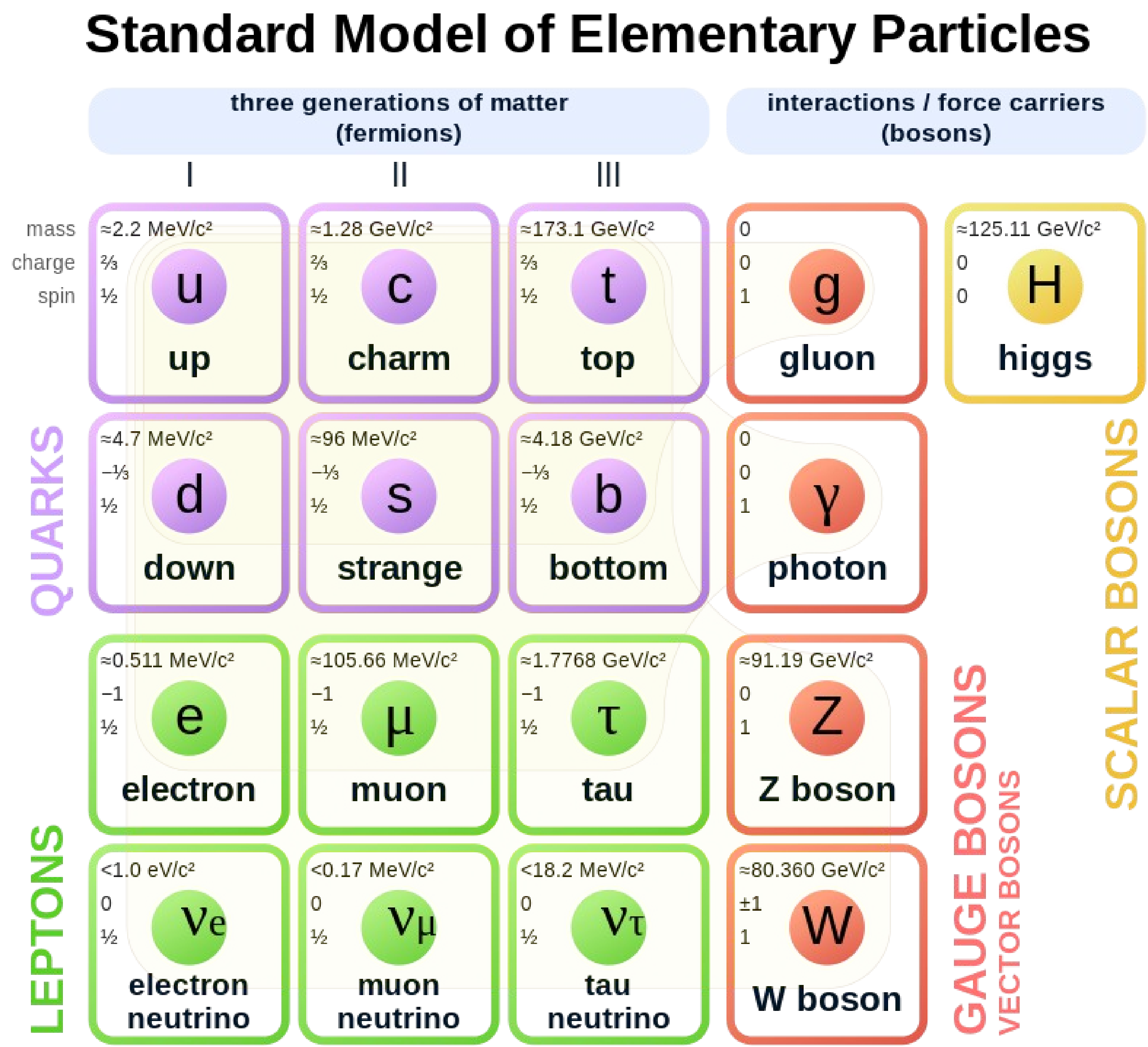
The two-pole nature of the Lambda(1405) from lattice QCD

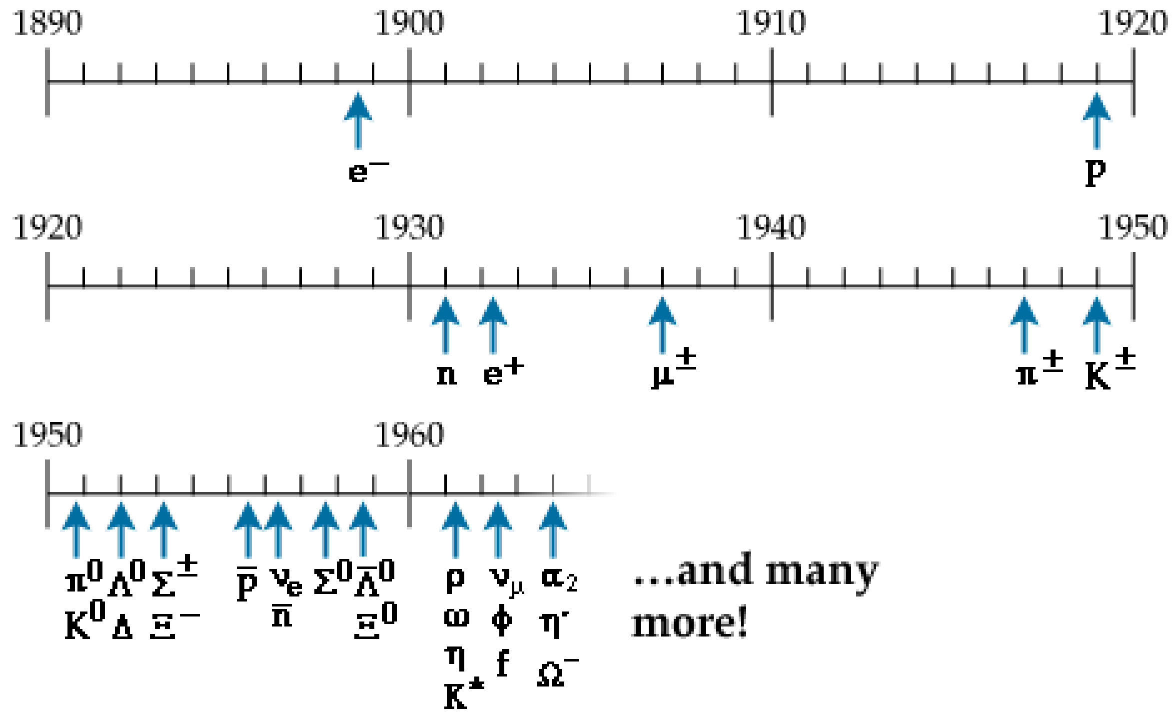
John Bulava

RUHR UNIVERSITÄT BOCHUM

PRISMA+ Colloquium, U. of Mainz May. 15th, 2024

- Quarks feel the strong nuclear force, are bound in hadrons
- ‘Everyday’ hadronic matter composed of up and down quarks
- A scoop of strange quarks has big effect





- 1950's: particle physics revolution
- New hadrons called 'hyperons' → 'strangeness'
- Particle Astrophysics: first hyperon $\Lambda(1115)$ discovered in cosmic rays

  V. D. Hopper, S. Biswas '50
- Organizing principle: up, down, and strange quarks are different 'flavors'

  M. Gell-Mann, Y. Ne'emann '61

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}$$ transforms like the '3' irrep of SU(3)

('isospin' for just u,d quarks) W. Heisenberg '32

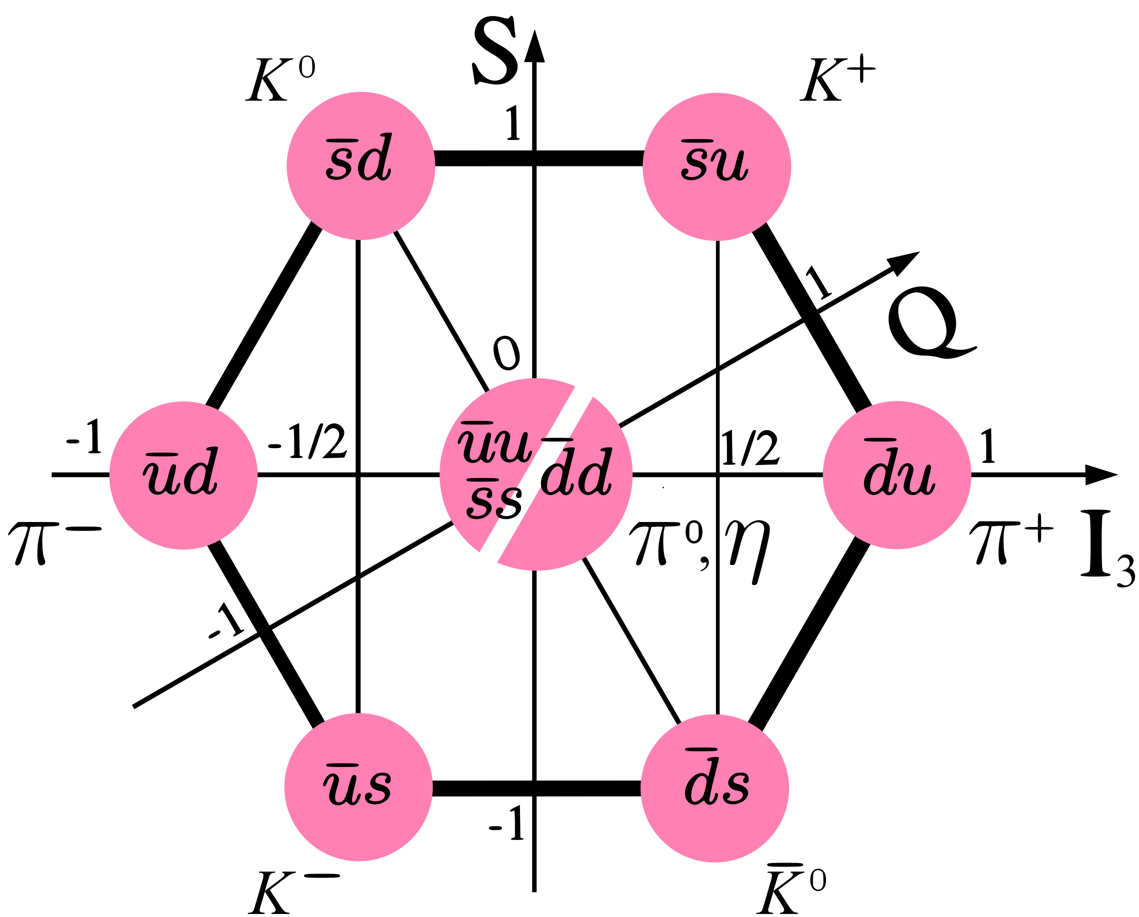
The Eightfold Way: meson singlet and octet

M. Gell-Mann, Y. Ne'emann '61

$$3 \otimes \bar{3} = 1 \oplus 8$$

Isospin analogy: addition of angular momentum for two spin-half states

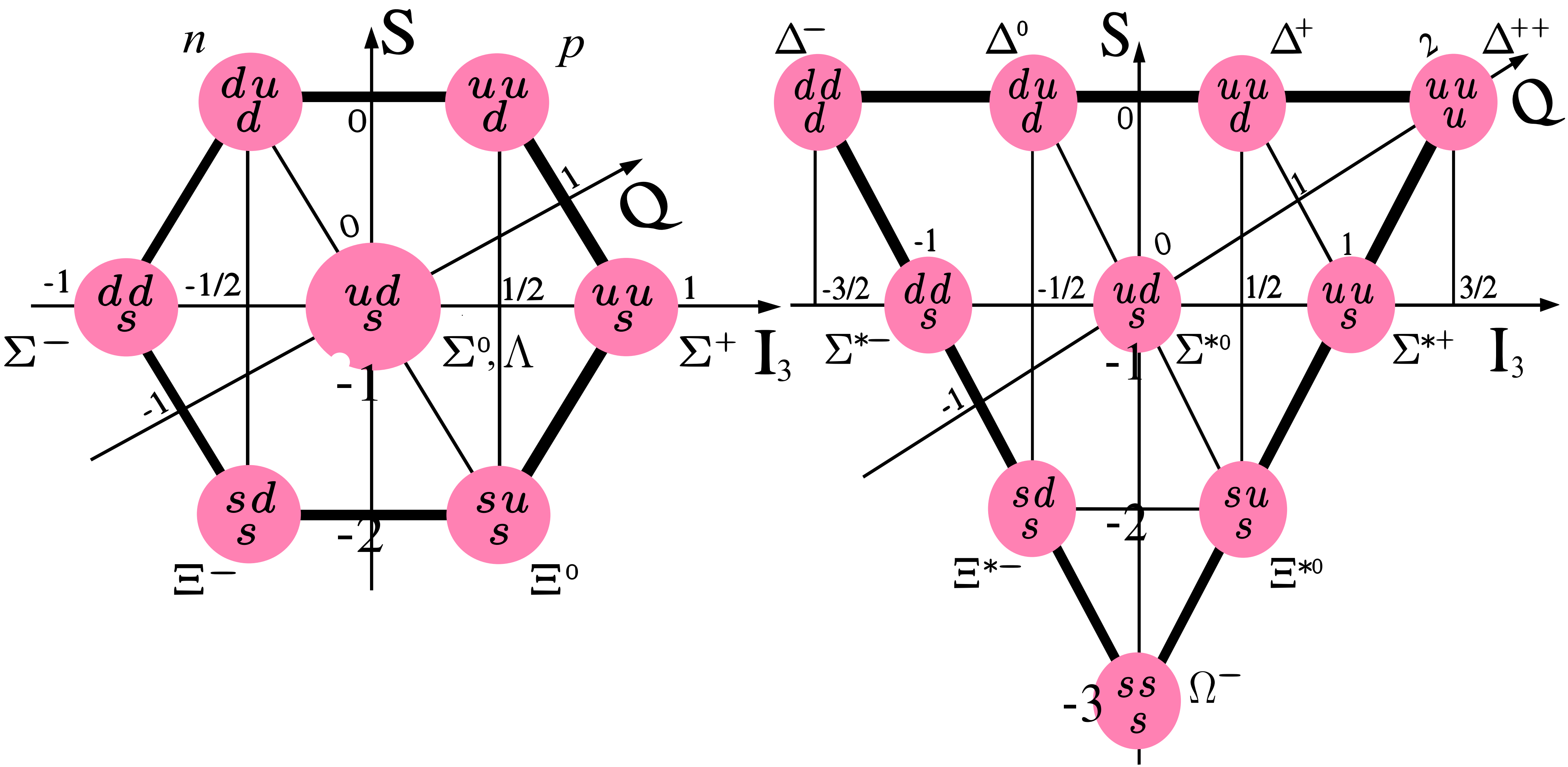
$$2 \otimes 2 = 1 \oplus 3$$



$$(\bar{u}u,\ \bar{d}d,\ \bar{s}s)$$
$$\eta'$$

The Eightfold Way: Baryon octet and decuplet:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus \cancel{1}$$



Constituent quark model:

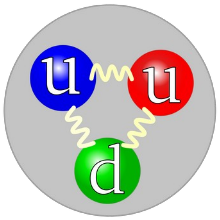
- Hadrons contain 'constituent quarks'

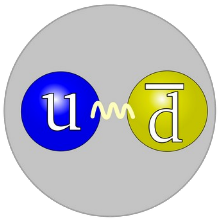
$$m_{\mathrm{u}} \approx m_{\mathrm{d}} \approx 390\,\mathrm{MeV} \qquad\qquad m_{\mathrm{s}} \approx 510\,\mathrm{MeV}$$

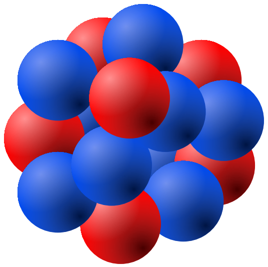
- Quarks interact via a central potential

- Quantum numbers from quarks + relative motion. Ground states $(\ell = 0)$:
  - Meson octets: $J^P = 0^-$ and 1^-
  - Baryon octet: $1/2^+$
  - Baryon decuplet: $3/2^+$
- Corresponding multiplets for excited states

- Classes of hadronic matter:
  - Mesons (quark-antiquark)
  - Baryons (three quarks)
  - Nuclei (protons and neutrons)
  - 'Exotics' ?





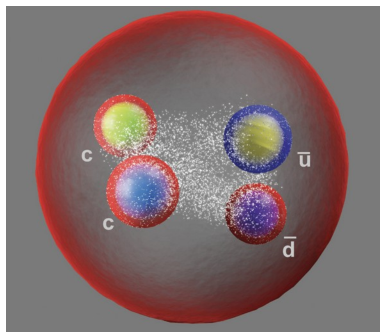


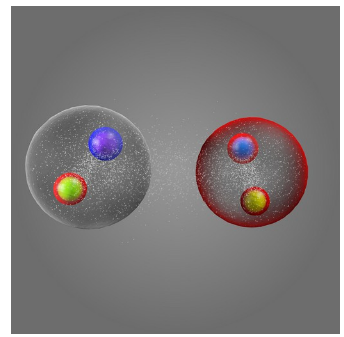
Aside: Exotic hadrons *do* exist!

- How do you know if a hadron is exotic?
- Need a failure of the quark model. Examples:
  - Boson with two charm quarks: tetraquark/molecule

LHCb Collaboration, Nature Physics (2022)

$T_{cc}^{+}(3875)$

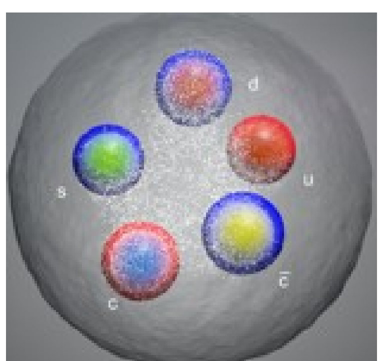


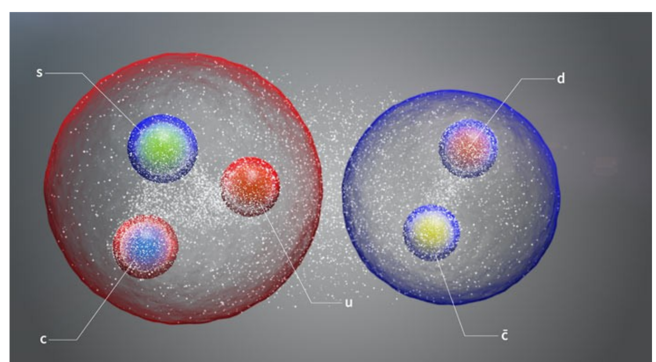


  - Fermion with charm-anticharm pair: pentaquark

LHCb Collaboration, Phys. Rev. Lett. 131, 031901 (2023)

$P_{\psi s}^{\Lambda}(4338)$





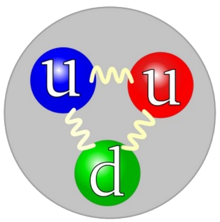
  - ***Extra/missing state compared to quark model multiplet

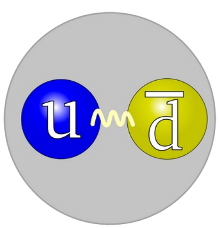
Quantum Chromodynamics (QCD): part of The Standard Model

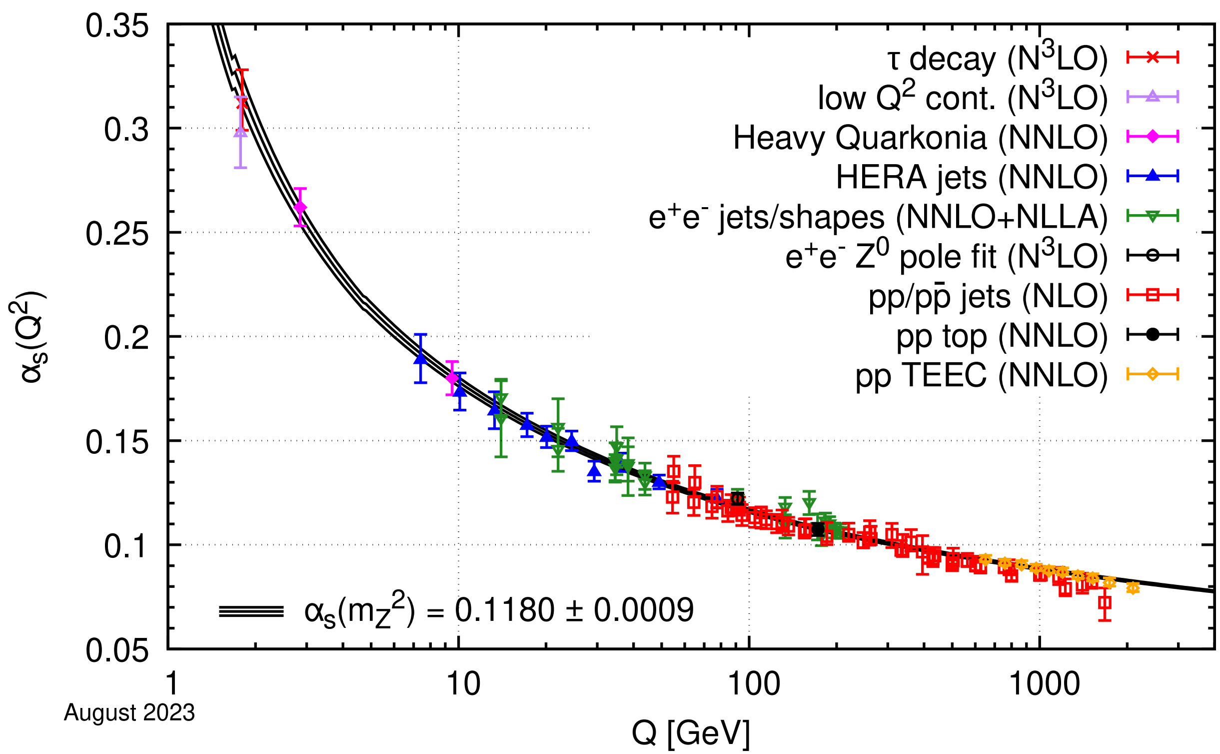
- ‘current’ quarks interact via gluon exchange, have ‘color’

$$m_{\mathrm{u}} \approx 2\,\mathrm{MeV}, \qquad m_{\mathrm{d}} \approx 5\,\mathrm{MeV}, \qquad m_{\mathrm{s}} \approx 95\,\mathrm{MeV}$$

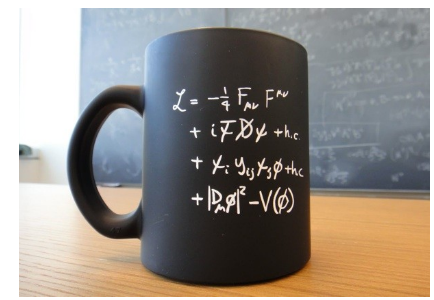
- QFT interaction strength depends on energy:

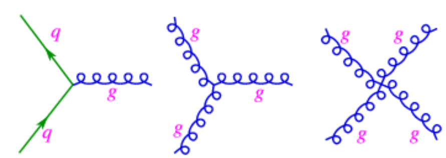






RIP Peter Higgs (1929-2024)





The Curious Case of the $\Lambda(1405)$: the first 'exotic' hadron?

- Low-energy K+p interactions → sub-threshold S=-1 resonance.

  R. Dalitz. S.F. Tuan '59

- Now well-established: $\Lambda(1405)$ with $I(J^P) = 0(1/2^-)$

- Three-quark interpretation difficult:
  - $1/2^-$ state should be ~500 MeV above $\Lambda(1115)$
  - Already have the $\Lambda(1670)$

- Is the $\Lambda(1405)$ an exotic meson-baryon bound state?

What are bound states and resonances?

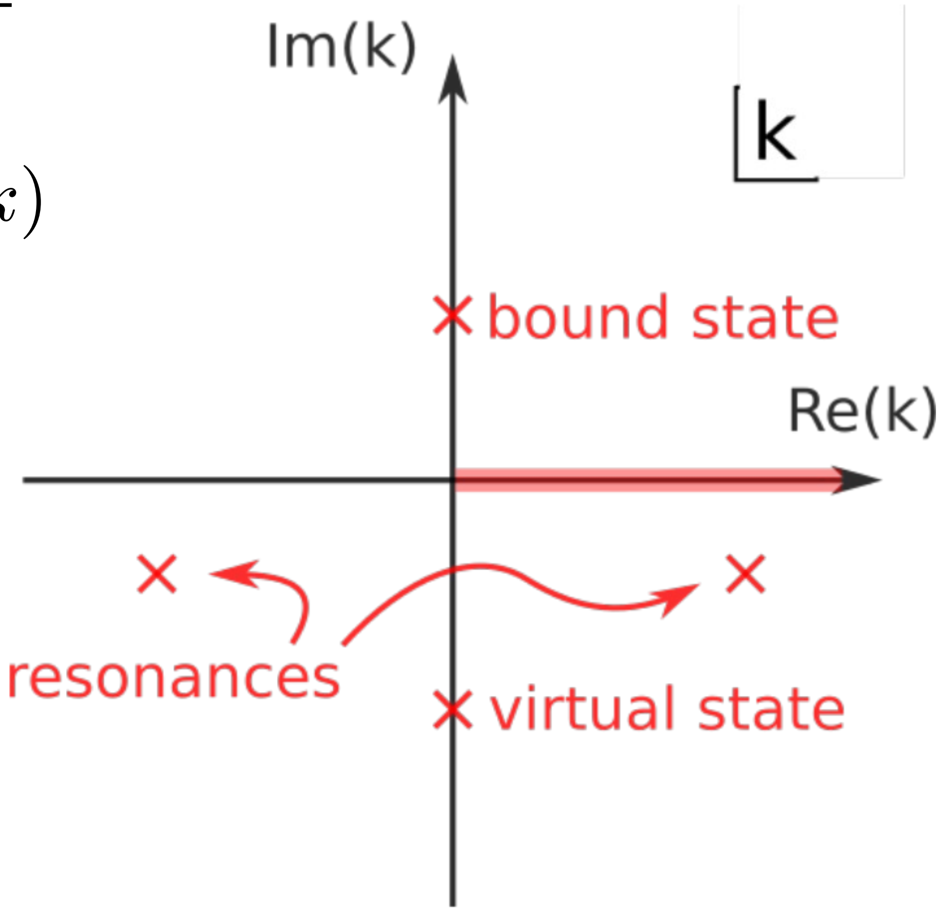
- Poles of (2-2) scattering amplitude $T(k)$ in the complex plane

- data on the positive real axis used for analytic continuation

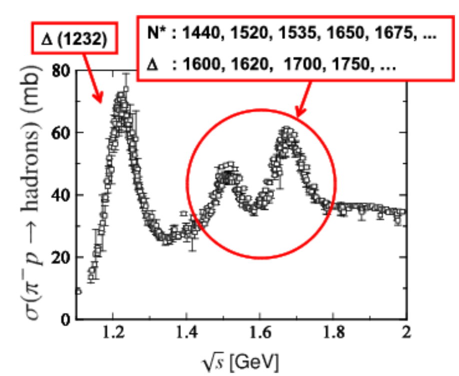
- Bound states occur below threshold

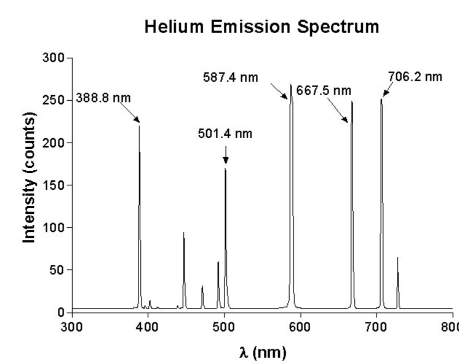
- Resonance poles come in pairs above threshold.

- Nearby resonances cause narrow 'peaks'



from I. Matuschek, V. Baru, F.-K. Guo, C. Hanhart Eur.Phys.J.A 57 (2021) 3, 101





Meson-baryon interactions in SU(3) flavor:

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

- Low-energy interaction governed by chiral symmetry. At LO: Weinberg, Tomozawa '66
  - attraction in 8 and 1
  - repulsion in 27
  - no interaction in (anti-)decuplet

→ Should be two states!

- Pole trajectories from chiral EFT + (model dependent) unitarization: U. Meissner, Symmetry 12 (2020) 6, 981; D. Jido, et al., Nucl.Phys.A 725 (2003) 181

Octet Baryons:

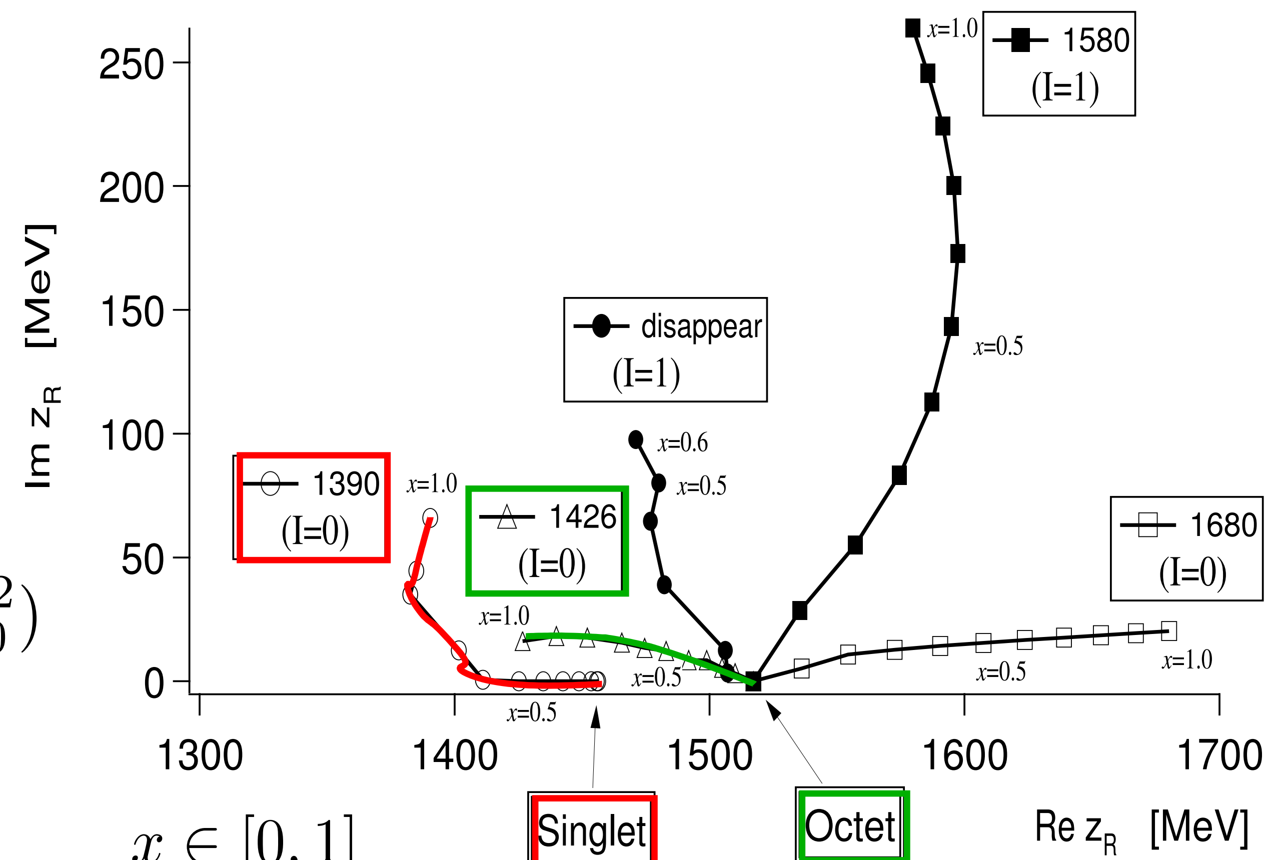
$$m_i(x) = m_0 + x(m_i - m_0)$$

$$m_0 = 1151\,\mathrm{MeV}$$

Octet mesons:

$$M_i^2(x) = M_0^2 + x(M_i^2 - M_0^2)$$

$$M_0 = 368\,\mathrm{MeV}$$

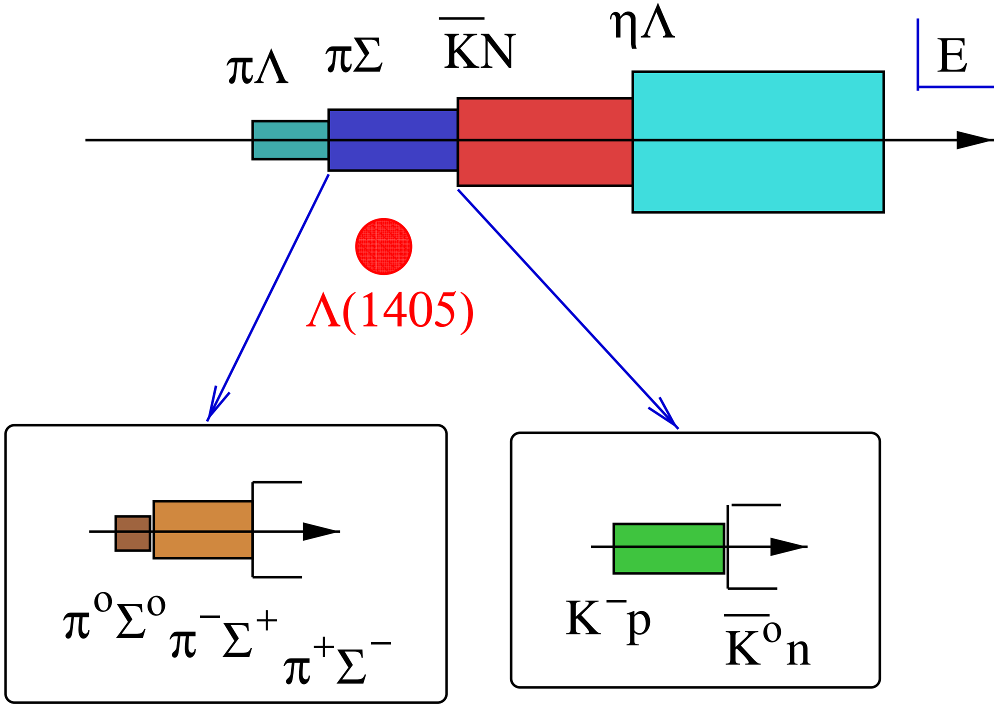


$x \in [0, 1]$

Quo Vadis?

- If three-quark: one state, but should be heavier
- If meson-baryon 'molecule': two states
- Particle Data Group lists $\Lambda(1405):\ ****$ $\qquad \Lambda(1380):\ **$
- Analytic continuation difficult. Many thresholds.

pole 1 [MeV]	pole 2 [MeV]
$1424^{+7}_{-23} - i\,26^{+3}_{-14}$	$1381^{+18}_{-6} - i\,81^{+19}_{-8}$
$1421^{+3}_{-2} - i\,19^{+8}_{-5}$	$1388^{+9}_{-9} - i\,114^{+24}_{-25}$
$1434^{+2}_{-2} - i\,10^{+2}_{-1}$	$1330^{+4}_{-5} - i\,56^{+17}_{-11}$
$1429^{+8}_{-7} - i\,12^{+2}_{-3}$	$1325^{+15}_{-15} - i\,90^{+12}_{-18}$

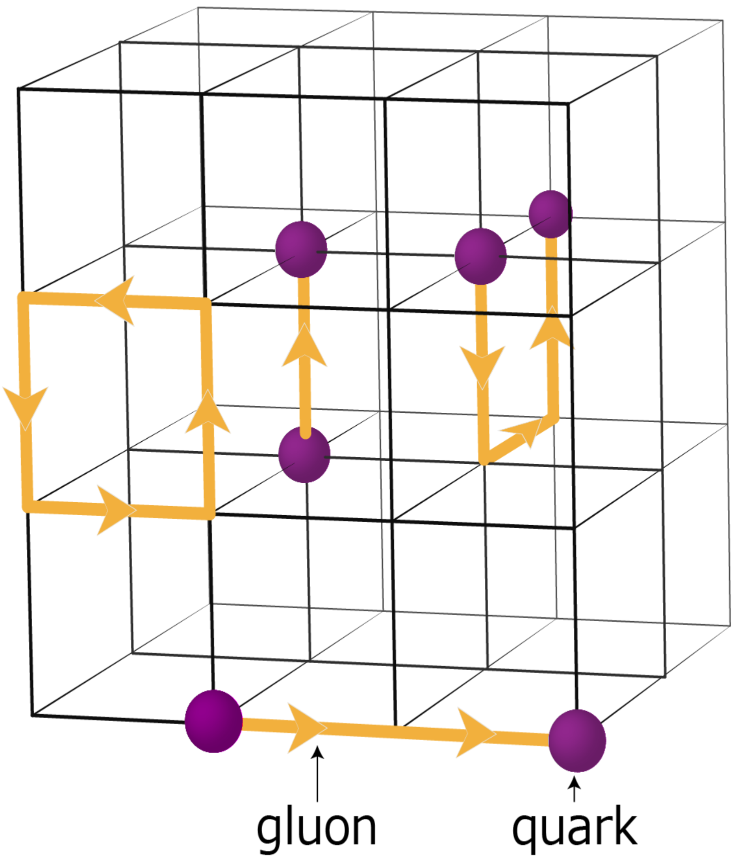


from T. Hyodo, U. Meissner, PDG '21

from U. Meissner, Symmetry 12 (2020) 6, 981

Lattice QCD: a bridge between high and low energies

- Monte Carlo sampling of QCD path integral



- Errors:
  - Monte Carlo statistics
  - Finite volume and lattice spacing
- Imaginary time required: $t \to i\tau$
- Advantages of lattice QCD:
  - Turn off electroweak interactions
  - Vary quark masses
  - If $m_{\mathrm{u}} = m_{\mathrm{d}}$ then only two channels: $\pi\Sigma - \bar{K}N$

Observables from lattice QCD: Euclidean correlation functions

- Large time separation: ground state saturation (Analogy: SHO)

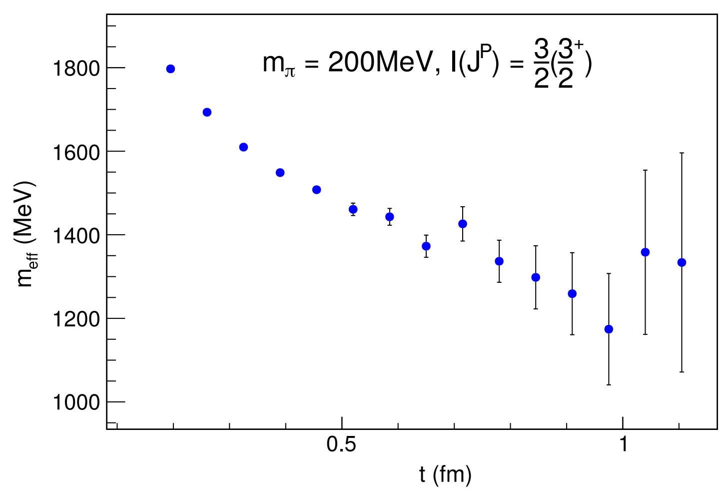
$$C(\tau) = \langle 0|\hat{x}(\tau)\,\hat{x}(0)|0\rangle = \langle 0|\hat{x}\,\mathrm{e}^{-\hat{H}\tau}\,\hat{x}|0\rangle = \sum_n |\langle 0|\hat{x}|n\rangle|^2 \mathrm{e}^{-E_n\tau}$$

- Low-lying states from large-time limit:

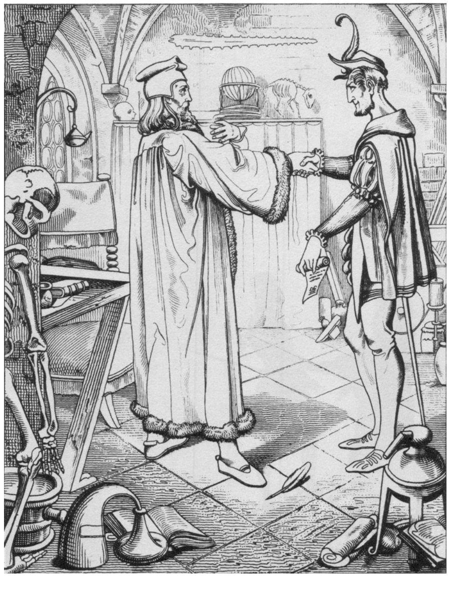
$$\lim_{\tau\to\infty} C(\tau) = A\mathrm{e}^{-E_1\tau} \times \left\{1 + \mathrm{O}(\mathrm{e}^{-(E_2-E_1)\tau})\right\}$$

- Signal-to-noise problem → ‘Teufelspakt’

$$m_{\mathrm{eff}}(\tau) = \log\left[\frac{C(\tau)}{C(\tau+1)}\right]$$



→



Plot courtesy of C. W. Andersen, apologies to J. W. von Goethe

What about scattering in lattice QCD?:

Asymptotic limit of $\langle 0|\hat{\mathcal{O}}'(\tau_f)\hat{\mathcal{O}}^\dagger(\tau_i)|0\rangle$ contains no info about on-shell amplitudes. → ~~Haag-Ruelle~~

L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585
M. Bruno, M. T. Hansen, JHEP 06 (2021) 043

- Finite volume method: two-hadron energies below $n \geq 3$ hadron thresholds:

$$\det[K^{-1}(E^L_{\rm cm}) - B(E^L_{\rm cm})] = 0$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531

For single channel:

$$K^{-1}_{\ell\ell'} = \delta_{\ell\ell'} \cot\delta_\ell$$

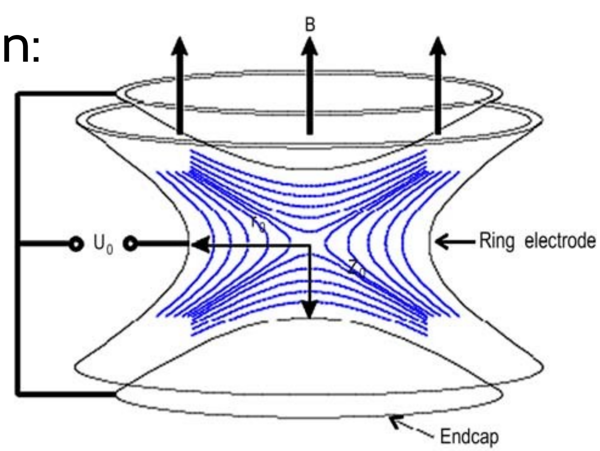
- Determinant over all partial waves and channels
  - Truncation at some $\ell_{\rm max}$ → systematic error

- Signal comes from interaction shift. Large-*L* threshold expansion:

$$\Delta E = E^{I=2}_{2\pi} - 2m_\pi = -\frac{4\pi a_0^{I=2}}{m_\pi L^3} + \mathrm{O}(L^{-4})$$

- For attractive *s*-wave interaction $E^L_{\rm cm} < E_{\rm thresh}$

→ direct constraints in complex plane!

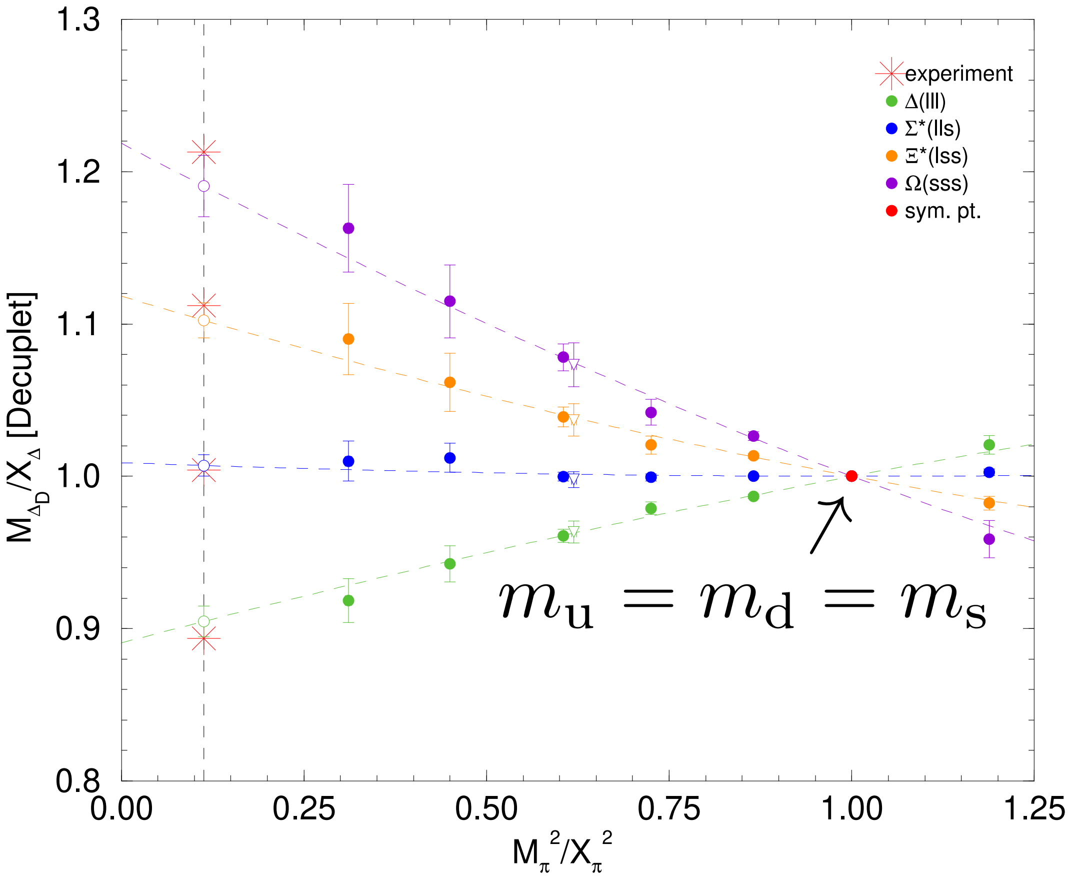


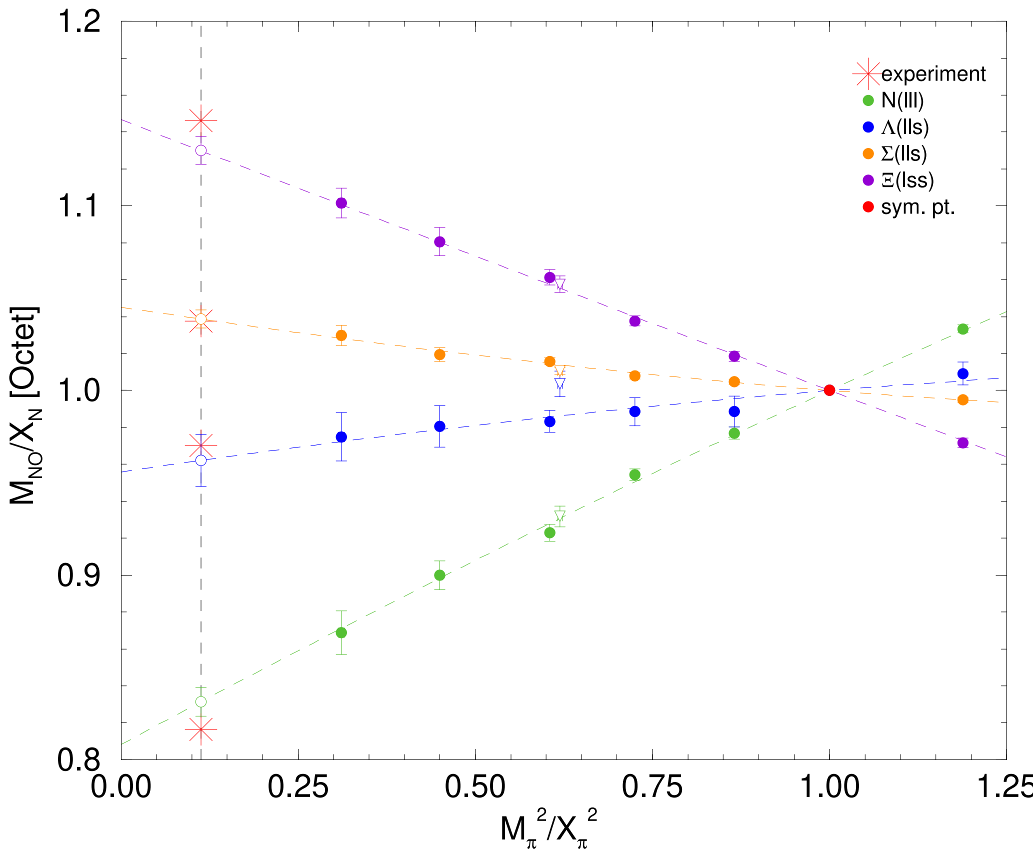
Analogy: two cold atoms in a trap
T. Busch, B.-G. Englert, K. Rzazewski, M. Wilkens, Found. Phys., 28 (1998)

Varying quark masses: connection to The Eightfold Way

Lattice QCD results for baryon masses:

$$m_{\rm u} = m_{\rm d}, \qquad m_{\rm u} + m_{\rm d} + m_{\rm s} = \text{phys.}$$





from QCDSF Collaboration, Phys.Rev.D 84 (2011) 054509

See also RQCD Collaboration, JHEP 05 (2023) 035

Recent lattice computation of the $\pi\Sigma - \overline{K}N$ amplitude:

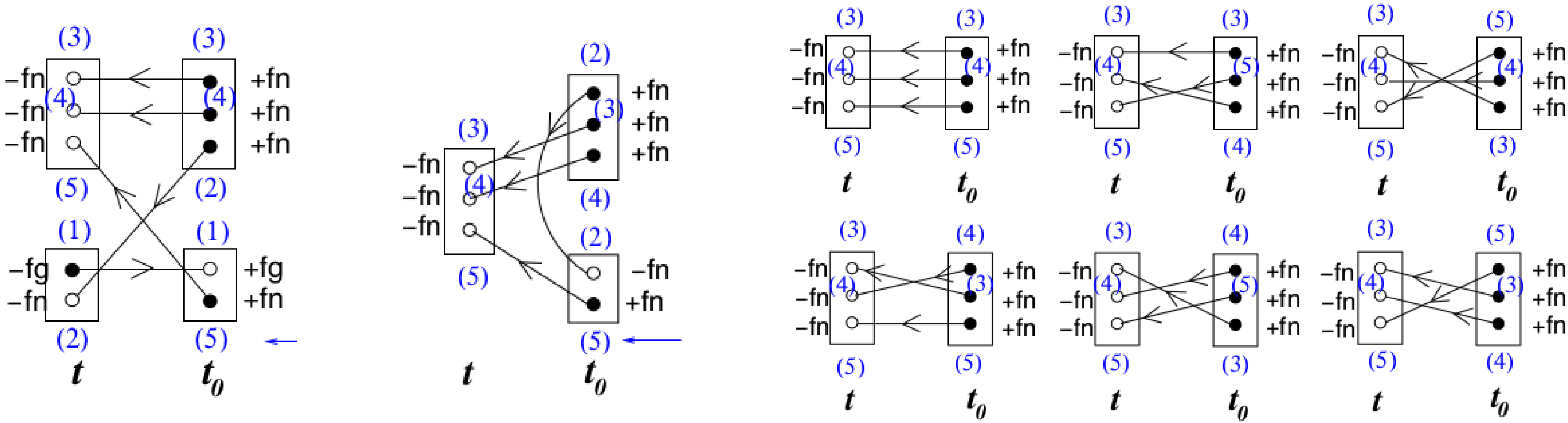
JB, B. Cid-Mora, A. Hanlon, B. Hoerz, D. Mohler, C. Morningstar, J. Moscoso, A. Nicholson, F. Romero-Lopez, A. Walker-Loud
(For the Baryon Scattering (BaSc) Collaboration) Phys.Rev.Lett. 132 (2024) 5, 051901 (Editor's Suggestion)

- CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\mathrm{fm},\ m_\pi = 200\mathrm{MeV},\ \frac{m_\pi^2}{m_X^2} = 0.23$$

$$m_\pi + m_\Sigma \approx 1380\,\mathrm{MeV}$$

- Correlation functions from tensor contraction: inverse of large sparse Dirac matrix



- Factorization enabled by the distillation/stochastic LapH algorithms for quark propagation

M. Peardon et al. Phys.Rev.D 80 (2009) 054506; C. Morningstar et al. Phys.Rev.D 83 (2011) 114505

Baryon Scattering Collaboration (BaSC): comprehensive study of many channels

Flavor channel	Number of Correlators
$I=0,\ S=0,\ NN$	8357
$I=0,\ S=-1,\ \Lambda, N\overline{K}, \Sigma\pi$	8143
$I=\frac{1}{2},\ S=0,\ N\pi$	696
$I=\frac{1}{2},\ S=-1,\ N\Lambda, N\Sigma$	17816
$I=1,\ S=0,\ NN$	7945
$I=\frac{3}{2},\ S=0,\ \Delta, N\pi$	3218
$I=\frac{3}{2},\ S=-1,\ N\Sigma$	23748
$I=0,\ S=-2,\ \Lambda\Lambda, N\Xi, \Sigma\Sigma$	16086
$I=2,\ S=-2,\ \Sigma\Sigma$	4589
Single hadrons (SH)	33

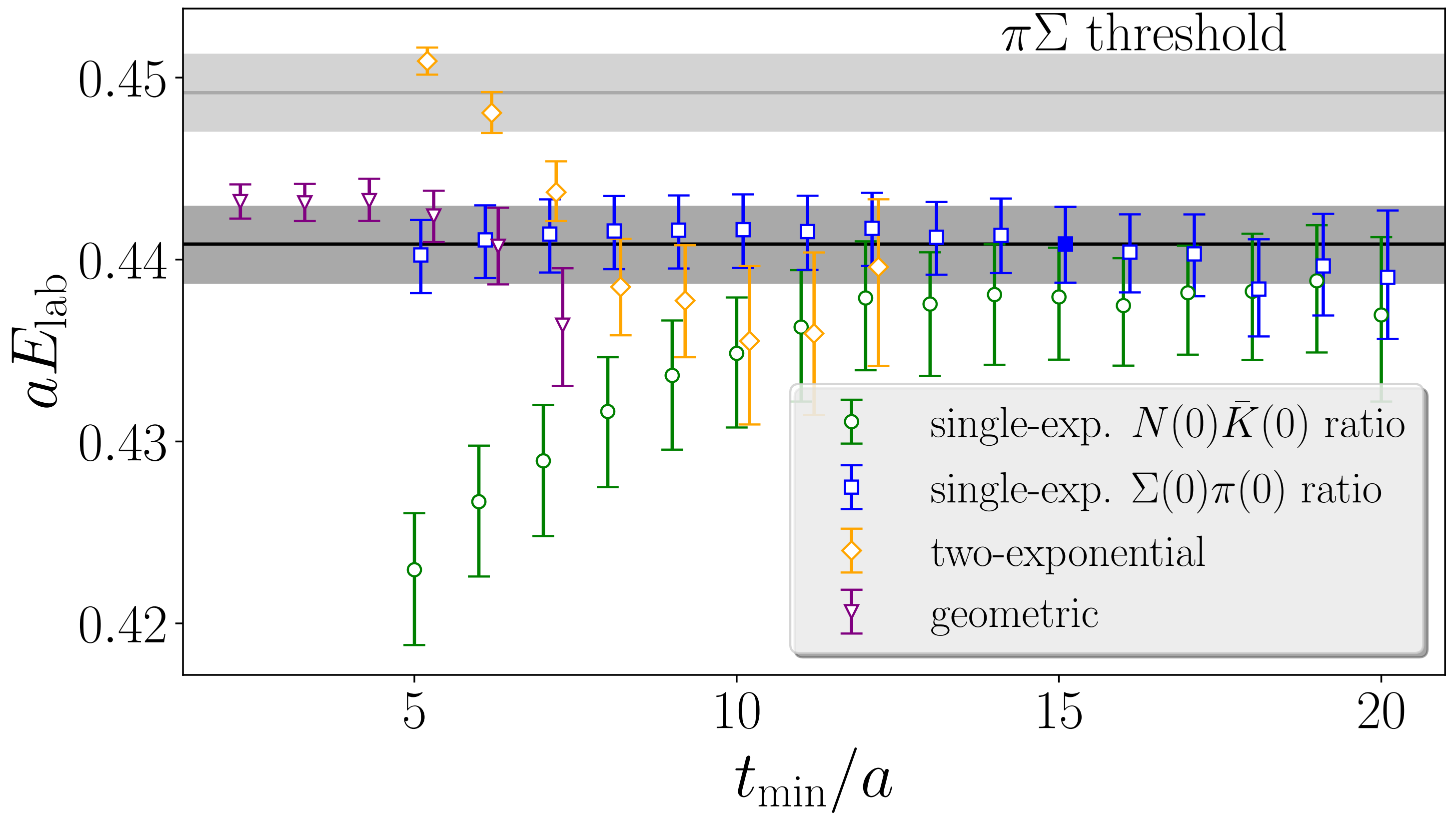
- 25M core-hr on JUQUEEN (JSC) and HIM
- 120M core-hr on Frontera (TACC)

Determination of finite-volume spectrum:

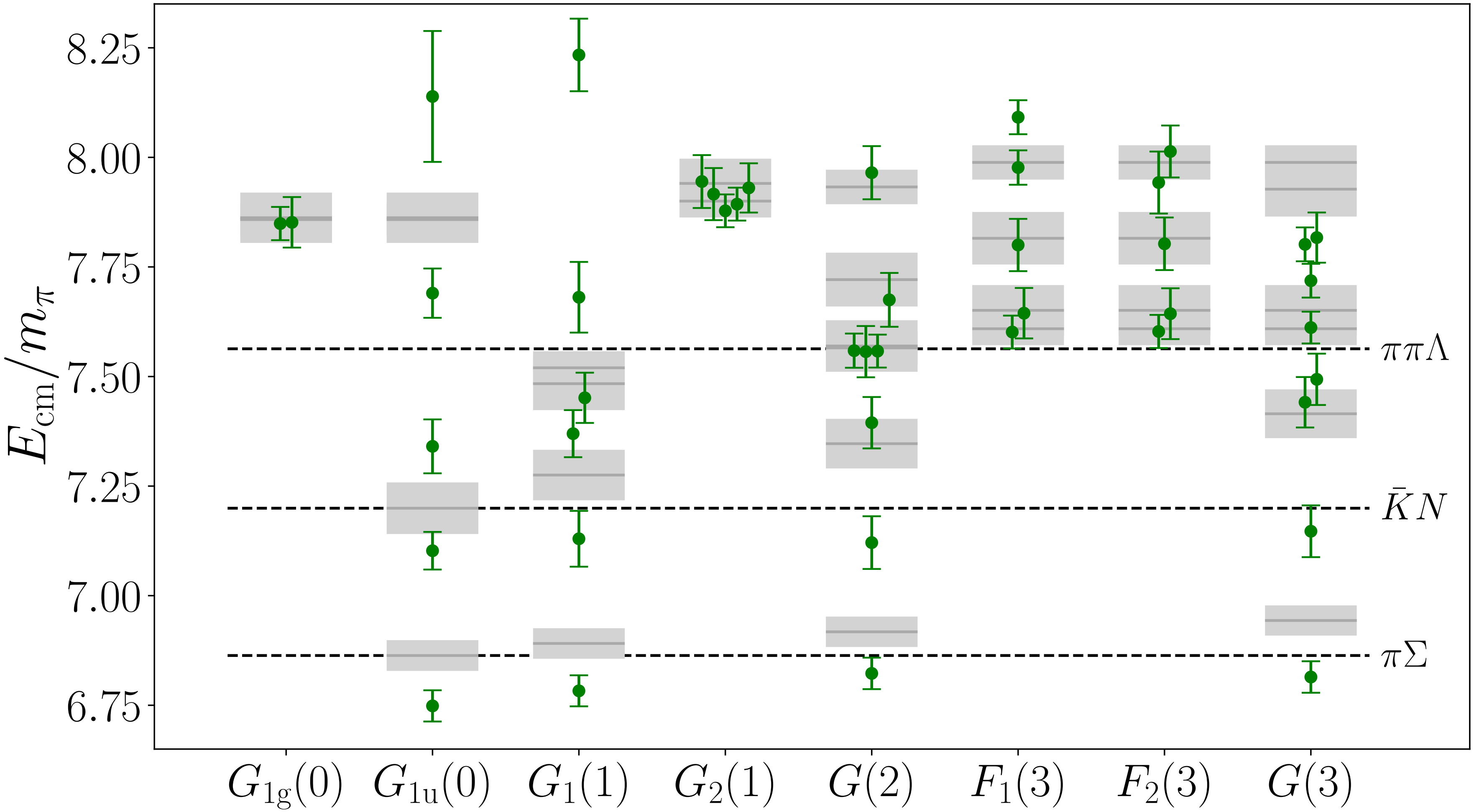
- Direct determination of interacting shifts from ratios:

$$R(t) = \frac{C(t)}{C_1(t)C_2(t)} \approx A\mathrm{e}^{-\Delta E t}$$

- Single-exp. fits over range $[t_{\min}, t_{\max}]$

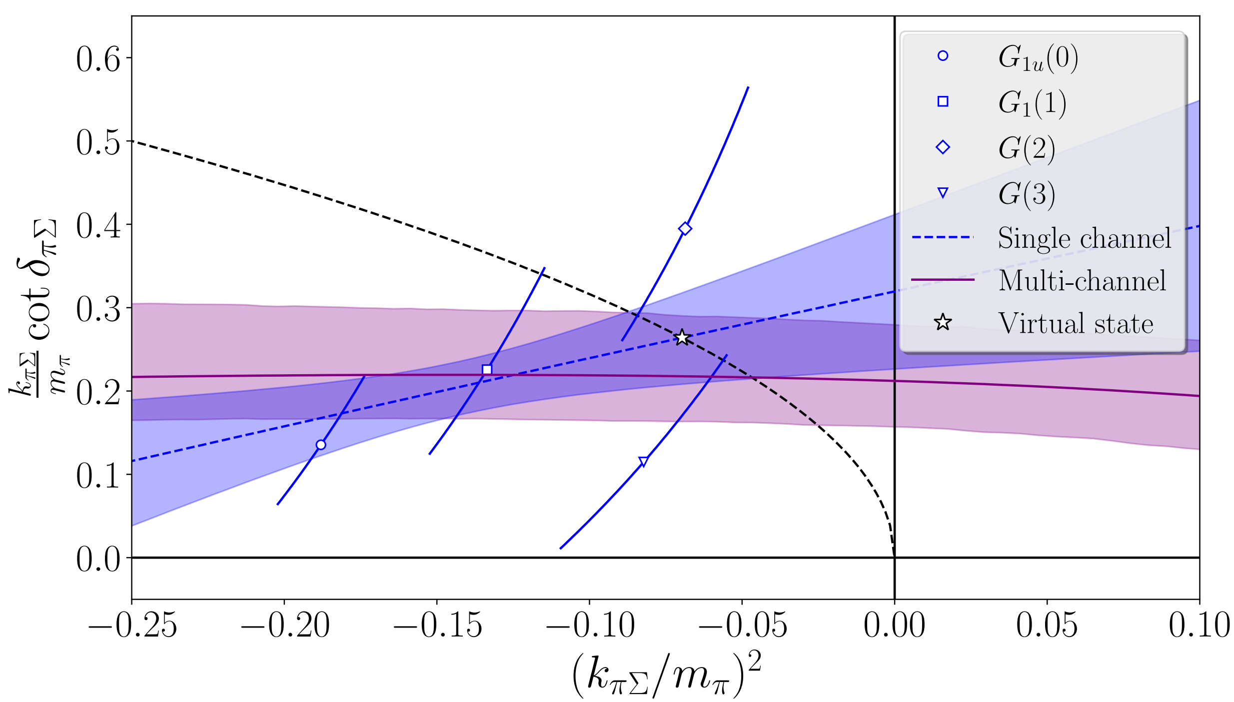


Determination of finite-volume spectrum:



Elastic Sigma-pion scattering: below nucleon-kaon threshold

- Single-partial wave approximation: $k \cot \delta = B(E_{\mathrm{cm}}^{L})$
- Effective range parametrization: $k \cot \delta = A + Bk^2$
- Analytic continuation: pole occurs when $k \cot \delta = ik$
- Pole for pure-imaginary, negative *k* → virtual bound state



Coupled-channel scattering: below three-hadron threshold

- Simple amplitude parametrizations (no chiral symmetry constraints)

  Variants of:

$$K_{ij}^{-1} = A_{ij} + B_{ij}\Delta(E_{\mathrm{cm}})$$

  as well as for *K* and Blatt-Biedenharn

- Analytic continuation: find zeroes of

$$t^{-1} = K^{-1} - i\hat{k}$$

- No nearby left- or right-hand cuts!

J. R. Green, et al., Phys.Rev.Lett. 127 (2021) 24, 242003

Coupled-channel scattering: below three-hadron threshold

- Pole 1

$$E_1 = [1455(13)(2)(17) - i\,11.5(4.4)(4)(0.1)]\,\mathrm{MeV}$$

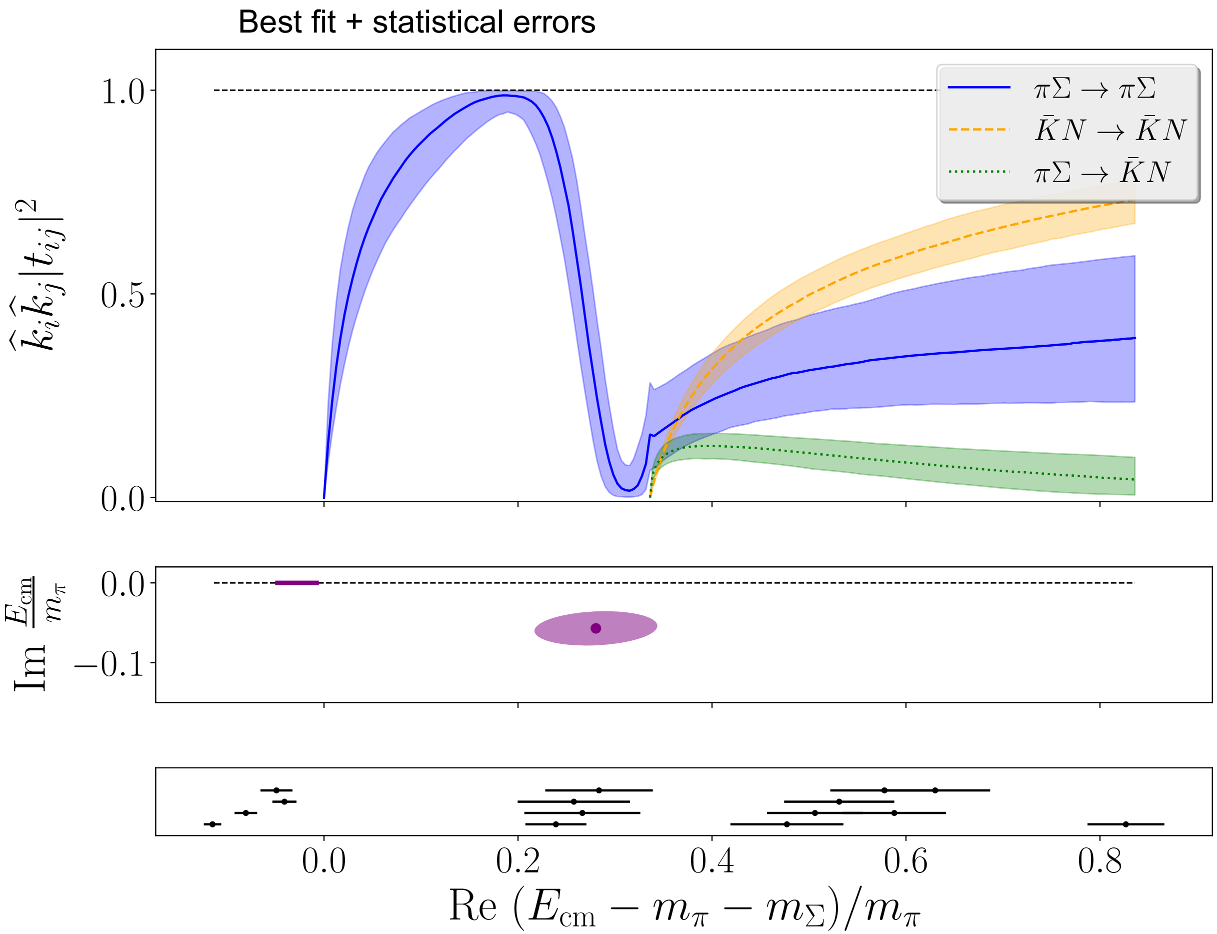
- Pole 2

$$E_2 = 1392(9)(2)(16)\,\mathrm{MeV}$$

- Residues:

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 1.9(4)(6)$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 0.53(9)(10)$$



Coupled-channel scattering: below three-hadron threshold

- Pole 1

$$E_1 = [1455(13)(2)(17) - i\,11.5(4.4)(4)(0.1)]\,\mathrm{MeV}$$

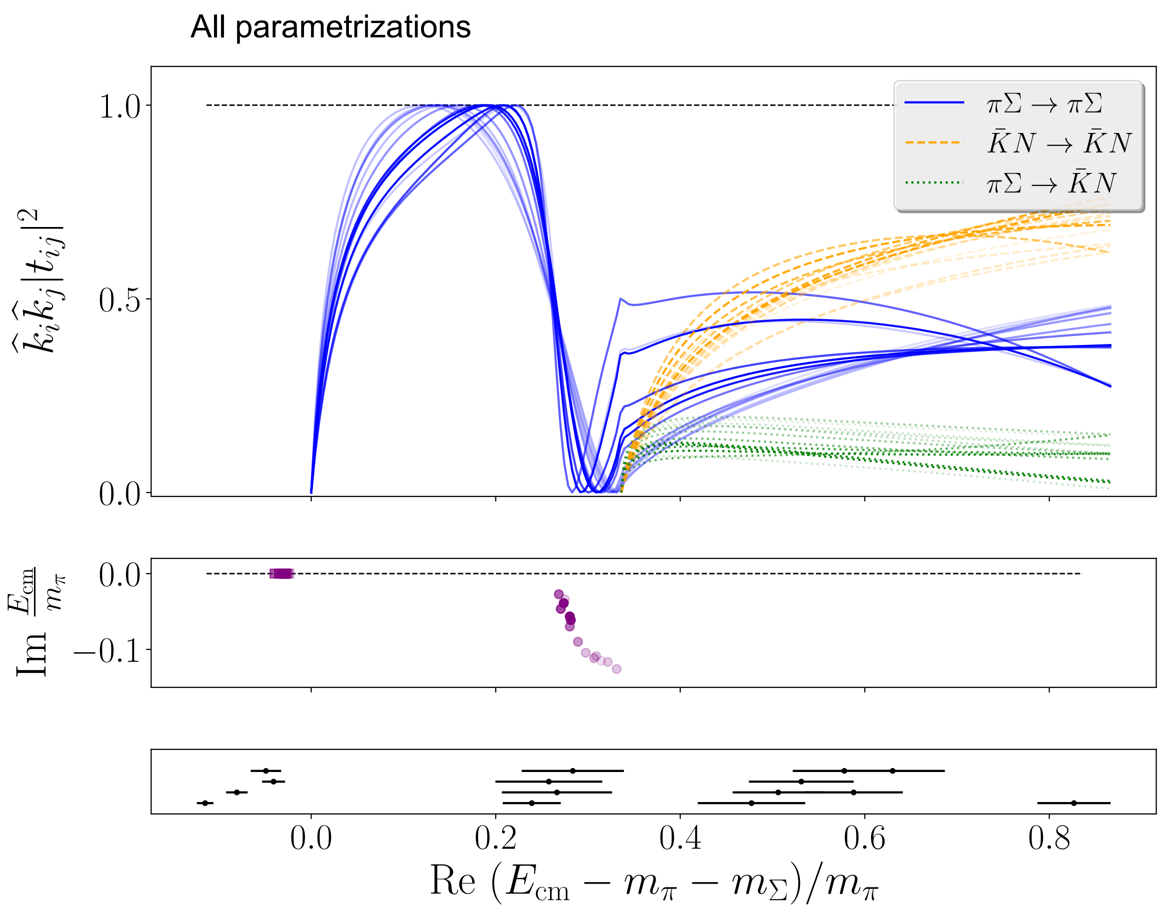
- Pole 2

$$E_2 = 1392(9)(2)(16)\,\mathrm{MeV}$$

- Residues:

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 1.9(4)(6)$$

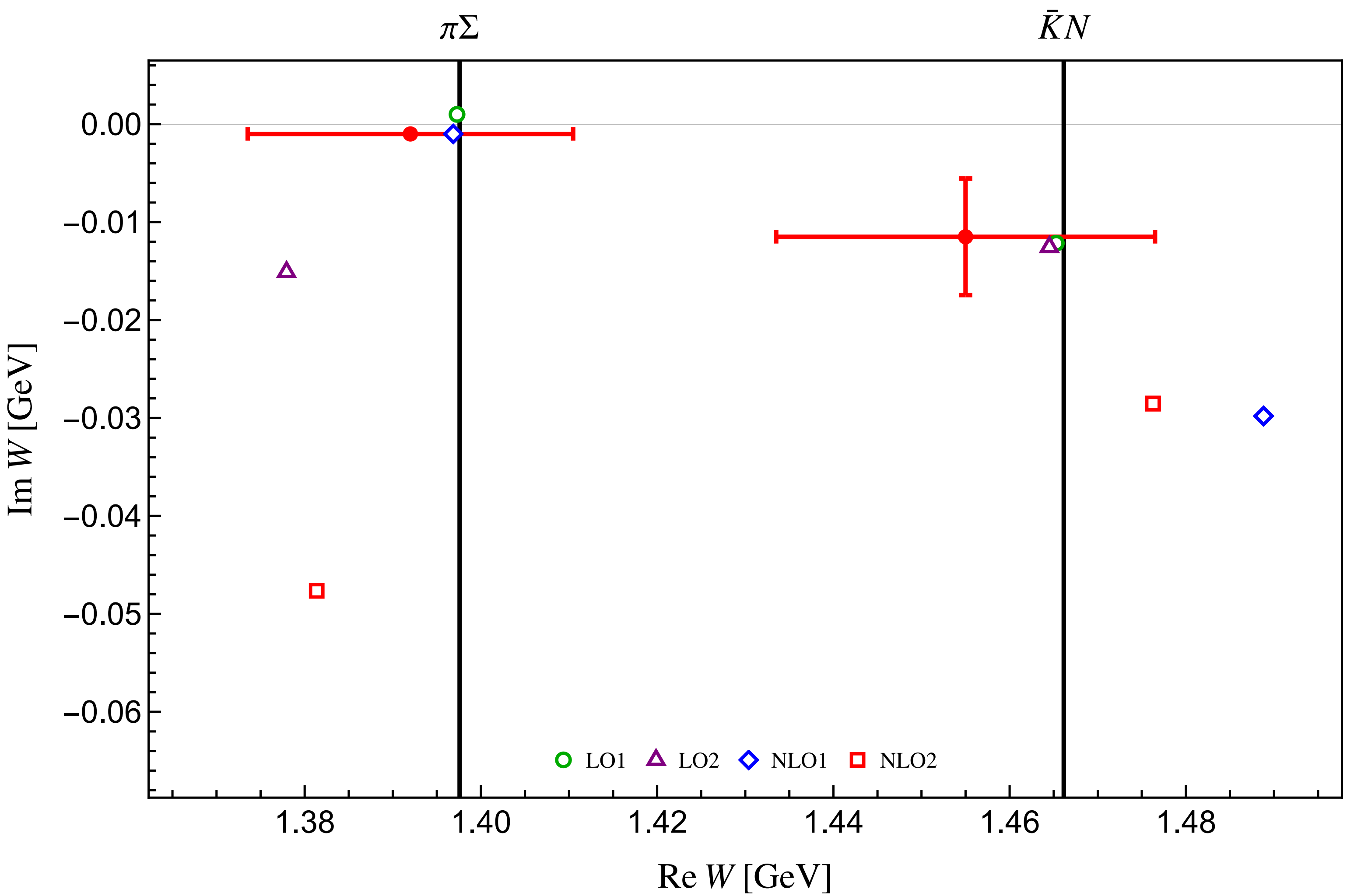
$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 0.53(9)(10)$$



Comparison with Unitarized Chiral EFT

- Consistent pole locations

F.-K. Guo, Y. Kamiya, M. Mai, U.-G. Meissner, 2308.07658 [hep-ph]

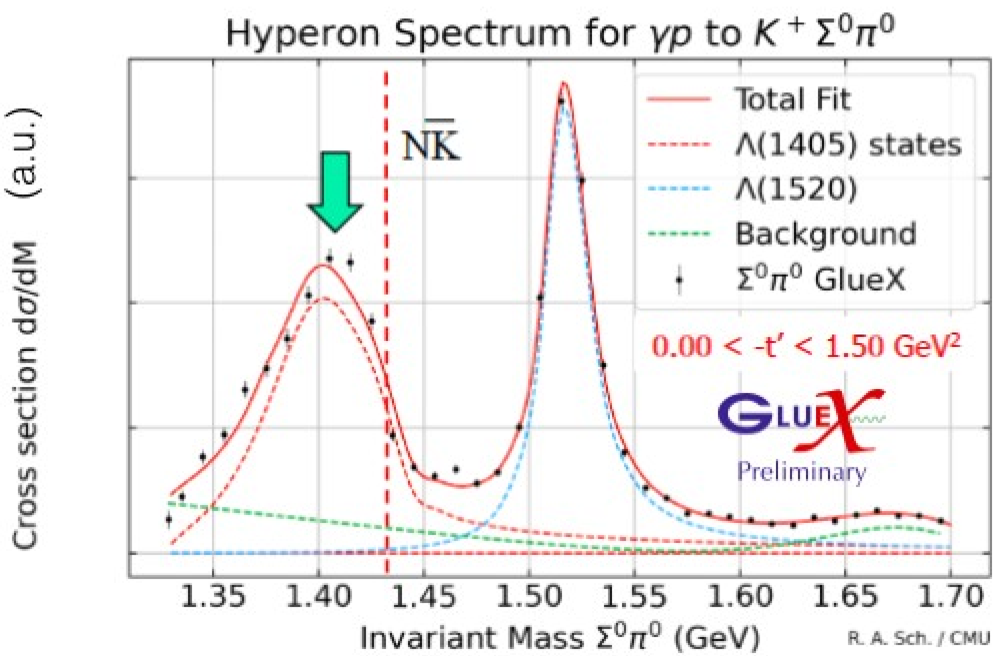


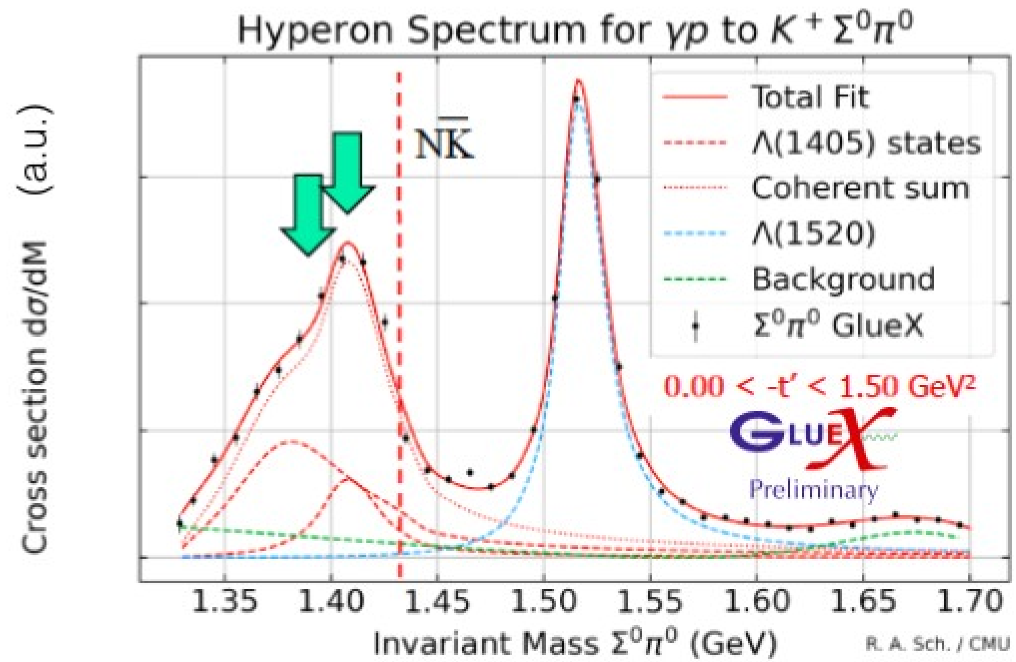
- Lattice QCD already provides useful constraints

Recent (preliminary) experimental results:

- Preliminary analysis from the Glue-X collaboration

  N. Wickramaarachchi, R. A. Schumacher, G. Kalicy, arXiv:2209.06230 [nucl-ex]





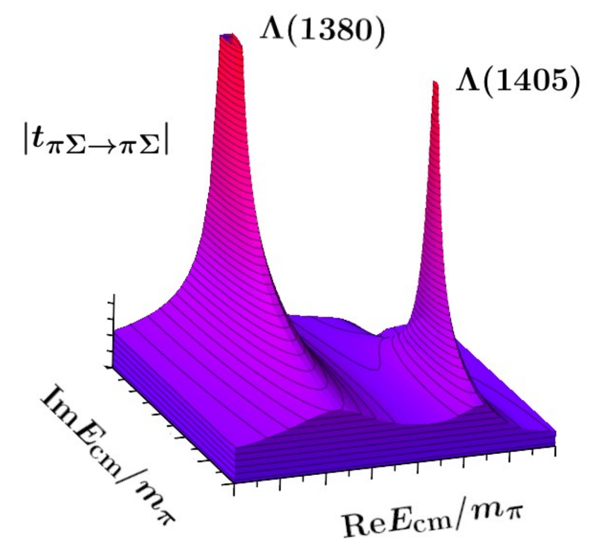
- First hint of two poles in experimental data?

Conclusions

- Exotic hadrons require an 'Eightfold Way 2.0' with new mechanisms
- First lattice QCD computation unambiguously confirms two states: $\Lambda(1405)$ and $\Lambda(1380)$
- Completed Glue-X analysis may also provide strong evidence
- Clearest evidence so far for an exotic hadron without heavy quarks

- TODO:
  - ➔ Complete quark mass trajectory: SU(3) point → real world
    - ➔ Separation of states at SU(3) point strong evidence for two-pole picture
  - ➔ Lattice spacing effects, left- and right-hand cuts

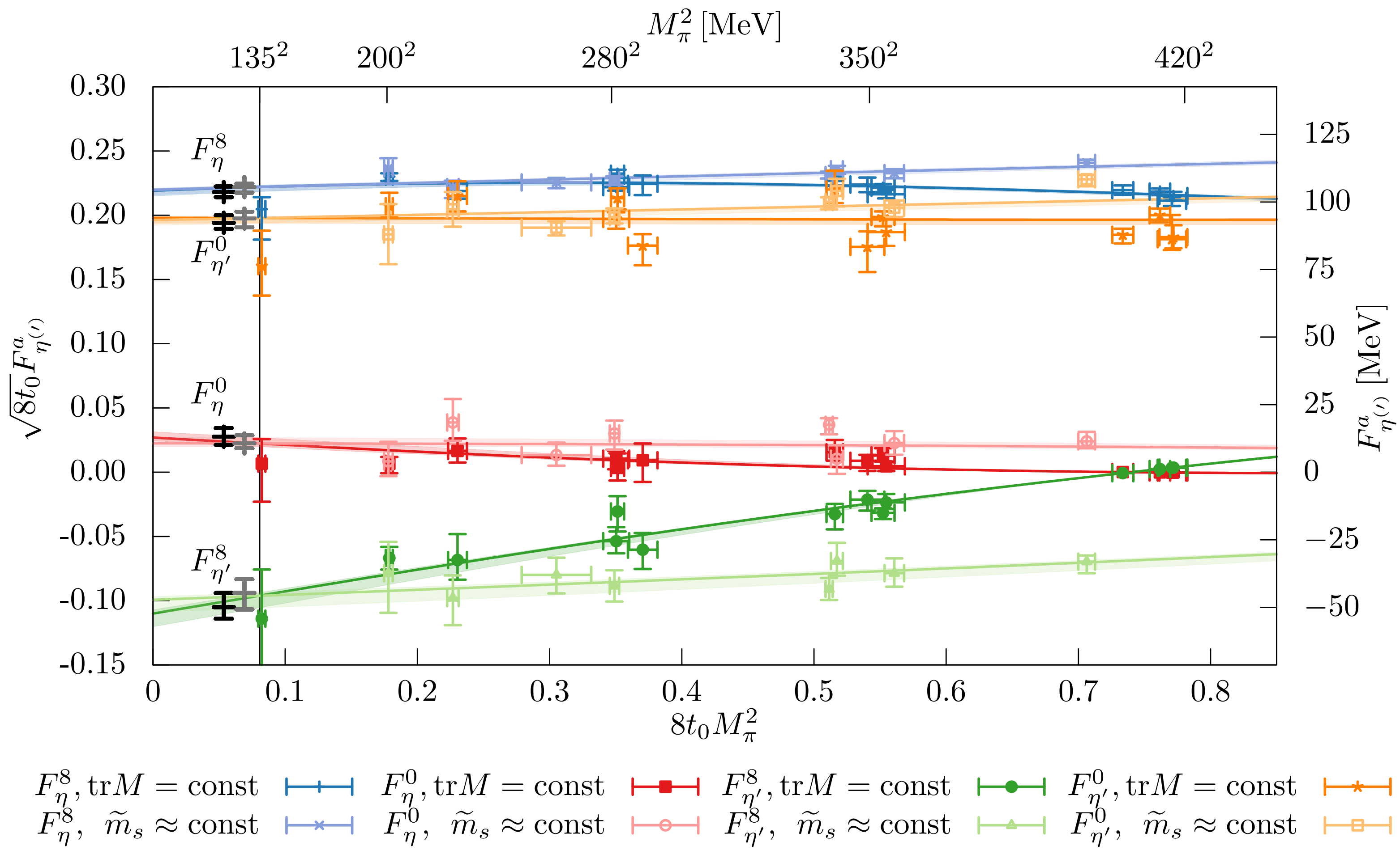
- Many more systems to study:
  - ➔ Another old mystery: $N(1440)$ 'Roper'
  - ➔ Higher strangeness channels have less experimental data
  - ➔ Meson octet – baryon decuplet
  - ➔ Baryon octet – baryon octet → Hyperon puzzle in neutron stars



A single-hadron analogy: eta-eta' mixing

- Recent computation from Regensburg group:

Bali, et al. JHEP 08 (2021) 137



$$\langle 0|A_\mu^a|m(p)\rangle = iF_m^a p_\mu, \qquad a = 0, 8, \qquad m = \eta, \eta'$$

- No singlet-octet mixing at SU(3) flavor symmetric point