Effective Field Theory for Nuclear Halo and Clustering

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2025.07.02

- Born in Nanjing
- B.Sc (Physics), Nanjing University (2002-2006)
- Ph.D, Ohio University (2006-2012)
- Postdoc, TRIUMF (2012-2015)
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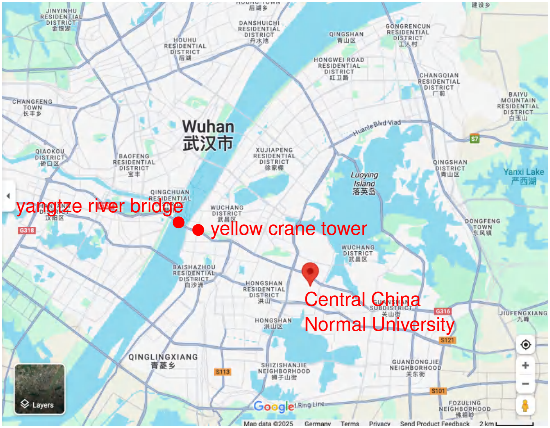
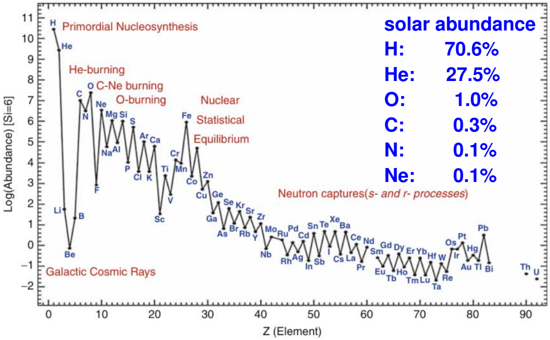
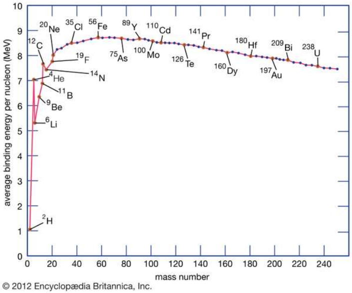


Institute of Particle Physics

- Faculty members: 47
- Research Areas:
  - Particle physics
  - Nuclear physics
  - Detector technology
  - Complex systems
  - Computational physics
- Platforms:
  - Pixel Laboratory at CCNU (PLAC)
  - Nuclear Science Computer Center at CCNU (NSC3)
  - Central China Center for Nuclear Theory (C3NT)

Nuclear abundance and nucleosythesis





- primordial/big-bang nucleosynthesis

  Alpher, Bethe, Gamow ($\alpha\beta\gamma$)

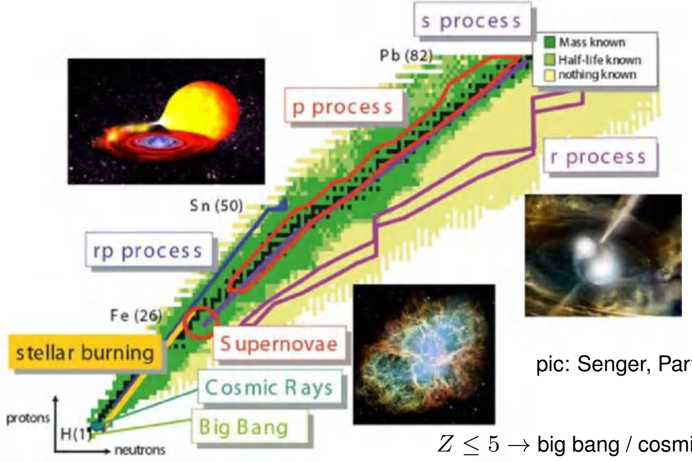
  Phys. Rev. 73 (1948) 803

- stellar nucleosynthesis

  Burbidge, Burbidge, Fowler, Hoyle (B^2FH)

  Rev. Mod. Phys. 29 (1957) 547

Nucleosynthesis & astrophysical processes



pic: Senger, Particles 3 (2020) 320

118 known elements

$\sim$3000 known isotopes

$\sim$4000 unknown isotopes

$Z \leq 5 \rightarrow$ big bang / cosmic ray

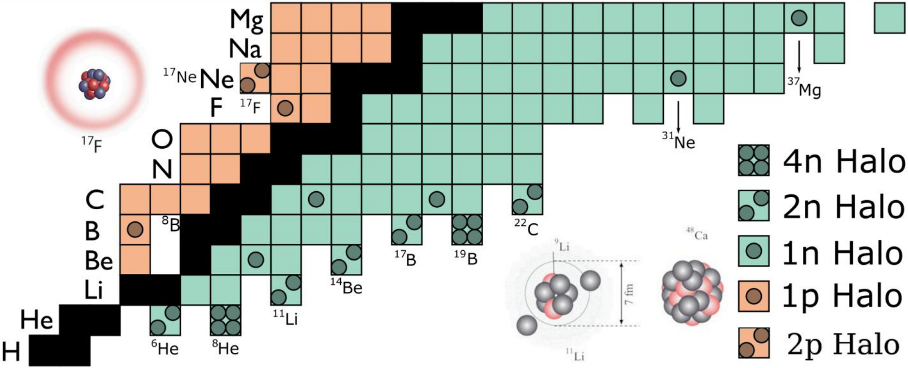
fusion $\rightarrow$ stellar burning

s process $\rightarrow$ AGB star

r process $\rightarrow$ supernovae & neutron-star merger

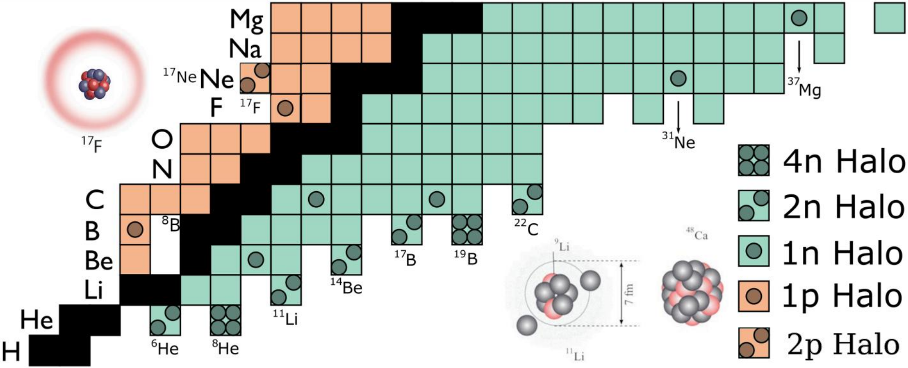
p & rp process $\rightarrow$ sun-like- and neutron-star binary

Introduction to halo nuclei



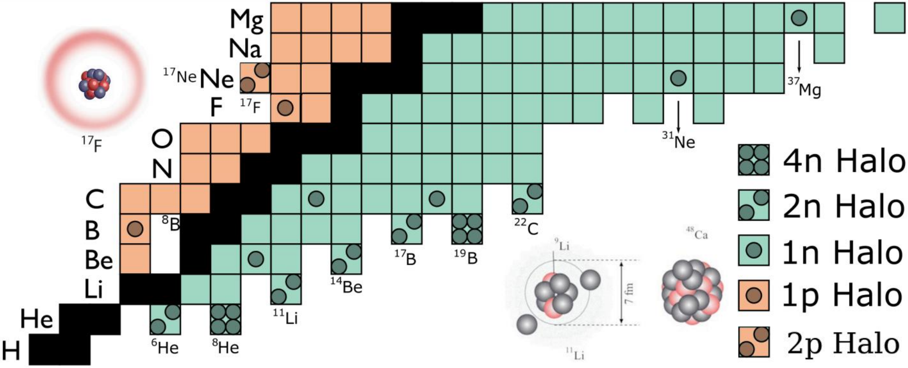
- **Definition**
  - diffuse neutron/proton clouds extending far beyond the core
- **Cluster structure**
  - tight core surrounded loosely by valence nucleon(s)

Introduction to halo nuclei



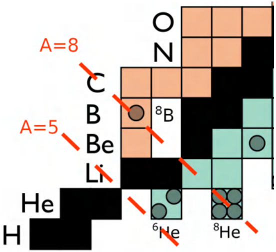
- **Characteristics**
  - Extremely large matter radii compared to $A^{1/3}$ scaling
  - Weak binding/resonance of the last few nucleons ($S_n \lesssim 1$ MeV)
  - Low angular momentum states (s or p waves) for valence nucleons
  - Enhanced cross section in astrophysical reaction at finite temperature

Introduction to halo nuclei

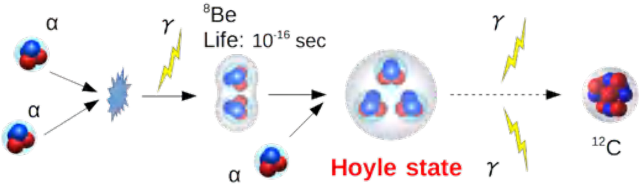


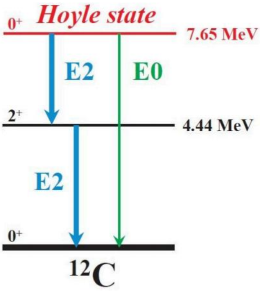
- **Classification**
  - 1n halos: ^{11}Be, ^{15}C, ^{19}C, ^{31}Ne, ^{37}Mg
  - 2n halos: ^{6}He, ^{11}Li, ^{14}Be, ^{17}B, ^{22}C
  - 4n halos: ^{8}He, ^{19}B
  - 1p halos: ^{8}B, ^{17}F
  - 2p halos: ^{17}Ne

α clustering



- Hoyle state in ^{12}C: $3\alpha\ 0^+$ resonance
- triple-α reaction is enhanced by Hoyle state
- it bridges the $A = 5$ and $A = 8$ gaps in primordial nucleosynthesis





Historical discovery

- **First Evidence (LBNL):**
  - Tanihata et al. measured anomalously large interaction cross sections in ^{6,8}He and ^{11}Li (PLB 1985; PRL 1985)

$$\sigma_I \approx \pi(R_I(p) + R_I(t))^2$$

  - much larger than expected from $A^{1/3}$ scaling

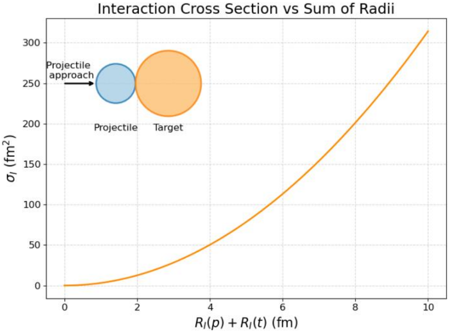


TABLE I. Interaction cross sections (σ_I) in millibarns.

Beam	Target Be	Target C	Target Al
^{6}Li	651 ± 6	688 ± 10	1010 ± 11
^{7}Li	686 ± 4	736 ± 6	1071 ± 7
^{8}Li	727 ± 6	768 ± 9	1147 ± 14
^{9}Li	739 ± 5	796 ± 6	1135 ± 7
^{11}Li		1040 ± 60	
^{7}Be	682 ± 6	738 ± 9	1050 ± 17
^{9}Be	755 ± 6	806 ± 9	1174 ± 11
^{10}Be	755 ± 7	813 ± 10	1153 ± 16

Historical discovery

- **Theoretical Interpretation**
  - Hansen & Jonson (1987) proposed ”neutron halo” structure

EUROPHYSICS LETTERS 15 August 1987

Europhys. Lett., 4 (4), pp. 409-414 (1987)

The Neutron Halo of Extremely Neutron-Rich Nuclei.

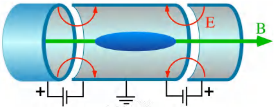
P. G. HANSEN (*) (§) and B. JONSON (**)

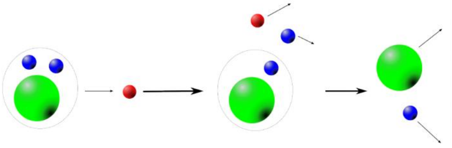
(*) *EP-Division, CERN, Geneva, Switzerland*
(**) *Department of Physics, Chalmers University of Technology, Göteborg, Sweden*

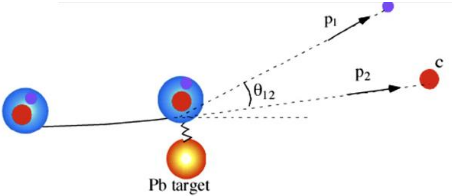
- **Subsequent Evidence**
  - Narrow momentum distributions (Kobayashi et al., PRL **60**, 2599, 1988)
  - Enhanced electromagnetic dissociation cross sections (Kobayashi et al., PLB **232**, 51, 1989)
  - Charge radius isotope shift measurements (atomic spectroscopy)

Experimental probes to halo nuclei

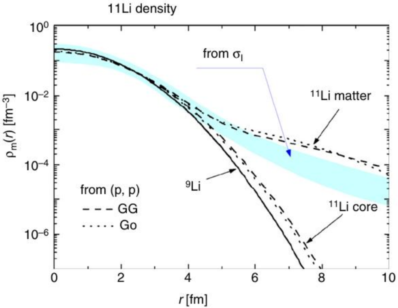
- static methods
  - ISOTRAP: atomic mass
  - laser spectroscopy: charge radius
  - β-NMR: μ_M & Q_E







- reaction methods
  - spectroscopy by breakup
    - nuclear breakup $p(^{11}\mathrm{Li}, pn)^{10}\mathrm{Li}$
    - Coulomb breakup $^{11}\mathrm{Be}(\gamma^*, n)^{10}\mathrm{Be}$
  - spatial/momentum configuration
    - elastic scattering (p, p)
    - interaction cross section
    - neutron/proton removal



review: Tanihata et al., PPNP 68 (2013) 215

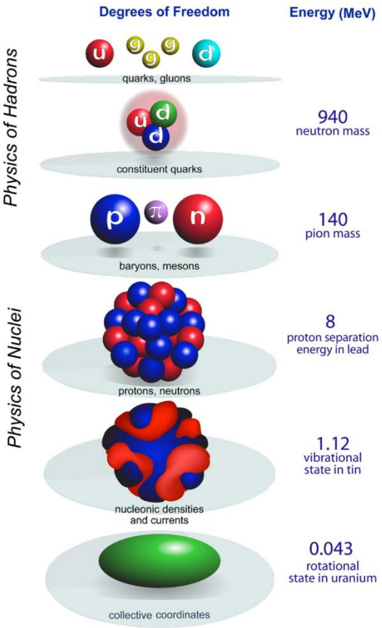
Emergence of halo effective field theory

- Three phases of halo theories
  - Back-of-the-envelope period (1985–1992)
    - "quick" estimates of halo properties by reproducing σ_R
    - gaussian spatial distribution $\rightarrow$ reproduce $\sigma_I \rightarrow R_m$ too small!
  - Few-body models period (1992–2000)
    - cluster structure models (core + valence nucleons)
    - few-body reaction models (Glauber, DWBA, CDCC,...)
    - unresolved model dependence
    - limited applicable regimes
  - Microscopic models period (2000–present)
    - ab initio structure theory
    - difficulties in computational power & extension to threshold physics
    - need to develop ab initio reaction theory (e.g. optical potential)

Halo Nuclei, Al-Khalili, Morgan & Claypool Publishers, 2017

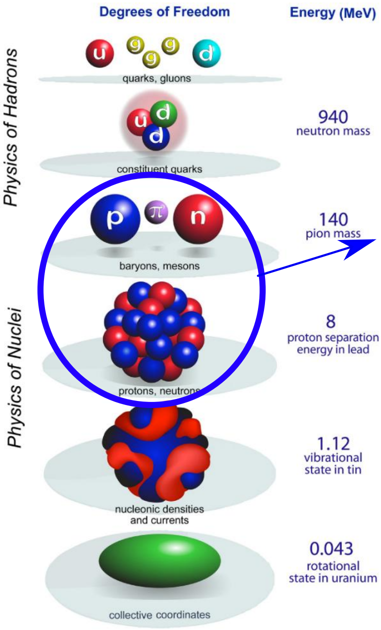
  - Halo effective field theory
    - systematically embed microscopic information into cluster model
    - provide guidance to build reaction theory

Introduction to effective field theory



scale hierarchy in nuclear physics

Introduction to effective field theory



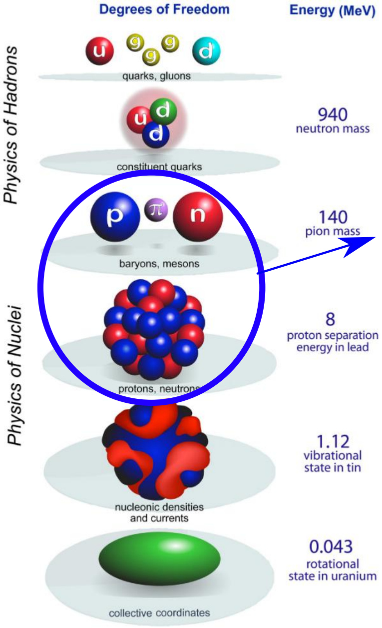
scale hierarchy in nuclear physics

underlying theory

high scale: Λ

low scale: Q

Introduction to effective field theory



scale hierarchy in nuclear physics

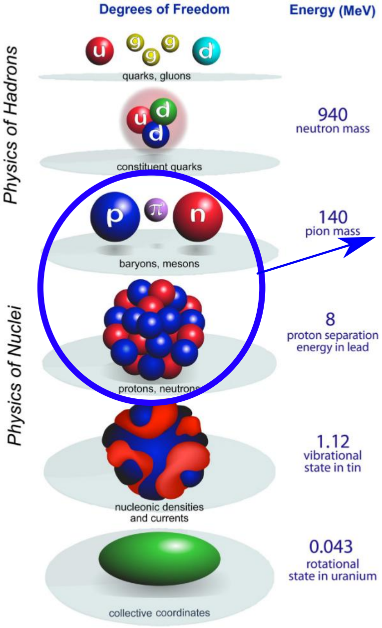
underlying theory

high scale: Λ

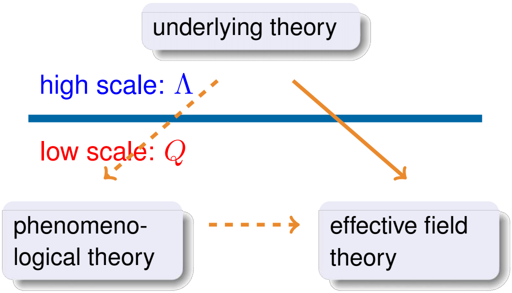
low scale: Q

phenomeno-
logical theory

Introduction to effective field theory



scale hierarchy in nuclear physics



- EFT emerges from underlying theory
- EFT “inherits” asymptotics from phenomenology

Key elements of an EFT

- Separation of scales $Q \ll \Lambda$:
  - low-energy observables $\rightarrow Q$
  - short-range interactions $\rightarrow \Lambda$

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- Systematic expansion of Lagrangian in Q/Λ:
  - order-by-order construction of effective interactions:

$$V_{\text{eff}} = \sum_n \hat{V}^{(n)}; \qquad \hat{V}^{(n)} \sim (Q/\Lambda)^{n-1}$$

  - prediction uncertainty is controlled by $(Q/\Lambda)^{(n+1)}$

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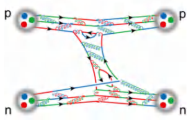
- Predict low-energy physics:
  - low-energy observables (Q) $\xrightarrow{\text{insensitive}}$ details of short-range interactions (Λ)
  - EFT unveils universal correlations

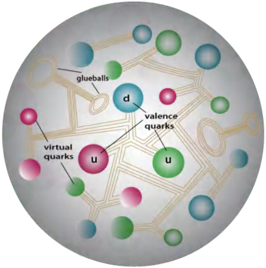
NN interaction in atomic nuclei

QCD

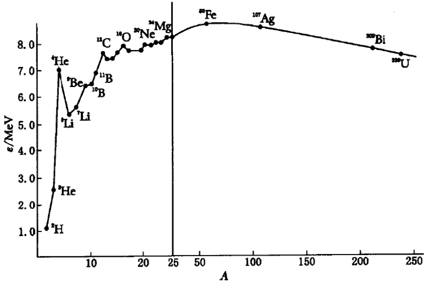


$\Lambda \sim 1$GeV

$Q \sim 100$ MeV



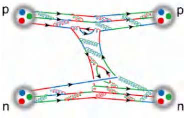
Λ: EFT breakdown scale



$Q \approx \sqrt{2M_N B/A}$: typical scale in EFT

NN interaction in atomic nuclei

QCD



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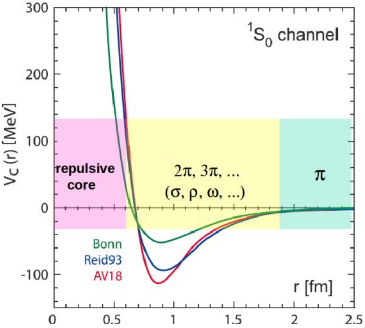
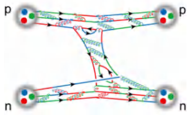
phenomenology



pic: Aoki et al. Comp. Sci. Disc. 2008

NN interaction in atomic nuclei



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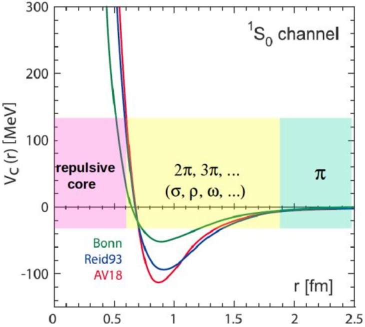
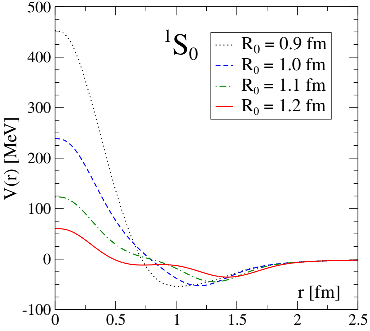


pic: Aoki et al. Comp. Sci. Disc. 2008

chiral EFT



pic: Gezerlis et al. Phys. Rev. C 2014

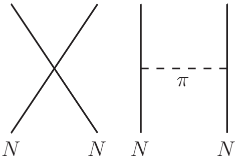
EFT with contact interactions

- Effective field theory with contact interactions originate from pionless EFT

chiral EFT NN force

- short range: $V_s = C_0$
- intermediate/long range:

$$V_{1\pi} \sim \frac{1}{q^2 + m_\pi^2}$$



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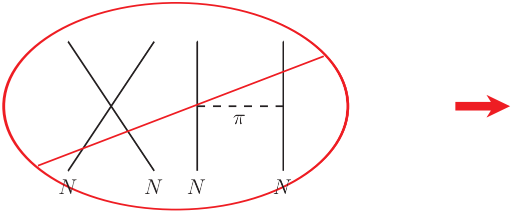
- short range: $V_s = C_0$
- intermediate/long range:

$$V_{1\pi} \sim \frac{1}{q^2 + m_\pi^2}$$

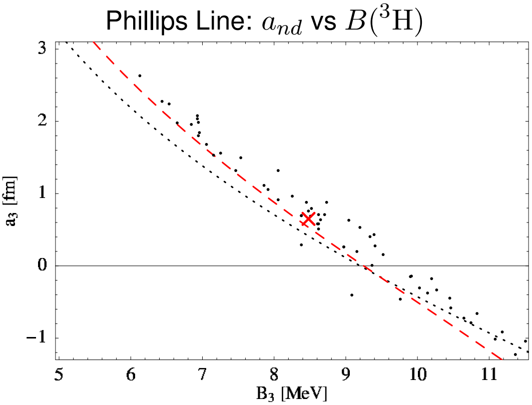
$\not\pi$EFT NN force

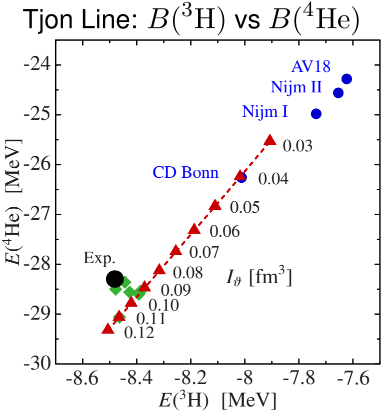
- NN momentum $q^2 \ll m_\pi^2$

$$V_{1\pi} \xrightarrow{q^2 \ll m_\pi^2} C_0 + C_2 q^2 + \cdots$$



Universality in $\pi\!\!\!/$EFT





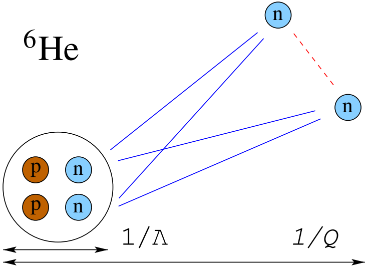
- $\pi\!\!\!/$EFT indicates universal correlations among few-body observables
- long-range (low-energy) physics is insensitive to details of short-range interactions

Halo physics near clustering threshold

ab initio theory

$\Lambda \sim \sqrt{m_N E^*_{\text{core}}}$

$Q \sim \sqrt{m_N S_N}$



Halo physics near clustering threshold

ab initio theory

$\Lambda \sim \sqrt{m_N E^*_{\text{core}}}$

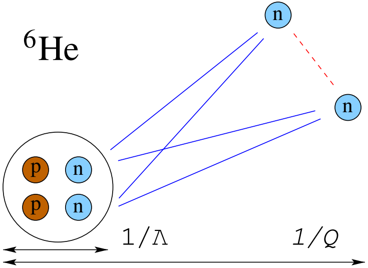
$Q \sim \sqrt{m_N S_N}$

halo physics is difficult for ab initio theories

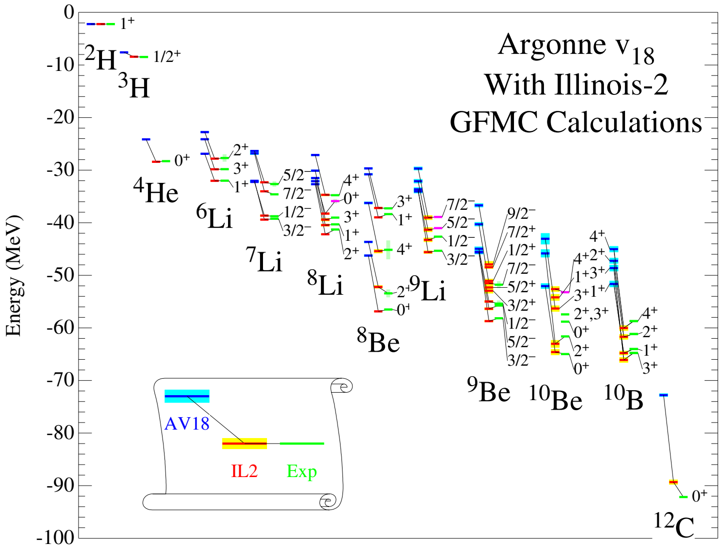
- continuum problem in many-body calculations
  NCSMC, GSM-Bergren, Lattice-EFT, LIT, $\cdots$
- uncertainty control in chiral potentials
  threshold observable converges slower in χEFT

$$\text{halo scale}: Q_{\text{halo}} \ll Q_{\chi\text{EFT}} \approx (2M_N B/A)^{1/2}$$

$$\text{uncertainty}: \ \Delta_{\text{halo}}\% \approx \frac{Q_{\chi\text{EFT}}}{Q_{\text{halo}}}\left(\frac{Q_{\chi\text{EFT}}}{\Lambda_{\chi\text{EFT}}}\right)^{(n+1)}$$



ab initio description of nuclear spectrum



microscopic description of nuclear spectrum is in general accurate

Pieper, Wiringa, Annu. Rev. Nucl. Part. Sci. 51 (2001) 53

ab initio description of halo features

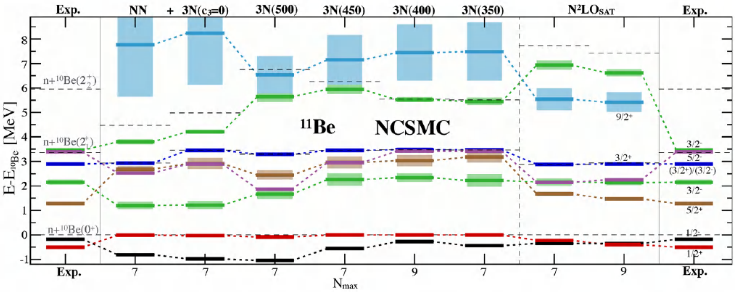


FIG. 2. NCSMC spectrum of ^{11}Be with respect to the n + ^{10}Be threshold. Dashed black lines indicate the energies of the ^{10}Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

- ab initio calculation of ^{11}Be has been done by NCSMC
- predictions of threshold properties rely significantly on the nuclear interactions

Calci et al. Phys. Rev. Lett. 117 (2016) 242501

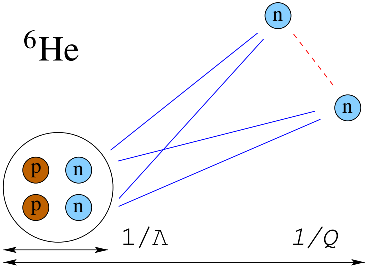
Halo physics near clustering threshold

ab initio theory

$\Lambda \sim \sqrt{m_N E^*_{\text{core}}}$

$Q \sim \sqrt{m_N S_N}$

cluster model



difficulties in cluster models:

- assess model dependence?
- assign theory uncertainty

Halo physics near clustering threshold

ab initio theory

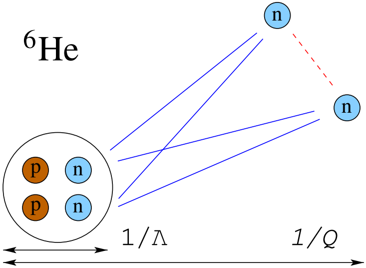
$\Lambda \sim \sqrt{m_N E^*_{\text{core}}}$

$Q \sim \sqrt{m_N S_N}$

cluster model

halo EFT

- cluster configuration in halo EFT:
  core $+$ valence nucleons d.o.f.
- separation of scales:
  $Q \ll \Lambda \rightarrow$ systematic expansion in observables
- short-range physics from underlying theory:
  anti-symmetrization of core nucleons is embedded in contact interactions



Halo Effective Field Theory Lagrangian

- Use EFT with **contact interactions** to describe clustering in halo nuclei
- **For s-wave interactions in 1n halo**, we introduce auxiliary dimer fields for bound/virtual states

$$
\begin{aligned}
\mathscr{L} &= \mathscr{L}_1 + \mathscr{L}_{nc}^s \\
\mathscr{L}_1 &= n^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_n} \right) n + c^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_c} \right) c \\
\mathscr{L}_{nc}^s &= \sigma^\dagger \left[w_\sigma \left(i\partial_0 + \frac{\nabla^2}{2M_\sigma} \right) + \Delta_\sigma \right] \sigma - g_\sigma \left(\sigma_{s,\beta}^\dagger [nc]_{s,\beta} + \text{h.c.} \right)
\end{aligned}
$$

where:

- d is the dimer field for nc states
- Δ_d, Δ_σ are residual masses
- $M_\sigma = m_n + m_c$ is the total mass of 1n halo
- $[]_{s,\beta}$ denotes spin coupling

$$[nc]_{s,\beta} = \sum_\delta \left(\frac{1}{2} \delta \varsigma_c \beta - \delta \middle| s\beta \right) n_\delta c_{\beta-\delta}.$$

One-neutron s-wave halos

- **scattering amplitude:** $t_0(k) = \frac{2\pi}{\mu}\left(\frac{1}{a_0} - \frac{r_0}{2}k^2 + ik\right)^{-1}$
  - in low-energy bound/virtual state: $a_0 \sim 1/Q$; $r_0 \sim 1/\Lambda$
  - expand t-matrix in r_0/a_0

One-neutron s-wave halos

- **scattering amplitude:** $t_0(k) = \frac{2\pi}{\mu}\left(\frac{1}{a_0} - \frac{r_0}{2}k^2 + ik\right)^{-1}$
  - in low-energy bound/virtual state: $a_0 \sim 1/Q$; $r_0 \sim 1/\Lambda$
  - expand t-matrix in r_0/a_0
- **Iterative summation at LO**

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$$a_0 = \left(\frac{2\pi\Delta_\sigma}{\mu_\sigma g_\sigma^2} + \Lambda\right)^{-1}; \quad w_\sigma = -\text{sgn}(r_0); \quad r_0 = -w_\sigma \frac{2\pi}{\mu_\sigma^2 g_\sigma^2}$$

One-neutron s-wave halos

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- **pole expansion**: $t_0(k) \approx \frac{2\pi}{\mu}\frac{\mathcal{Z}_R}{\gamma_0 + ik}$
  - ANC: $\psi_0(\boldsymbol{r}) = \frac{C_\sigma}{\sqrt{4\pi}r}\exp(-\gamma_{0,\sigma}r)$
  - LO: $C_{\sigma,LO} = \sqrt{2\gamma_0}$ NLO: $C_{\sigma,NLO} = \sqrt{\frac{2\gamma_0}{1-\gamma_0 r_0}}$
  - renormalization constant $\mathcal{Z}_R = \frac{C^2_{\sigma,NLO}}{C^2_{\sigma,LO}} = \frac{1}{1-\gamma_0 r_0}$

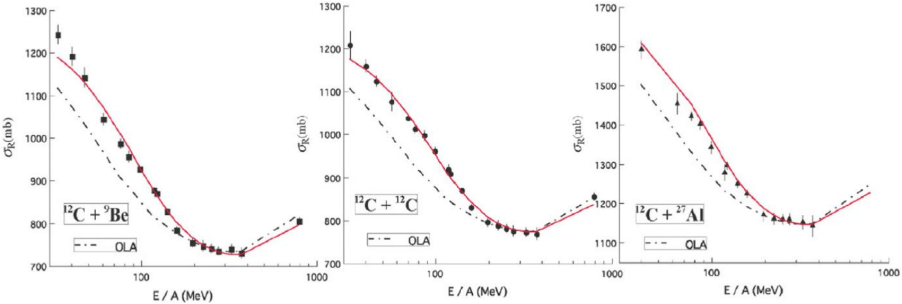
Probe density distribution - reaction cross sections

- σ_R **of heavy-ion collision is described by Glauber model**

$$\sigma_R = \int [1 - T(\mathbf{b})] d\mathbf{b}$$

transmission function: $$T(\mathbf{b}) \approx \exp\left[-\sum_{i,k} \sigma_{ik} \int\int \rho^z_{Pi}(\mathbf{s}) \rho^z_{Tk}(\mathbf{b}+\mathbf{s}) d\mathbf{s}\right]$$

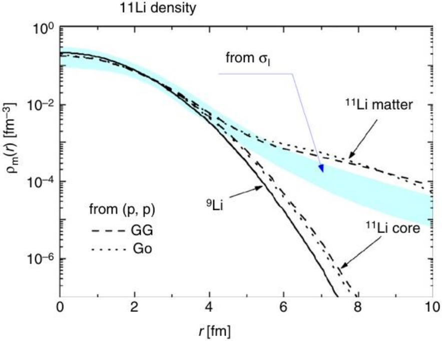
- **Fit to** σ_R**'s energy dependence to extract** ρ_P



Nucleon density and matter radius

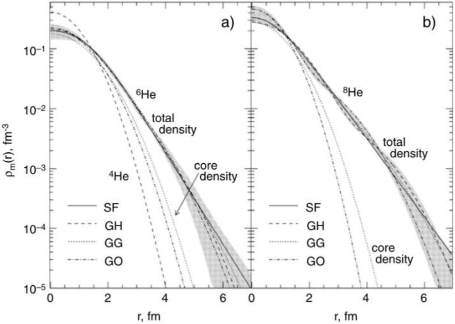
- ρ_m for halo nuclei exhibits an extended tail compared to ρ_m of the core
- **matter radius:**
  - $R_m^2 = \int \rho_m(r) r^2 dr$
  - indicates total nucleon distribution
  - normal nuclei: $R_m \approx 1.2A^{1/3}$ fm

^{11}Li nucleon density



Egelhof et al., EPJA 15 (2002) 27

^{6,8}He nucleon density



Alkhazov et al., NPA 712 (2002) 269

Isotope-shift measurements and charge radius

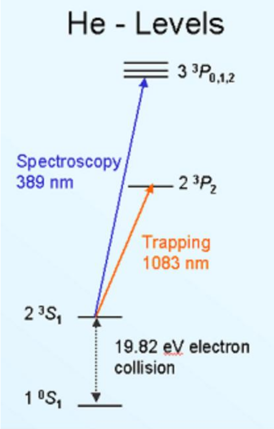
- **Charge Radii Measurements:**
  - Laser spectroscopy of trapped ions/atoms
  - Atomic levels for ${}^{A}_{Z}\mathrm{X}$ in j state:

$$E_j({}^{A}_{Z}\mathrm{X}) = E_{j,\mathrm{NR}} + E_{j,\mathrm{rel}} + \alpha^3 E_{j,\mathrm{QED}} + C_j R_c^2({}^{A}_{Z}\mathrm{X})$$

  - atomic transition $j \to k$: $\delta\nu = E_j - E_k$
  - Isotope shift:

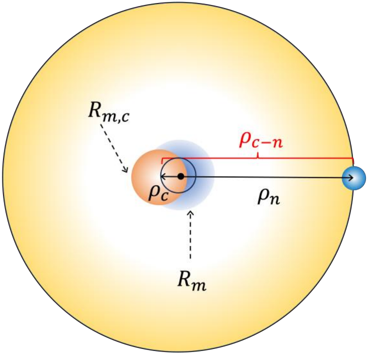
$$\begin{aligned} \nu_{A'A} &= \delta E({}^{A}_{Z}\mathrm{X}) - \delta E({}^{A'}_{Z}\mathrm{X}) \\ &= \delta\nu_{MS} + \delta\nu_{FS} \end{aligned}$$

  - mass shift $\delta\nu_{MS}$:
    shifts in nuclear mass and QED correction
  - field shift :
    $\delta\nu_{FS} = (C_j - C_k)\left[R_c^2({}^{A'}_{Z}\mathrm{X}) - R_c^2({}^{A}_{Z}\mathrm{X})\right]$



Decouple core and halo neutron

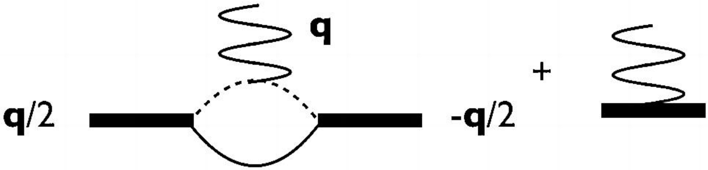
- $1n$ **halo coordinates:**
  - Valence neutron orbits at large distances from the core
  - Changes in R_c or R_m between the halo and core nuclei are due to shifts in C.M.
- **Relations:**
  - $\rho_c = \sqrt{\frac{1}{A} R_m^2 - \frac{1}{A+1} R_{m,c}^2}$
  - $\rho_c = \sqrt{R_c^2 - R_{c,c}^2 - \frac{1}{Z} r_n^2}$
  - $\rho_{c-n} = (A+1)\rho_c$



n-c coordinates in C.M frame

Electric form factor for 1n-halos

Electric form factor (Breit frame) → radius ρ_c^2



$$G_c(q) = \frac{C_\sigma^2}{fq} \arctan \frac{fq}{2\gamma_0} + 1 - \frac{C_\sigma^2}{2\gamma_0}$$

- $f = \mu_\sigma / M_c = 1/(A+1)$
- photon-dimer term restores normalization at NLO: $G_c(0) = G_c^{LO}(0) = 1$
- determine ρ_c by $G_c(q) = 1 - \frac{\rho_c^2}{6} q^2 + \cdots$

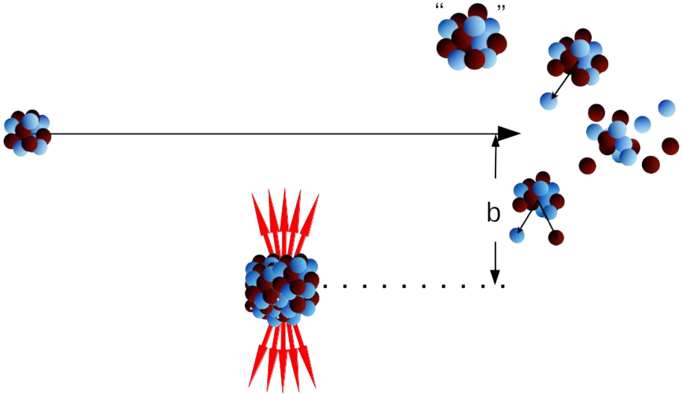
$$\rho_c = \sqrt{\frac{f^2 C_\sigma^2}{4\gamma_0^3}} = \frac{1}{A+1} \frac{C_\sigma / C_{\sigma,0}}{\sqrt{2}\gamma_0}$$

One-neutron s-wave halos

	^{2}H	^{11}Be	^{15}C	^{19}C
Experiment				
S_{1n} [MeV]	2.224573(2)	0.50164(25)	1.2181(8)	0.58(9)
E_c^* [MeV]	140	3.36803(3)	6.0938(2)	1.62(2)
ρ_{c-n} [fm]	3.936(12)	6.05(23)	4.15(50)	6.6(5)
	3.95014(156)	5.7(4)	7.2±4.0	6.8(7)
		5.77(16)	4.5(5)	5.8(3)
Halo EFT				
Q/Λ	0.33	0.39	0.45	0.6
r_0/a_0	0.32	0.32	0.43	0.33
$\sqrt{\mathcal{Z}_R}$	1.295	1.3	1.63	1.3
ρ_{c-n} [fm]	3.954	6.85	4.93	5.72

Coulomb dissociation in $1n$ halos

- Coulomb dissociation
  - breakup by colliding a halo nucleus with a high-Z nucleus
  - the halo dynamics dominates when $E \sim S_{1n}$



EFT on Coulomb dissociation

- **E1 transition: direct breakup from s-wave 1n halo**

- Amplitude for Coulomb dissociation:

$$\mathcal{M}_{E1}^{(j=3/2)} = \sqrt{2}\mathcal{M}_{E1}^{(j=1/2)} = 4\sqrt{\gamma_0}\frac{C_\sigma}{C_{\sigma,LO}} f Z e \frac{p}{\left(\gamma_0^2 + p^2\right)^2}$$

EFT on Coulomb dissociation

- Differential E1 transition strength:

$$\frac{dB(E1)}{dE} = \frac{1}{(2\pi)^3}\left(|\mathcal{M}_{E1}^{(J=1/2)}|^2 + |\mathcal{M}_{E1}^{(J=3/2)}|^2\right)\frac{d^3p}{dE}$$
$$= \frac{12}{\pi^2}\mu_\sigma (fZ)^2 e^2 \gamma_0 \frac{C_\sigma^2}{C_{\sigma,LO}^2}\frac{p^3}{\left(\gamma_0^2 + p^2\right)^4}$$

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- Re-express into universal scaling function
  - $p^2/\gamma_0^2 \to E/S_{1n}$
  - $Q_{\mathrm{eff}} = fZ = Z/(A+1)$

$$\frac{d\mathrm{B(E1)}}{dE} = \frac{Q_{\mathrm{eff}}^2}{\mu_\sigma S_{1n}^2}\frac{C_\sigma^2}{C_{\sigma,LO}^2}\frac{3e^2}{\pi^2}\frac{(E/S_{1n})^{3/2}}{(1+E/S_{1n})^4},$$

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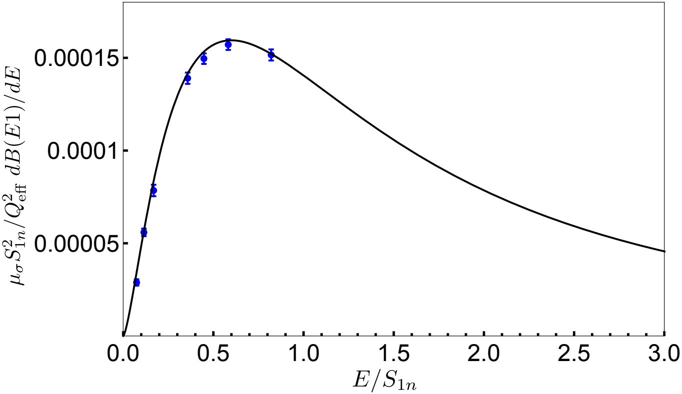
- **Universal scaling function** for $dB(E1)/dE$
- Only the prefactor $Q_{\mathrm{eff}}^2/(\mu_\sigma S_{1n}^2)$ is system dependent

E1 transition in deuteron

- Divide by the prefactor

$$\frac{\mu_\sigma S_{1n}^2}{Q_{\rm eff}^2}\frac{d\mathrm{B(E1)}}{dE} = \frac{C_\sigma^2}{C_{\sigma,LO}^2}\frac{3e^2}{\pi^2}\frac{(E/S_{1n})^{3/2}}{(1+E/S_{1n})^4}$$

- The rest of the function only depends on the ratio E/S_{1n}
- $C_\sigma^2/C_{\sigma,LO}^2 = 1/(1-\gamma_0 r_0)$ introduces corrections to the universality

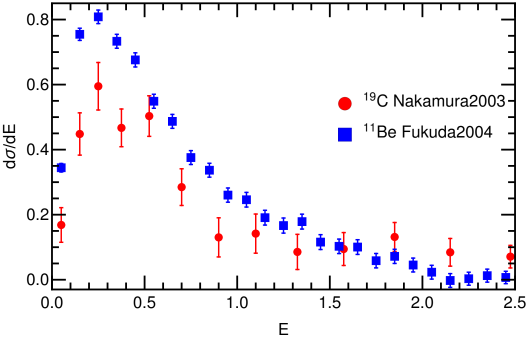


Coulomb dissociation in ^{11}Be and ^{19}C

- E1 strength to Coulomb dissociation cross section:

$$\frac{d\sigma}{dE} = \frac{16\pi^3}{9} N_{E1}(E,R) \frac{dB(E1)}{dE}$$

- Rescale $d\sigma/dE \quad \rightarrow \quad$ universal function



Coulomb dissociation energy spectrum in ^{11}Be and ^{19}C

Coulomb dissociation in ^{11}Be and ^{19}C

- E1 strength to Coulomb dissociation cross section:

$$(1-\gamma_0 r_0)\frac{\mu_\sigma S_{1n}^2}{Q_{\text{eff}}^2}\frac{d\sigma}{dE} = \frac{16\pi^3}{9} N_{E1}(E,R)(1-\gamma_0 r_0)\frac{\mu_\sigma S_{1n}^2}{Q_{\text{eff}}^2}\frac{d\text{B(E1)}}{dE}$$

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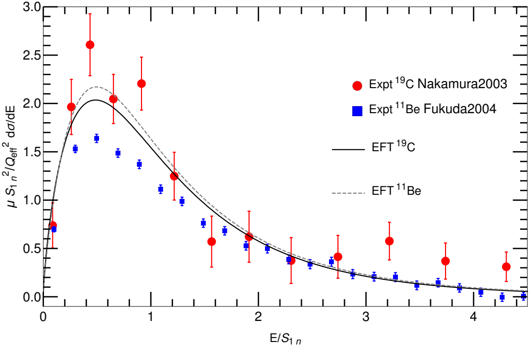

Coulomb dissociation energy spectrum in ^{11}Be and ^{19}C

Hammer, Phillips, NPA '11
Acharya, Phillips, NPA '13
Hammer, CJ, Phillips, JPG '17

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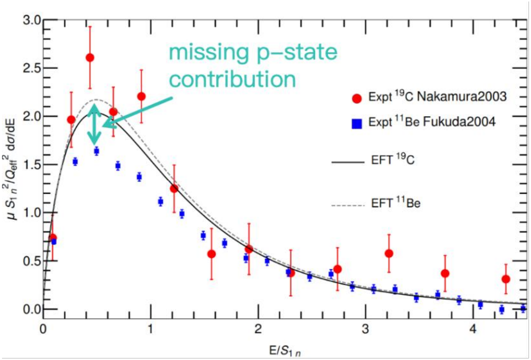


Coulomb dissociation energy spectrum in ^{11}Be and ^{19}C

Hammer, Phillips, NPA '11
Acharya, Phillips, NPA '13
Hammer, CJ, Phillips, JPG '17

Halo EFT with Coulomb

- In halo/clustering systems with Coulomb interactions, a new scale $k_c = Q_c e^2 \mu$ enters
  - $k_c \gtrsim Q$: Coulomb interaction is nonperturbative

  - Coulomb Green's function (non-perturbative)

$$(\boldsymbol{r}|G_C(E)|\boldsymbol{r}') = \int \frac{d^3p}{(2\pi)^3} \frac{\psi_{\boldsymbol{p}}(\boldsymbol{r})\psi^*_{\boldsymbol{p}}(\boldsymbol{r}')}{E - \boldsymbol{p}^2/(2\mu_{nc}) + i\epsilon}$$

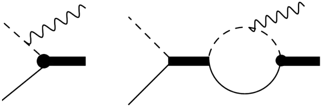
$$\psi_{\boldsymbol{p}}(\boldsymbol{r}) = \sum_{l=0}^{\infty} (2l+1) i^l \exp(i\sigma_l) \frac{F_l(\eta,\rho)}{\rho} P_l(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{r}})$$

    p-p scattering [Kong, Ravndal, PLB '99; NPA '10]
    p-α and α-α scattering [Higa, Hammer, van Kolck, NPA '08; Higa, FBS '11]
    ^{17}F* [Ryberg, Forssén, Hammer, Platter, PRC '14; AnnPhys '16]

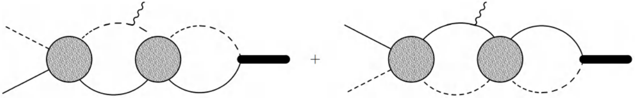
  - $k_c \ll Q$: Coulomb interaction is perturbative
    ^{3}H and ^{3}He [König, Grießhammer, Hammer, van Kolck, JPG '16]

Radiative Nucleon Captures

neutron captures



proton captures



$$\frac{d\sigma}{d\Omega} = \frac{\mu_{nc} E}{8\pi^2 p} \sum_{i=1}^{2} \left| \epsilon_i \cdot \frac{\mathcal{M}}{\sqrt{\Sigma'(-B)}} \right|^2$$

$^7\mathrm{Li} + n \to {}^8\mathrm{Li} + \gamma$

Rupak, Higga, PRL ’11; Fernando, Higa, Rupak, EPJA ’12;
Zhang, Nollett, Phillips, PRC ’14

$^{14}\mathrm{C} + n \to {}^{15}\mathrm{C} + \gamma$

Rupak, Fernando, Vaghani, PRC ’12

$^7\mathrm{Be} + p \to {}^8\mathrm{B} + \gamma$

Zhang, Nollett, Phillips, PRC ’14; Ryberg, *et al.* EPJA ’14

$^{16}\mathrm{O} + p \to {}^{17}\mathrm{F}^*(1/2^+) + \gamma$

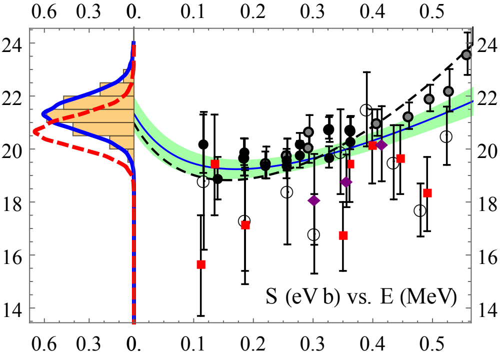
Ryberg, Forssén, Hammer, Platter, PRC ’14; AnnPhy ’16

Radiative Nucleon Captures

astrophysical S-factor for ^{7}Be$(p,\gamma)^8$B

EFT(NLO): VMC ANC + Bayesian error analysis

Zhang, Nollett, Phillips, Phys. Lett. B 751 (2015) 535

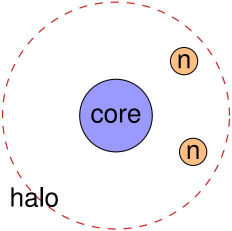


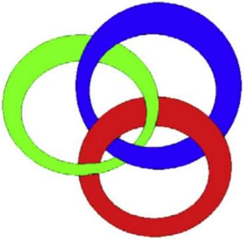
$$S(E) = E e^{2\pi\eta(E)}\sigma(E)$$

$$S(0) = 21.3 \pm 0.7 \text{ eV} \cdot \text{b}$$

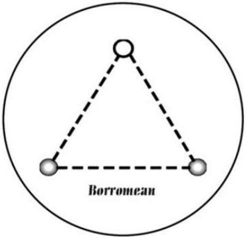
Two-Neutron Halos

- **n-n-core three-body clustering:**
  - n-core bound
    e.g., ^{12}Be, ^{20}C, ^{62}Ca
  - n-core unbound (Borromean system)
    e.g., ^{6}He, ^{11}Li, ^{14}Be, ^{22}C, ^{62}Ca
  - Borromean systems show universal features (halo nuclei, cold atoms, etc.)





Borromean rings



Two-body unbound - - - - -
Three-body bound

Symbol for house of Borromeo

Pallas and the Centaur

Painting by Sandro Botticelli (Uffizi Gallery in Florence, Italy)

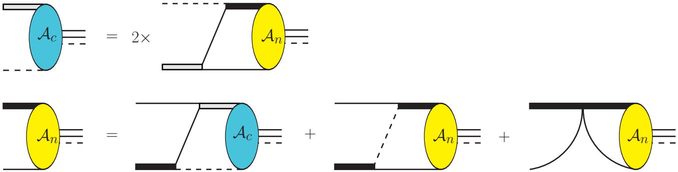
3-body borromean rings

4-body borromean rings

Symbol for house of Medici

$2n$ halos in Faddeev formalism

- **solving transition amplitudes $\mathcal{A}_c$ and $\mathcal{A}_n$**



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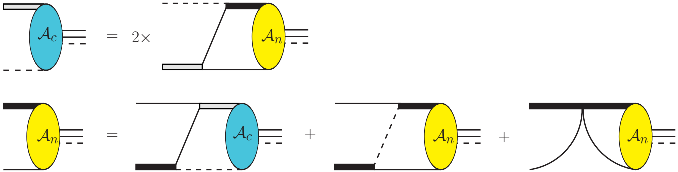
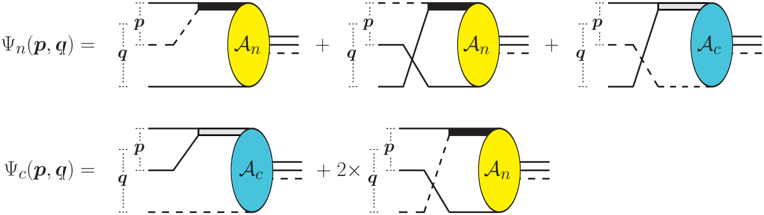


- **three-body wave functions**

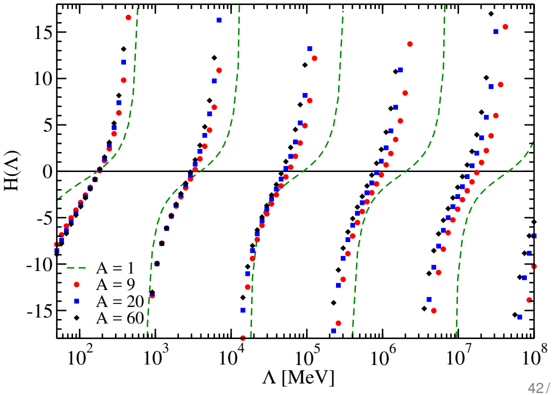


Three-body renormalization

- **running of three-body coupling**
  - tune $H(\Lambda) = \Lambda^2 h/2mg^2$:
    reproduce one observable in a $2n$-halo

Hammer, CJ, Phillips,

JPG 44 (2017) 103002



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Hammer, CJ, Phillips,

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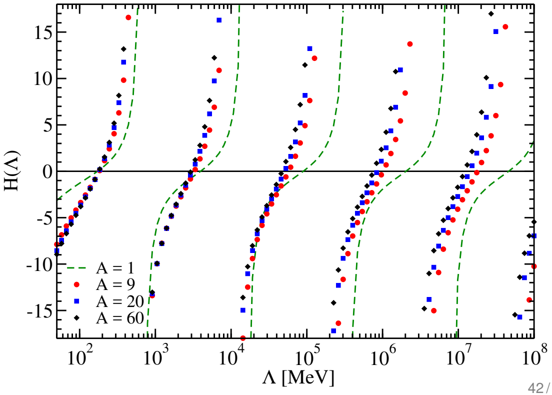

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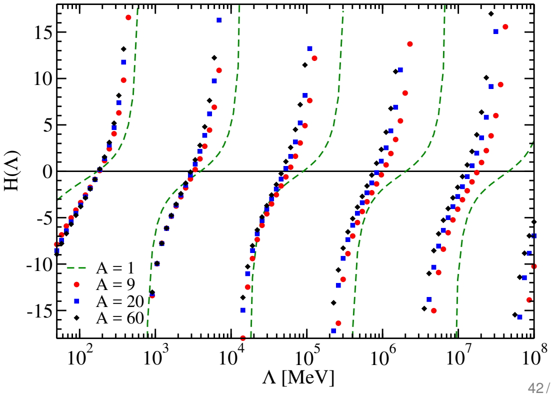
    reproduce one observable in a $2n$-halo
  - $H(\Lambda)$ periodic for $\Lambda \to \lambda\Lambda$ [$A = 1$ Bedaque *et al.* '00]
  - $H(\Lambda)$ appears as RG limit cycle [Mohr et al., AnnPhys '06]
  - discrete scale invariance $\to$ Efimov physics

Hammer, CJ, Phillips,

JPG 44 (2017) 103002



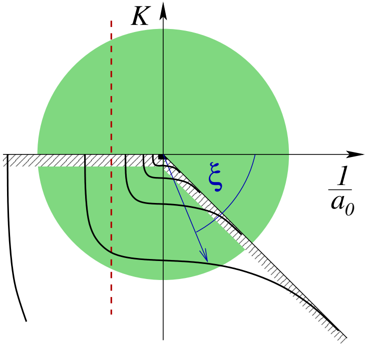
Efimov physics

- a universal spectrum of three-body bound states

$$B_3 = -\frac{1}{ma_0^2} + \left[e^{-2\pi n} f(\xi)\right]^{1/s_0} \frac{\kappa_*^2}{m}$$

Braaten, Hammer, Phys. Rept. '06

- atomic physics: vary a_0 through Feshbach resonance
- nuclear physics: fixed a_0



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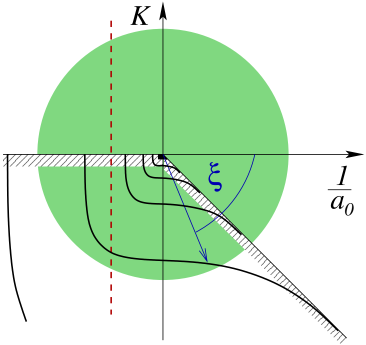
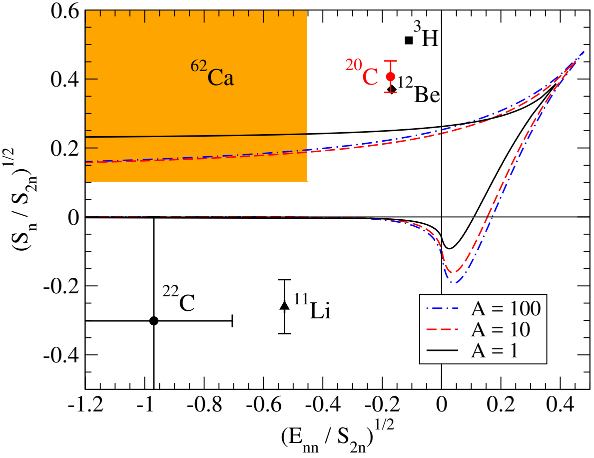


- unitary limit ($a \to \infty$): $B_3 = e^{-2\pi n/s_0} \frac{\kappa_*^2}{m}$
- discrete scale invariance: $\kappa_* \to \kappa_*, \quad a_0 \to e^{\pi n/s_0} a_0$
- exploring Efimov physics in halo nuclei is an important subject

Efimov universality in $2n$ s-wave halo

- contour constraints on ground-state energy S_{2n} if the excited-state energy $B_3^* = \max\{0, E_{nn}, S_{1n}\}$



Canham, Hammer, EPJA '08; Frederico *et al.* PPNP '12;

Hammer, CJ, Phillips, JPG 44 (2017) 103002

^{6}He: $2n$ Halo with p-wave nc interactions

- nc interaction in a p-wave bound/resonance state

$$= \frac{2\pi}{\mu} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

  - $a_1 < 0$: shallow resonance: ^{5}He $(3/2^{-})$
  - $a_1 > 0$: shallow bound state: ^{11}Be $(1/2^{-})$, ^{8}Li (2^{+}), ^{8}Li* (1^{+})

- p-wave power counting
  - resum ik^3: $1/a_1 \sim Q^3$, $r_1 \sim Q$ [Bertulani, Hammer, van Kolck NPA '02]
  - perturbative ik^3: $1/a_1 \sim Q^2\Lambda$, $r_1 \sim \Lambda$ [Bedaque, Hammer, van Kolck PLB '03]

Unconventional momentum-dependent $n\alpha$ interaction

- If $a_0 \sim r_0 \sim Q^{-1}$ in s-wave interaction, we need to tune both a_0 and r_0 at LO

$$V_0(p, p') = -\frac{2\pi}{\mu} \frac{\lambda}{\sqrt{p'^2 + 2\mu\Delta}\sqrt{p^2 + 2\mu\Delta}}$$

Peng, Lyu, König, Long (2021), PRC 105, 054002 (2022); Beane, Farrell, FBS 63, 45 (2022); van Kolck, Symmetry 14, 1884 (2022)

- For $n\alpha$ p-wave interaction, both a_1 and r_1 are at LO

$$V_1(p, p') = -\frac{2\pi}{\mu} \frac{\lambda p p'}{\sqrt{p'^2 + 2\mu\Delta}\sqrt{p^2 + 2\mu\Delta}}$$

- on-shell t-matrix:

$$T^{(0)}(k, k; k) = -\frac{2\pi}{\mu} \frac{k^2}{-\frac{1}{a_1} + \frac{1}{2} r_1 k^2 - ik^3} \qquad V(E) : k^2 \to pp'$$

$$\to \text{resonance} + \text{spurious pole}$$

- off-shell t-matrix:

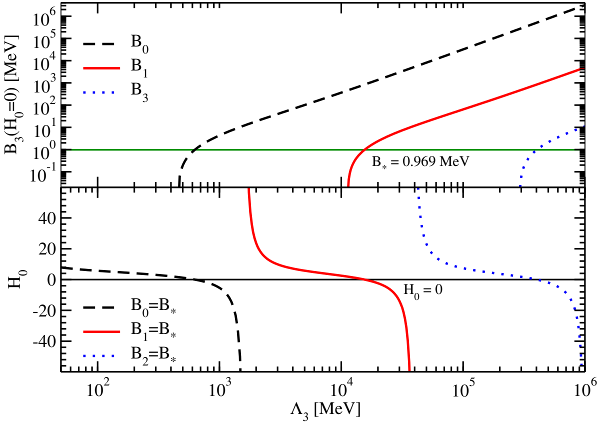
$$T_1(p', p; k) = -\frac{2\pi}{\mu} \frac{p'}{\sqrt{p'^2 + \gamma^2}} \frac{ik - \gamma}{k^2 + i(\gamma + \frac{r_1}{2})k + \frac{1}{a_1 \gamma}} \frac{p}{\sqrt{p^2 + \gamma^2}}$$

$$\gamma + \frac{r_1}{2} + a_1^{-1}\gamma^{-2} = 0 \qquad \text{spurious pole vanishes}$$

Li, Lyu, CJ, Long, arXiv:2303.17292 (accepted at PRC)

Unconventional $n\alpha$ interaction in ^{6}He

- Implementing unconventional p-wave potential in $nn\alpha$ system
- Running of $H(\Lambda)$ and $B(\Lambda)$

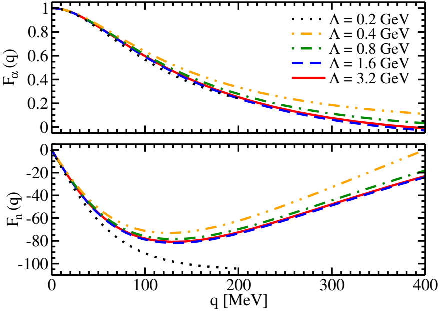


Li, Lyu, CJ, Long, arXiv:2303.17292 (accepted at PRC)

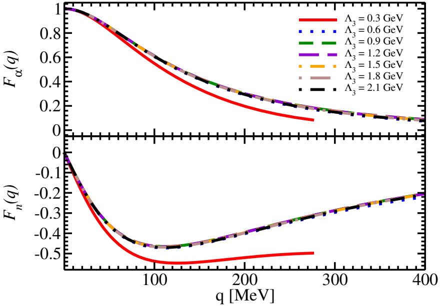
Unconventional $n\alpha$ interaction in ^{6}He

- Implementing unconventional p-wave potential in $nn\alpha$ system
- Running of $H(\Lambda)$ and $B(\Lambda)$
- Faster cutoff convergence of Faddeev components with unconventional potential

energy-dependent $n\alpha$ interaction

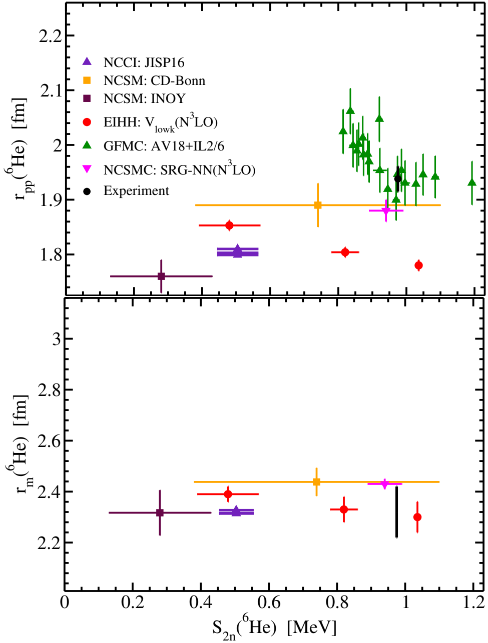


unconventional $n\alpha$ interaction



Li, Lyu, CJ, Long, arXiv:2303.17292 (accepted at PRC)

Universal Correlations Among Radii & S_{2n} in ^{6}He



- **He-6 charge radius**
- **He-6 matter radius**

compare with ab initio calculations

NCCI: Caprio, Maris, Vary, PRC ’14

NCSM: Caurier, Navratil, PRC ’06

EIHH: Bacca, Barnea, Schwenk, PRC ’12

GFMC: Pieper, RNC ’08

NCSMC: Romero et al., PRL ’16

Halo EFT: preliminary (uncertainty)

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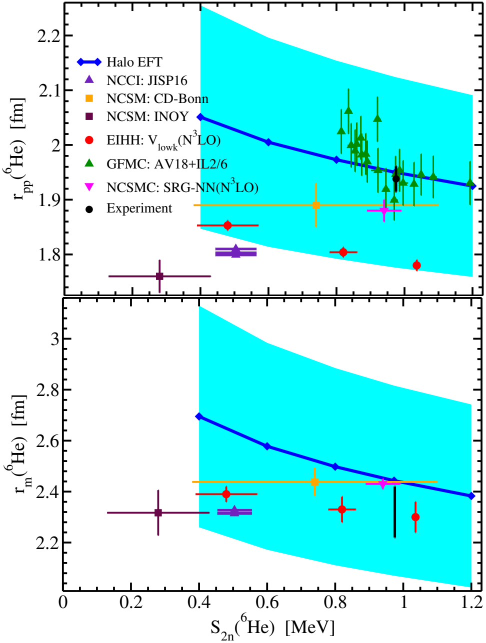

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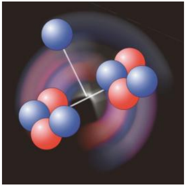
Halo EFT: preliminary (uncertainty)

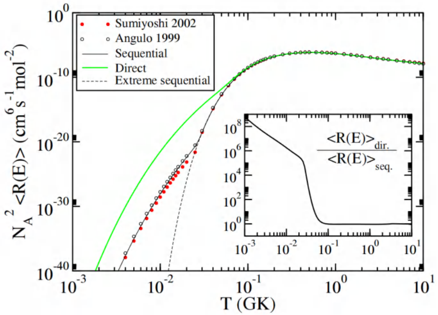
α-clustering in nuclei

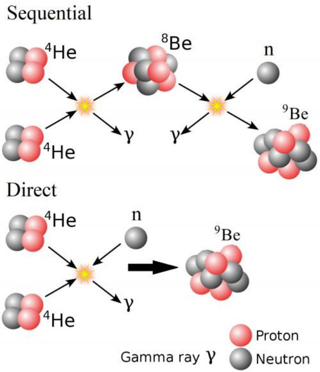
- ^{9}Be $\alpha - \alpha - n$ Borromean structure
  - Formation of ^{9}Be strongly influences r-process nucleosynthesis in neutron-rich environment
  - α-clustering drives ^{9}Be's fusion mechanism:

    sequential: $\alpha + \alpha \leftrightharpoons {}^{8}\text{Be}(n,\gamma)^{9}\text{Be}$

    direct: $\alpha(\alpha n,\gamma)^{9}\text{Be}$
  - reaction rate of ^{9}Be formation is sensitive to fusion mechanism



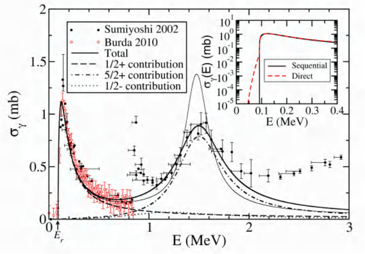




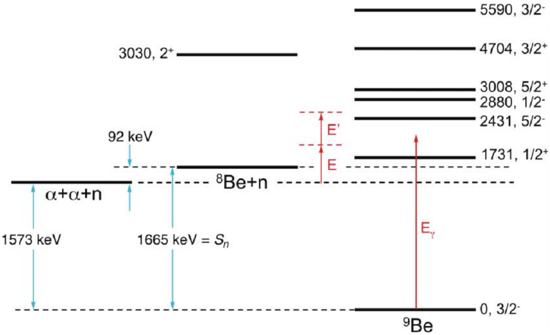
Garrido, et al., EPJA 47 (2011) 102

^{9}Be photodisintegration

- direct measurement of ^{9}Be formation: difficult
- indirect measurement: photodisintegration (inverse process)
  - $1/2+$ resonance: difficult to separate two- and three-body breakup
  - $1/2+$ resonance is close to ^{8}Be resonance
  - requires accurate theoretical analysis

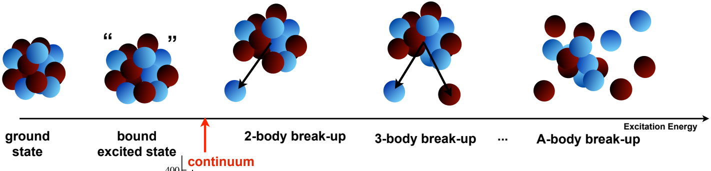


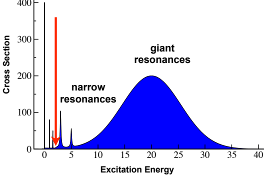
Garrido, et al., EPJA 47 (2011) 102



^{9}Be photodisintegration in Halo EFT at NLO

- construct α-α and n-α interaction from halo EFT
- calculate $\alpha\alpha n$ quantumm three-body problem:
  $\rightarrow$ hyperspherical harmonics expansion
- calculate photoabsorption cross section
  $\rightarrow$ Lorentz integral transform continuum $\rightarrow$ bound-state





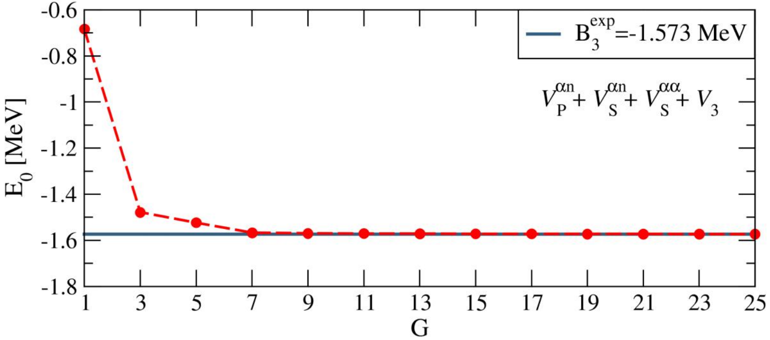
$$\sigma \propto \left|\langle \Psi_f | J^\mu | \Psi_0 \rangle\right|^2$$

Exact knowledge limited in energy and mass number

Capitani, Filandri, CJ, Orlandini, Leidemann arXiv:2506.05040

^{9}Be photodisintegration in Halo EFT

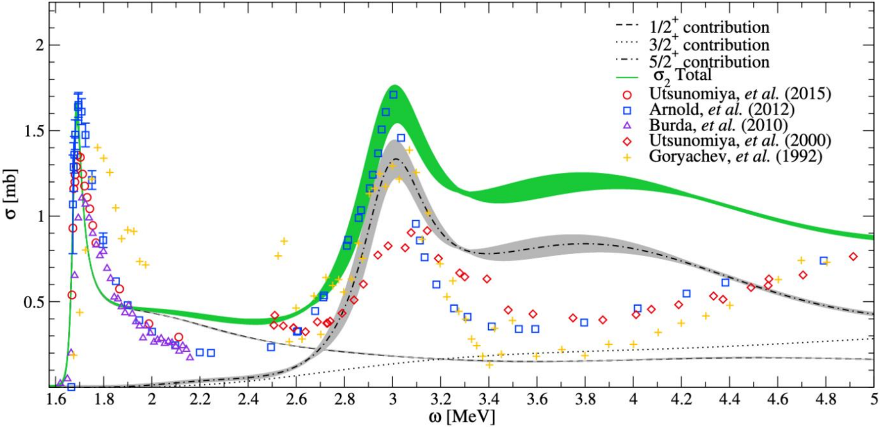
- B_3 of ^{9}Be converges when expanding the model space



Capitani, Filandri, CJ, Orlandini, Leidemann arXiv:2506.05040

^{9}Be photodisintegration in Halo EFT

- EFT calculation of photo-disintegration cross section
- focus on E1 transition (using Siegert theorem)
- $3/2^- \to 1/2^+; 3/2^+; 5/2^+$



Capitani, Filandri, CJ, Orlandini, Leidemann arXiv:2506.05040

Summary

- Effective field theory comes with limited powers determined by Q and Λ, different EFTs are efficient at different energy regimes
- Development of Halo EFT provides tools to tackle low-energy few-body structure and reaction physics
- Halo EFT
  - describes near-threshold physics in a controlled expansion of Q/Λ
  - rejuvenates cluster models with a systematic uncertainty estimates
  - Combine with *ab initio* calculations and experiments to extract correlations in threshold physics
  - helps to understand nucleosynthesis in astrophysical processes