Beyond the Higgs with Amplitude Methods

Yael Shadmi, TECHNION

mainly based on work with former & current students & postdocs at Technion:

Yaniv Weiss, Jared Goldberg, Julian Northey

Gauthier Durieux, Teppei Kitahara, Reuven Balkin, Michael Waterbury, Teng Ma, Hongkai Liu

PRISMA colloquium

Feb 25

short bio:

BSc Physics+ Math, Tel-Aviv U

PhD Stanford U (SLAC, Lance Dixon)

Fermilab postdoc

Weizmann and Princeton postdoc

Technion faculty

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QCD amplitudes (birth of modern amplitude program)

model building: supersymmetry,

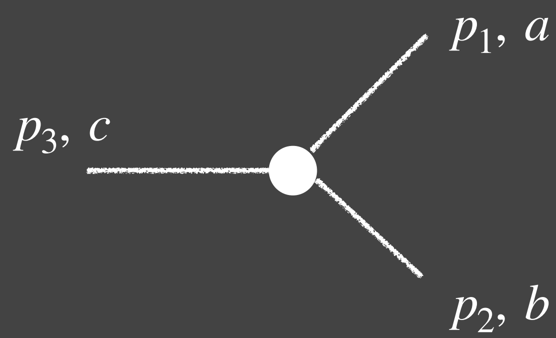
dynamical supersymmetry breaking, RS..

amplitudes for BSM

PRELUDE

what are the most general interactions of spin-1 particles?

eg, photons?



(massless) Benincasa Cachazo '08
Durieux Kitahara YS Weiss '19
Liu Yin '22

Lorentz: most general amplitude:

$$C^{abc}\,\left(\langle\mathbf{12}\rangle[\mathbf{23}]\langle\mathbf{31}\rangle+[\mathbf{12}]\langle\mathbf{23}\rangle[\mathbf{31}]\ +\ \text{perm}\right)/M^2 \qquad +\ \mathcal{O}(\text{mass-splittings})$$

$$+C'^{abc}\,\langle\mathbf{12}\rangle\langle\mathbf{23}\rangle\langle\mathbf{31}\rangle/\Lambda^2+\mathbf{C''^{abc}}\,[\mathbf{12}][\mathbf{23}][\mathbf{31}]/\Lambda^2$$

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Lorentz part ($p_1\ \ p_2\ \ p_3$) written just in terms of 2-component spinors

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completely antisymmetric

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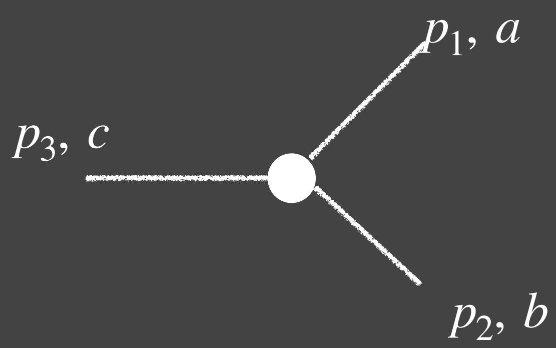
—> C^{abc} completely antisymmetric

what are the most general interactions of spin-1 particles?



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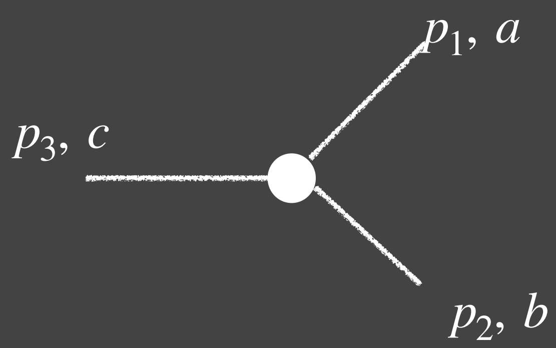

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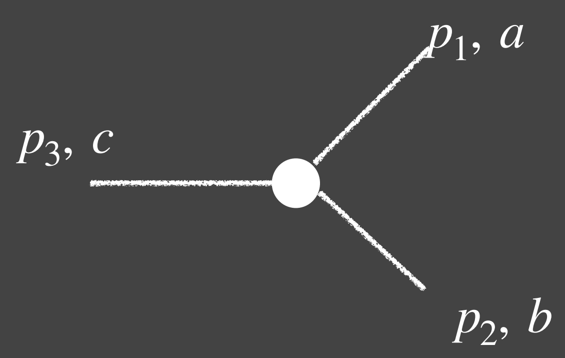
so:

coupling of 3 photons = 0 (indeed)

power of Lorentz

need at least 3 “photons” for a nonzero interaction (indeed realized in nature: W^+W^-Z)

what are the most general interactions of spin-1 particles?



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and probably rings a bell..

Lie groups

$$[T^a, T^b] = i f^{abc} T^c$$

$$\left([J^a, J^b] = i \, \varepsilon^{abc} T^c \right)$$

f^{abc} totally antisymmetric

obey Jacobi identity: $f^{abc} f^{ade} + \cdots = 0$

—> classification of all Lie algebras

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—> classification of all Lie algebras

Sophus Lie ~ 1870s

something that physicists will never lay their hands on..

vector self-coupling

↓

math: Lie algebras; full classification

physics: the structure of gauge field theories

all (almost) you learn in QFT1

* Jacobi identity:

consistent factorization of 4-vector amplitude on 3-vector amplitude

come back to this

plan

1) Amplitudes & the amplitude bootstrap

2) The Higgs (and electroweak symmetry breaking)

what we know; what we dont know

3) Effective Theories (EFTs)

in practice: all the physics we do..

plan

1) Amplitudes & the amplitude bootstrap

2) The Higgs (and electroweak symmetry breaki

what we know; what we dont know

putting it

all together

3) Effective Theories (EFTs)

in practice: all the physics we do..

scattering amplitudes & the amplitude bootstrap

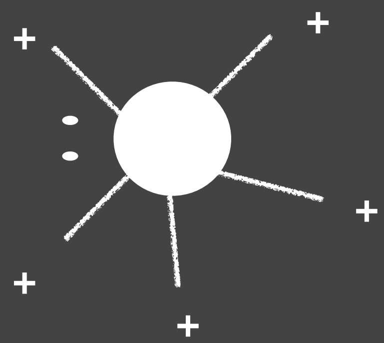
scattering amplitudes & the amplitude bootstrap

Münchhausen zieht sich am Zopf aus dem Sump, Distelli

Why amplitudes?

1st clue: amplitudes: the whole is SMALLER than the sum of its parts:

gauge boson amplitudes: many Feynman diagrams (~10 million for tree 10-gluon):



$= \quad 0$

1986: Parke & Taylor: expressions for the **amplitudes-squared** of n-gluons of definite helicities: (one page PRL)

$$|\mathcal{M}_n(+++++\dots)|^2 = c_n(g,N)\,[\,0 + O(g^4)\,] \quad (1)$$

$$|\mathcal{M}_n(-++++\dots)|^2 = c_n(g,N)\,[\,0 + O(g^4)\,] \quad (2)$$

$$|\mathcal{M}_n(--+++\dots)|^2 = c_n(g,N)\,\Big[\,(1\cdot 2)^4 \sum_P \frac{1}{(1\cdot 2)(2\cdot 3)(3\cdot 4)\dots(n\cdot 1)} + O(N^{-2}) + O(g^2)\,\Big] \quad (3)$$

where $c_n(g,N) = g^{2n-4}N^{n-2}(N^2-1)/2^{n-4}n$ and $(i\cdot j) = p_i\cdot p_j$. The sum is over all permutations, P, of $1\dots n$. Eqn(3) has the correct dimensions for a

Why so much simpler?

Feynman diagram calculation comes from Lagrangian:

gluon (massless spin-1, just like photon) described by vector field $(\phi(x), \overrightarrow{A}(x))$

4 degrees of freedom $x \equiv (t, \vec{x})$

amplitudes more efficient: focus on physical dof's only: 2 (gluon polarizations)

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for manifest Lorentz invariance

amplitude is function of:

- momenta
- polarizations for external particles of nonzero spin

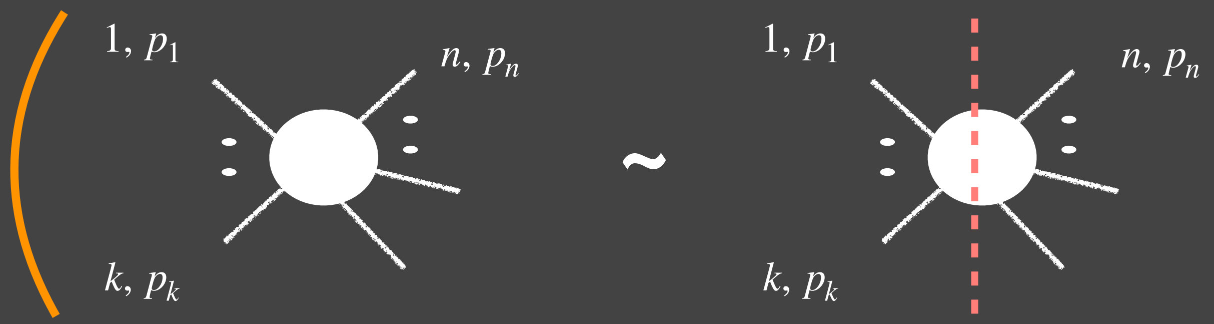
$$A(p_1, p_2, \ldots, p_n) \qquad\qquad p^2 = (p^0)^2 - \vec{p}^2 = m^2$$

function of the momenta (complex plane)

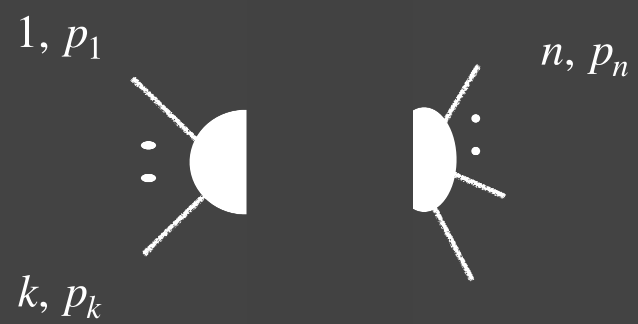
singularities encode physical spectrum:

poles:

$$(p_1 + \cdots + p_k)^2 = m^2_{particle}$$



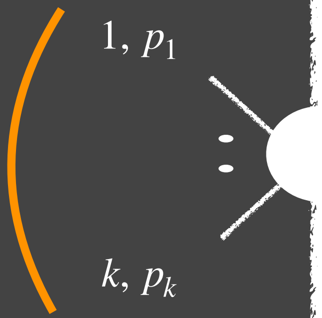
$$\sim \frac{1}{(p_1 + \cdots + p_k)^2 - m^2_{particle}}$$



some propagator goes ``on-shell" (on mass-shell)

poles:

$$(p_1 + \cdots + p_k)^2 = m^2_{particle}$$



know all 3-point amplitudes

—> know all residues of 4-point amplitudes

—> determine 4-point amplitude

...

$$\sim \frac{\ldots}{(p_1 + \cdots + p_k)^2 - m^2_{particle}}$$

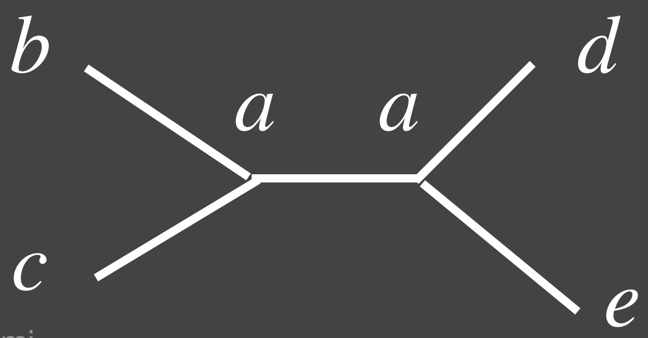
vector self-coupling

↓

Lie algebras

Jacobi identity:

consistent factorization of 4-vector amplitude on 3-vector amplitude



$$+\cdots \qquad \propto f^{abc} f^{ade} + \cdots$$

bootstrap

construct amplitudes recursively from the bottom up: w/out Lagrangian

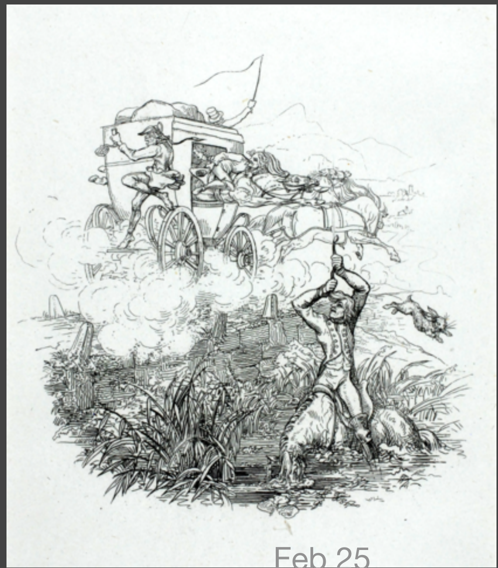
start with 3-point amplitudes

determine from: Lorentz, global symmetries, Bose/Fermi statistics

factorization —> higher point amplitudes (almost)

[rediscover QFT: Lie groups, gauge theory massless + massive: Higgsing]

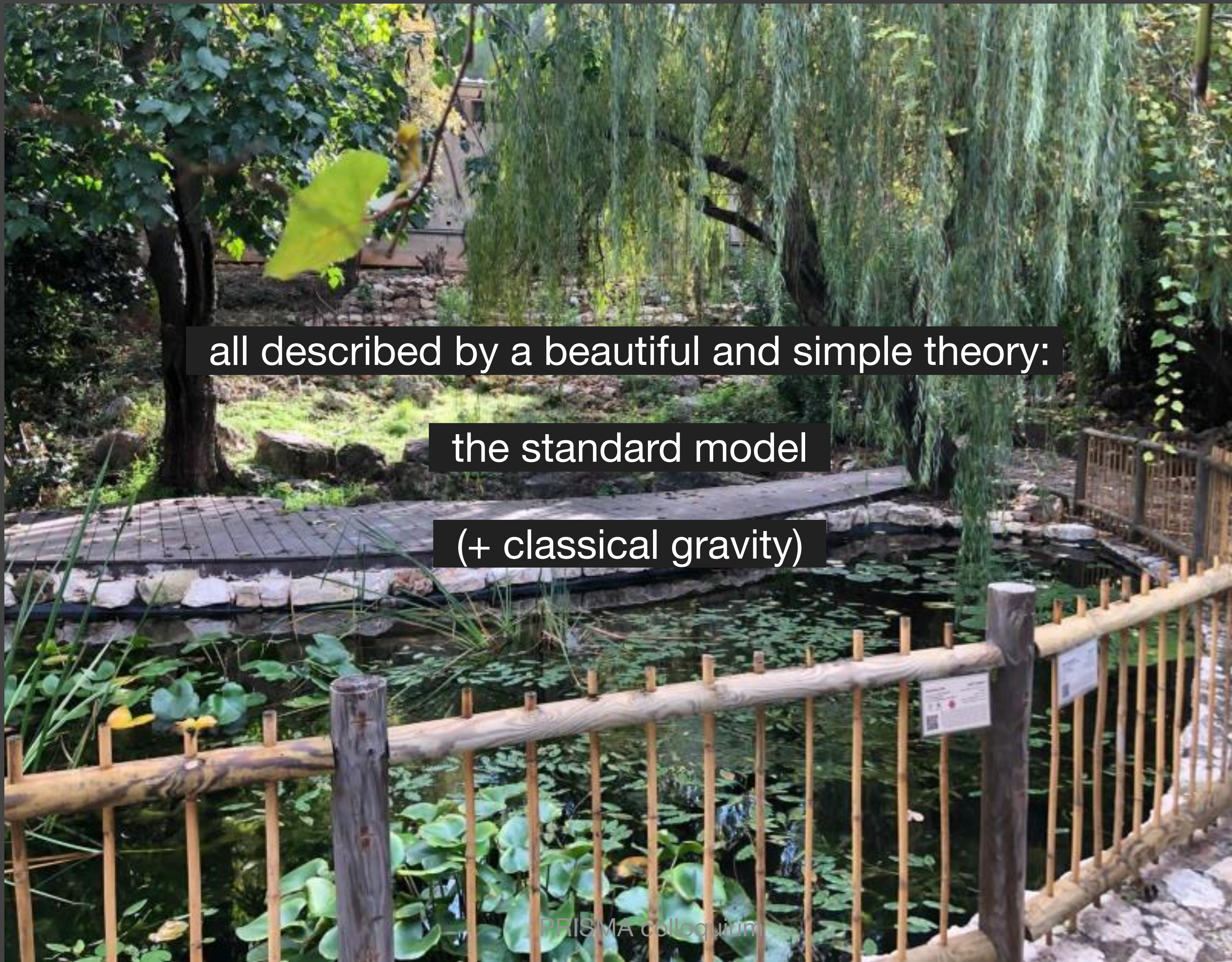
interested in electroweak symmetry breaking



and now to something completely different

The Higgs

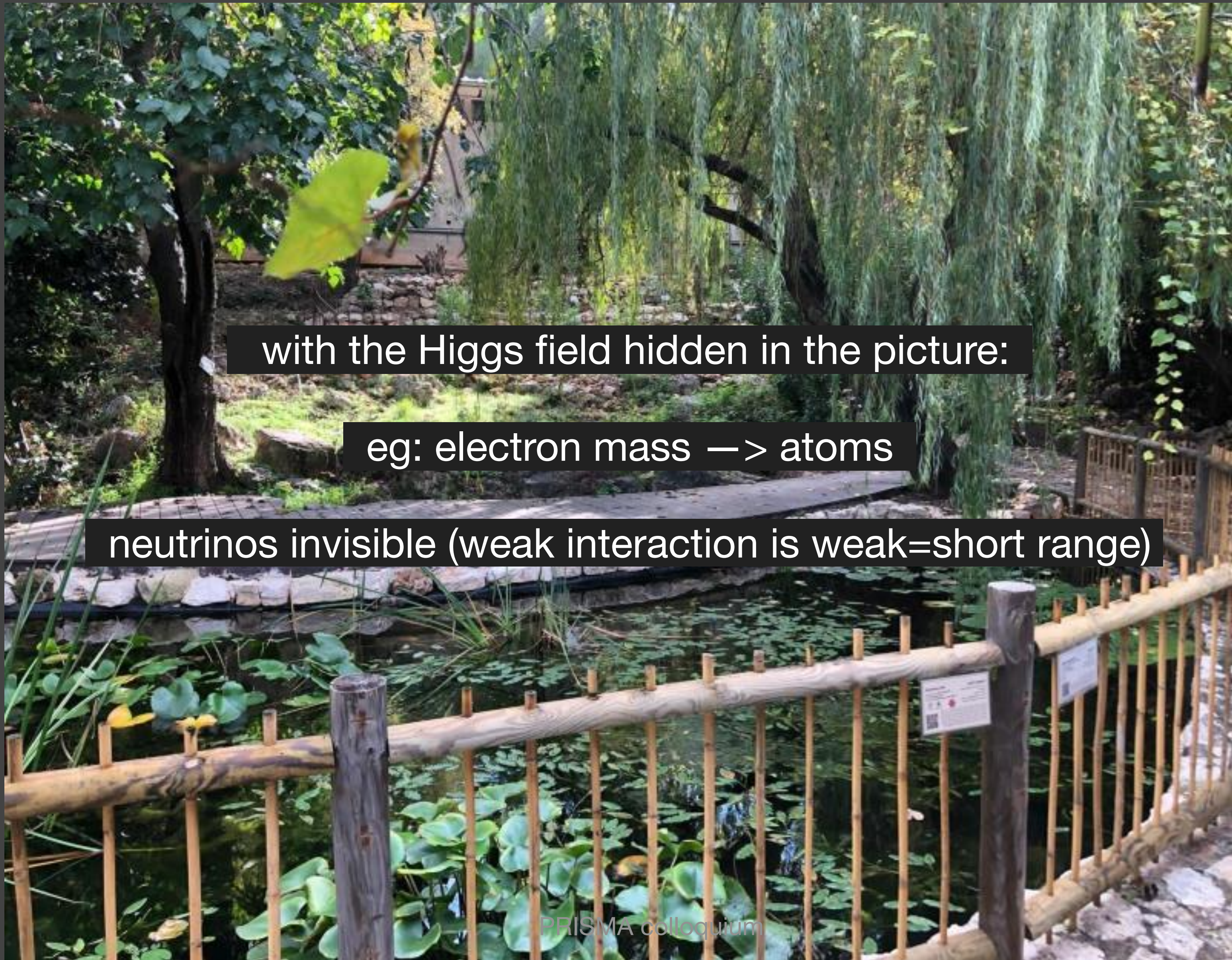
(and the standard model)



all described by a beautiful and simple theory:

the standard model

(+ classical gravity)



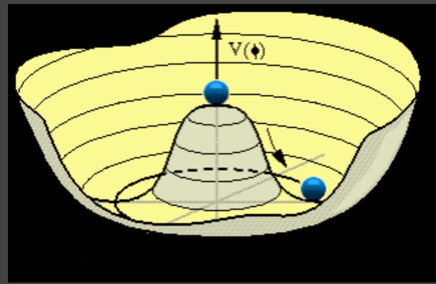
with the Higgs field hidden in the picture:

eg: electron mass —> atoms

neutrinos invisible (weak interaction is weak=short range)

SM Higgs:

simple parametrization in terms of Higgs doublet



$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \qquad v = \sqrt{-\mu^2/\lambda}$$

correct description?

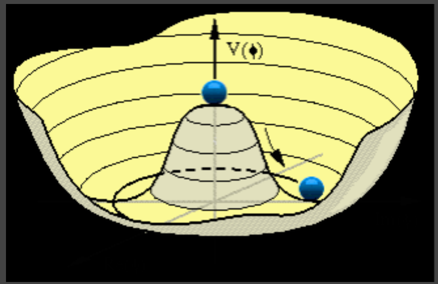
$$\rightarrow \quad V(h) = \frac{1}{2}m_h^2 h^2 + \#\lambda v h^3 + \lambda^4 h^4$$

v measured (W, Z masses)

higgs mass determines λ

$$m_h^2 = \frac{1}{2}\lambda v^2$$

parametrized by



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potential predicts:

cubic Higgs self-coupling $\propto \lambda v$

quartic Higgs self-coupling $\propto \lambda$

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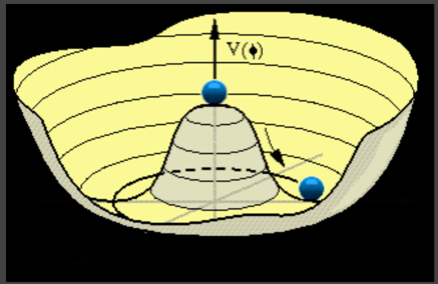

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??

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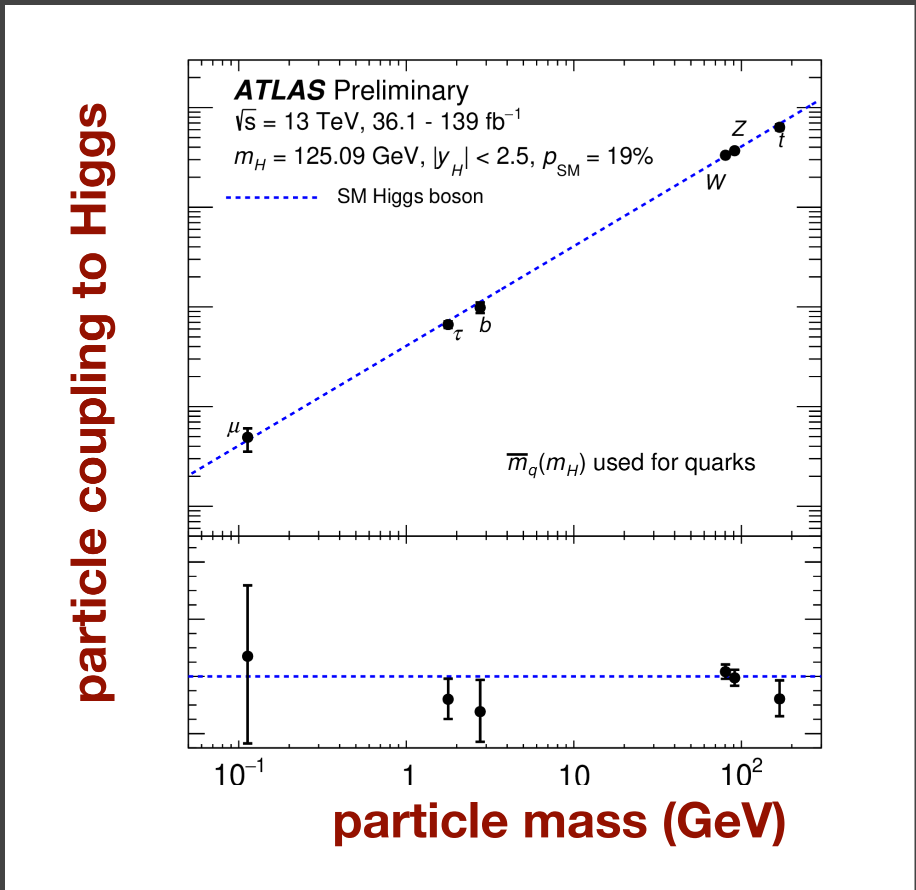
quartic Higgs self-coupling $\propto \lambda$

the ultimate quantum machine

every possible initial state —> every possible final state

Higgs produced and decays in many different processes so far: learned that the heavier particles get their mass from the Higgs:

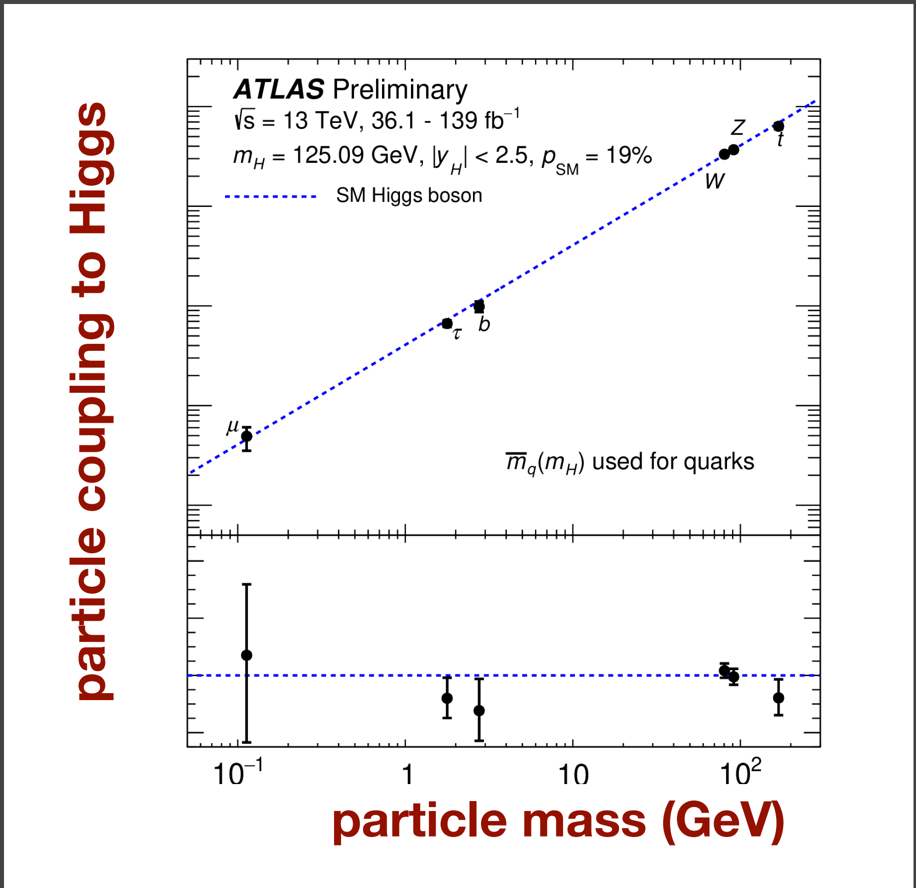


just like particle in medium: effective mass

$\propto$ size of the particle interaction with the Higgs

$\propto v = 246$ GeV

so far: learned that the heavier particles get their mass from the Higgs:



??

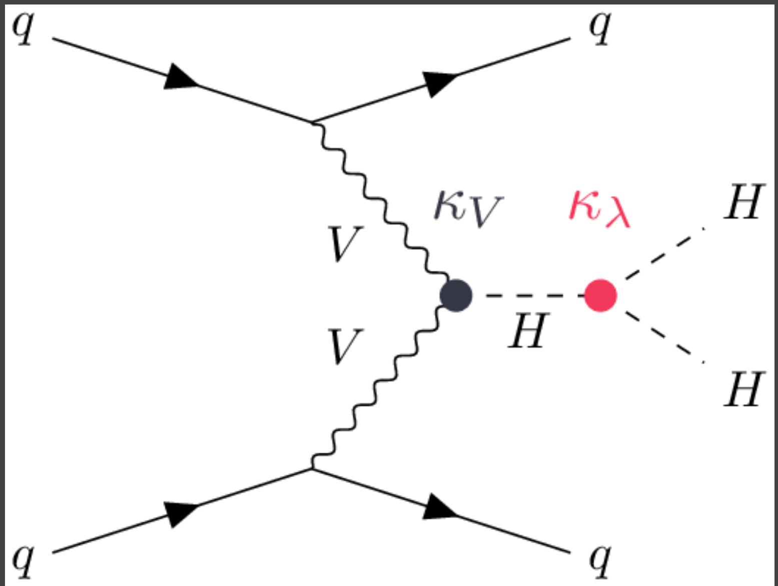
electron

up, down,

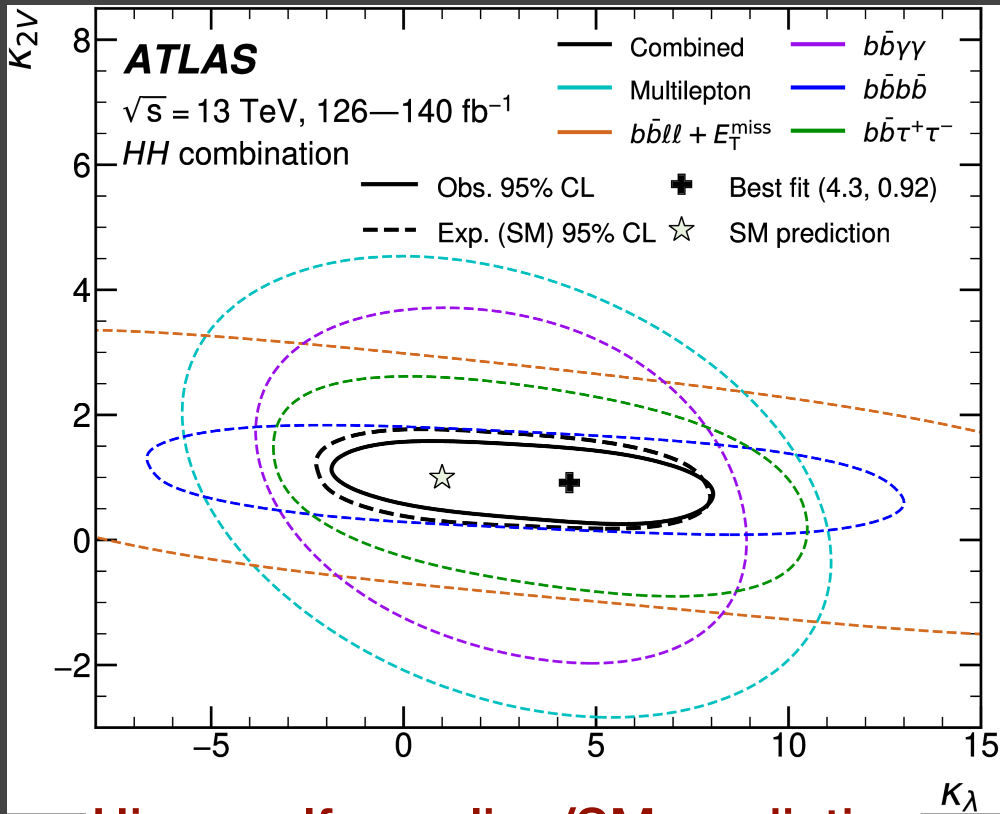
strange, charm

Higgs self coupling:

one contribution to production

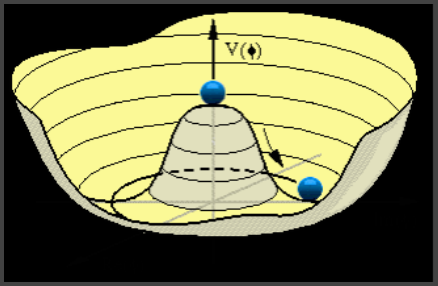


ATLAS Phys. Rev. Lett. 133 (2024)



Higgs self coupling/SM prediction

+ another layer of questions (theory)

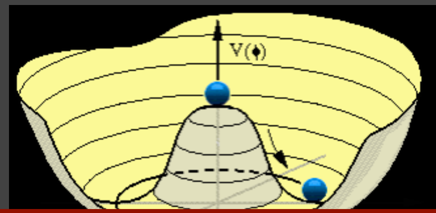


“by hand:”

? minimum away from origin

? 246 GeV scale

? stable against radiative corrections



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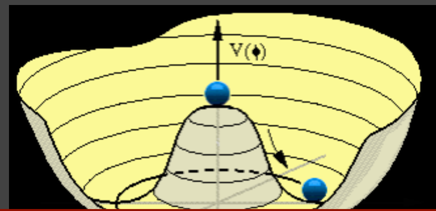
eg (weakly coupled): supersymmetric extensions of SM

stop mass + top Yukawa —> minimum away from origin

origin of scale: new dynamics: dynamical supersymmetry breaking simple *parametrization* in terms of Higgs mechanism



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(weakly coupled): supersymmetric extensions of SM

stop mass + top Yukawa —> minimum away from origin

origin of scale: new dynamics: dynamical supersymmetry breaking

How do you look for a theory you don’t know?

bottom-up Effective Theories (EFTs)

taking a step back:

Higgs was first discovered long ago.. 1890’s: beta decay

Higgs discovery timeline

1890s	1982	2012
beta decay	W	h

EFT footprints of W

Higgs discovery timeline

What is the origin of the Higgs potential?

Is there new physics involved?

1890s

beta decay

EFT footprints of W

1982

W

2012

h

??

EFT

parametrize our ignorance

the beauty of the **quantum world: sensitivity to high energy scales**

even though not directly accessible

$$E_n(\lambda) = E_n^{(0)} + \lambda \left\langle n^{(0)} \middle| V \middle| n^{(0)} \right\rangle + \lambda^2 \sum_{k \neq n} \frac{\left| \left\langle k^{(0)} \middle| V \middle| n^{(0)} \right\rangle \right|^2}{E_n^{(0)} - E_k^{(0)}} + O(\lambda^3)$$

parametrize our ignorance: effective theory

main two versions (two levels of ignorance)

M_{heavy} ???

$$\mathscr{L}_{effective} = \sum_i c_i \mathcal{O}_i(\phi_1, \ldots, \phi_n)$$

standard model **SU(3)xSU(2)xU(1)**

(known) fields **Higgs is part of a doublet**

parametrize our ignorance: effective theory

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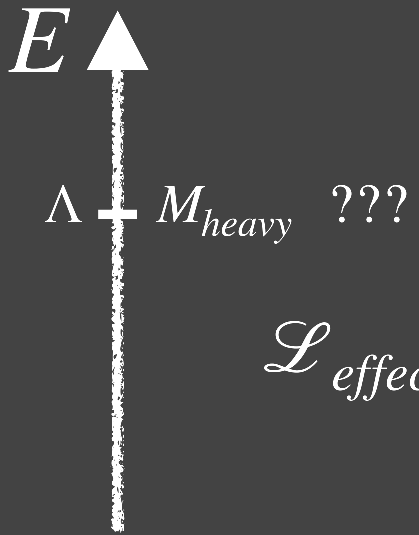

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SM-EFT or SMEFT

parametrize our ignorance: effective theory

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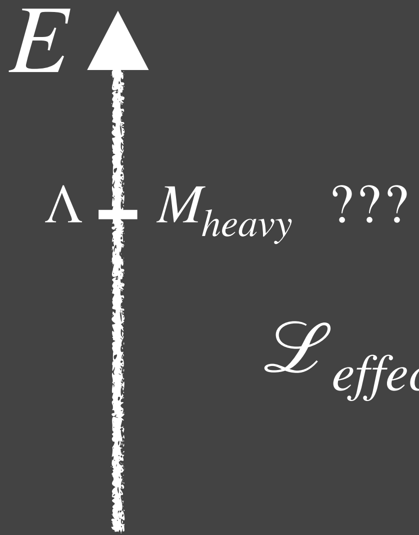

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just **SU(3)xU(1)$_{\text{EM}}$**

(really known) fields

Higgs not part of a doublet

parametrize our ignorance: effective theory

main two versions (two levels of ignorance)

E

Λ M_{heavy} ???

$$\mathscr{L}_{effective} = \sum_i c_i \mathcal{O}_i(\phi_1, \ldots, \phi_n)$$

Generic EFT extension of SM

"HEFT"

just **SU(3)xU(1)$_{\mathbf{EM}}$**

(really known) fields

Higgs not part of a doublet

- write down the most general Lagrangian: impose
  - global symmetries
  - gauge symmetry
  - [Lorentz, locality]

- need: **basis of operators:** complete set of independent operators:

operators can be traded for each other via:

field redefinitions, integration by parts, use of equations of motion
~1000 operators at leading order (dim-6)..

- using a complete basis is important: various operators affect multiple processes; each process typically affected by multiple operators (global fits)

- write down the most general Lagrangian: impose
  - global symmetries
  - gauge symmetry
  - [Lorentz, locality]

- need: **basis of operat** dependent operators:

operators can b r via:

hard problem

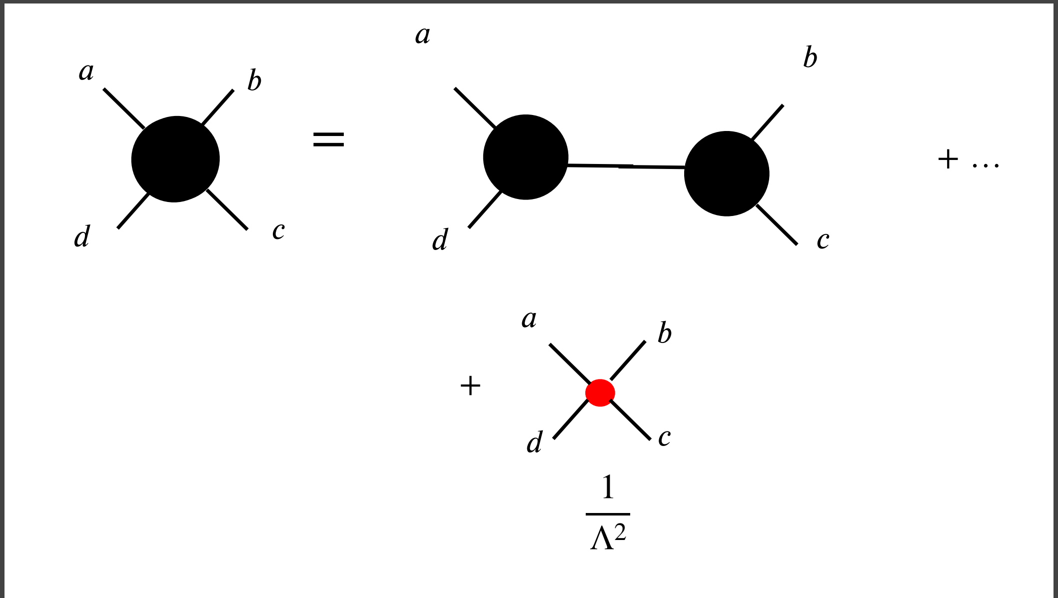
field redefinitions, in of equations of motion
~1000 operators at l g order (dim-6)..

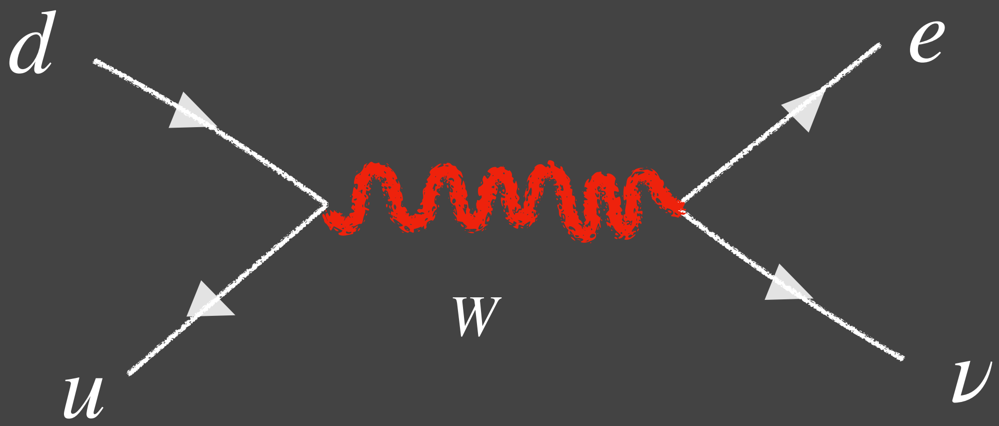
- using a complete basis is important: various operators affect multiple processes; each process typically affected by multiple operators (global fits)

made easier by amplitudes:

bootstrap

only possibly missing pieces: new “contact” interactions: exactly what we are after

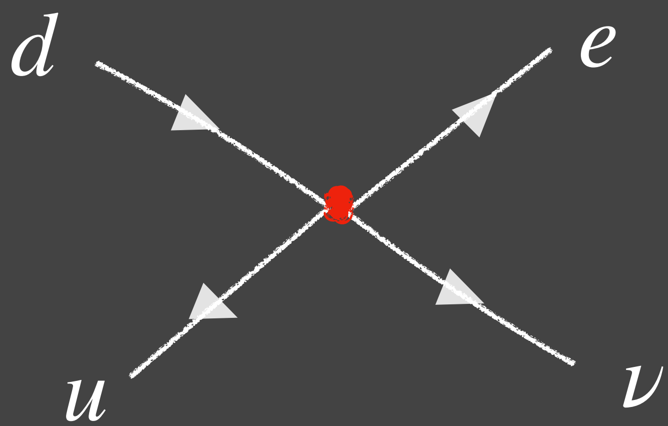




$$\propto \frac{1}{(p_d - p_u)^2 - M_W^2}$$

$(M_W \sim 80\, m_{proton})$

$$\propto \frac{1}{\cancel{(p_d - p_u)^2} - M_W^2}$$



CONSTANT !

$$\propto \frac{1}{M_W^2} \sim G_F$$

local "contact term"

EFT via on-shell bootstrap

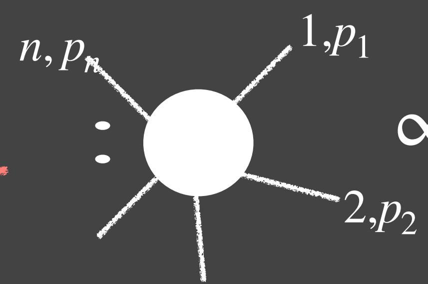
YS Weiss ‘18
…

usually: start with SM fields: most general $\mathscr{L}$
consistent with symmetries (global, gauge)

on-shell: start with SM particles: most general $\mathscr{A}$
consistent with symmetries (global, gauge)

$$\mathscr{L} = \sum_i c_i \mathscr{O}_i(\phi_1, \ldots, \phi_n)$$

1-1 correspondence



$\propto c_i$

local “contact term”

- no redundancies, field redefinitions, physical dof’s only

- + theory-wise: we are looking for the theory of electroweak symmetry breaking
  —> back to basics: physical dof’s

  —> on-shell/amplitude understanding of Higgs mechanism

EFT applications

On-shell applications to EFTs (massless)

- selection rules: explain zeros in
  - matrix of anomalous dimensions of EFT operators (loop cuts & generalized cuts)

    Cheung Shen ’15
    Bern Parra-Martinez Sawyer ’20
  - interference of SM x EFT amplitudes (tree)

    Azatov Contino Machado Riva ‘16
- derive anomalous dimensions of EFT operators (loop cuts & generalized cuts)

Barratella Fernandez von Harling Pomarol ’20
Bern Parra-Martinez Sawyer ’20
Jiang Ma Shu ’20
De Angelis Accettulli-Huber ’21
Barratella ’22
…

On-shell applications to EFTs (massless + massive)

- count (& construct) bases of EFT operators:

YS Weiss '18
Ma Shu Xiao '19
Remmen Rodd '19
Li Ren Shu Xiao Yu Zheng '20
Durieux Machado '20
…
also used in Henning Melia Murayama '15

- UV matching

…

De Angelis Durieux '23

in many of these:

amplitude

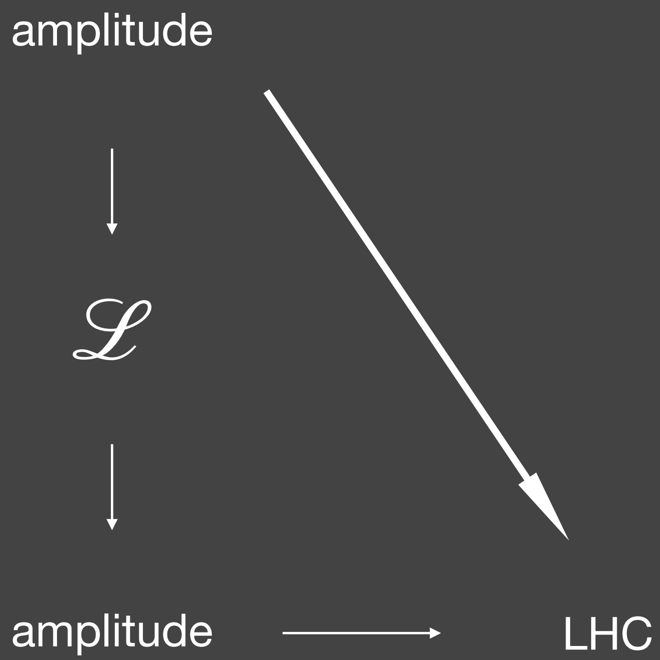
↓

$\mathscr{L}$

↓

amplitude ⟶ LHC

Instead:



SMEFT: to derive predictions:

- basis of operators in unbroken theory
- turn on Higgs VEV —> Lagrangian in broken theory: SM fields, couplings shift
- derive Feynman rules of broken theory in some gauge
- redefine parameters from “input” physical masses, couplings

amplitudes: working with physical dof’s, couplings only

HEFT:

“sick” EFT : eg, integrated out fields with masses from EWSB

<—> no scale separation

UV matching ambiguous   Dawson Fontes Quezada-Calonge Sanz-Cillero ‘23

amplitudes: make concrete

amplitude construction: bottom-up:

—> starting with the massive (and massless) **particles** we know:
construct **most general** amplitudes

- 3-points (renormalizable + higher-dim): dictated by little group, symmetries
- factorizable parts of higher-point amplitudes (determined by 3-pts..)
- higher-point contact terms: dictated by little group, symmetries

contact-term part of amplitude:

YS Weiss ‘18
Durieux Kitahara YS Weiss ’19
Durieux Kitahara Machado YS Weiss ’20
...

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}}\, P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

local: no poles

$$\mathscr{A} = \frac{[\,\cdot\,\cdot\,]\cdot\cdot\langle\,\cdot\,\cdot\,\rangle}{\Lambda^{\#}}\, P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; “stripped” off

all Lorentz invariants s_{ij}

“stripped contact term”   SCT

different SCTs can come from integrating out different UV fields — different suppressions

Chang Chen Liu Luty ’22

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; "stripped" of
all Lorentz invariants s_{ij}
"stripped contact term" SCT

polynomial in Lorentz
invariants s_{ij}
subject to kinematical constraints,
eg, $s_{12} + s_{13} + s_{23} = \sum m^2$

derivative expansion

structure of 2 to 2 contact-terms:

$$\mathscr{A} = \underbrace{\frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}}}_{\text{SCT}} \; \underbrace{P\left(\frac{s}{\Lambda^2}, \frac{t}{\Lambda^2}\right)}$$

scattering
angle
and
decay angles

scattering
angle

? construct observables to isolate novel SCTs not appearing in SM

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; "

all Lorentz invar

"stripped contact t

raints,

m^2

bottom up construction; input: physical particles
SU(3)xU(1)
higgs = gauge singlet

gives **HEFT** amplitudes

What about **(low-energy) SMEFT** amplitudes?

use on-shell Higgsing

Balkin Durieux Kitahara YS Weiss ‘21

EFT: new fields Λ

EWSB $m \sim v$

massless $\mathscr{A}$ impose full SU(3)xSU(2)xU(1)

↓

derive massive $\mathscr{M}$

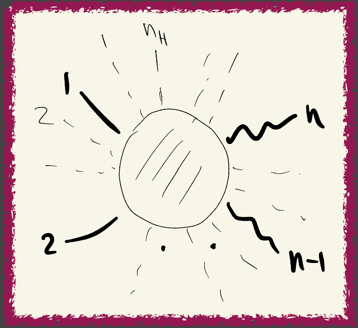
(contact term part only)

anatomy of on-shell Higgsing

Balkin Durieux Kitahara YS Weiss ‘21

massless amplitudes of unbroken theory —> “Higgs” to get low-energy massive amplitudes

extra Higgs legs non-dynamical: soft: $H(q_i) \quad q_i \to 0$



probe field space

+ Cheung Helset Parra-Martinez’23

matching at high energy:

$$E \gg q \sim m \; (\sim VEV \;\; v)$$

$$M_n(1,\ldots,n) \;=\; A_n(1,\ldots,n) + v \lim_{q\sim v\to 0} A_{n+1}(1,\ldots,n;H(q)) + \cdots$$

results: HEFT,   SMEFT

HEFT inventory *(**observables**; many more results on operators, anomalous dim's via on-shell)*

- *all HEFT 3-points (+matching to SMEFT)* — *Durieux Kitahara YS Weiss '19*
- *[all generic 3-points for spins up to 3*
- *all generic 4-pt SCTs for spins 0, 1/2, 1]* — *Durieux Kitahara Machado YS Weiss'20*
- *HEFT 4-points: hggg, Zggg, ffVh, WWhh* — *Shadmi et al '18, Durieux et al '19, Balkin et al '21*

  + some full amplitudes (factorizable + contact terms): ffWh, ffZh, WWhh
- *5V (4W+Z etc)* — *De Angelis '21*
- *Higgs, top 4pts in terms of momenta+polarizations* — *Chang et al '22, '23*
- *all HEFT 4pts up to d=8* ← — *Liu Ma YS Waterbury '23*
- *SMEFT 4pts up to d=8 for VV* — *Goldberg Liu YS '24*

all HEFT 4-pts up to d=8

Liu Ma YS Waterbury ’23

[Dong Ma Shu Zhou ’22 HEFT operators]

- most relevant for collider studies: 2 to 2
- dimension counting: classify contact terms by energy growth

full set of EFT contact terms with E^2 growth: (mostly dim-6 operators)

LOW ENERGY AMPLITUDES

Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(\gamma\gamma hh)$	$C^{\pm\pm}_{\gamma\gamma hh}(12)^2$
$\mathcal{M}(\gamma Zhh)$	$C^{\pm}_{\gamma Zhh}(1\mathbf{2})^2$
$\mathcal{M}(hhhh)$	C_{hhhh}
$\mathcal{M}(f^c fhh)$	$C^{\pm\pm}_{ffhh}(\mathbf{12})$
$\mathcal{M}(f^c fWh)$	$C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c fZh)$	$C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c f\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(q^c qgh)$	$C^{\pm\pm\pm}_{qqgh}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(f^c f f^c f)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34})$, $C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}]$, $C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}]$, $C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24})$, $C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle$, $C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle$, $C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

$(12) = [12]$ or $\langle 12\rangle$

C's: Wilson coefficients

most suppressed by $\bar{\Lambda}^2$
(amplitude dim-less)

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- similarly: derived full set of CTs with E^3, E^4 growth
- corresponding to $d \leq 8$ HEFT operators
- clear identification of operator dimension from dim-analysis:

factors of $p]p\rangle$ (external massive vector) $\rightarrow p]p\rangle/M$

any extra powers of E compensated by powers of Λ

—> read off dimension of operator

but recall $\Lambda \sim v$; E/v terms in amplitudes reflect non-locality of HEFT

(cancel in SMEFT amplitudes: gauge invariance <—> perturbative unitarity)

SMEFT 4-pts

full list of CTs from $d \leq 6$ SMEFT

Massive $d=6$ amplitudes	SMEFT Wilson coefficients
$\mathcal{M}(W_L^+ W_L^- hh) = C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{WWhh} = (c^{(+)}_{(H^\dagger H)^2} - 3c^{(-)}_{(H^\dagger H)^2})/2$
$\mathcal{M}(W_\pm^+ W_\pm^- hh) = C^{\pm\pm}_{WWhh}(\mathbf{12})^2$	$C^{\pm\pm}_{WWhh} = 2c^{\pm\pm}_{WWHH}$
$\mathcal{M}(Z_L Z_L hh) = C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{ZZhh} = -2c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{M}(Z_\pm Z_\pm hh) = C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$	$C^{\pm\pm}_{ZZhh} = c_W^2 c^{\pm\pm}_{WWHH} + s_W^2 c^{\pm\pm}_{BBHH} + c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(g_\pm g_\pm hh) = C^{\pm\pm}_{gghh}(12)^2$	$C^{\pm\pm}_{gghh} = c^{\pm\pm}_{GGHH}$
$\mathcal{M}(\gamma_\pm \gamma_\pm hh) = C^{\pm\pm}_{\gamma\gamma hh}(12)^2$	$C^{\pm\pm}_{\gamma\gamma hh} = s_W^2 c^{\pm\pm}_{WWHH} + c_W^2 c^{\pm\pm}_{BBHH} - c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(\gamma_\pm Zhh) = C^{\pm}_{\gamma Zhh}(1\mathbf{2})^2$	$C^{\pm}_{\gamma Zhh} = s_W c_W c^{\pm\pm}_{WWHH} - s_W c_W c^{\pm\pm}_{BBHH} + \frac{1}{2}(s_W^2 - c_W^2)c^{\pm\pm}_{BWHH}$
$\mathcal{M}(hhhh) = C_{hhhh}$	$C_{hhhh} = -3c_{(H^\dagger H)^2} + 45\, v^2 c_{(H^\dagger H)^3}$
$\mathcal{M}(f^c_\pm f_\pm hh) = C^{\pm\pm}_{ffhh}(\mathbf{12})$	$C^{\pm\pm}_{ffhh} = 3c^{\pm\pm}_{\Psi\psi HHH}v/(2\sqrt{2})$
$\mathcal{M}(f^c_+ f'_- W_L h) = C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{ffWh} = (c^{+-,(+)}_{\Psi\Psi HH} - c^{+-,(-)}_{\Psi\Psi HH})/2$
$\mathcal{M}(f^c_- f'_+ W_L h) = C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0}_{ffWh} = c^{-+}_{\psi_R \psi'_R HH}$
$\mathcal{M}(f^c_\pm f'_\pm W_\pm h) = C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffWh} = c^{\pm\pm\pm}_{\Psi\psi WH}/2$
$\mathcal{M}(f^c_+ f_- Z_L h) = C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{e_L e_L Zh} = -i\sqrt{2}c^{+-,(+)}_{\Psi\Psi HH},\ C^{+-0}_{\nu_L \nu_L Zh} = -i(c^{+-,(+)}_{\Psi\Psi HH} + c^{+-,(-)}_{\Psi\Psi HH})/\sqrt{2}$
$\mathcal{M}(f^c_- f_+ Z_L h) = C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0,\mathrm{CT}}_{ffZh} = -i\sqrt{2}c^{-+}_{\psi\psi HH}$
$\mathcal{M}(f^c_\pm f_\pm Z_\pm h) = C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffZh} = -(s_W c^{\pm\pm\pm}_{\Psi\psi BH} + c_W c^{\pm\pm\pm}_{\Psi\psi WH})/\sqrt{2}$
$\mathcal{M}(f^c_\pm f_\pm \gamma_\pm h) = C^{\pm\pm\pm}_{ff\gamma h}(13)(\mathbf{2}3)$	$C^{\pm\pm\pm}_{ff\gamma h} = (-s_W c^{\pm\pm\pm}_{\Psi\psi WH} + c_W c^{\pm\pm\pm}_{\Psi\psi BH})/\sqrt{2}$
$\mathcal{M}(q^c_\pm q_\pm g^A_\pm h) = C^{\pm\pm\pm}_{qqgh}\lambda^A(13)(\mathbf{23})$	$C^{\pm\pm\pm}_{qqgh} = c^{\pm\pm\pm}_{\Psi\psi GH}/\sqrt{2}$

Table 3: The low-energy E^2 contact terms (left column) and their $d=6$ coefficients in the SMEFT (right column). $c_{(H^\dagger H)^2}$ without a superscript is the renormalizable four-Higgs coupling. The mapping for four fermion contact terms is trivial, so we do not include them here.

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to get these:

start with **massless** dim-6 SMEFT amplitudes

and Higgs these to get massive amplitudes

Amplitude	Contact term	Warsaw basis operator	Coefficient
$\mathcal{A}(H_i^c H_j^c H_k^c H^l H^m H^n)$	$T_{ijk}^{+\,lmn}$	$\mathcal{O}_H/6$	$c_{(H^\dagger H)^3}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$s_{12} T_{ij}^{+\,kl}$	$\mathcal{O}_{HD}/2 + \mathcal{O}_{H\Box}/4$	$c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$(s_{13} - s_{23}) T_{ij}^{-\,kl}$	$\mathcal{O}_{HD}/2 - \mathcal{O}_{H\Box}/4$	$c^{(-)}_{(H^\dagger H)^2}$
$\mathcal{A}(B^\pm B^\pm H_i^c H^j)$	$(12)^2 \delta_i^j$	$(\mathcal{O}_{HB} \pm i\mathcal{O}_{H\tilde{B}})/2$	$c^{\pm\pm}_{BBHH}$
$\mathcal{A}(B^\pm W^{I\pm} H_i^c H^j)$	$(12)^2 (\sigma^I)_i^j$	$\mathcal{O}_{HWB} \pm i\mathcal{O}_{H\tilde{W}B}$	$c^{\pm\pm}_{BWHH}$
$\mathcal{A}(W^{I+} W^{J+} H_i^c H^j)$	$(12)^2 \delta^{IJ} \delta_i^j$	$(\mathcal{O}_{HW} \pm i\mathcal{O}_{H\tilde{W}})/2$	$c^{\pm\pm}_{WWHH}$
$\mathcal{A}(g^{A\pm} g^{B\pm} H_i^c H^j)$	$(12)^2 \delta^{AB} \delta_i^j$	$(\mathcal{O}_{HG} \pm i\mathcal{O}_{H\tilde{G}})/2$	$c^{\pm\pm}_{GGHH}$
$\mathcal{A}(L_i^c e H_j^c H^k H^l)$	$[12] T_{ij}^{+\,kl}$	$\mathcal{O}_{eH}/2$	c^{++}_{LeHHH}
$\mathcal{A}(Q_{a,i}^c d^b H_j^c H^k H^l)$	$[12] T_{ij}^{+\,kl} \delta_a^b$	$\mathcal{O}_{dH}/2$	c^{++}_{QdHHH}
$\mathcal{A}(Q_{a,i}^c u^b H_j^c H_k^c H^l)$	$[12] \varepsilon_{im} T_{jk}^{+\,ml} \delta_a^b$	$\mathcal{O}_{uH}/2$	c^{++}_{QuHHH}
$\mathcal{A}(e^c e H_i^c H^j)$	$\langle 142] \delta_i^j$	$\mathcal{O}_{He}/2$	c^{-+}_{eeHH}
$\mathcal{A}(u_a^c u^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hu}/2$	c^{-+}_{uuHH}
$\mathcal{A}(d_a^c d^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hd}/2$	c^{-+}_{ddHH}
$\mathcal{A}(u_a^c d^b H^i H^j)$	$\langle 142] \epsilon^{ij} \delta_a^b$	$\mathcal{O}_{Hud}/2$	c^{-+}_{udHH}
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T_{ik}^{+\,jl}$	$\left(\mathcal{O}_{HL}^{(1)} + \mathcal{O}_{HL}^{(3)}\right)/8$	$c^{+-,(+)}_{LLHH}$
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T_{ik}^{-\,jl}$	$\left(\mathcal{O}_{HL}^{(1)} - \mathcal{O}_{HL}^{(3)}\right)/8$	$c^{+-,(-)}_{LLHH}$
$\mathcal{A}(Q_{a,i}^c Q^{b,j} H_k^c H^l)$	$[142\rangle T_{ik}^{+\,jl} \delta_a^b$	$\left(3\mathcal{O}_{HQ}^{(1)} + \mathcal{O}_{HQ}^{(3)}\right)/8$	$c^{+-,(+)}_{QQHH}$
$\mathcal{A}(Q_{a,i}^c Q^{b,j} H_k^c H^l)$	$[142\rangle T_{ik}^{-\,jl} \delta_a^b$	$(\mathcal{O}_{HQ}^{(1)} - \mathcal{O}_{HQ}^{(3)})/8$	$c^{+-,(-)}_{QQHH}$
$\mathcal{A}(L_i^c e B^+ H^j)$	$[13][23] \delta_i^j$	$-i\mathcal{O}_{eB}/(2\sqrt{2})$	c^{+++}_{LeBH}
$\mathcal{A}(Q_{a,i}^c d^b B^+ H^j)$	$[13][23] \delta_i^j \delta_a^b$	$-i\mathcal{O}_{dB}/(2\sqrt{2})$	c^{+++}_{QdBH}
$\mathcal{A}(Q_{a,i}^c u^b B^+ H_j^c)$	$[13][23] \epsilon_{ij} \delta_a^b$	$-i\mathcal{O}_{uB}/(2\sqrt{2})$	c^{+++}_{QuBH}
$\mathcal{A}(L_i^c e W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j$	$-i\mathcal{O}_{eW}/(2\sqrt{2})$	c^{+++}_{LeWH}
$\mathcal{A}(Q_{a,i}^c d^b W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j \delta_a^b$	$-i\mathcal{O}_{dW}/(2\sqrt{2})$	c^{+++}_{QdWH}
$\mathcal{A}(Q_{a,i}^c u^b W^{I+} H_j^c)$	$[13][23] (\sigma^I)_{ik} \epsilon_j^k \delta_a^b$	$-i\mathcal{O}_{uW}/(2\sqrt{2})$	c^{+++}_{QuWH}
$\mathcal{A}(Q_{a,i}^c d^b g^{A+} H^j)$	$[13][23] \delta_i^j (\lambda^A)_a^b$	$-i\mathcal{O}_{dG}/(2\sqrt{2})$	c^{+++}_{QdGH}
$\mathcal{A}(Q_{a,i}^c u^b g^{A+} H_j^c)$	$[13][23] \epsilon_{ij} (\lambda^A)_a^b$	$-i\mathcal{O}_{uG}/(2\sqrt{2})$	c^{+++}_{QuGH}
$\mathcal{A}(W^{I\pm} W^{J\pm} W^{K\pm})$	$(12)(23)(31) \epsilon^{IJK}$	$(\mathcal{O}_W \pm i\mathcal{O}_{\tilde{W}})/6$	$c^{\pm\pm\pm}_{WWW}$
$\mathcal{A}(g^{A\pm} g^{B\pm} g^{C\pm})$	$(12)(23)(31) f^{ABC}$	$(\mathcal{O}_G \pm i\mathcal{O}_{\tilde{G}})/6$	$c^{\pm\pm\pm}_{GGG}$

Table 2: Massless $d = 6$ SMEFT contact terms [34] and their relations to Warsaw basis operators [3]. For each operator (or operator combination) $\mathcal{O}$ in the third column, $c\,\mathcal{O}$ generates the structure in the second column with the coefficient c given in the fourth column. c-superscripts denote charge conjugation.

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for completeness provide full mapping of 4-pt $d \leq 6$ EFT amplitudes to Warsaw basis

Ma Shu Xiao '19

VV pair production from dim=8 SMEFT: $V = W, Z, \gamma, g$

Goldberg Liu YS 2407.07945

derived all low-energy 4-pt CTs generated by dim-8 SMEFT

$VV \to VV$ $\bar{f}f \to VV$ .. (massless fermions)

- nonzero mass "resurrect" vanishing SM-SMEFT interference $\propto M_W, M_Z$
- good at $M_V \sim E \ll \Lambda$ (not just high-E where EFT not reliable)
- sensitivity to anomalous Higgs self couplings
- up/down quark SU(2) relations broken (first happens at dim-8)

VV pair production from dim=8 SMEFT: $V = W, Z, \gamma, g$

+ distinguish HEFT vs SMEFT:

- various coupling relations in SMEFT
- some SMEFT zeros (due to hypercharge or accidental)

EFT of electroweak precision measurements & spurion analysis

Northey, YS, Soreq, Ueda, 2502.?????

Z- and W-pole measurements: 3-points — simple & "exact" (no kinematic expansion)

$$M(\bar{Q}^i\, Q^j\, V) = C^i_j\, \frac{[1\mathbf{3}]\langle\mathbf{23}\rangle}{M_V}$$

EFT of electroweak precision measurements & spurion analysis

Northey, YS, Soreq, Ueda, 2502.?????

SU(2) structure **to all orders** via “spurion” analysis

spurion = normalized Higgs VEV

$$\mathcal{M}(\overline{Q}^i, Q_j, B) \sim c_{Q1}\delta_j{}^i + c_{Q2}(\tau^a)_j{}^i(\mathcal{H}^\dagger\tau^a\mathcal{H}) ,$$

$$\mathcal{M}(\overline{Q}^i, Q_j, W^a) \sim c_{Q3}(\tau^a)_j{}^i + c_{Q4}\delta_j{}^i(\mathcal{H}^\dagger\tau^a\mathcal{H}) + c_{Q5}i\varepsilon^{abc}(\tau^b)_j{}^i(\mathcal{H}^\dagger\tau^c\mathcal{H}) ,$$

5 structures

C’s functions of $H^\dagger H$

simple-minded (amplitude!) version of GeoSMEFT Helset Martin Trott ‘20

EFT of electroweak precision & spurion analysis

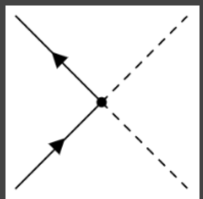
SU(2) structure **to all orders** via "spurion" analysis   spurion = Higgs VEV

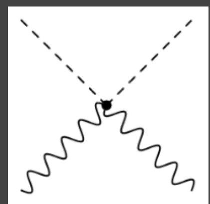
$$\mathcal{M}(\overline{Q}^i, Q_j, B) \sim c_{Q1}\delta_j{}^i + c_{Q2}(\tau^a)_j{}^i(\mathcal{H}^\dagger\tau^a\mathcal{H})\,,$$

$$\mathcal{M}(\overline{Q}^i, Q_j, W^a) \sim c_{Q3}(\tau^a)_j{}^i + c_{Q4}\delta_j{}^i(\mathcal{H}^\dagger\tau^a\mathcal{H}) + c_{Q5}i\varepsilon^{abc}(\tau^b)_j{}^i(\mathcal{H}^\dagger\tau^c\mathcal{H})\,,$$

examine on-shell Higgsing to see:

start @ dim-6





EFT of electroweak precision & spurion analysis

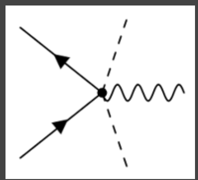
SU(2) structure **to all orders** via "spurion" analysis   spurion = Higgs VEV

$$\mathcal{M}(\overline{Q}^i, Q_j, B) \sim c_{Q1}\delta_j{}^i + c_{Q2}(\tau^a)_j{}^i(\mathcal{H}^\dagger\tau^a\mathcal{H}) \,,$$

$$\mathcal{M}(\overline{Q}^i, Q_j, W^a) \sim c_{Q3}(\tau^a)_j{}^i + c_{Q4}\delta_j{}^i(\mathcal{H}^\dagger\tau^a\mathcal{H}) + \boxed{c_{Q5} i\varepsilon^{abc}(\tau^b)_j{}^i(\mathcal{H}^\dagger\tau^c\mathcal{H})} \,,$$

examine on-shell Higgsing to see:

start @ dim-8



to conclude:

- now in the process of understanding electroweak symmetry breaking at LHC experiments
  ? uncover origin of Higgs mechanism (at this point: ad-hoc parametrization)

- mature(ing) methods for on-shell derivation of low-energy EFT amplitudes:

  - clear distinction between HEFT, SMEFT
  - alow for an interpretation of LHC measurements directly in terms of observables

- re-learn QFT from physical amplitudes: start to develop an *on-shell* understanding of field space — Higgs mechanism (power of Lorentz)

what will the LHC experiments tell us? hundreds of measurements never done before!

Thank you!

on to dim-8 SMEFT

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can have interesting effects (eg example here)

~ 1000 operators; with amplitudes, easy to concentrate on the relevant ones for a given observable

example: **WW, ZZ .. production** (sensitive probe of EWSB)

all relevant 4-pt CTs first generated at dim-8 (dim-6 SMEFT merely corrects SM-3pts)

from VVVV, VVHH etc: easy to see at amplitude level: 8 powers of $p]$ (or $p\rangle$) —> Λ^4

or 6 powers in ffVV —> SMEFT: Λ^4