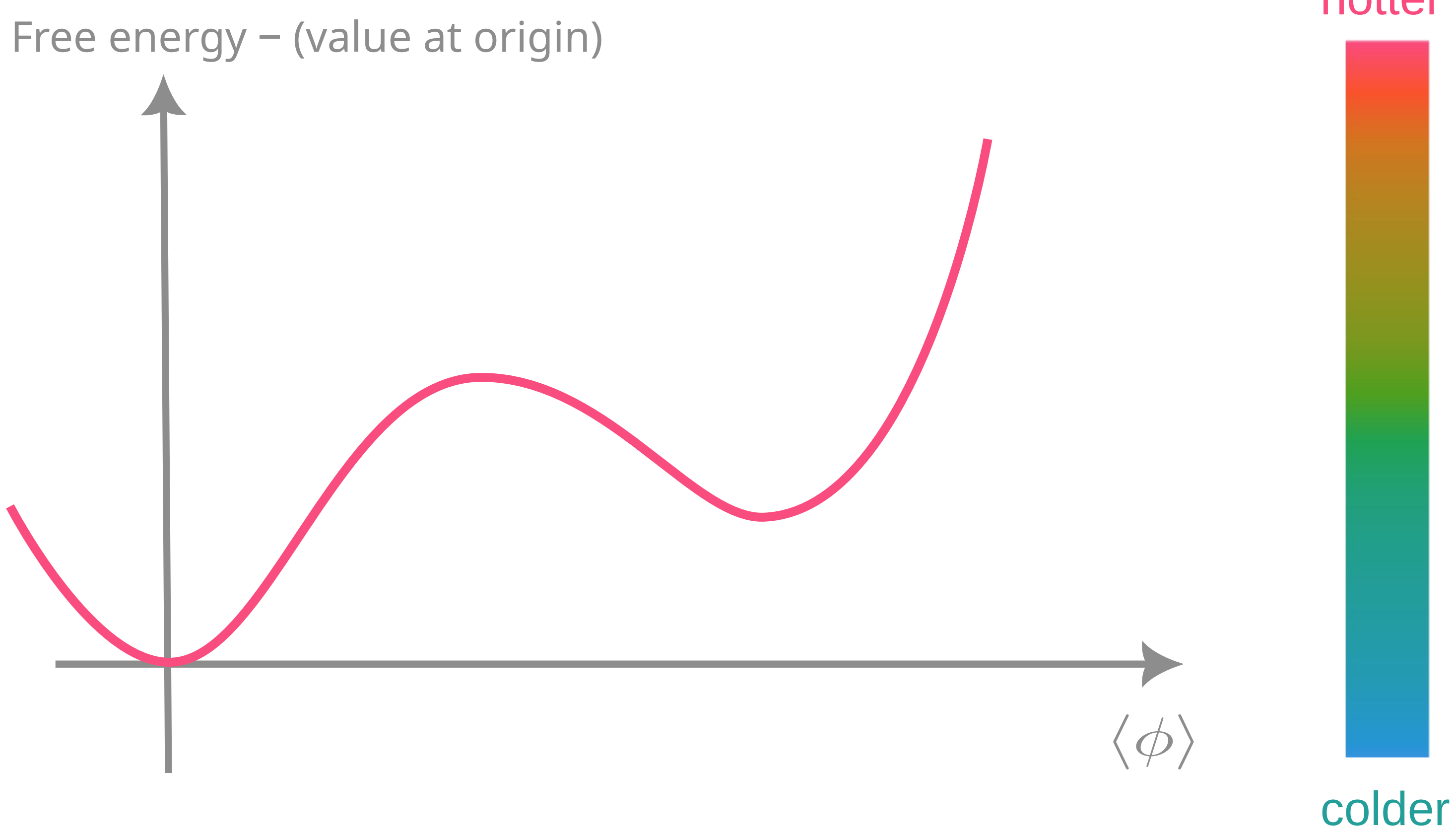
First-order phase transition

There are **two local minima** separated by a free-energy **barrier**

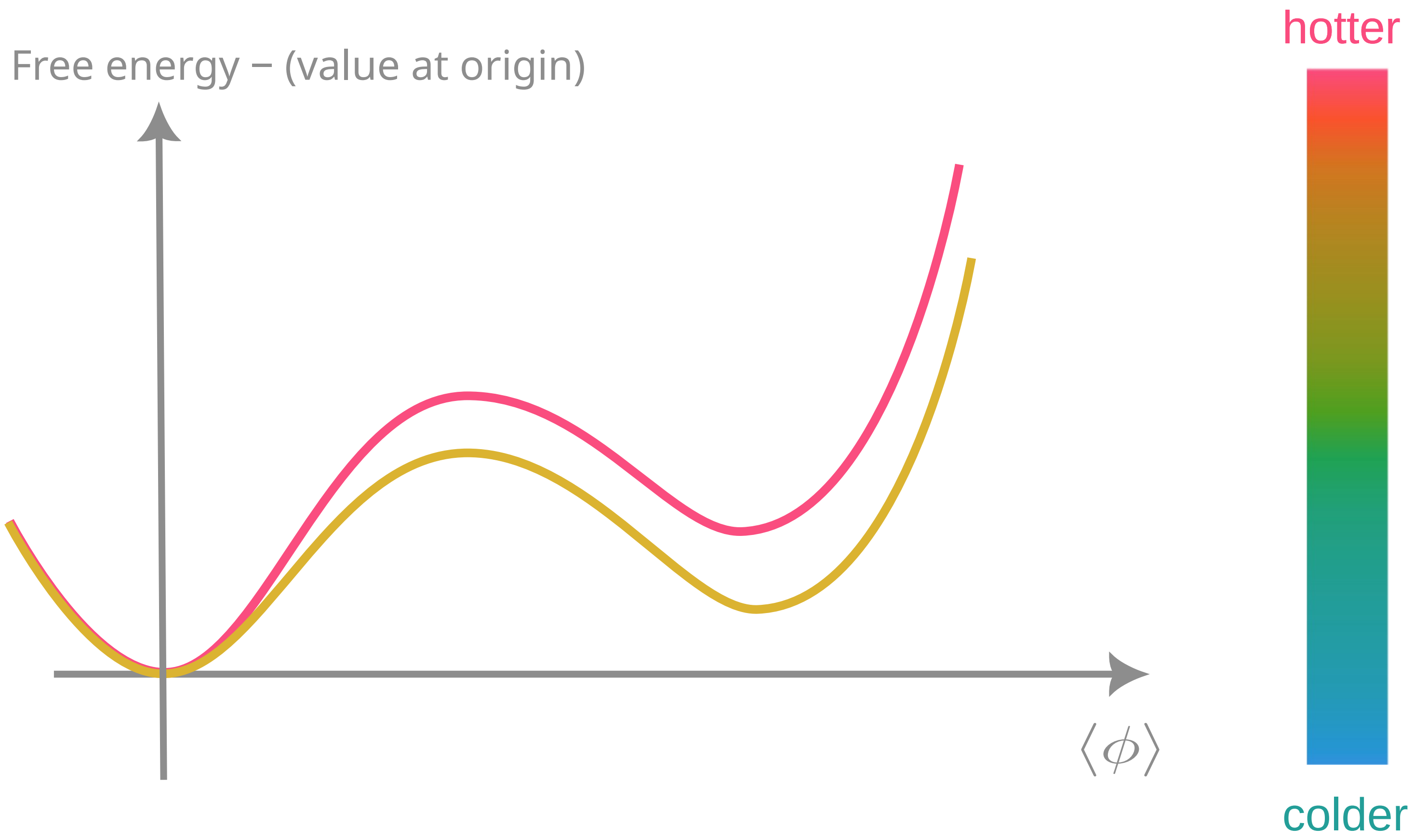
Second-order phase transitions / crossover

There is a **single minimum** whose **location changes** with temperature

First-order phase transitions



First-order phase transitions



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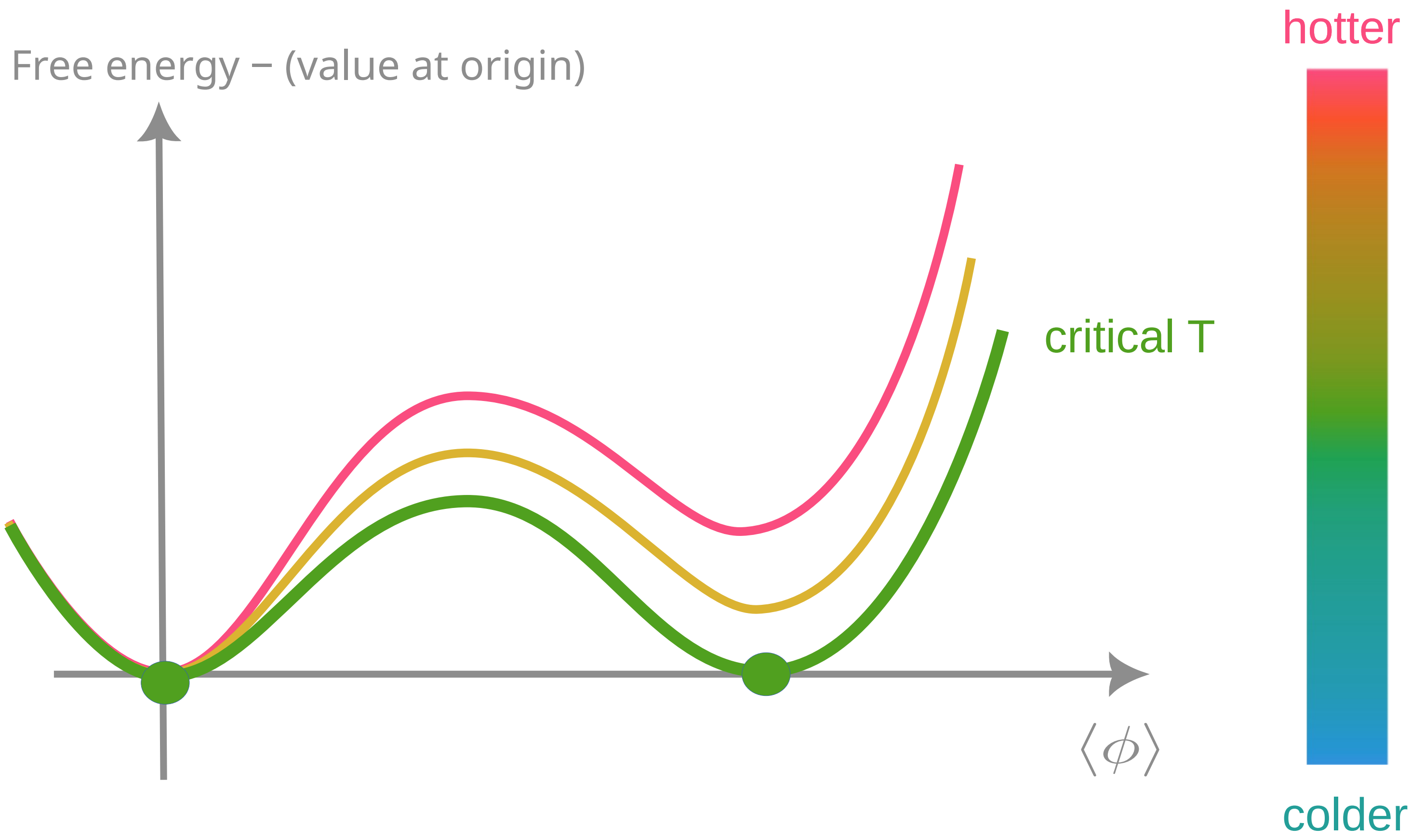

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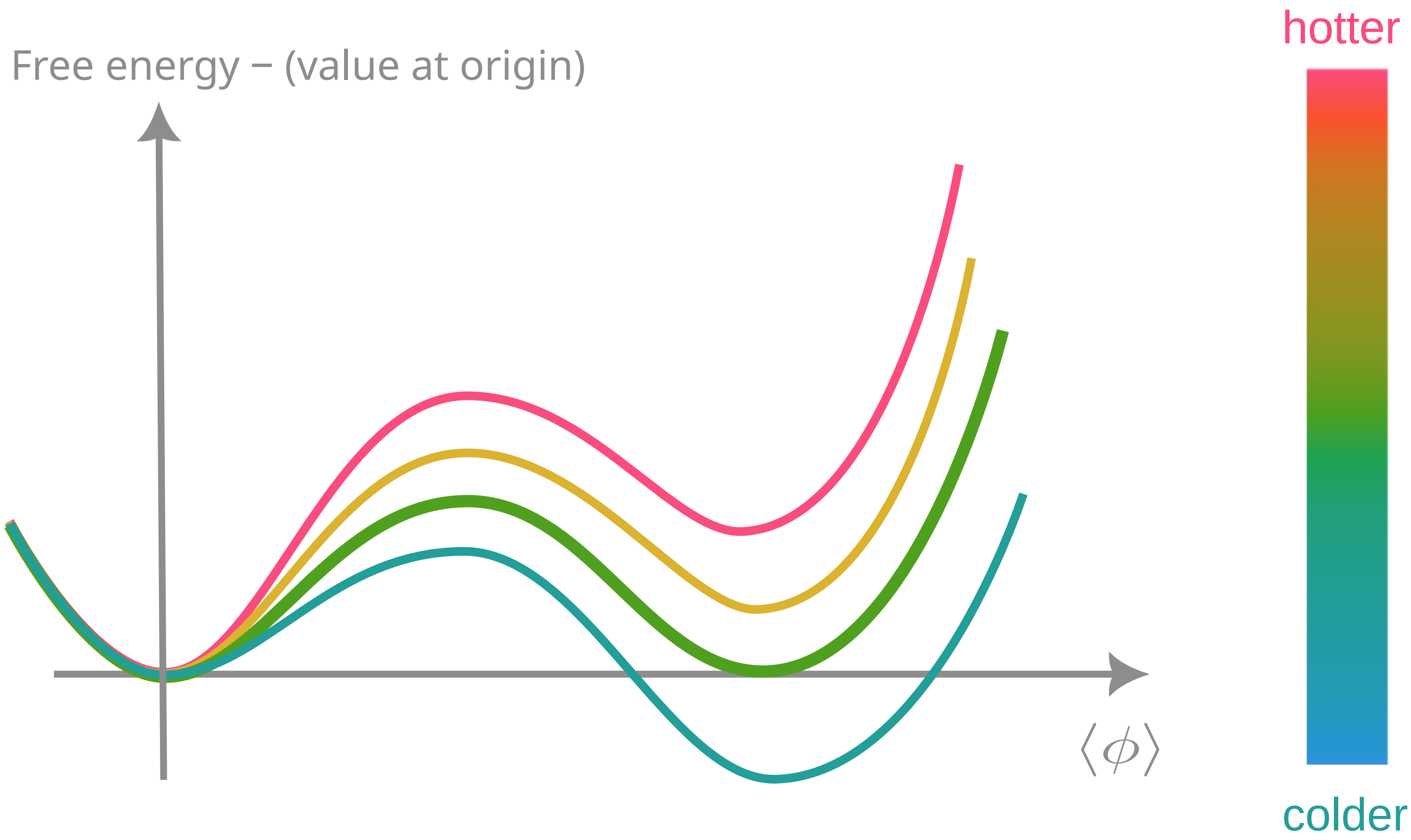

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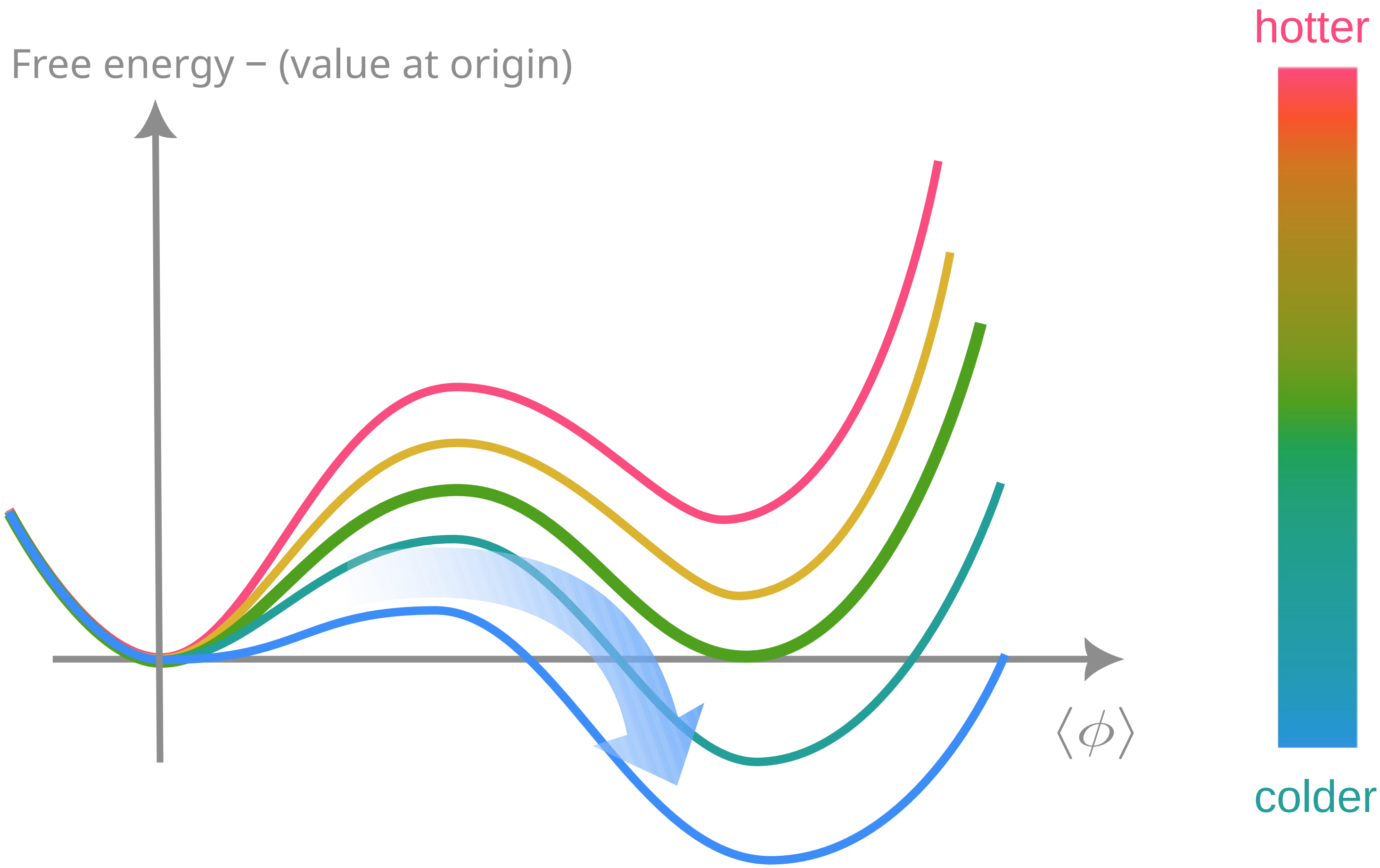
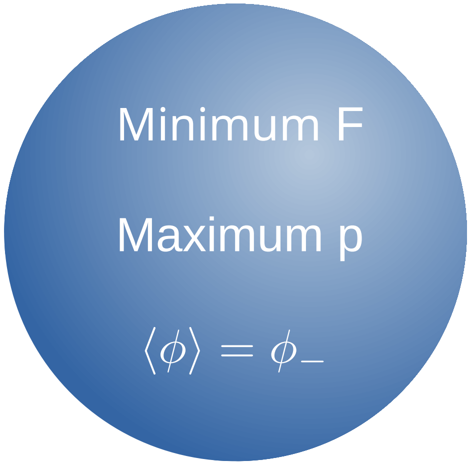


First-order phase transitions

Proceed through the **spontaneous nucleation** of bubbles of the **preferred phase**



Higher F

Lower p

$\langle \phi \rangle = \phi_+$

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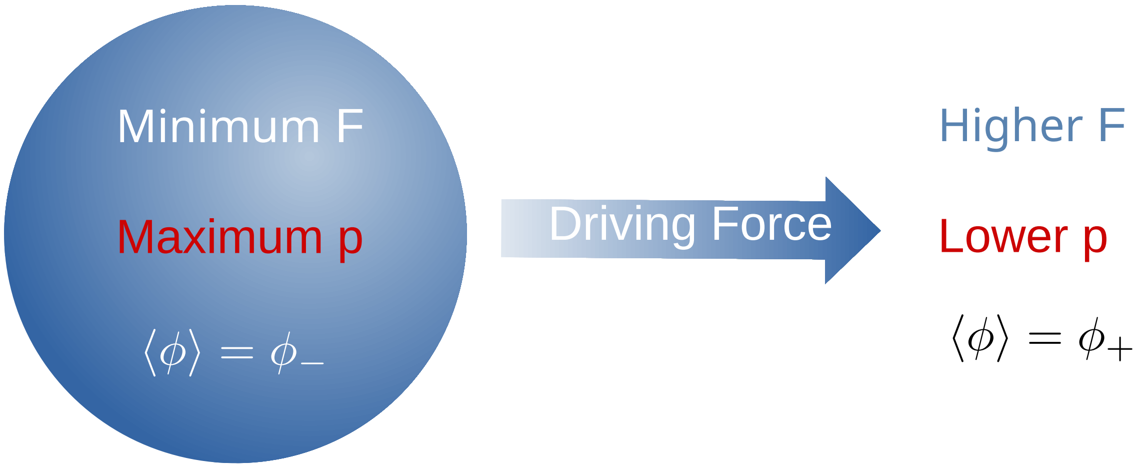

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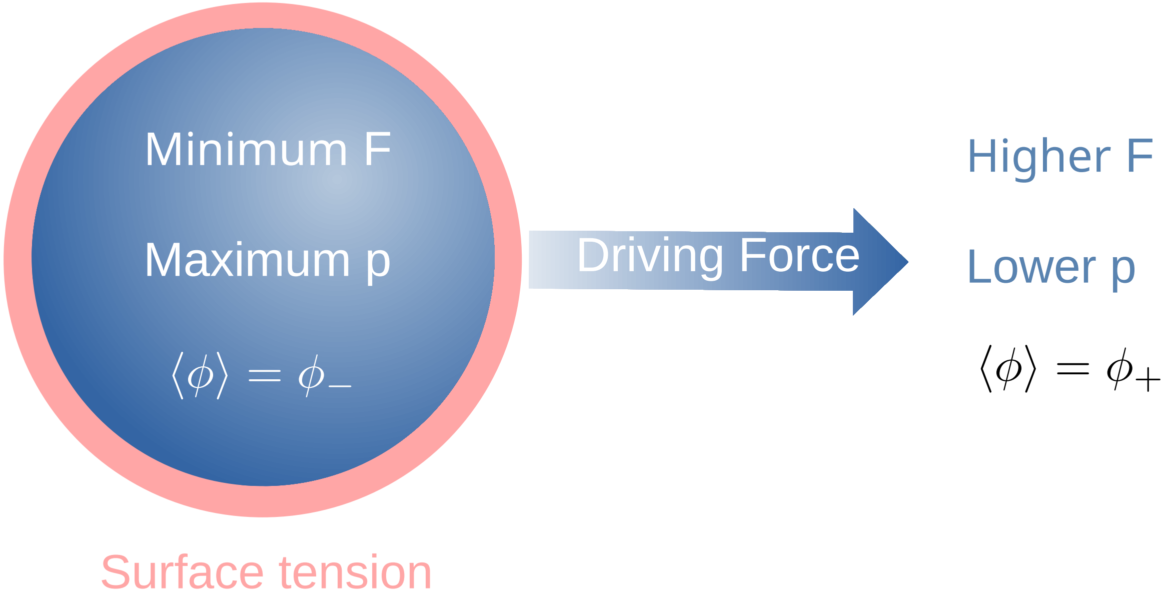

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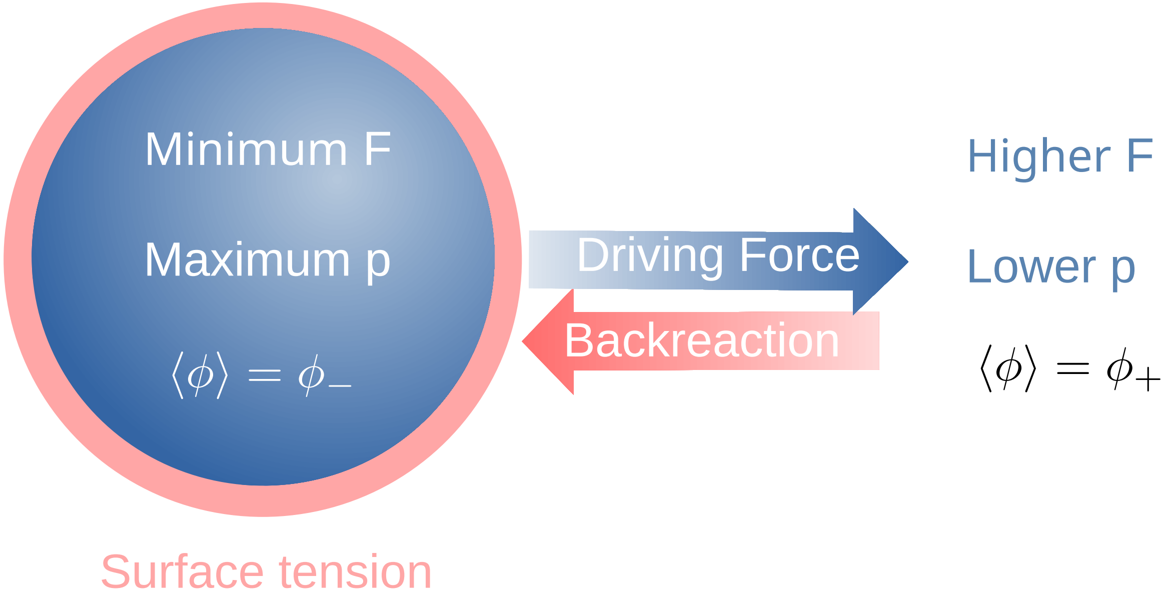
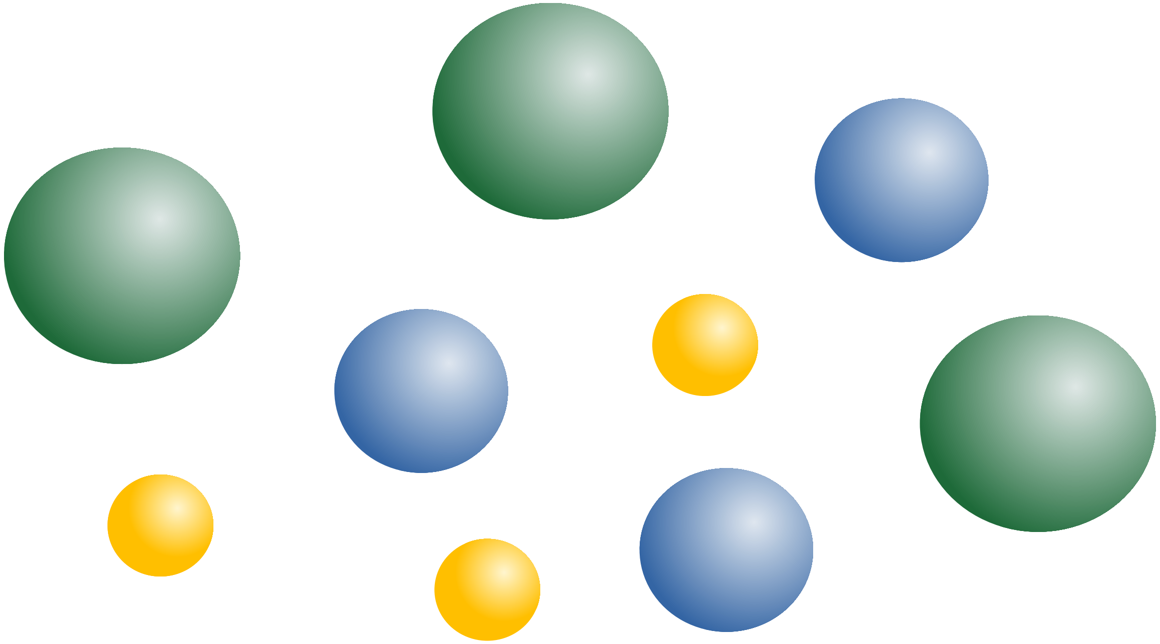


First-order phase transitions

Only for bubbles above a certain **critical size** does the driving force win



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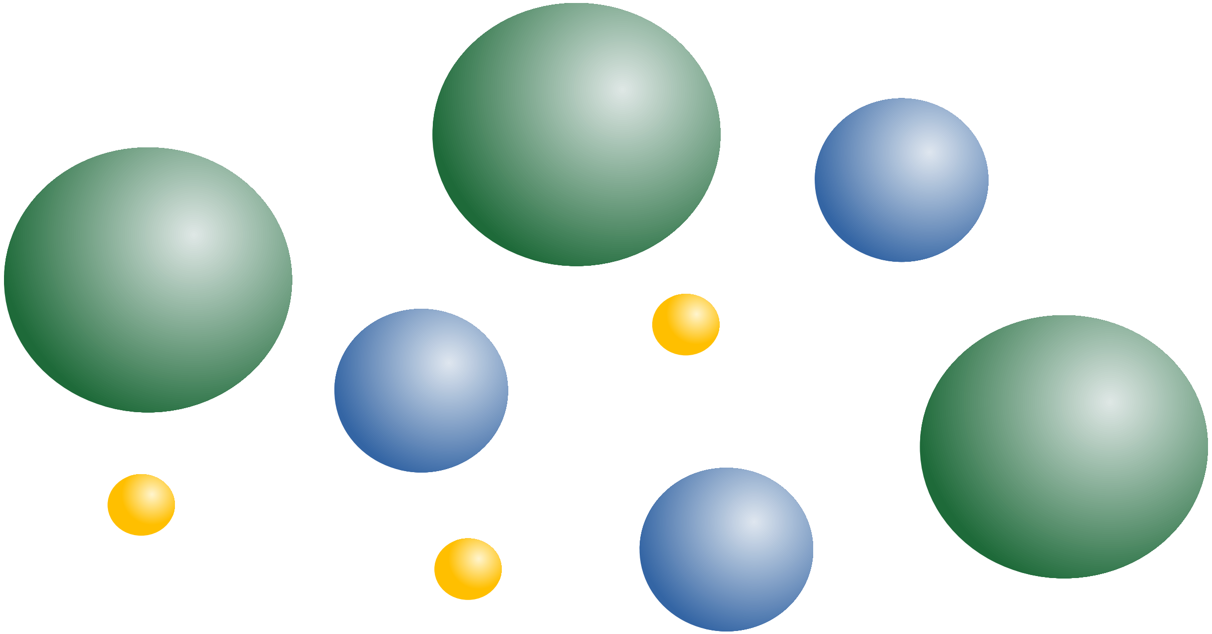

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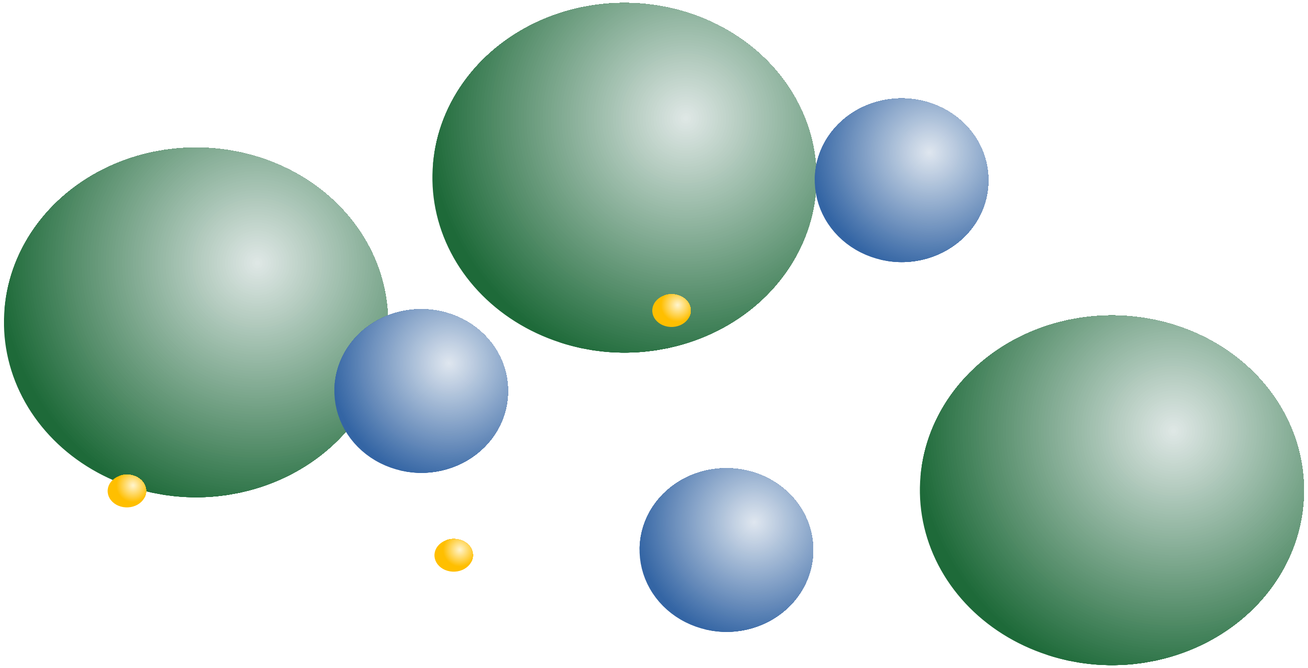

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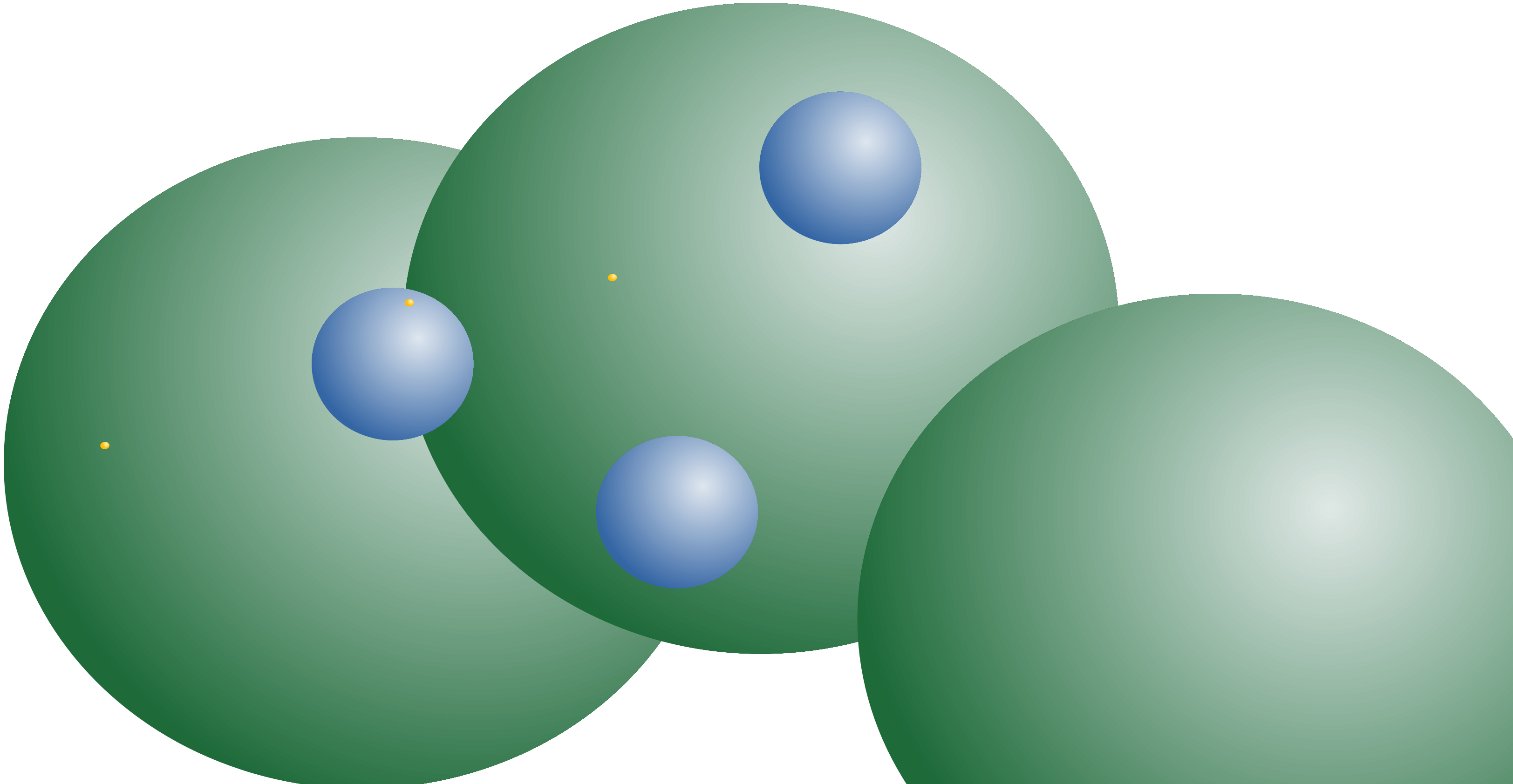
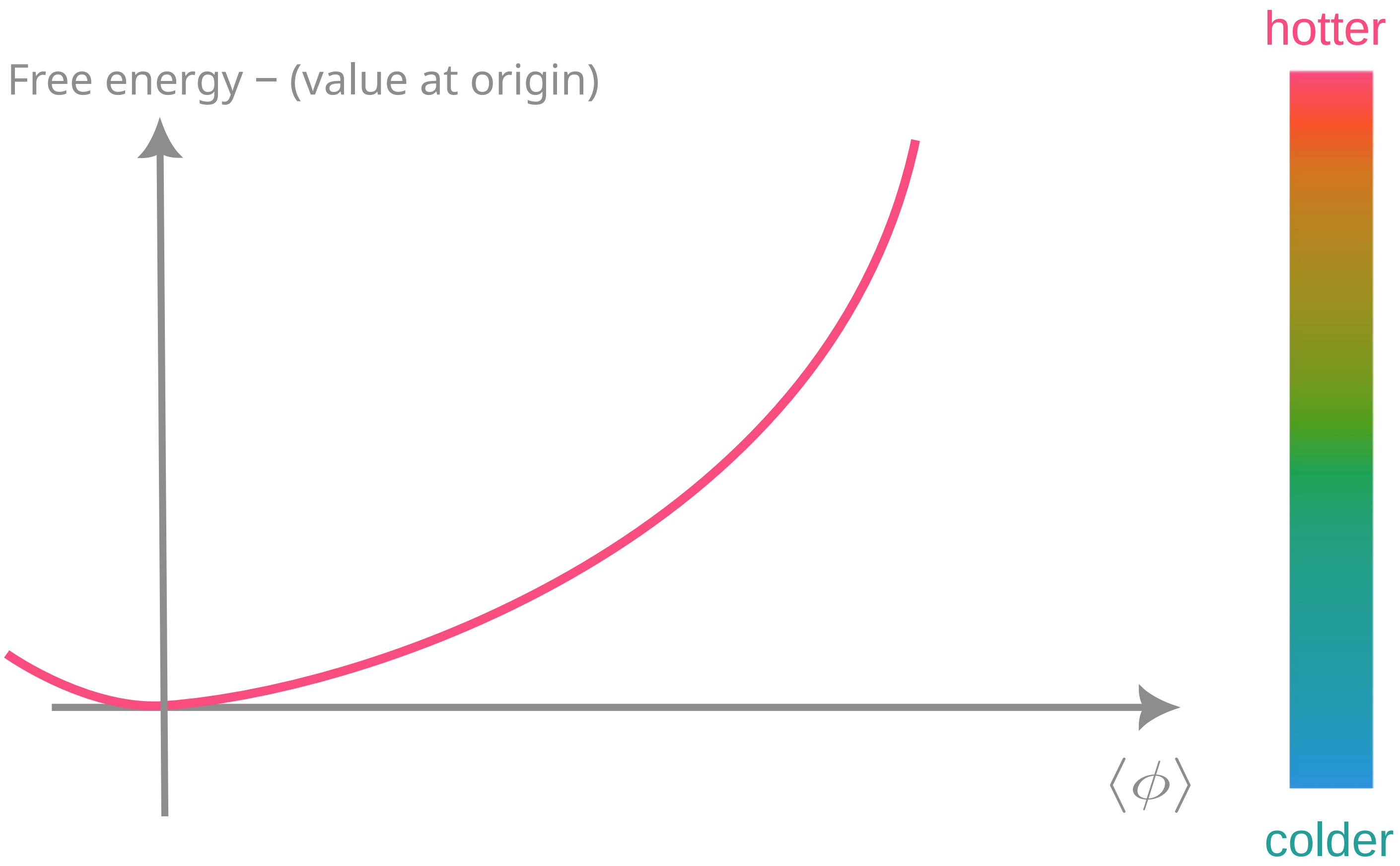
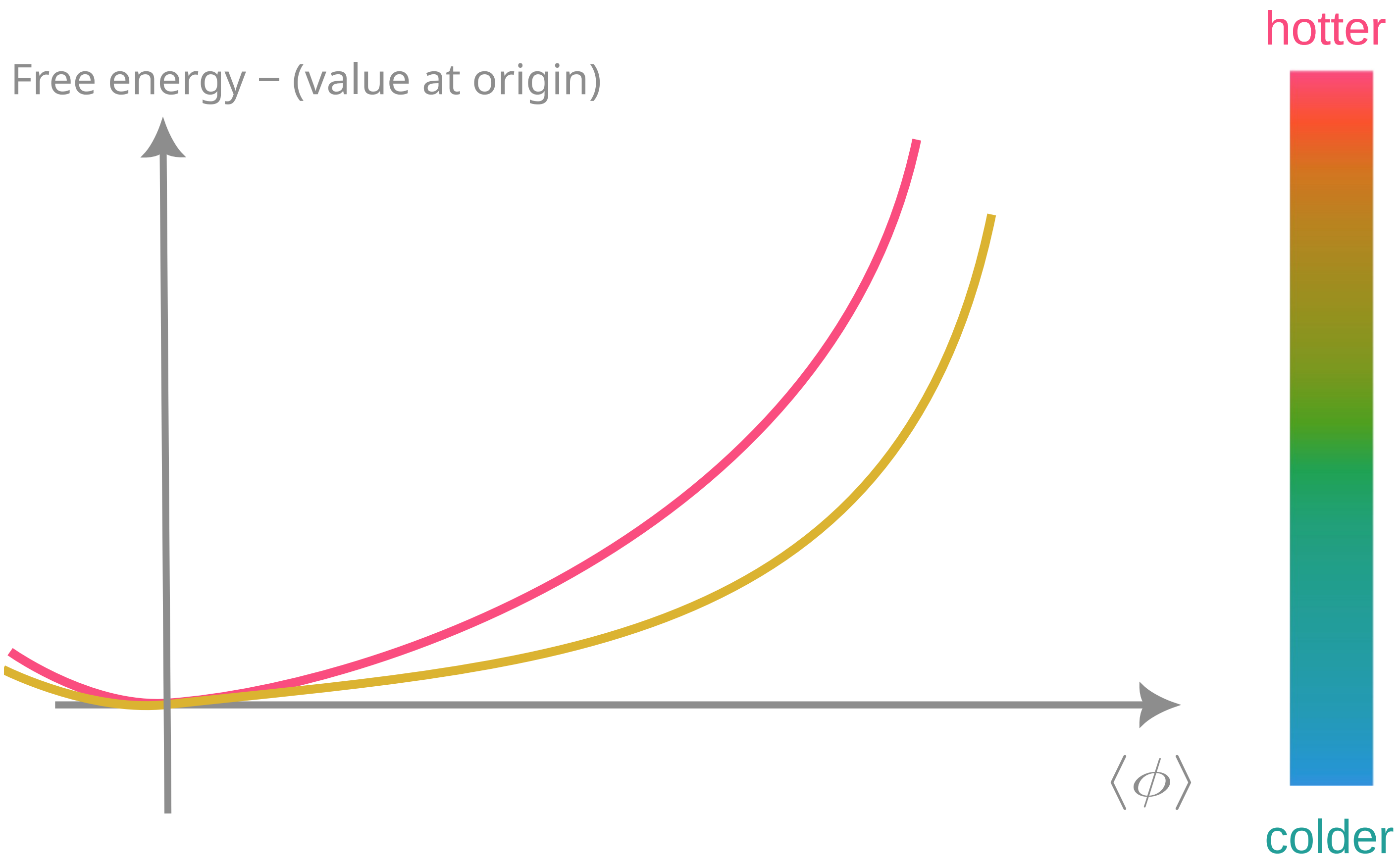


Second-order phase transitions



Second-order phase transitions



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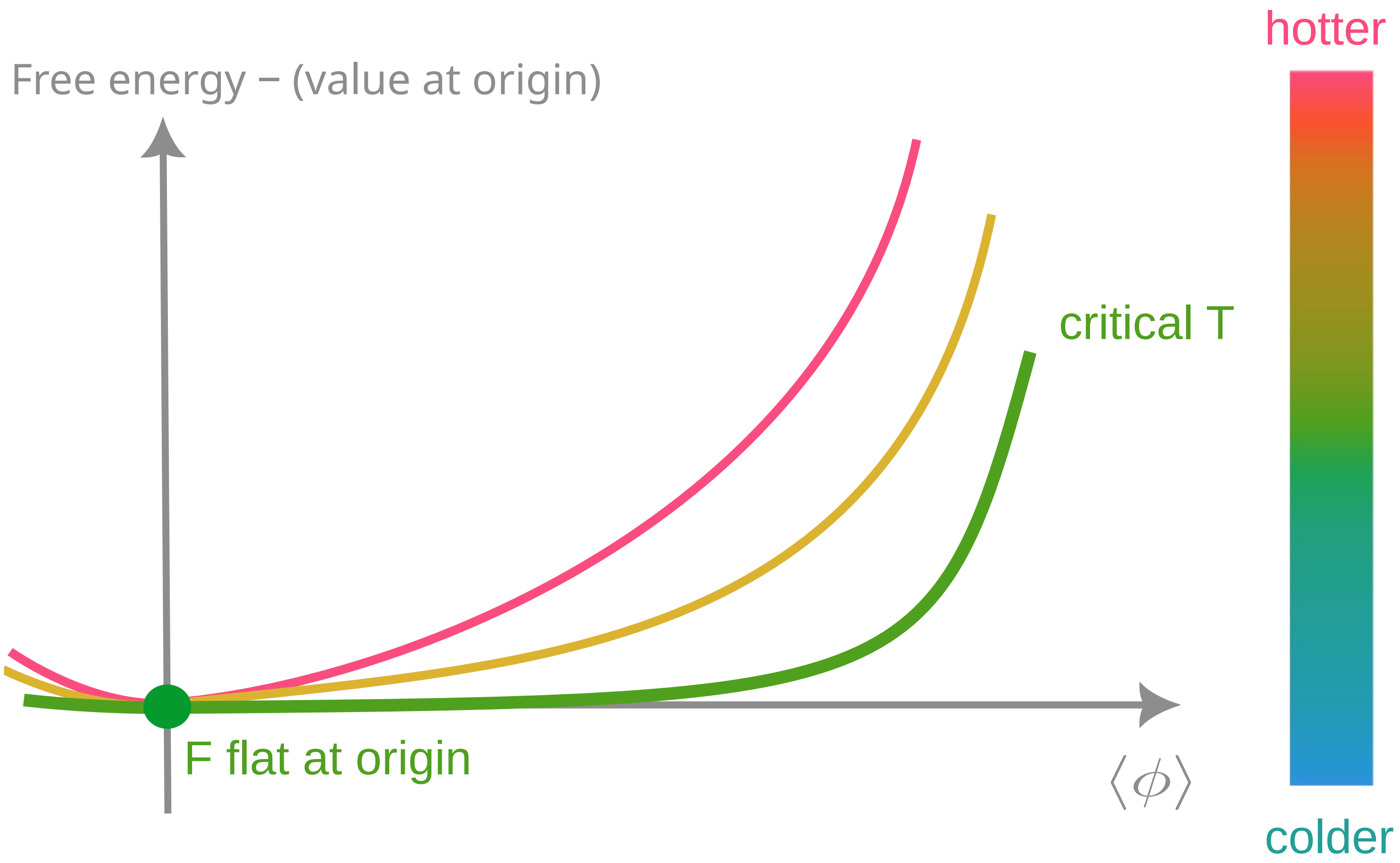

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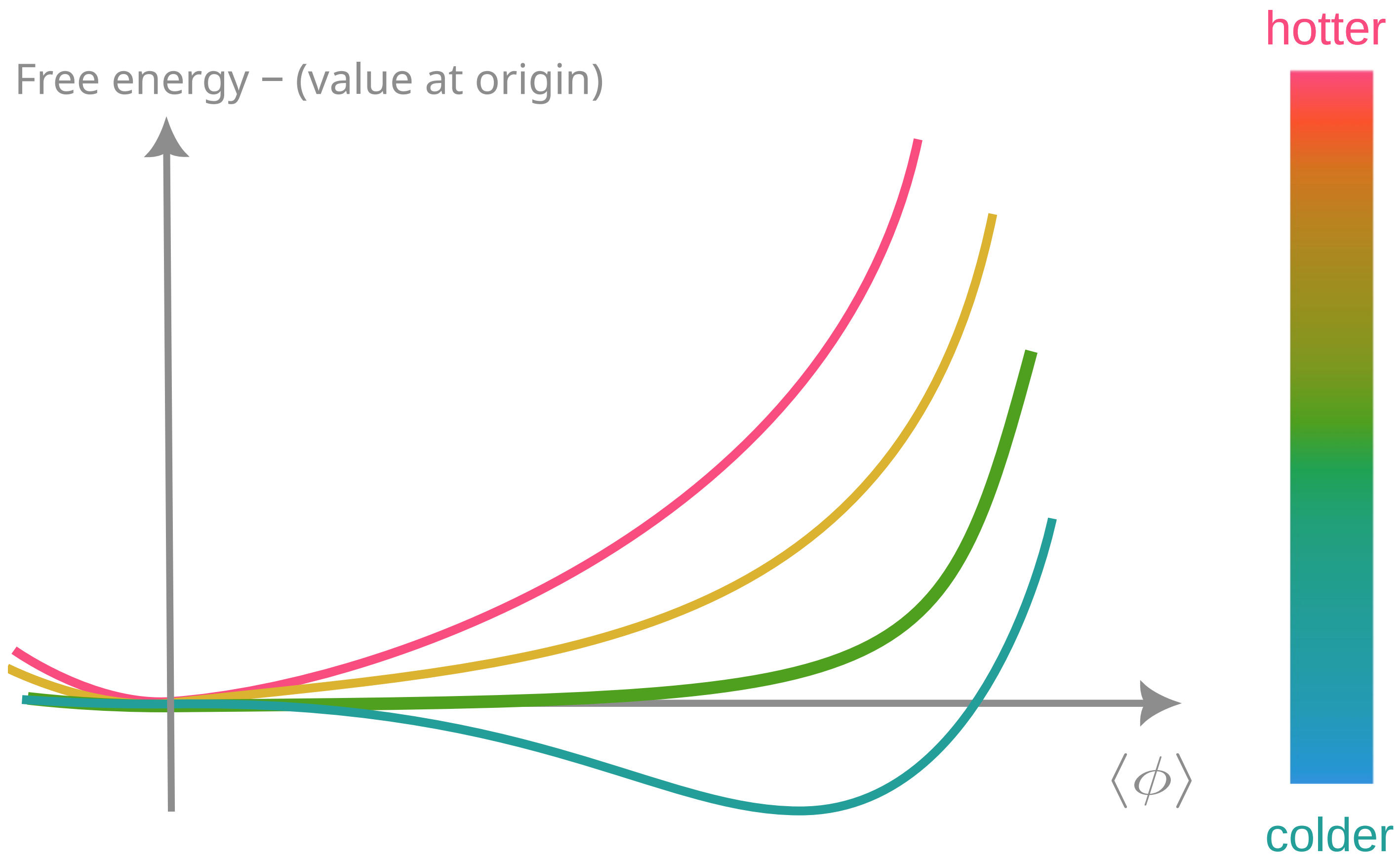
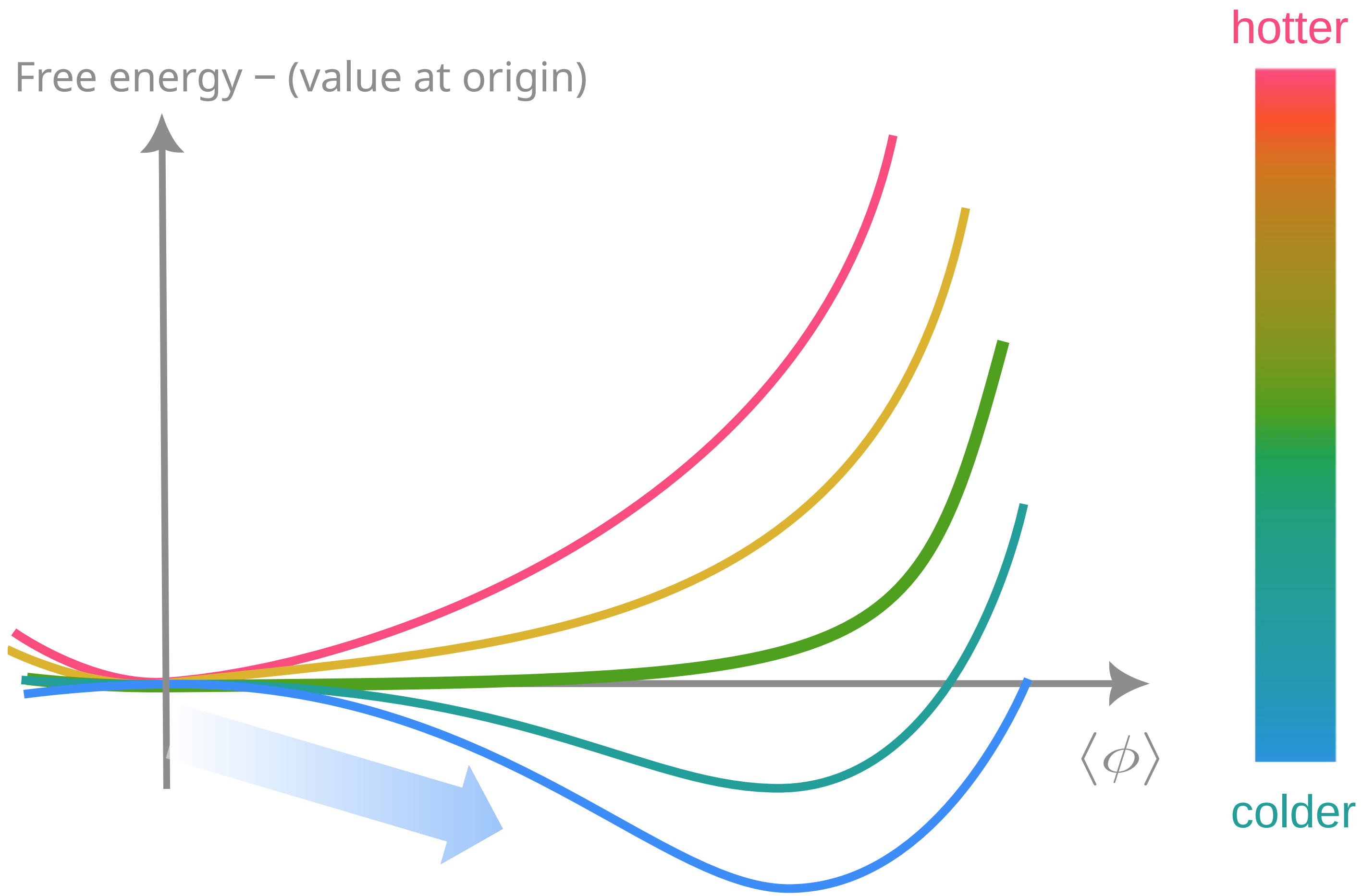


Second-order phase transitions



Possible phase transitions in the SM

Electroweak phase transition

T_c=160 GeV [Kajantie, Laine, Rummukainen, Shaposhnikov]

Triggers nonzero **Higgs vacuum expectation value**

2nd order / cross-over

QCD phase transition

T_c = 164 MeV [Fodor, Katz]

Triggers nonzero **fermion condensates**

2nd order / cross-over

Gravitational waves from 1st order transitions

GWs are sourced by non-isotropic, inhomogeneous distributions of stress-energy

$$ds^2 \supset -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j\,,$$
$$\partial^i h_{ij} = 0$$

$$\Box\left(h_{ij} - \frac{1}{2}\delta_{ij}h\right) = \frac{2}{M_P^2}T^{\rm TT}_{ij}\,,$$
$$T^{{\rm TT}\,i}{}_{i} = 0,$$
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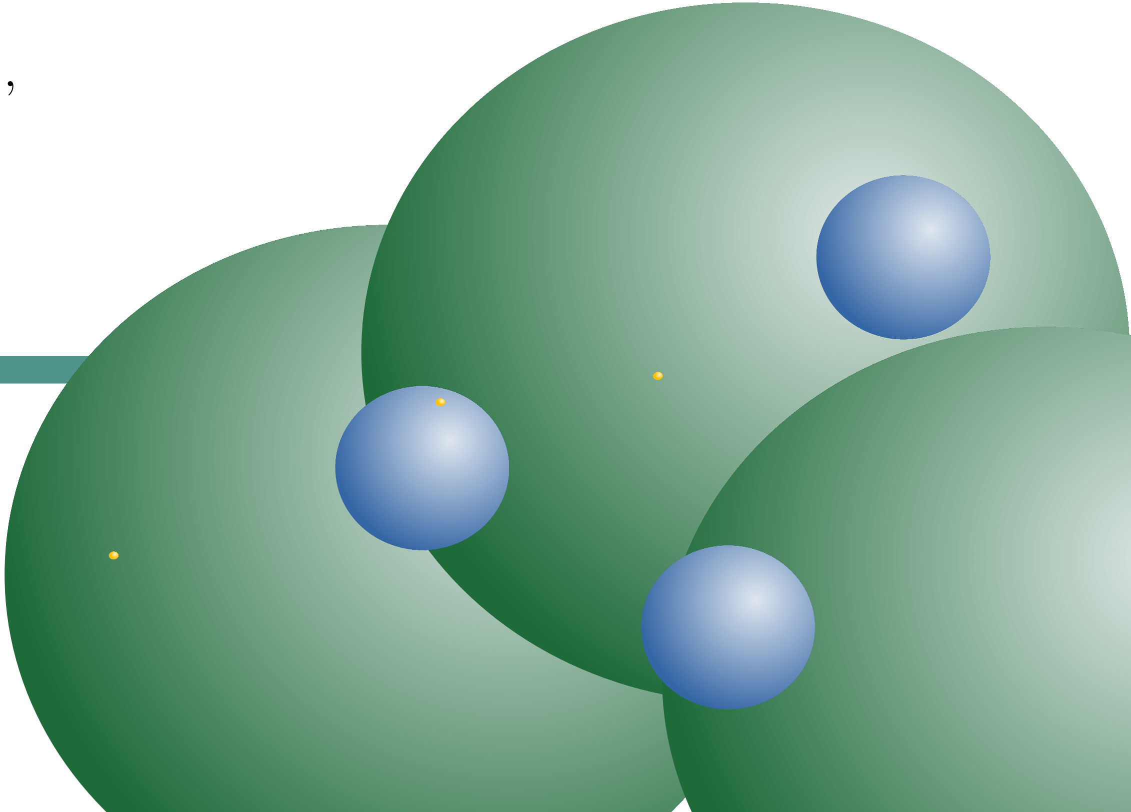
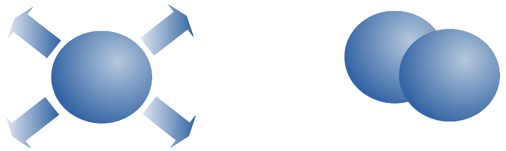


Baryogenesis from 1st order transitions

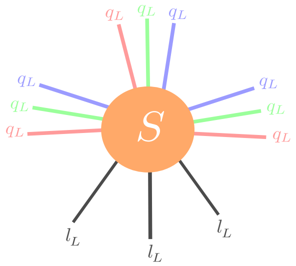
[Kuzmin, Rubakov, Shaposhnikov]

The SM + new physics ensuring a 1st order EW transition and augmenting CP-odd phases complies with the Sakharov conditions for baryogenesis

Departure from equilibrium

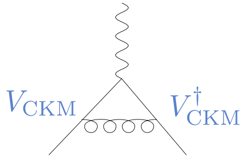


Baryon number violation

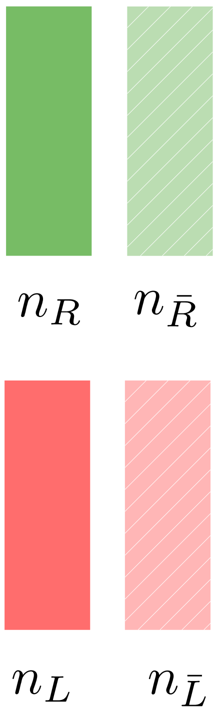


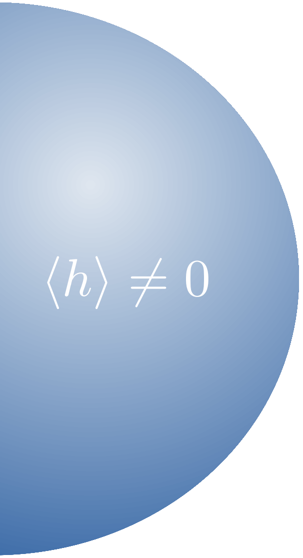
sphaleron $\Delta B = \Delta L = 3$

C and CP violation

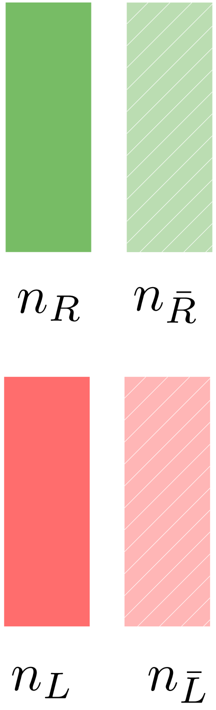


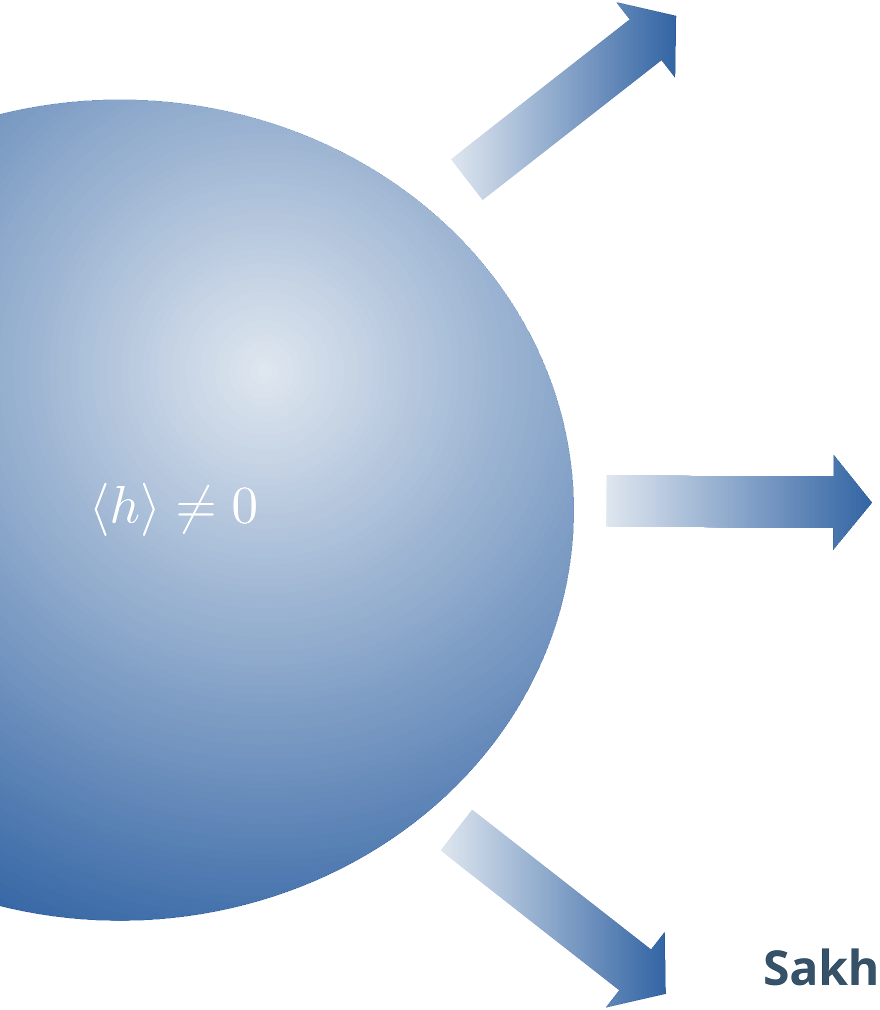
Thermal plasma, symmetric phase $\langle h \rangle = 0$



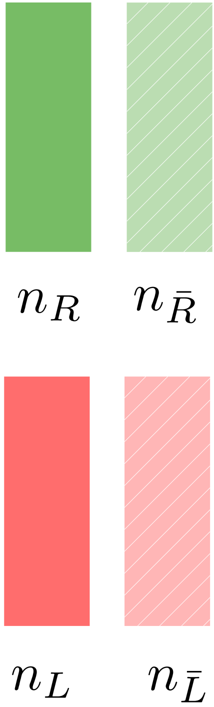


$$\langle h \rangle = 0$$



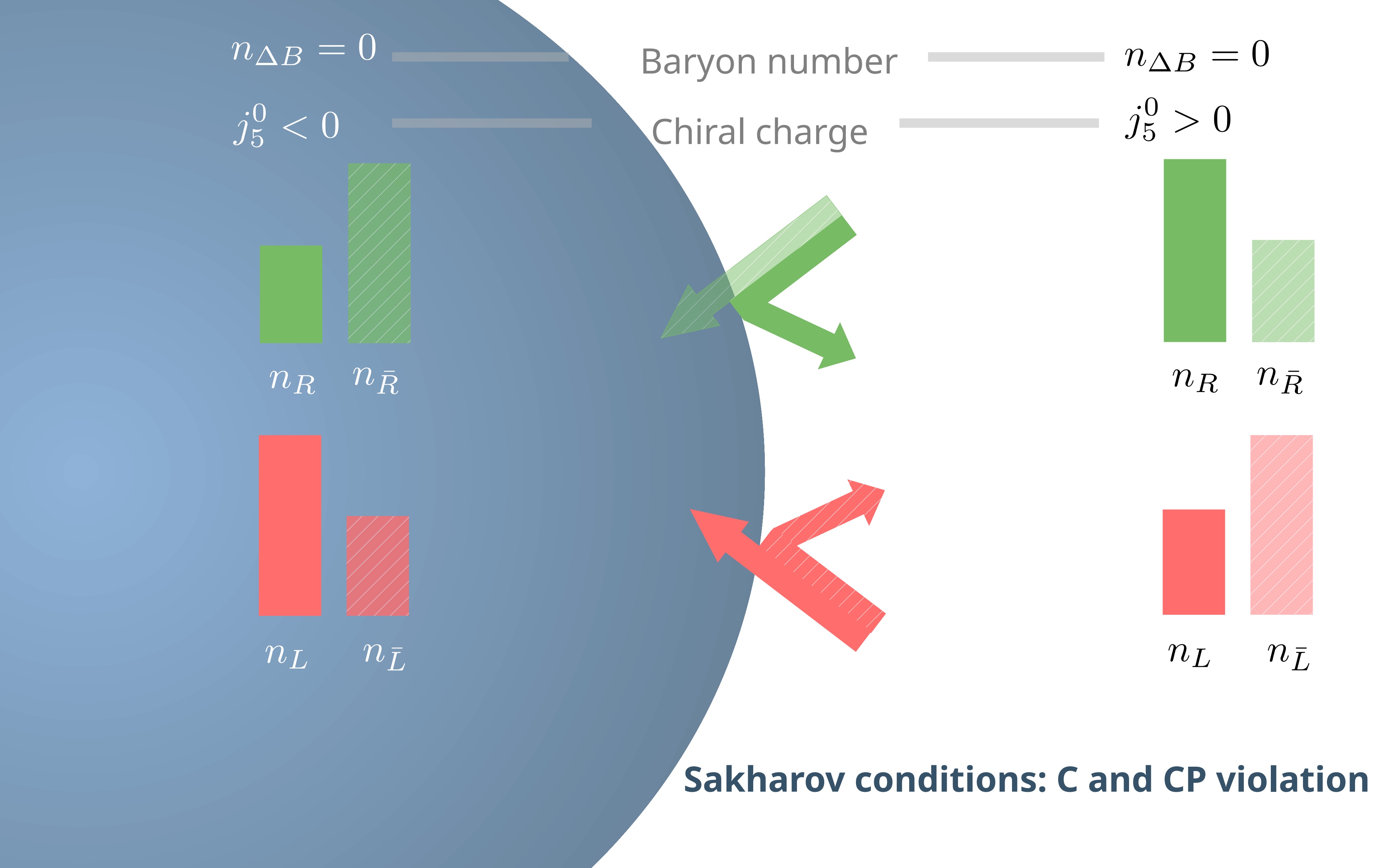


$\langle h \rangle = 0$

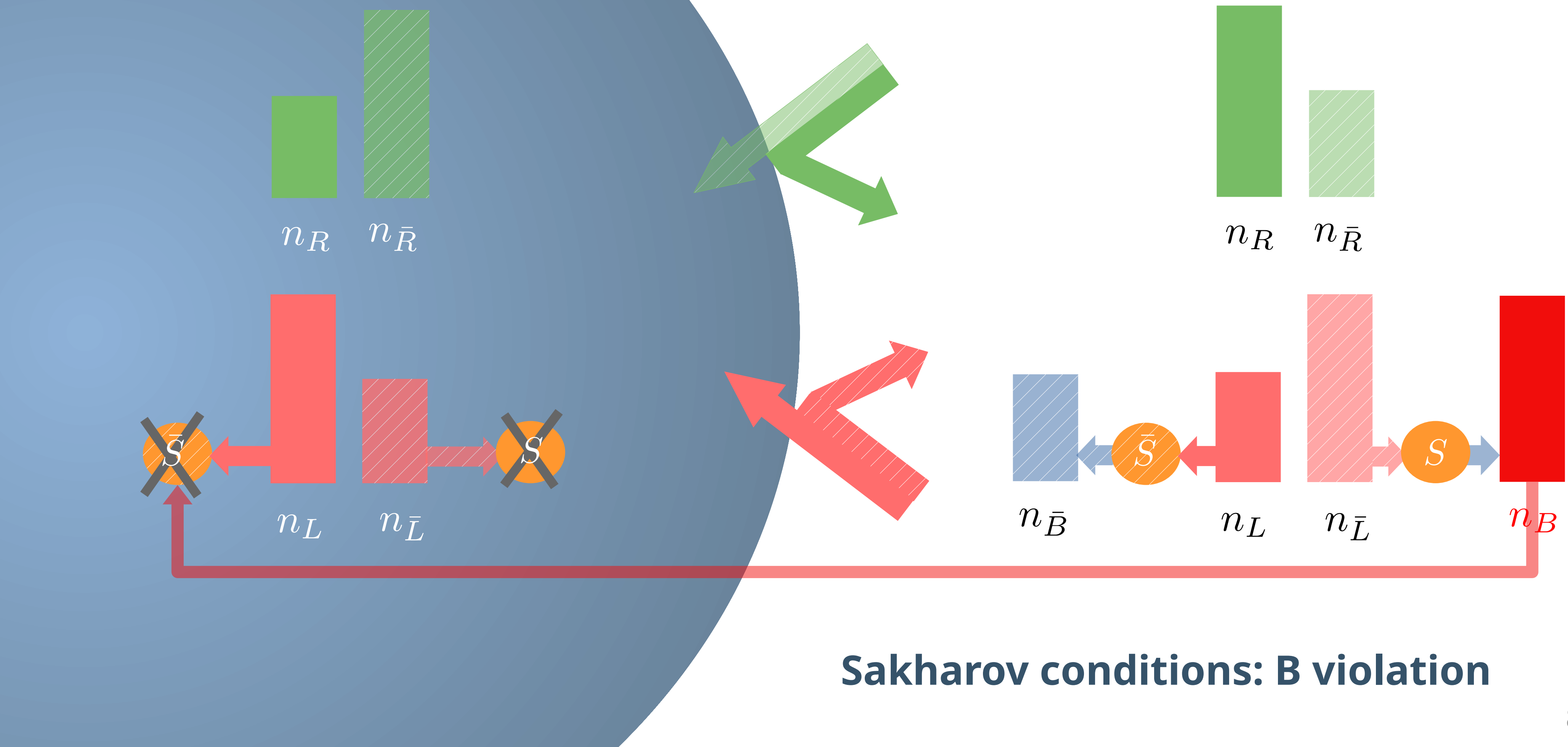


Sakharov conditions: Departure from equilibrium

Sakharov conditions: C and CP violation



[Kuzmin, Rubakov, Shaposhnikov]



Sakharov conditions: B violation

Questions relevant to current research

What is the velocity achieved by bubbles after nucleation?

Faster speeds lead to **more GWs**: Colliding bubbles carry more energy that can be converted into GWs

Smaller speeds preferred in **electroweak baryogenesis**, as they faciltate diffusion of CP asymmetry in front of wall

Can there be resonantly-enhanced CP violating sources in EW baryogenesis?

Would help to build models compatible with EDM constraints

2. The beauty and the challenges of thermal field theory

Path integrals for finite T computations

In **thermal equilibrium** with temp $T = 1/\beta$, averages are computed from the **partition function**

$$Z = \sum_n e^{-\beta E_n} = \sum_n \langle n | e^{-\beta H} | n \rangle$$

This is a **transition amplitude** that is equivalent to a **path integral**. Two **differences** with respect to the usual **vacuum case** in QFT:

$e^{-\beta H}$ vs $e^{i\Delta t H}$ → **Euclidean time!**

$\langle n | \cdot | n \rangle$ vs $\langle q' | \cdot | q \rangle$ → **Periodic boundary conditions!**

$$Z = \int_{\phi_i = \pm \phi_f} \mathcal{D}\phi e^{-\int_0^\beta S_E}$$

Feynman diagrams at finite T

One can define a **Feynman-diagram** based **perturbative expansion** based on **modified propagators** and **Feynman rules**, e.g.

$$\longleftrightarrow \quad \left.\frac{i}{p^2 - m^2}\right|_{p^0 = 2ni\pi T}, \qquad n \in \mathbb{Z}$$

In regards to the **free-energy** , one has

$$Z = e^{-\beta \int \frac{\partial F}{\partial V} d^3x} = e^{\beta \int p \, d^3x} = \sum (\text{"Vacuum" diagrams})$$

The **pressure/free energy** is **computable** from **Feynman diagrams**!!

Free energy from Feynman diagrams

Consider a **potential** with a minimum at T=0:

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

The total $\partial F/\partial V$ including the T=0 contribution from $V(\phi)$ is

$$\text{Total}\ \frac{\partial F}{\partial V} = -\frac{\pi^2}{30}g_* T^4 + \frac{1}{2}\left(-m^2 + \sum_{X,\text{bos}} \frac{dm_X^2}{d\phi^2}\frac{T^2}{12}\right)\phi^2 - \frac{T}{12\pi}\sum_{X,\text{bos}}\left(\frac{dm_X^2}{d\phi^2}\right)^{3/2}\phi^3 + \cdots$$

Pressure from ideal gas

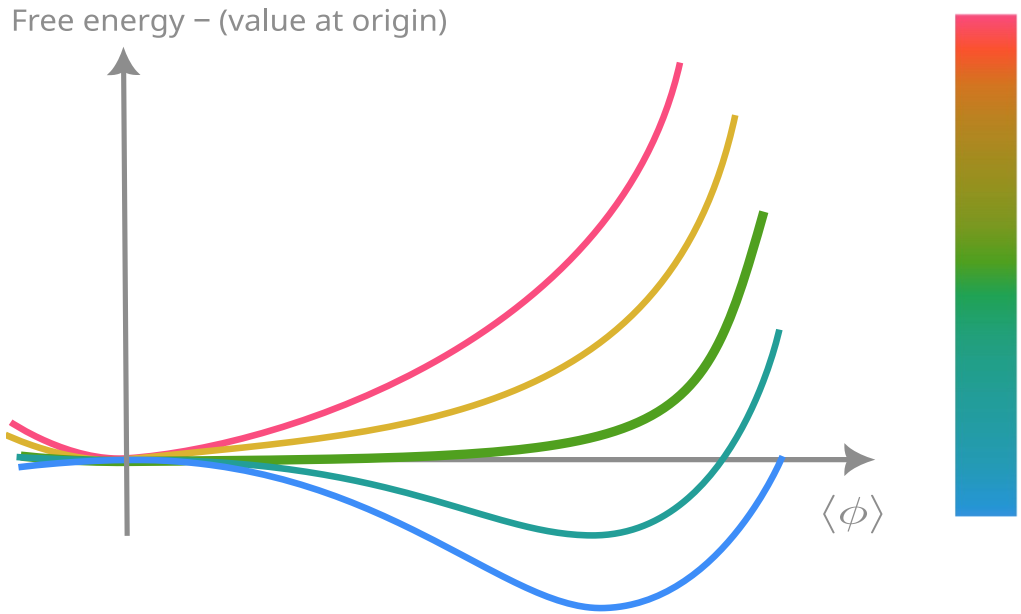
Positive thermal correction to the mass

Cubic term ~ possible barrier

Symmetry restoration at high T

The **pressure** from an **ideal gas** is recovered in the **relativistic limit**

The shift in m^2 leads to **symmetry restoration** at high T! [Kirzhnits & Linde]



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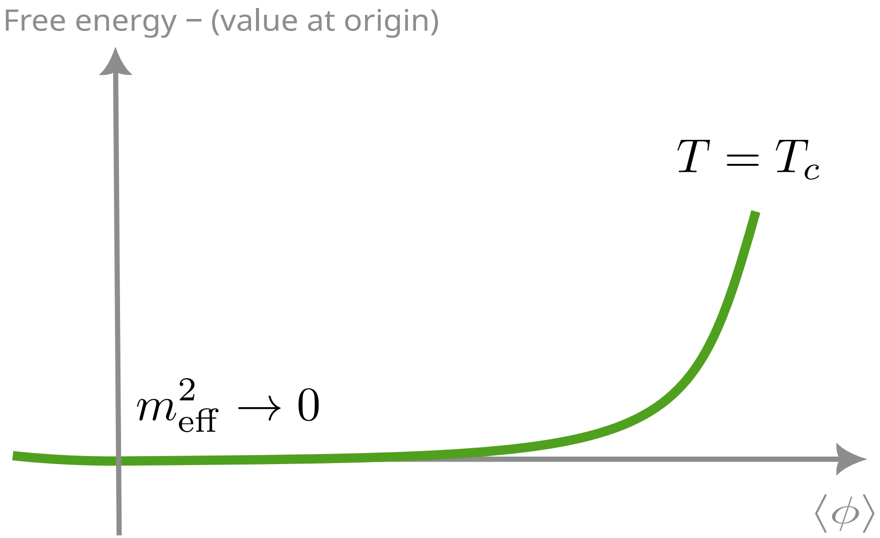
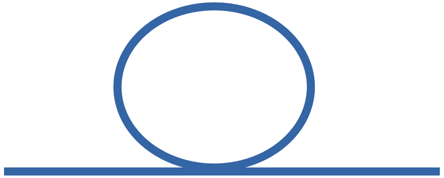


$m^2_{\text{eff}} \to 0$ ▶ $T_c \sim \mathcal{O}\left(\frac{m}{\sqrt{\lambda}}\right)$

Trouble with the perturbative expansion

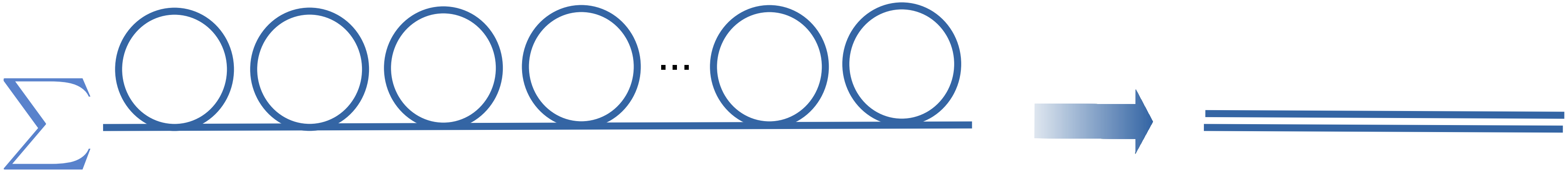


$$\approx (\text{Tree-level}) \times \mathcal{O}\left(\frac{\lambda T^2}{p^2}, \frac{\lambda T^2}{m^2}\right)$$

For *F*, we are interested in $p{=}0$, and the **perturbative expansion breaks down** if

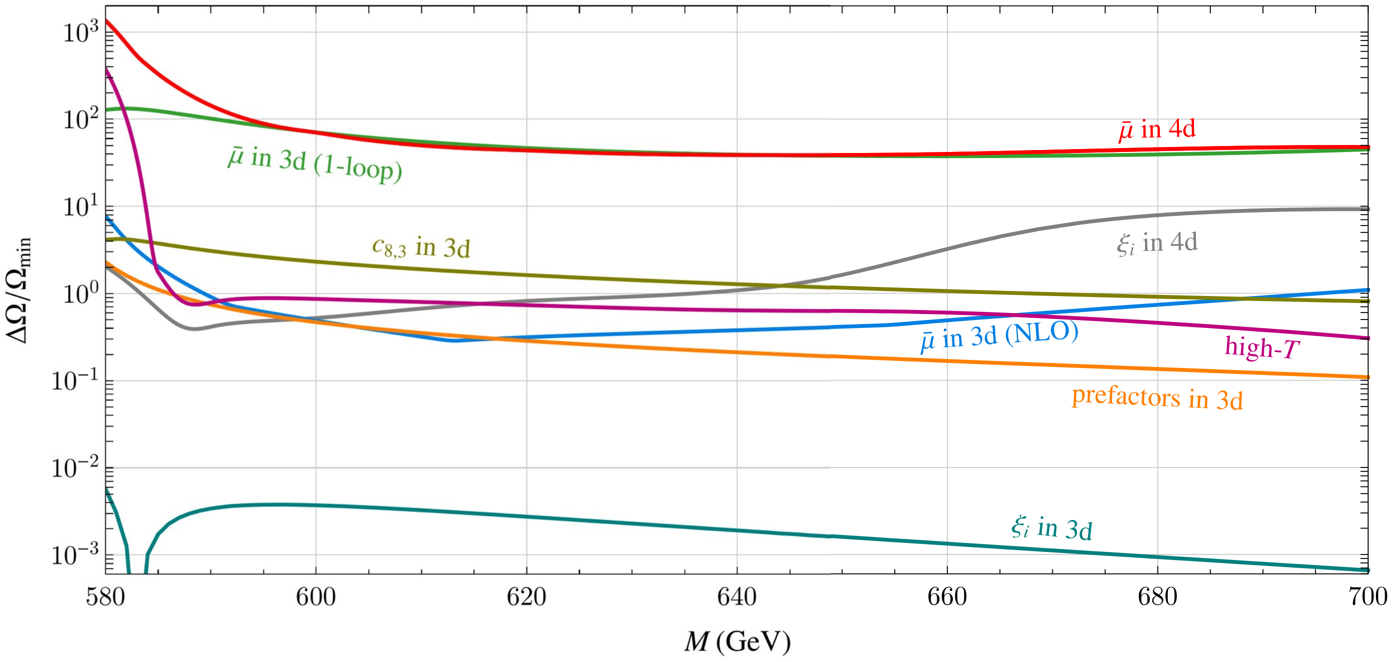
$$T \gtrsim \frac{m}{\sqrt{\lambda}} \sim T_c$$

Accuracy during p.t. requires **resummation / nonperturbative techniques**!



Trouble with the perturbative expansion

[Croon, Gould, Schicho,Tenkanen,White 2020]



GW spectrum: **Up to 3 orders of magnitude uncertainty** from RG **scale dependence**

Trouble with the perturbative expansion

This motivates **nonperturbative finite T studies**, e.g. using

Lattice techniques

The path integral is evaluated numerically [In Mainz: Harvey Meyer]

Holography

Strongly coupled field theories are mapped to weakly coupled gravitational theories in a higher dimensional space [In Mainz: Pedro Schwaller]

Going out of equilibrium

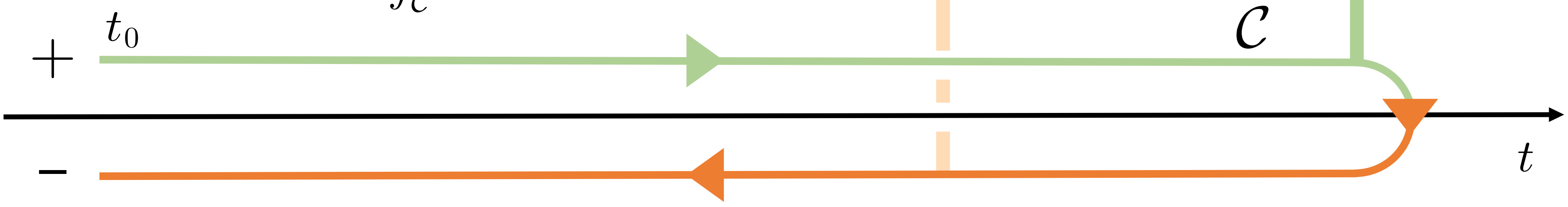
The equilibrium partition function cannot describe **time-dependent phenomena**.
For this one needs a different path integral with a **closed time-path (CTP)**

$$\langle S(t)|\hat{O}(t)|S(t)\rangle = \int \mathcal{D}\phi(t)\langle S(t_0)|e^{-i\hat{H}(t-t_0)}\hat{O}(t)|\phi\rangle_t \, {}_t\langle\phi|e^{i\hat{H}(t-t_0)}|S(t_0)\rangle.$$

Path integral from t to t_0

Path integral from t_0 to t

$$= \int_{\mathcal{C}} \mathcal{D}\phi e^{\mathrm{i}S[\phi]}$$



CTP propagators

- **Propagators** carry indices a,b= ± from the time branches of the field insertions

$$iS_{ab}(x,y) = \langle T_{\mathcal{C}} \psi_a(x) \bar{\psi}_b(y) \rangle$$

- Contain **info** about the **shell** and **number densities** of propagating d.o.f.s

$$iS^{+-}_{\rm tree}(x,k) \equiv iS^{<}_{\rm tree}(x,k) = -2\pi\delta(k^2-m^2)(\not{k}+m)\left[\theta(k^0) f(x,\mathbf{k}) - \theta(-k^0)(1-\bar{f}(x,-\mathbf{k})\right]$$

- They satisfy quantum equations of motion: **Schwinger-Dyson** eqs. in contour *C*

$$(i\not{\partial}-m)\, iS^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y) - i\sum_{c=\pm} c\int^4 z\, i\Sigma^{ac}(x,z)\, iS^{cb}(z,y)$$

Self energy (1PI)

▶ This leads to **Boltzmann / fluid equations from first principles**!

[For other uses of CTP techniques in Mainz: See work by Harz, Schwaller]

4. Bubble velocities

Bubble “friction” in local equilibrium

- It was **generally accepted** that a **constant velocity** could only achieved in the presence of **friction** due to **out-of-equilibrium effects** [Bödeker-Moore]

- There were however **hints** that this was **not necessarily the case** [No & Konstandin][Barroso-Mancha et al]

- My work has firmly established that **constant velocity** can be achieved even in **local thermal equilibrium** (LTE) [Balaji, Spannowsky, Tamarit] [Ai,Garbrecht, Tamarit]

- The **LTE effects** can **dominate** over nonequilibrium ones, simplifying computations!

The governing equations in LTE

Equation of motion of the scalar

$$\Box\phi + \frac{\partial V_T(\phi,T)}{\partial\phi} = 0 = \Box\phi + \frac{\partial(V(\phi) - p(\phi,T))}{\partial\phi}$$

Stress-energy conservation

$$\nabla_\mu(T^{\mu\nu}_{\text{scalar}} + T^{\mu\nu}_{\text{plasma}}) = 0 \qquad T^{\mu\nu}_{\text{plasma}} = (\rho + p)u^\mu u^\nu - p\eta^{\mu\nu} \equiv \omega u^\mu u^\nu - p\eta^{\mu\nu}$$

Thermodynamics relates everything to **pressure** (=- sum of “vacuum” diagrams)

$$\omega = T\frac{\partial p}{\partial T}$$

From “vacuum” diagrams one can fix all relevant thermodynamic quantities / $T_{\mu\nu}$

Equations of state: naive vs first principles

In the literature, it is common to use a **simplified parametrization** instead of the **full result** of **vacuum diagrams**

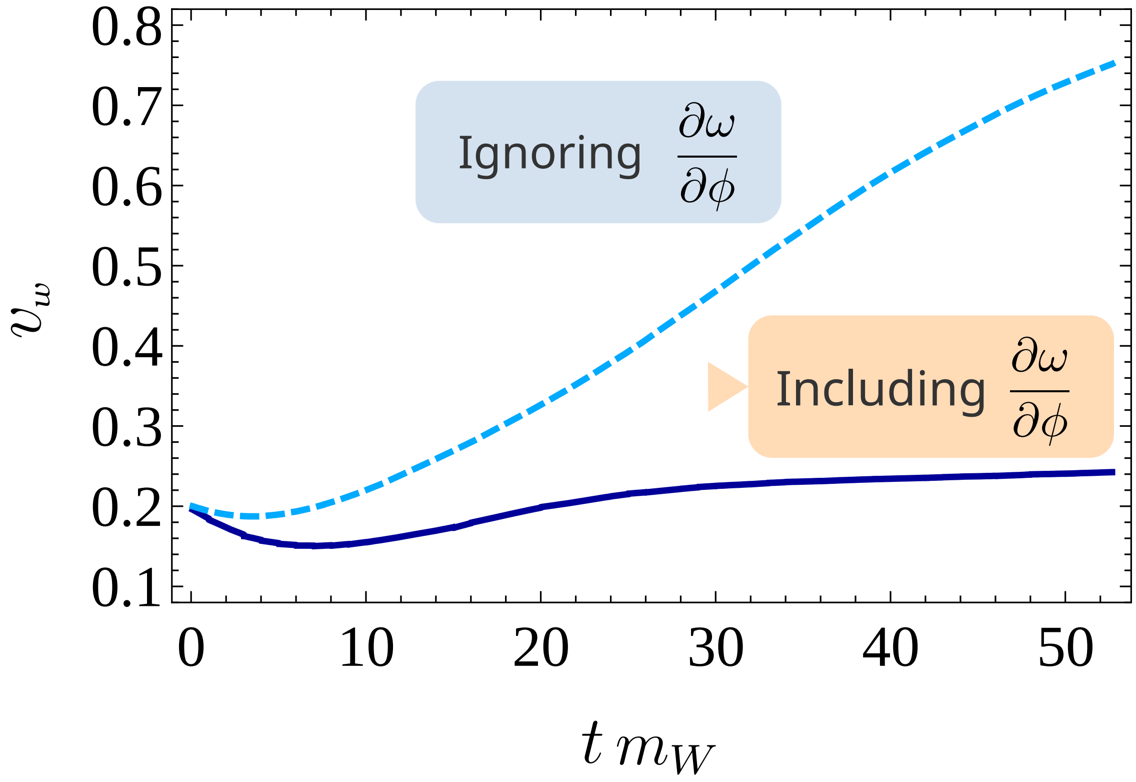
"Bag" equations of state

$$p = aT^4,$$
$$\rho = 3aT^4 + \epsilon$$

While this can be used away from the wall (where $\phi \to const$), when used across the bubble it leads to **ignoring hydrodynamic effects** that slow down bubbles even in LTE

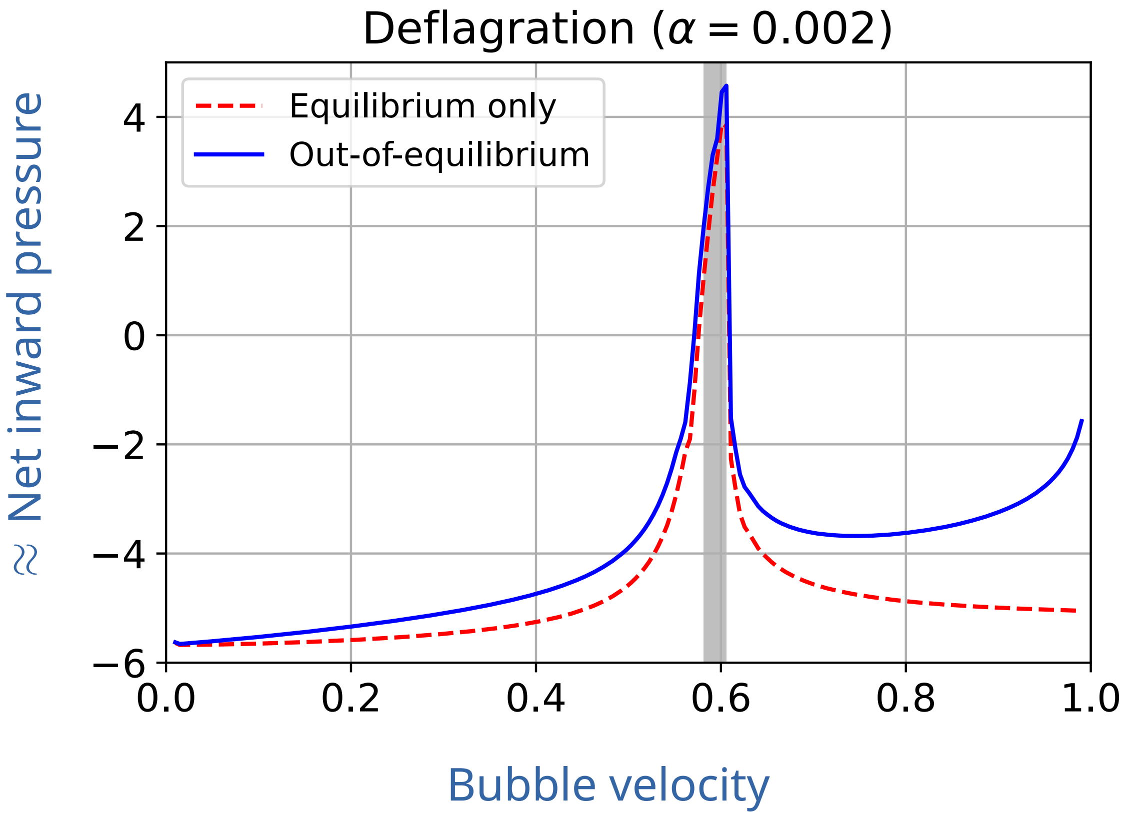
Friction in local equilibrium from $\partial\omega/\partial\phi$

[Balaji, Spannowsky, Tamarit]



Local equilibrium effects can be dominant

[Cline, Laurent]



Bubble velocities from minima of a potential

When going **beyond the bag equation of state**, one must **solve** for the field **equation of motion**, e.g. in a static frame

It is common to avoid this by adopting a **simplified Ansatz** [Cline, Laurent]

$$\phi(z) = a \tanh\left(b(z-c)\right)$$

And the parameters a,b,c are fixed with a minimization procedure

We have developed a **new method** which **bypasses solving the equation of motion** and does not rely on an ad-hoc Ansatz

Bubble velocities from minima of a potential

We introduce a **"pseudopotential"** $\hat{V}$

$$\hat{V}(\phi) = \int_0^{\phi} d\phi' \frac{\partial V_T}{\partial \phi}(\phi', T(\phi', v_w))$$

Potential including thermal corrections

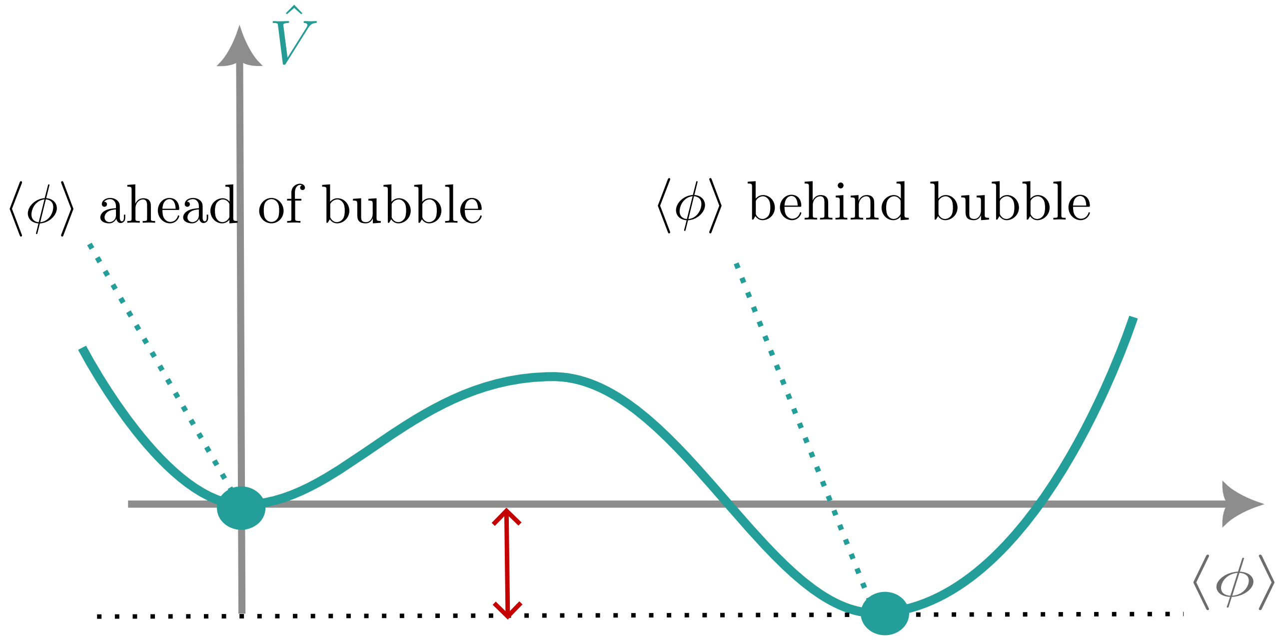
Temperature expressed in terms of ϕ, v_w thanks to **hydrodynamic equations**

Fields settle to minima of $\hat{V}$ in front / behind the bubble

$$\text{Net outward pressure } = \hat{V}(\text{ahead}) - \hat{V}(\text{behind})$$

Bubble velocities from minima of a potential

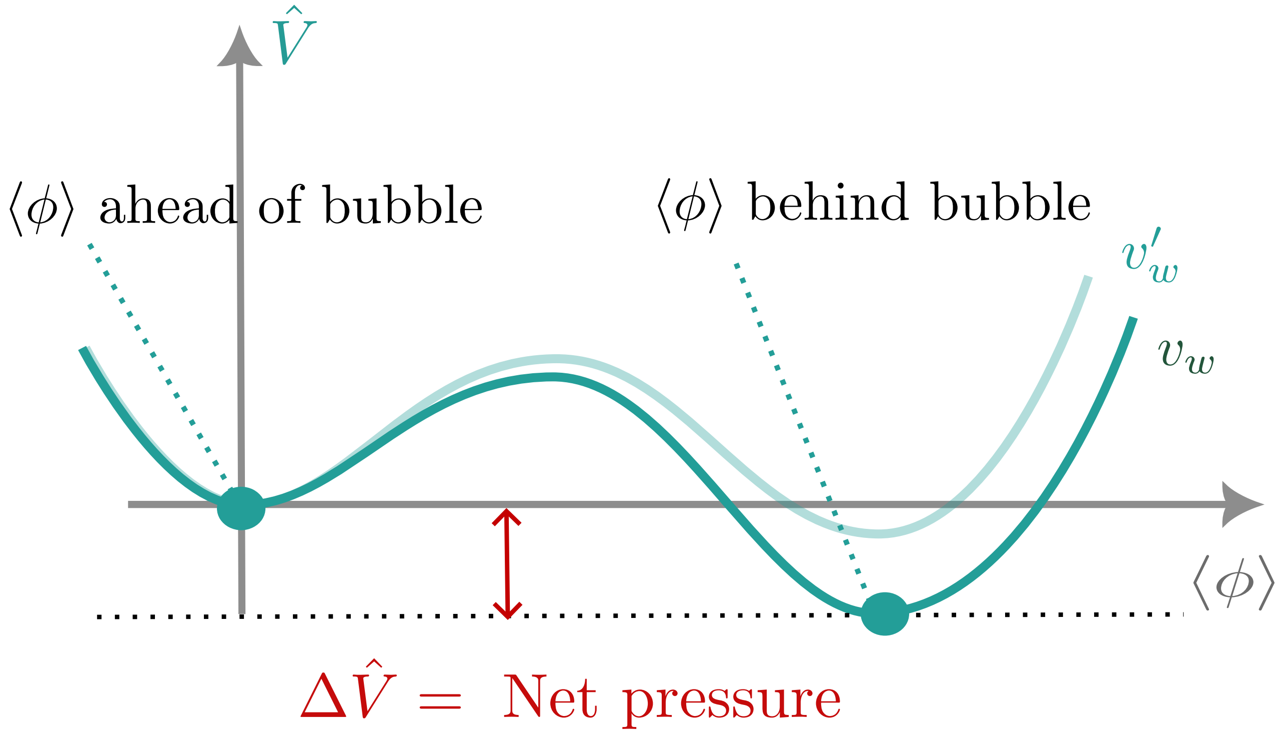
[Münzenberg, Tamarit, to appear]



$\Delta\hat{V} =$ Net pressure

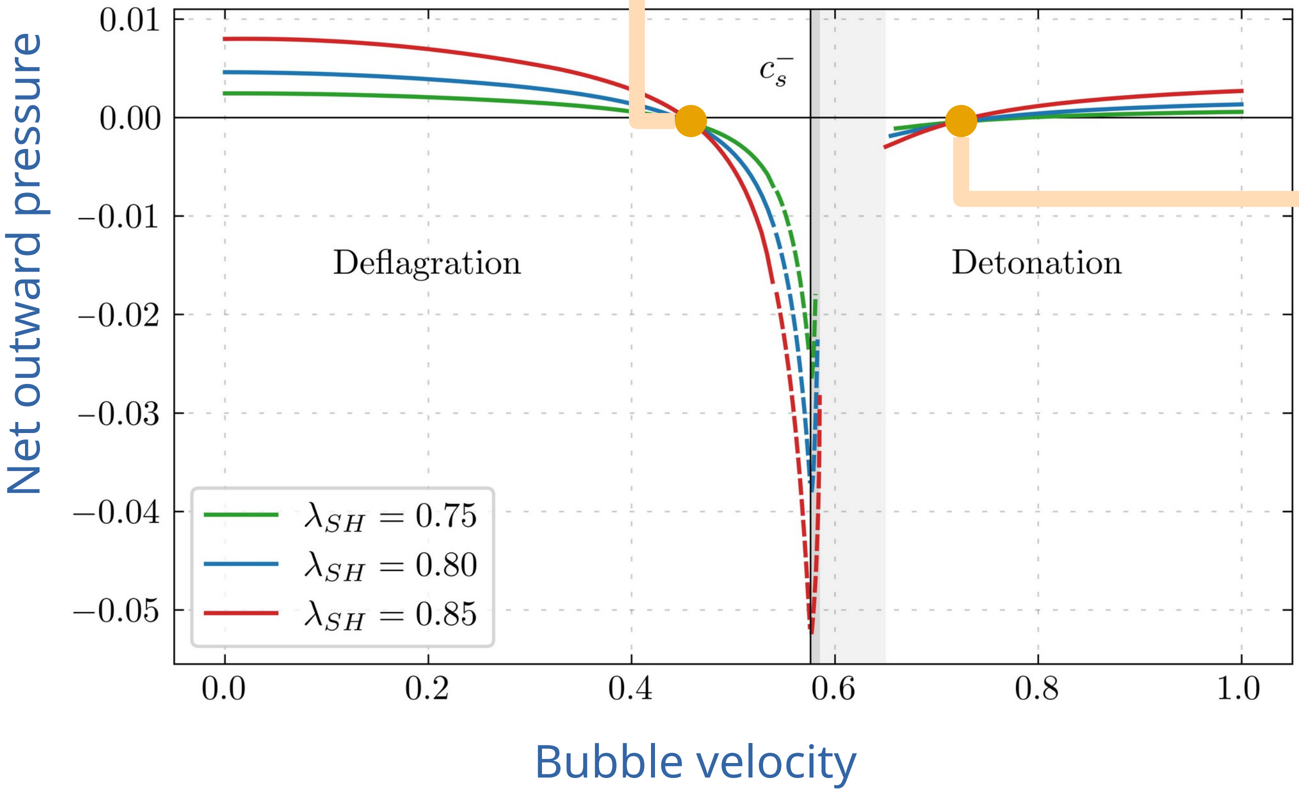
Bubble velocities from minima of a potential

[Münzenberg, Tamarit, to appear]



Bubble velocities from minima of a potential

[Münzenberg, Tamarit, to appear]



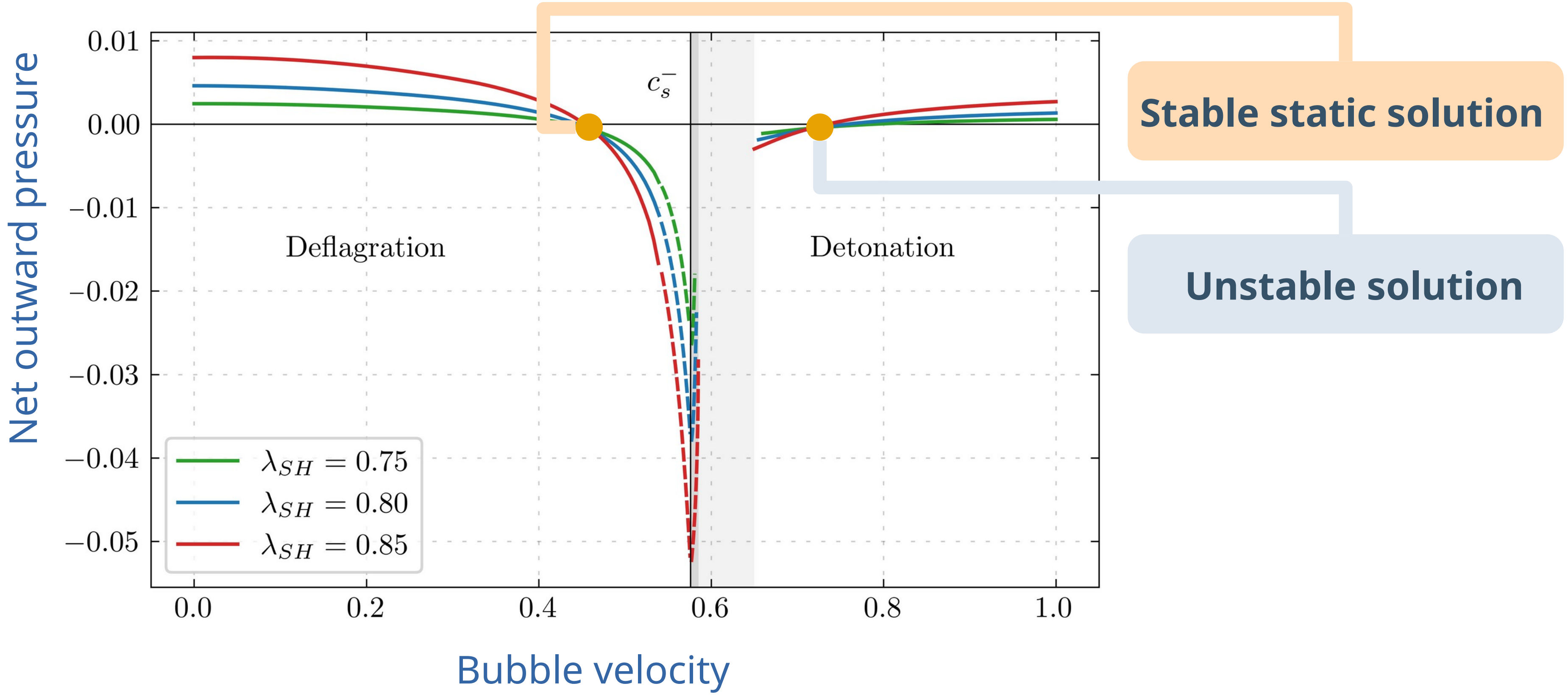
Net pressure = 0

⟷

Terminal velocities

Bubble velocities from minima of a potential

[Münzenberg, Tamarit, to appear]



5. Electroweak baryogenesis

Many methods, many CPV sources

$$\text{CPV source} \Leftrightarrow \partial_z(\text{CP-odd current}) = 0$$

Source type	Methods	Resonant enhancement?	Gradient order	
Single flavour	WKB	No	2 (AGREE!)	[Cline, Joyce, Kainulainen, Prokopec]
	spin dec.	No	2 (AGREE!)	[Kainulainen, Prokopec, Schmidt, Weinstock]
Multi-flavour	WKB	No	2	[Cline, Joyce, Kainulainen]
	spin dec.	Diag. sources are resonant but effect compensated by flavour oscillations	1+2	[Konstandin, Prokopec, Schmidt, Seco]
	VIA	Yes	1	[Riotto][Carena, Moreno, Quiros, Seco, Wagner] [Lee,Cirigliano, Ramsey-Musolf 05]

Derivation of CPV sources from the CTP

There has been a **20 year discussion** on whether there is a **resonant CPV source** at **order one in derivatives** of the **scalar background**

- Using two **CTP-based approaches** (spin decomposition and VIA) we found **agreeing expressions** for $\partial_z j_5^z$, with **resonant enhancement**, but of **order 2 in gradients**

- Going further to derive equations for **number densities** $f(\mathbf{x}, \mathbf{k})$ we have discovered that **unambiguous results** require computing the **modified dispersion relation** in the bubble background → **new consistency check**!

The model

- We consider a **2 fermion system** with **CP-odd phases** present in **mixing terms**

$$M = \begin{bmatrix} m_1 & e^{i\varphi} v_b(z) \\ v_a(z) e^{i\gamma} & m_2 \end{bmatrix}$$

scalar VEVs

$$\mathcal{L} \supset -\bar{\psi}(\hat{M} P_R + \hat{M}^\dagger P_L)\psi$$

- The **goal** is to compute a **source** for the CP-odd **axial current** j_5^μ in a static wall:

$$\langle \partial_z j_5^z \rangle, \quad j_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$$

- The source should be expressed in terms of **ordinary number currents**

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

2 fermion mixing: CPV source

Nonresonant

Involve CP odd phases, resonant

$$(\partial_z j_5^z)_{1,1} = -\frac{(v_a v_a' - v_b v_b')}{m_1^2 - m_2^2}\left(j_{1,1}^z - j_{2,2}^z\right) + \frac{\sin(\varphi+\gamma)\, m_1 m_2}{k_z(m_1^2 - m_2^2)}(2v_a' v_b' + v_b v_a'' + v_a v_b'')\left[\frac{1}{2k_z^2}\left(j_{1,1}^z - k_z\partial_{k_z} j_{1,1}^z\right) - \frac{1}{m_1^2 - m_2^2}\left(j_{2,2}^z - j_{1,1}^z\right)\right] + \mathcal{O}(v^3, vv''', v'v''),$$

$$(\partial_z j_5^z)_{2,2} = \frac{(v_a v_a' - v_b v_b')}{m_1^2 - m_2^2}\left(j_{1,1}^z - j_{2,2}^z\right) - \frac{\sin(\varphi+\gamma)\, m_1 m_2}{k_z(m_1^2 - m_2^2)}(2v_a' v_b' + v_b v_a'' + v_a v_b'')\left[\frac{1}{2k_z^2}\left(j_{2,2}^z - k_z\partial_{k_z} j_{2,2}^z\right) + \frac{1}{m_1^2 - m_2^2}\left(j_{1,1}^z - j_{2,2}^z\right)\right] + \mathcal{O}(v^3, vv''', v'v'')$$

Nonresonant

Same result with 2 different methods!

[Garbrecht, Ilyas, Tamarit, White, to appear]

Obtaining Boltzmann equations

The **currents** are of the form $\langle\bar{\psi}(\cdot)\psi\rangle$ and so can be related to **CTP propagators** which as we saw have the structure

$$\langle\bar{\psi}(\cdot)\psi\rangle \sim \delta(k^2 - m^2) f(\mathbf{x}, \mathbf{k})$$

We can get **Boltzmann equations** for $f(\mathbf{x}, \mathbf{k})$ by integrating over k^0

$$\int dk^0 \left(\frac{k^0}{\sqrt{\mathbf{k^2 + m^2}}}\right)^n (\text{EQ. for } \partial_z j_5^z)$$

The problem is: contributions with $\delta'()$ lead to ambiguous $(n$-dependent) results!

Solving the δ' problem

We have realized that the problem is solved when accounting for the fact that the **background changes the dispersion relation**

$$\langle \bar{\psi}(\cdot)\psi \rangle \sim \delta(k^2 - m^2) f(\mathbf{x}, \mathbf{k}) \quad \rightarrow \quad \langle \bar{\psi}(\cdot)\psi \rangle \sim \delta(k^2 - m^2 - \delta m^2) f(\mathbf{x}, \mathbf{k})$$

This effect turns out to be **calculable** from the **Schwinger-Dyson equations**, e.g.

$$\delta m_1^s = -\frac{s m_2 \sin(\gamma + \phi)\,(v_b v_a' + v_a v_b')}{2\,(m_1^2 - m_2^2)\sqrt{{k_3}^2 + m_1^2}} + \frac{m_1 (v_a^2 + v_b^2)}{2(m_1^2 - m_2^2)} + \frac{m_2 \cos(\gamma + \phi) v_a v_b}{m_1^2 - m_2^2} + \frac{k_3 m_2 \sin(\gamma + \phi)(v_a v_b' - v_a' v_b)}{(m_2^1 - m_2^2)^2}$$

spin

When including this effect, the δ' **terms “magically” cancel**

[Garbrecht, Ilyas, Tamarit, White, to appear]

7. Conclusions

- **Phase transitions** are a **generic prediction** of QFT at finite *T*: at high *T* one has **symmetry restoration,** and new phases appear at lower energies

- **First-order phase transitions** are particularly interesting because they can produce **gravitational waves** and generate the **baryon asymmetry**

- The **terminal velocity** of bubbles is relevant for the production of GWs and baryon number, and there has been **recent progress** in understanding the **importance of hydrodynamic effects** in **local equilibrium**

- **CP-violating sources** in electroweak baryogenesis have been **debated over 20 years**, with promising recent results

Thank you!