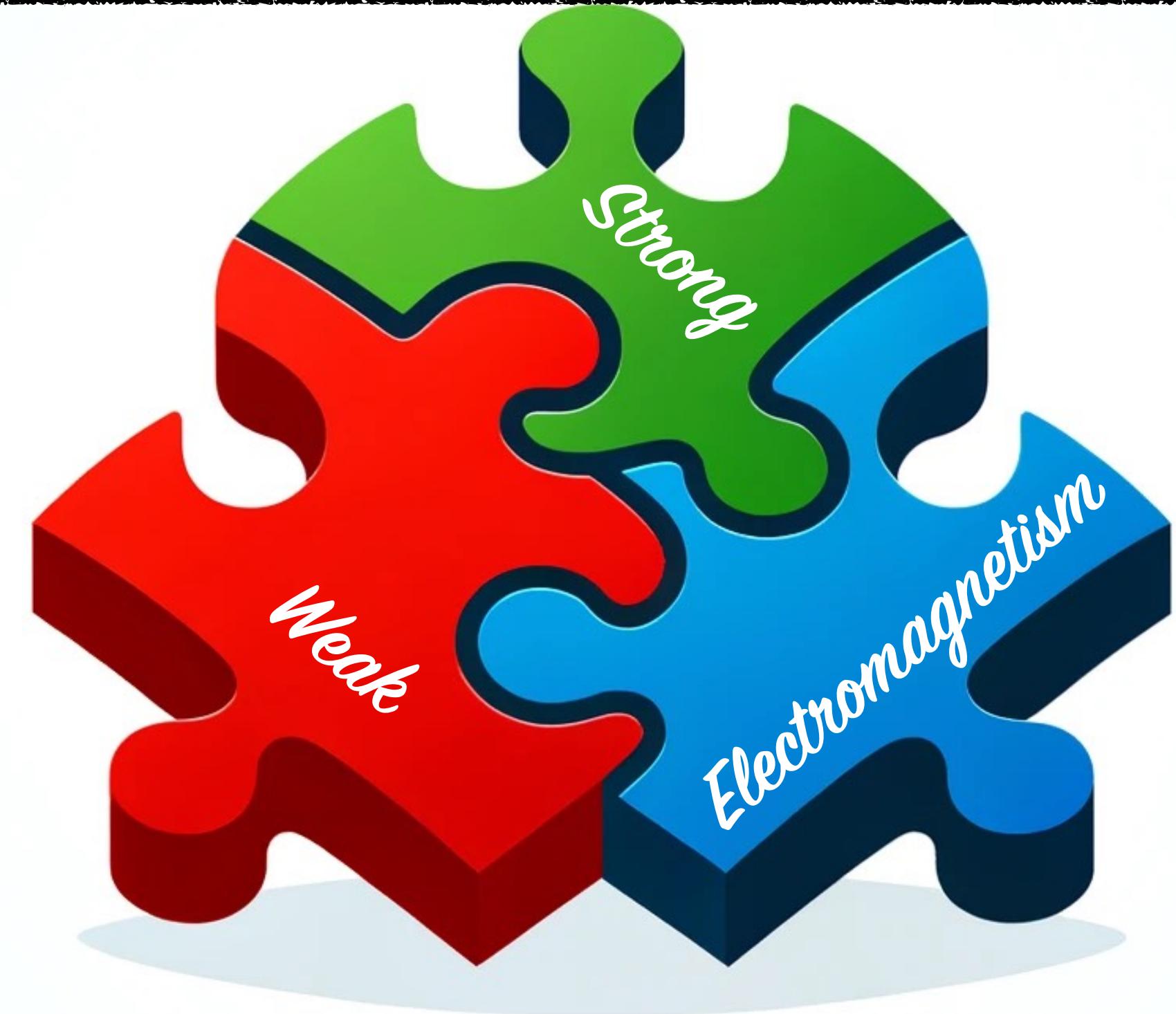


Proton Decay and Gravitational Waves as Complementary Tests of Grand Unification



Jessica Turner

Institute for Particle Physics Phenomenology, Durham University

PRISMA Colloquium, 12 June 2024

Career Progression & Motivation

- Studied Maths & Physics at Glasgow University (Scotland: 4 +1 years)



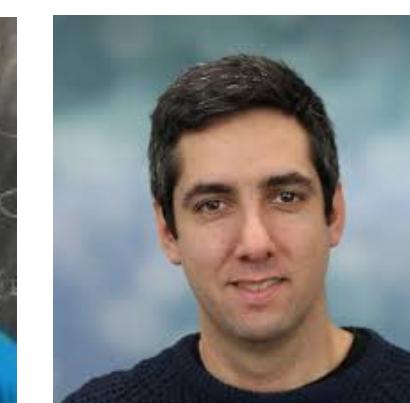
- PhD in theoretical particle physics in Durham University (North East England)



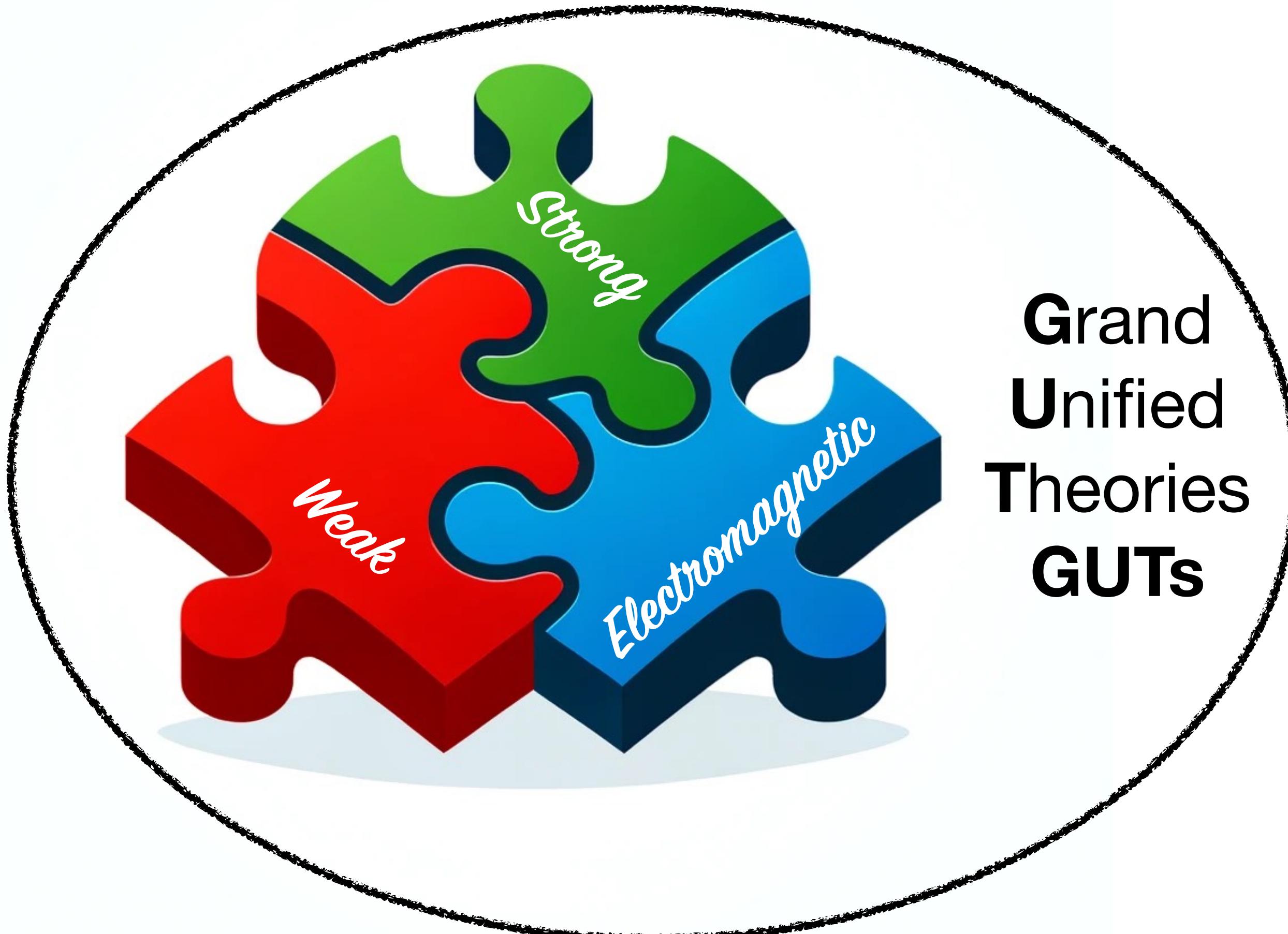
- Postdoc at Fermilab



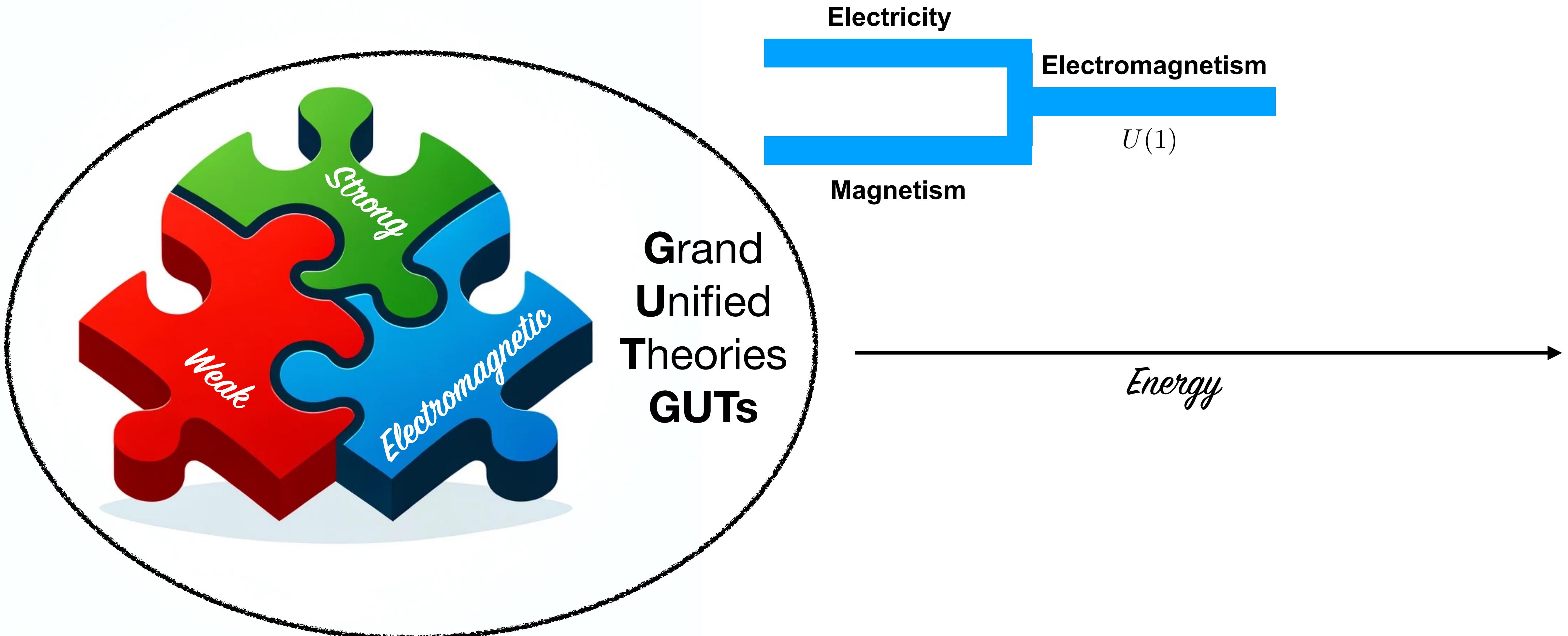
- Assistant Professor at Durham University since 2020



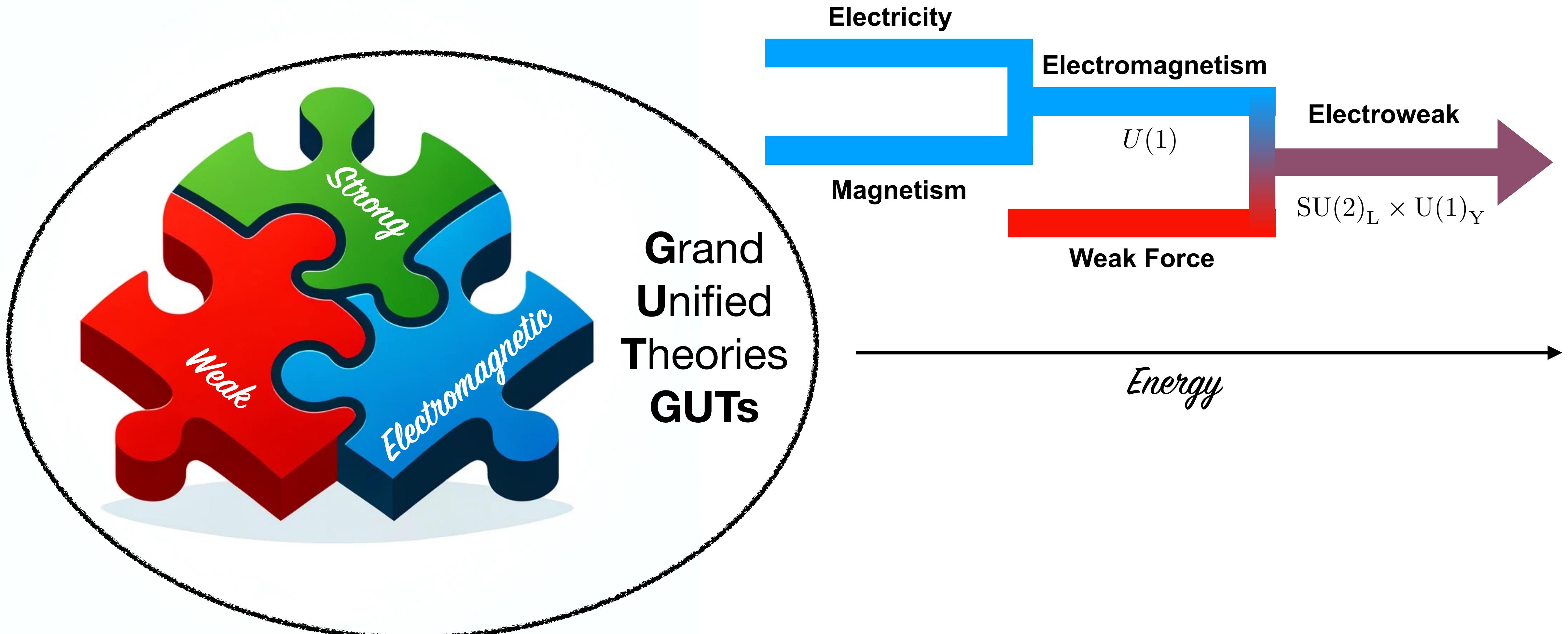
Motivation for Grand Unification



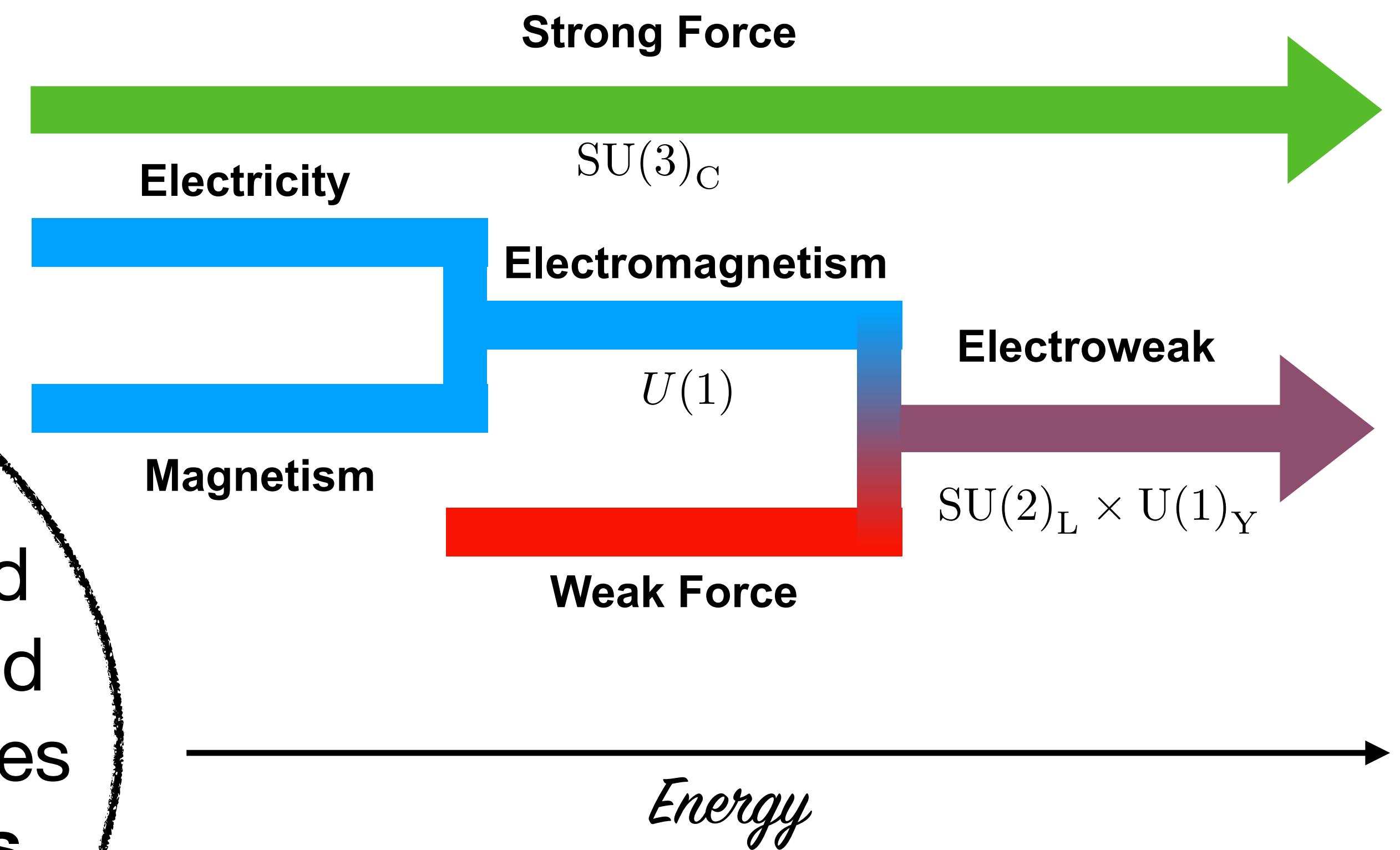
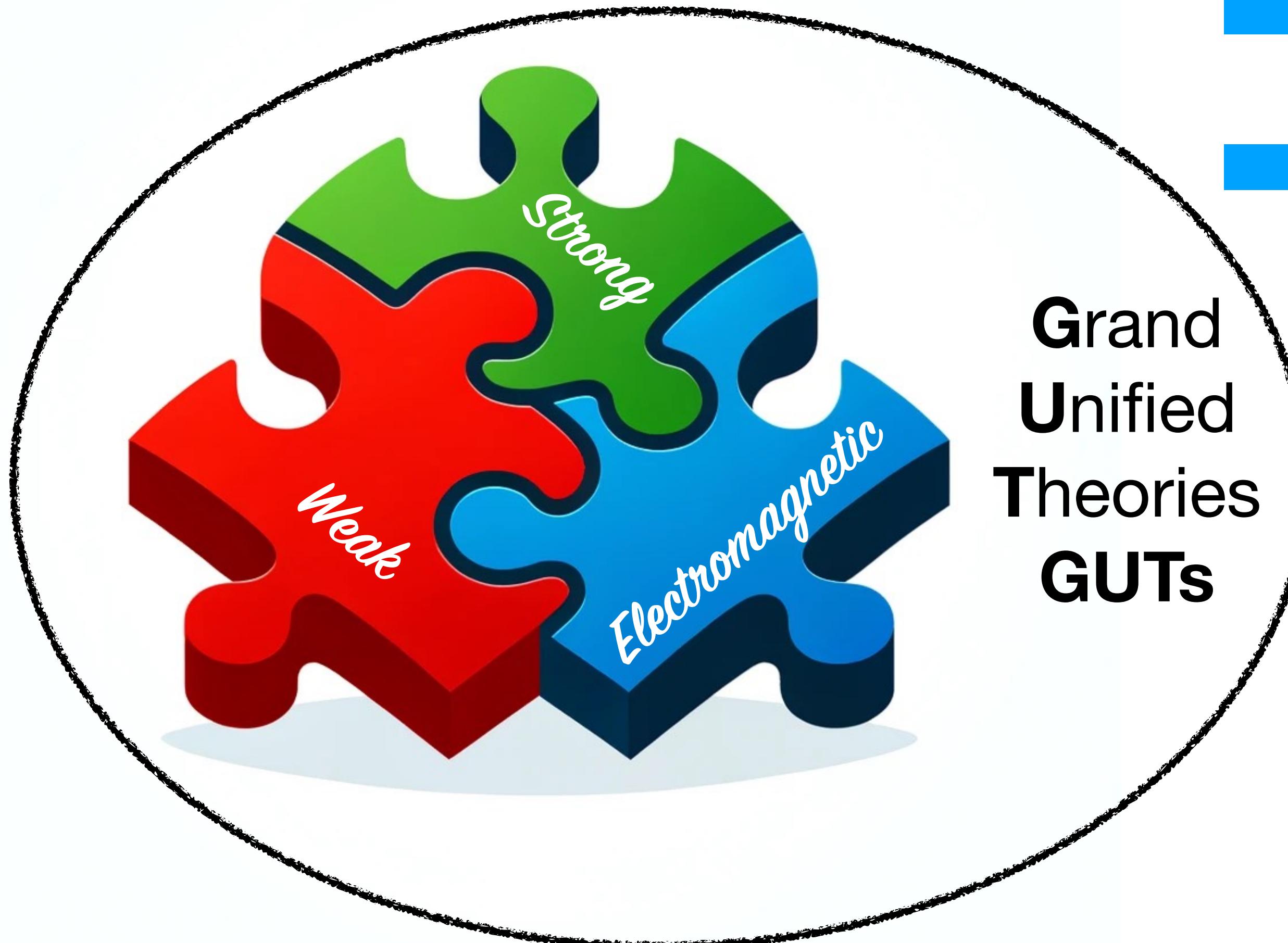
Motivation for Grand Unification



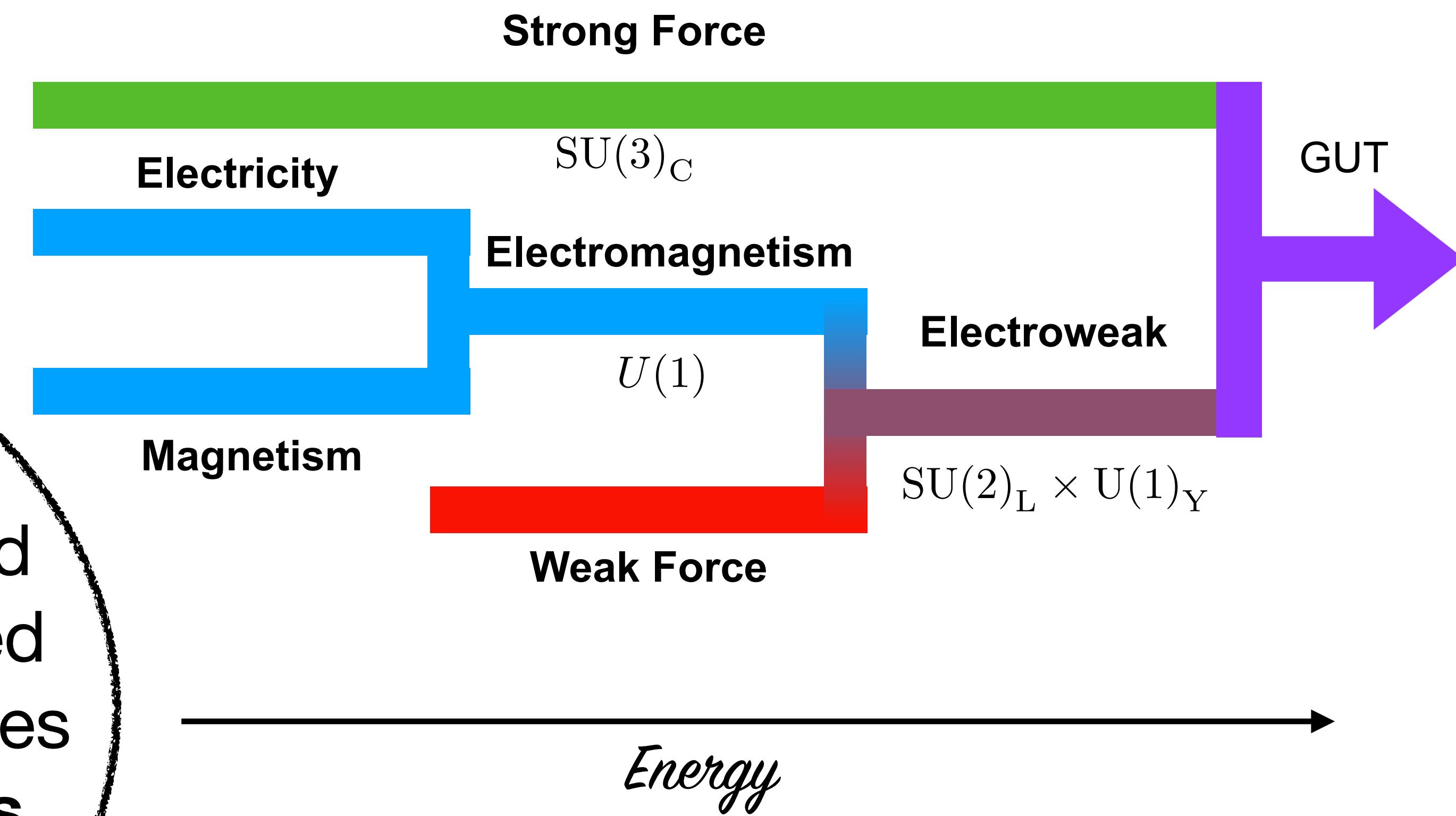
Motivation for Grand Unification



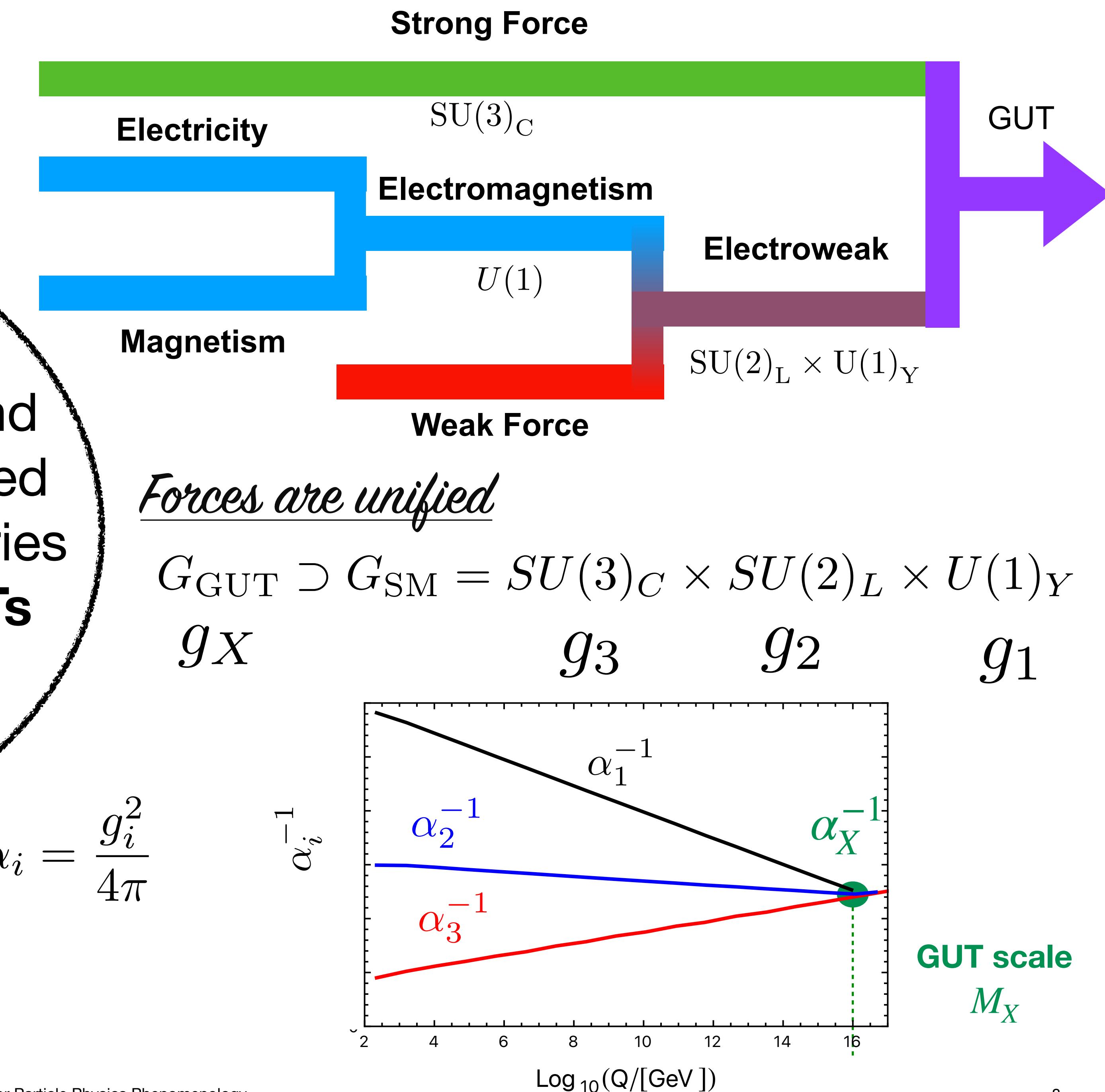
Motivation for Grand Unification



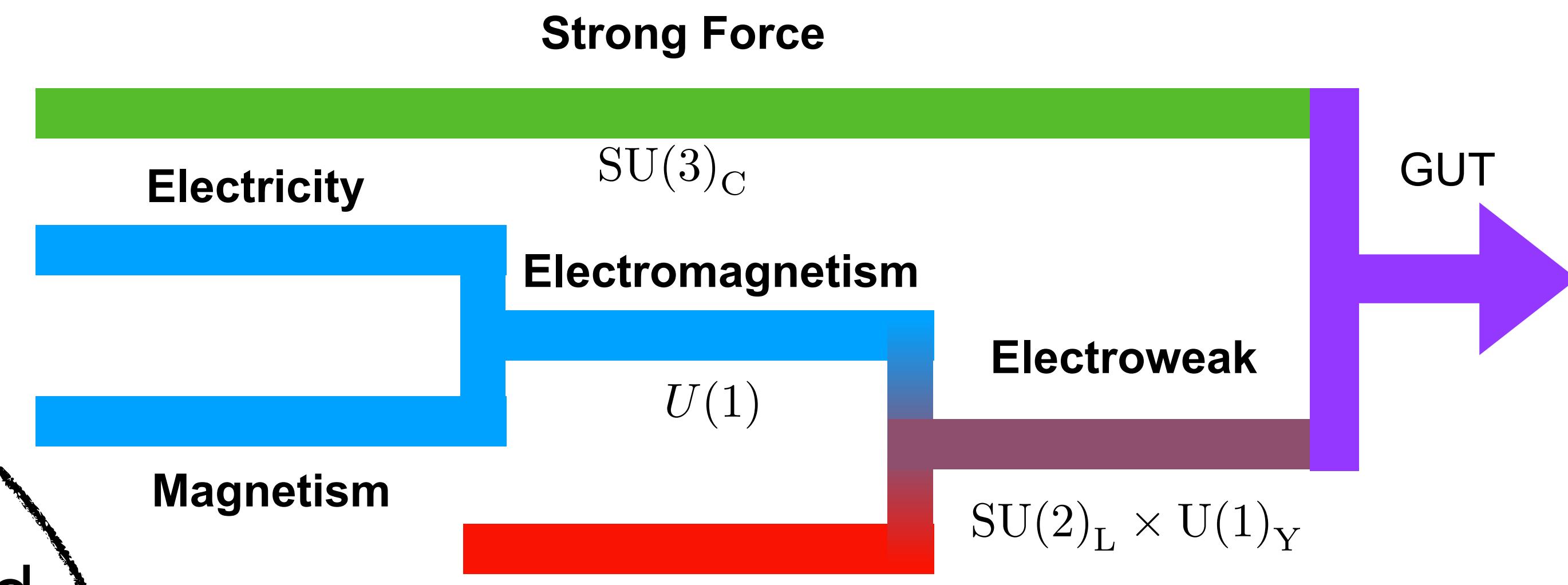
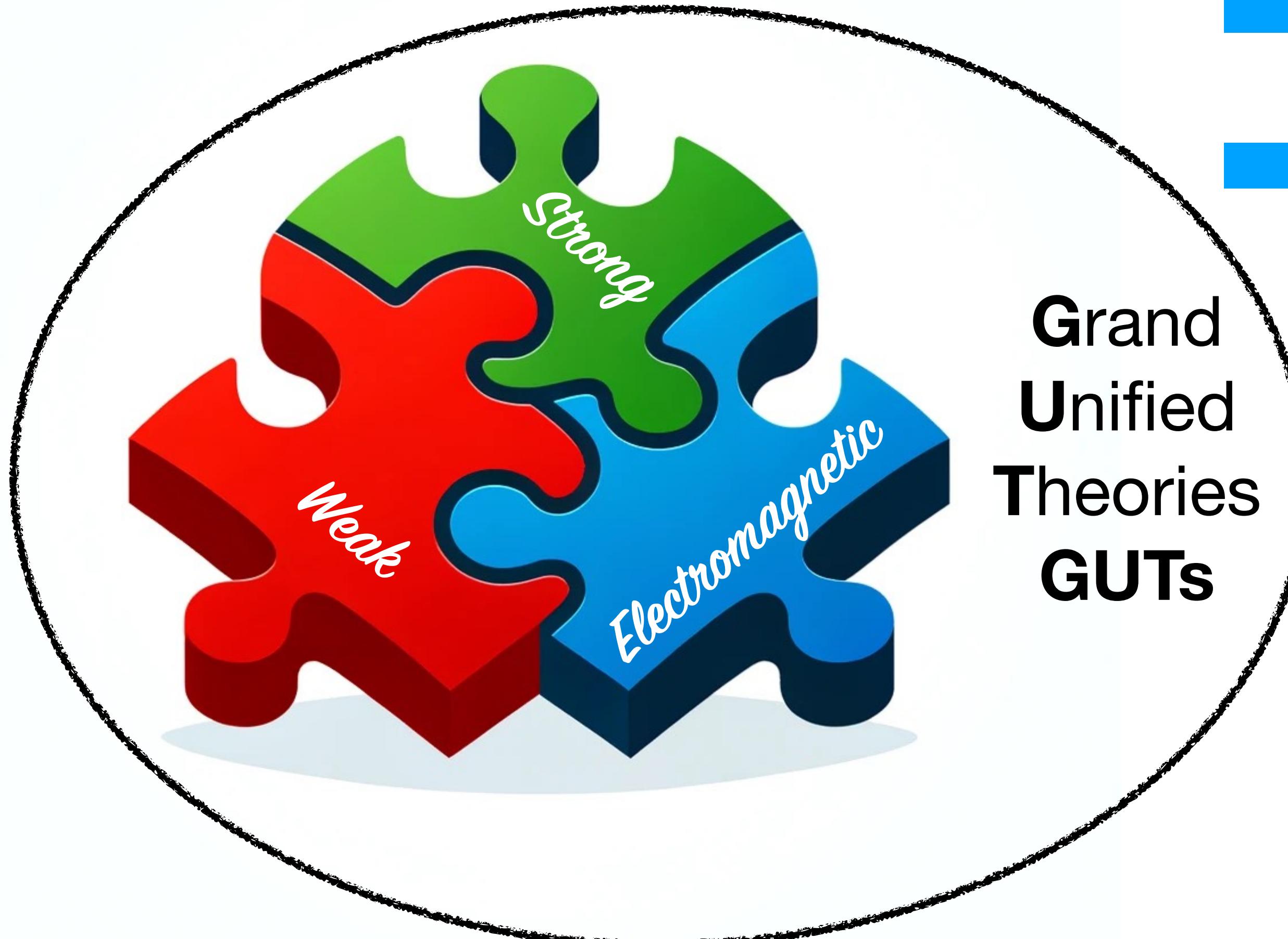
Motivation for Grand Unification



Motivation for Grand Unification



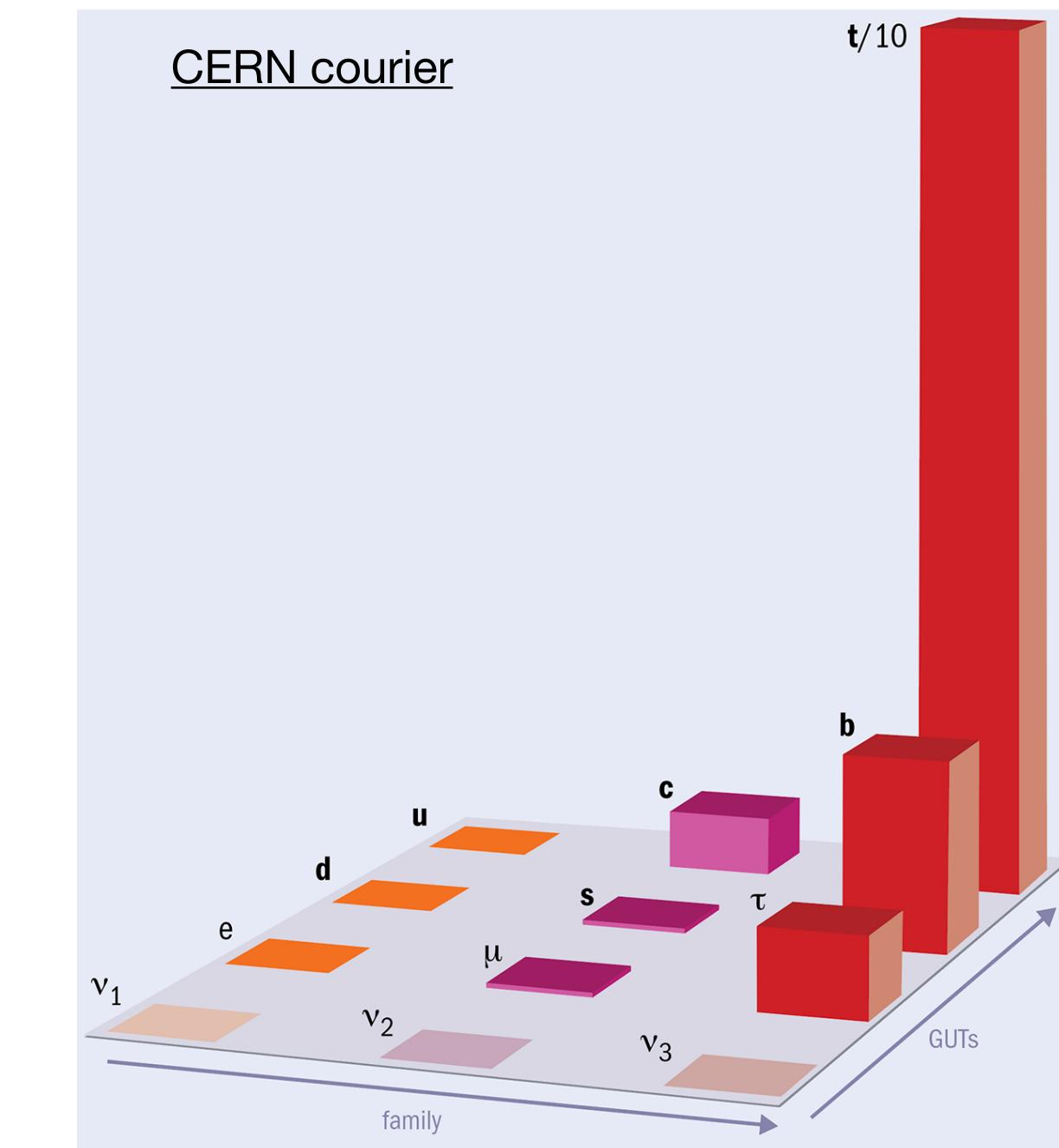
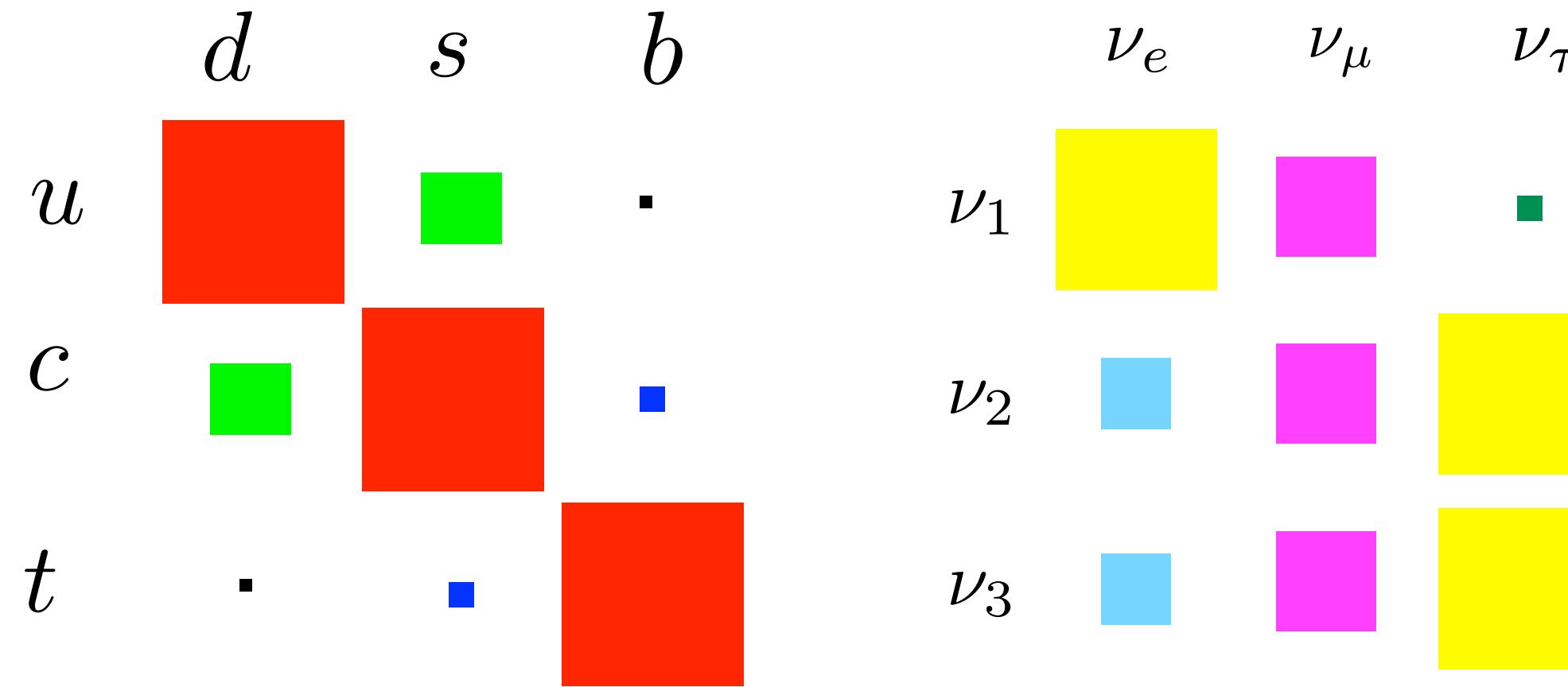
Motivation for Grand Unification



$$\bar{5} : \begin{pmatrix} d_r \\ d_g \\ d_b \\ e \\ \nu_e \end{pmatrix}$$

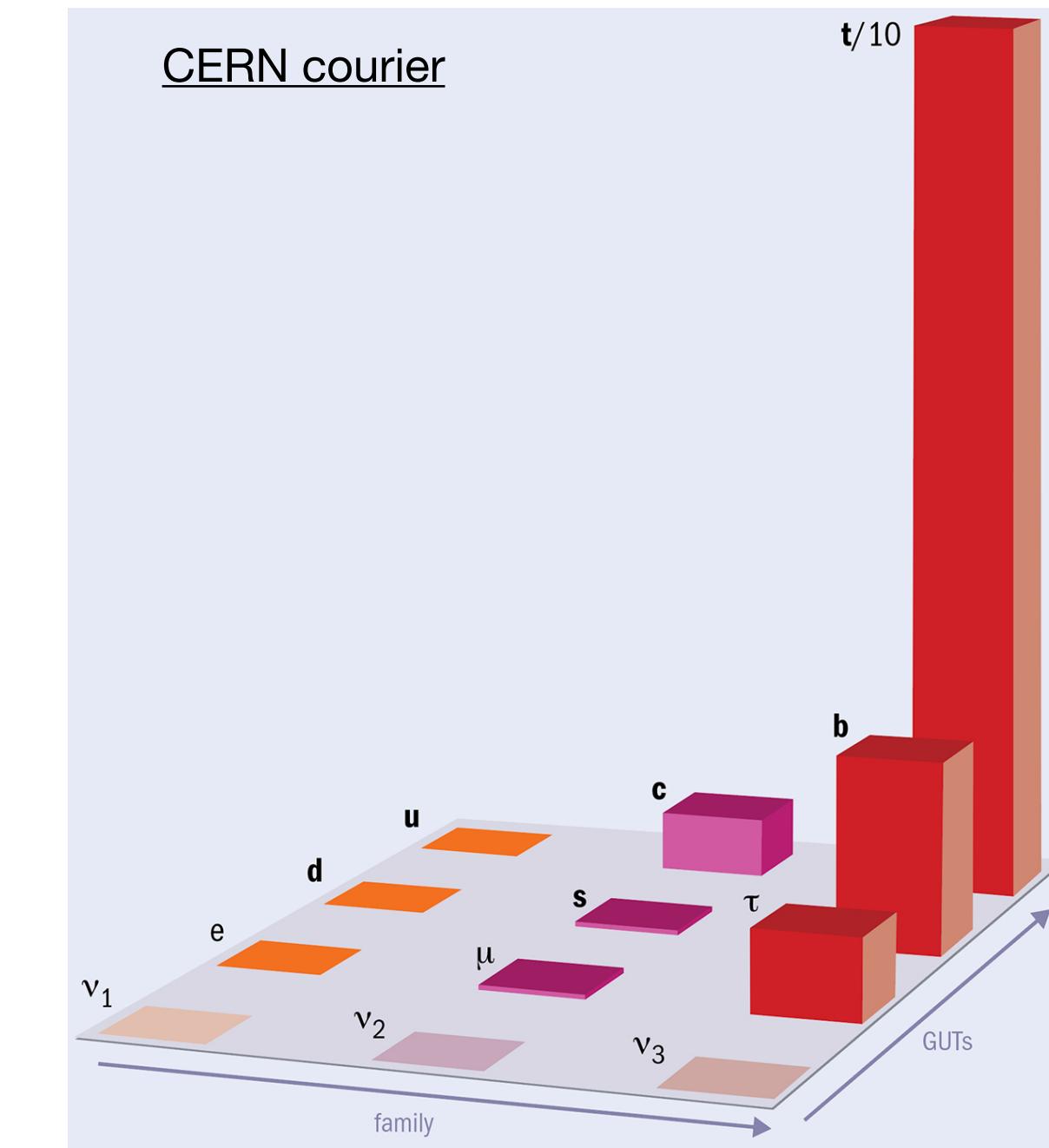
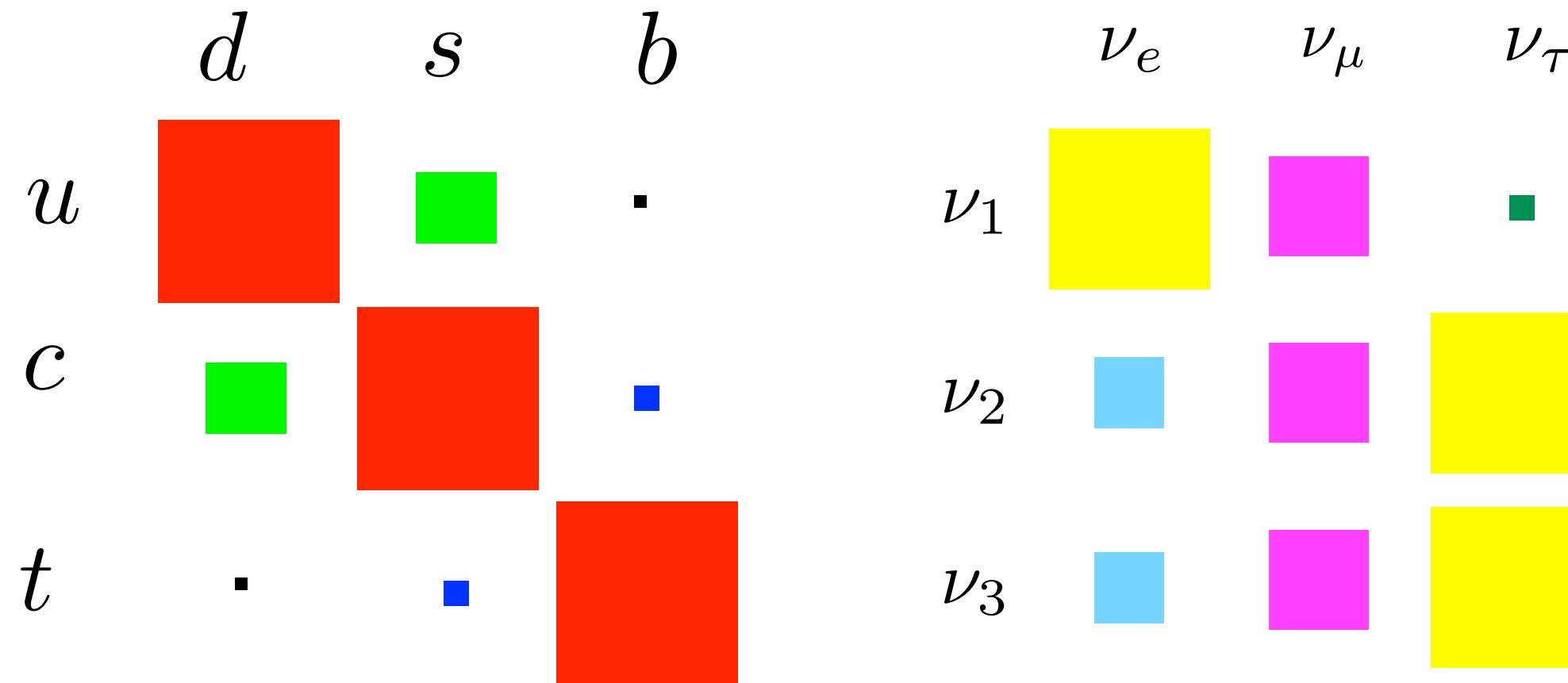
Motivation for Grand Unification

Flavour Puzzle

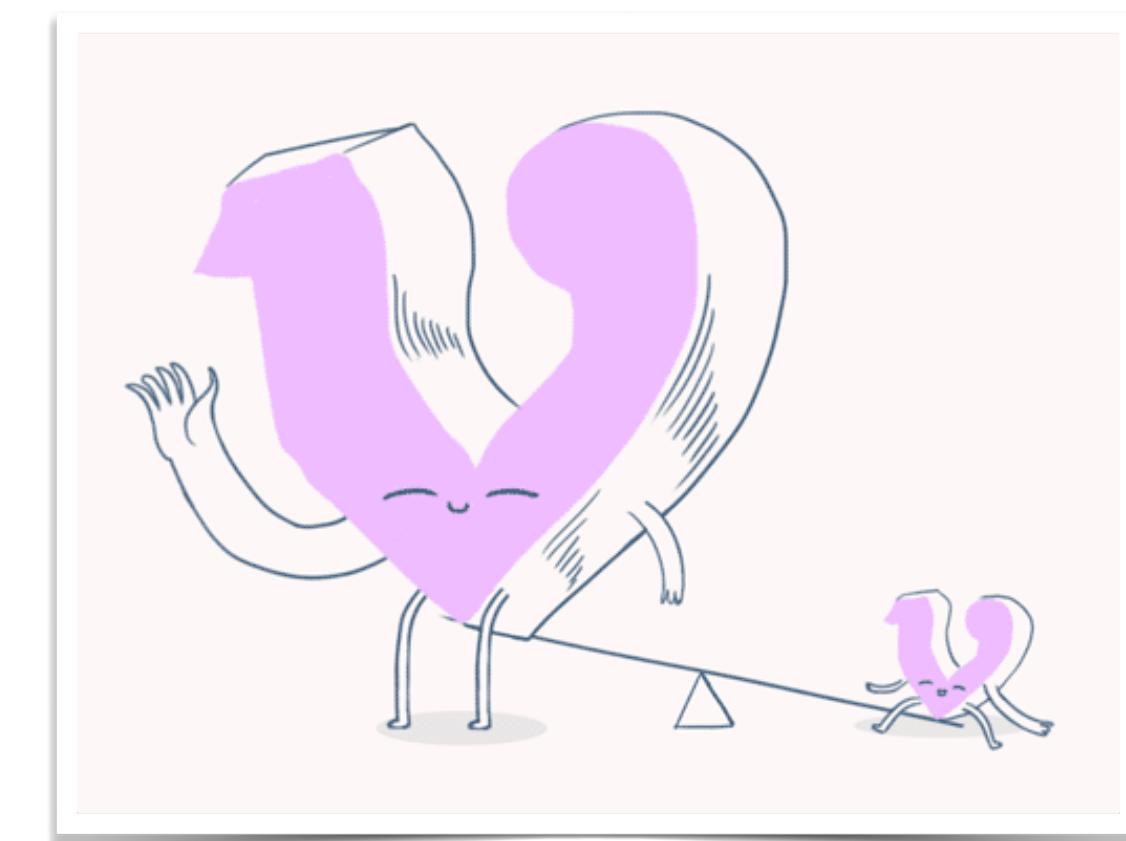


Motivation for Grand Unification

Flavour Puzzle

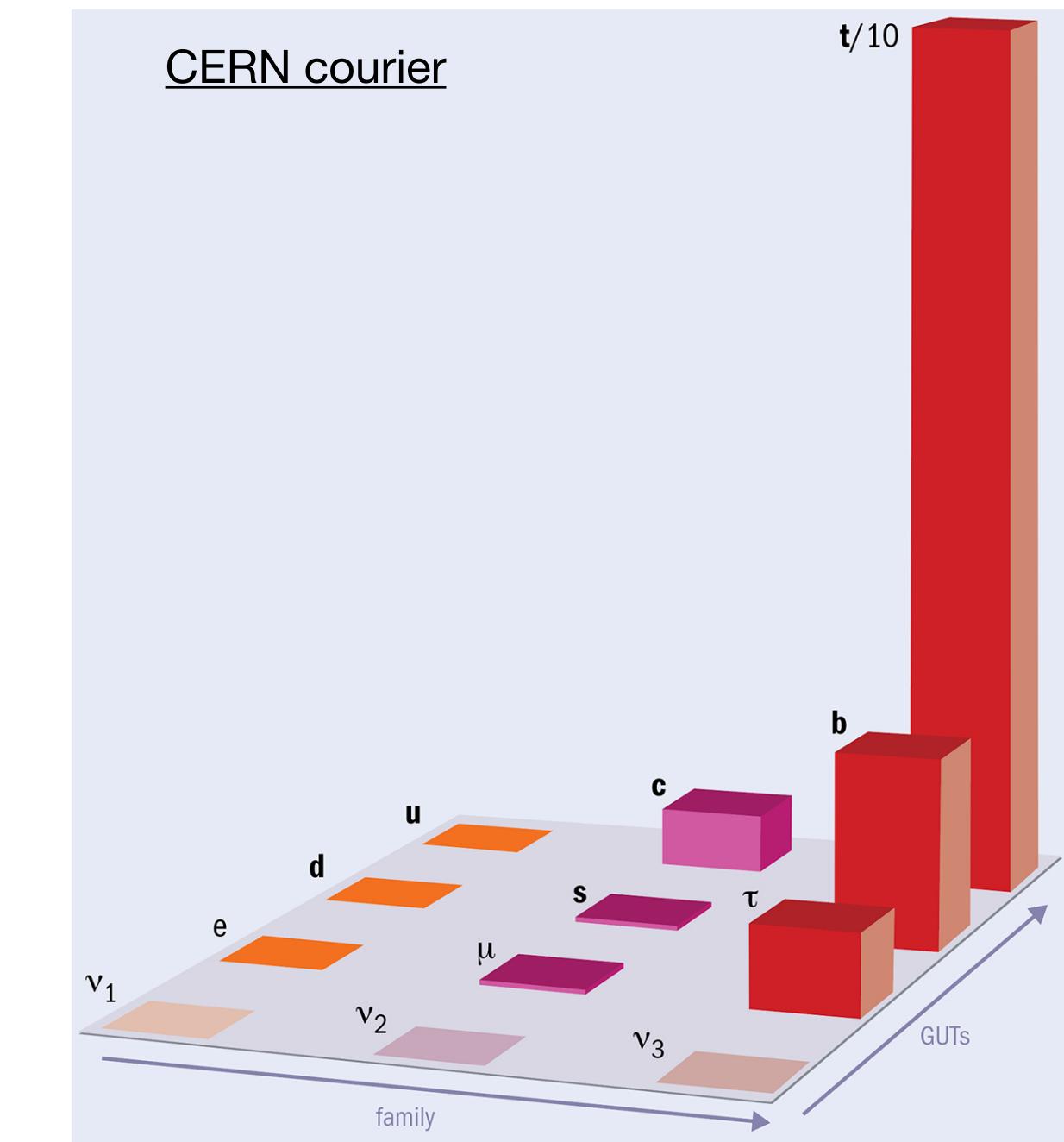
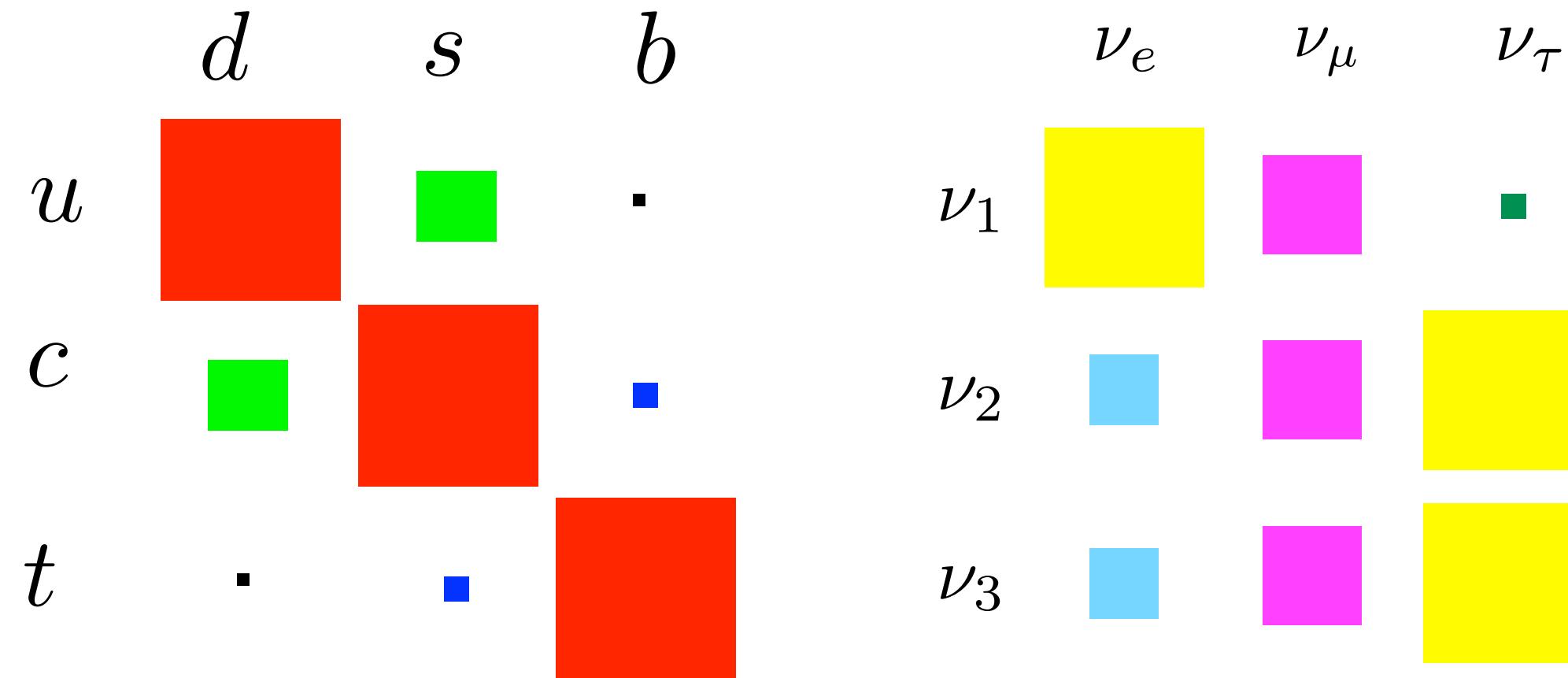


Neutrino Masses



Motivation for Grand Unification

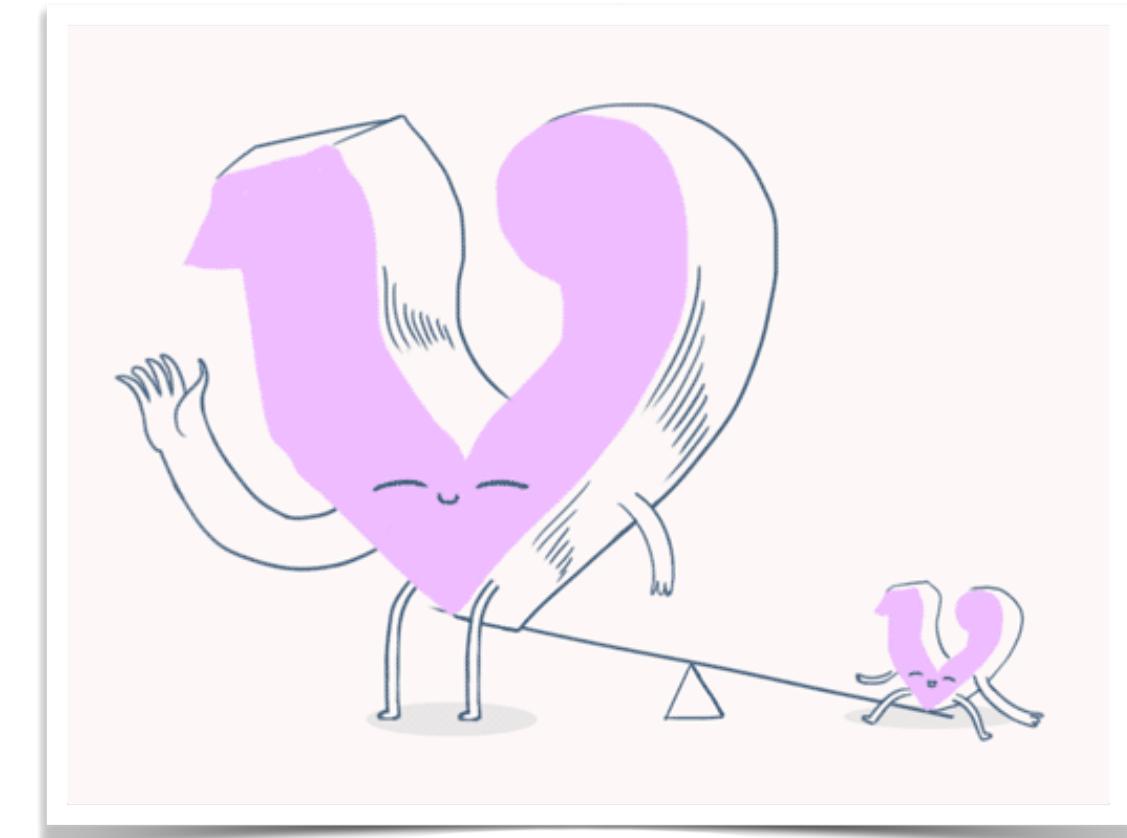
Flavour Puzzle



Matter-Antimatter Asymmetry

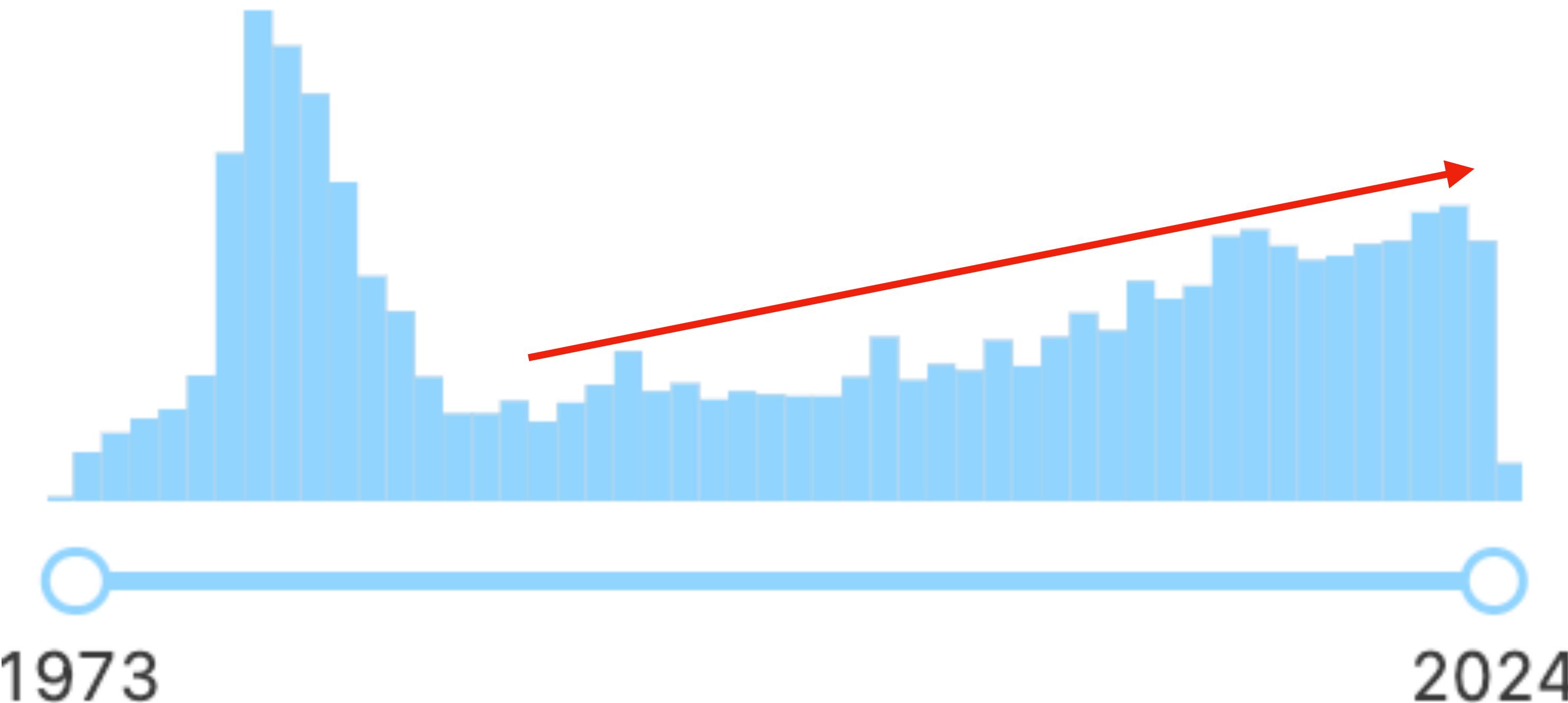


Neutrino Masses



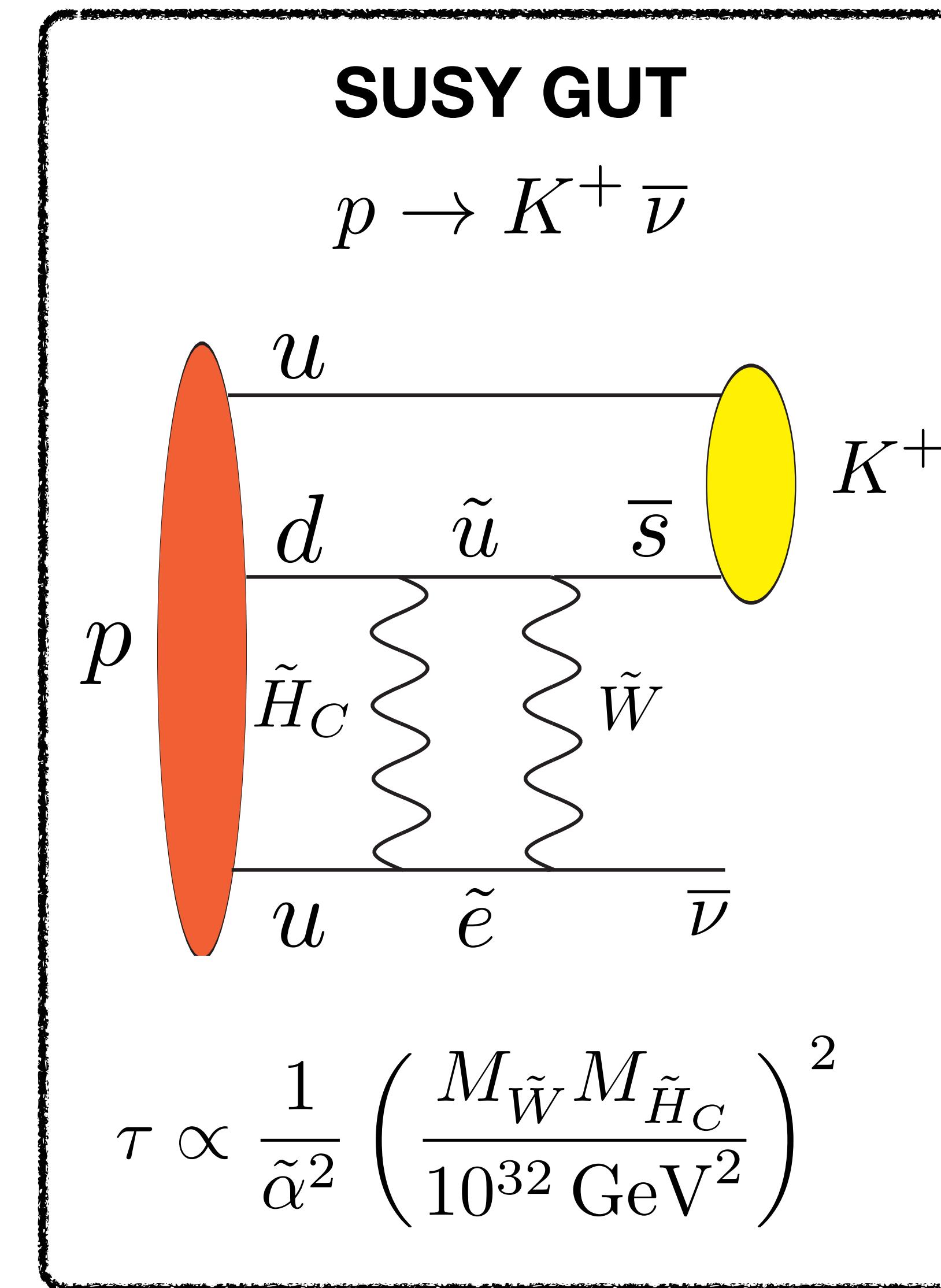
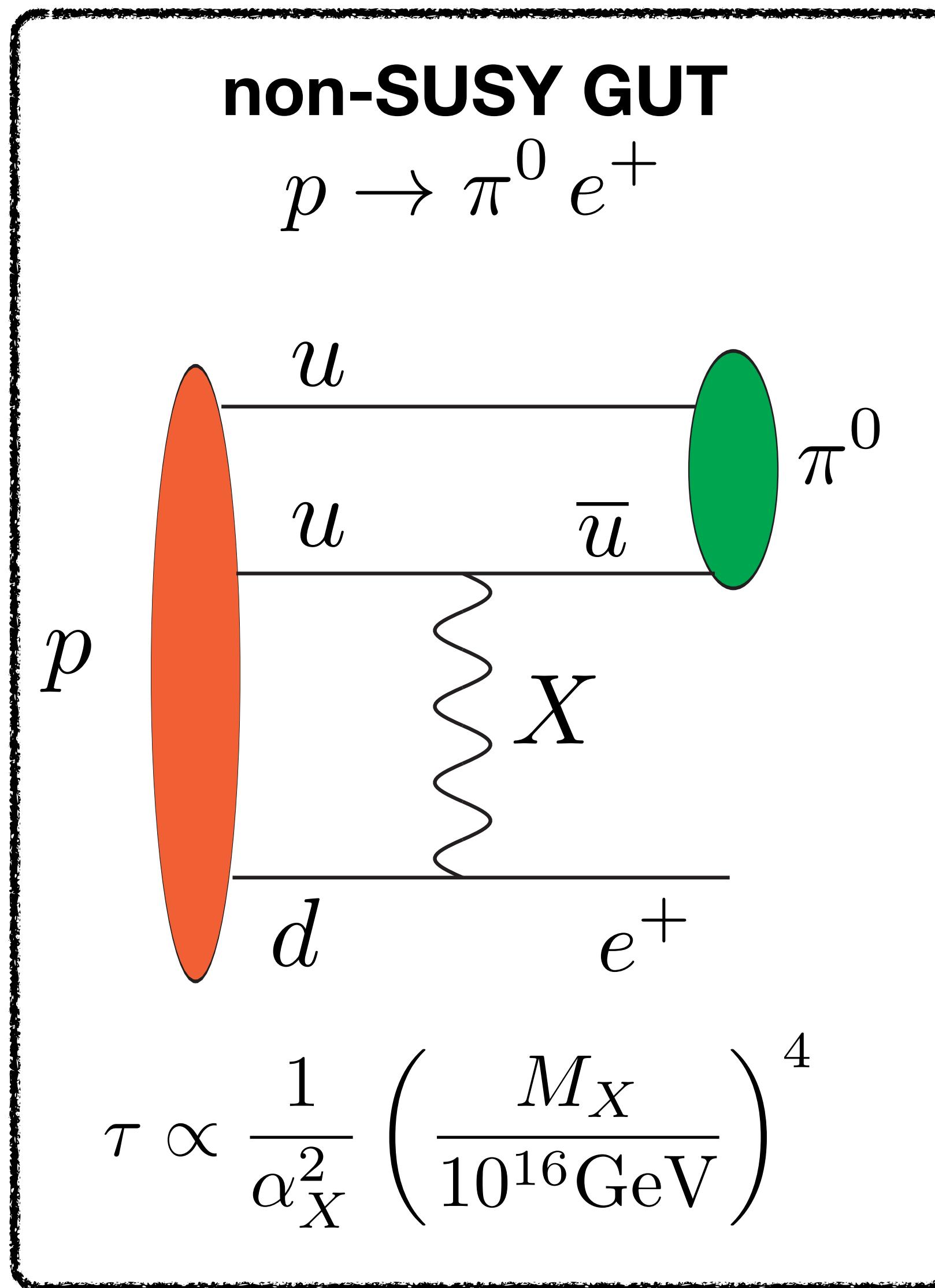
Motivation for Grand Unification

Georgi & Glashow 1974



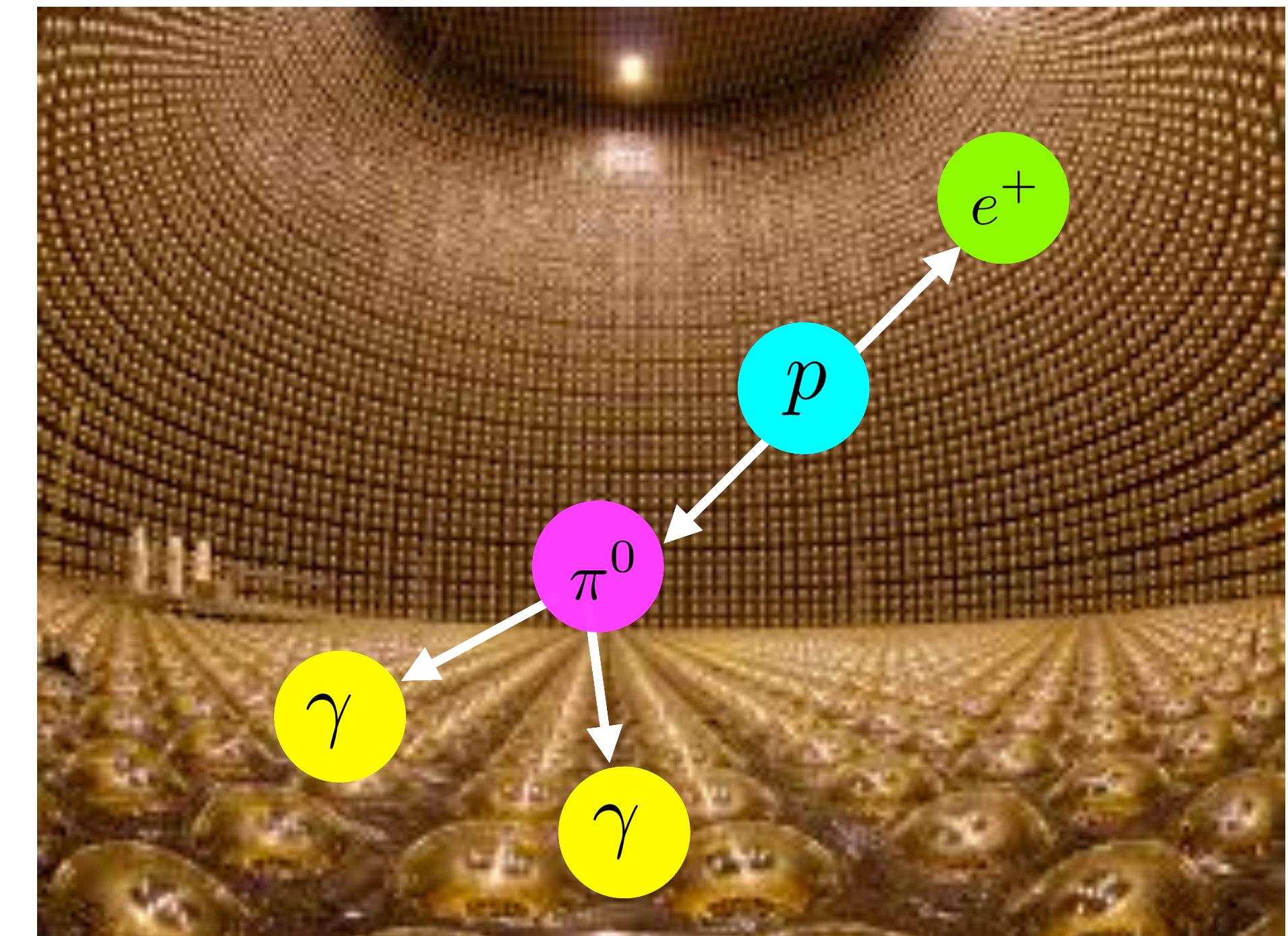
Reason 1: Proton Decay from GUTs

- GUTs unify leptons and quarks into common multiplets \Rightarrow B & L not conserved

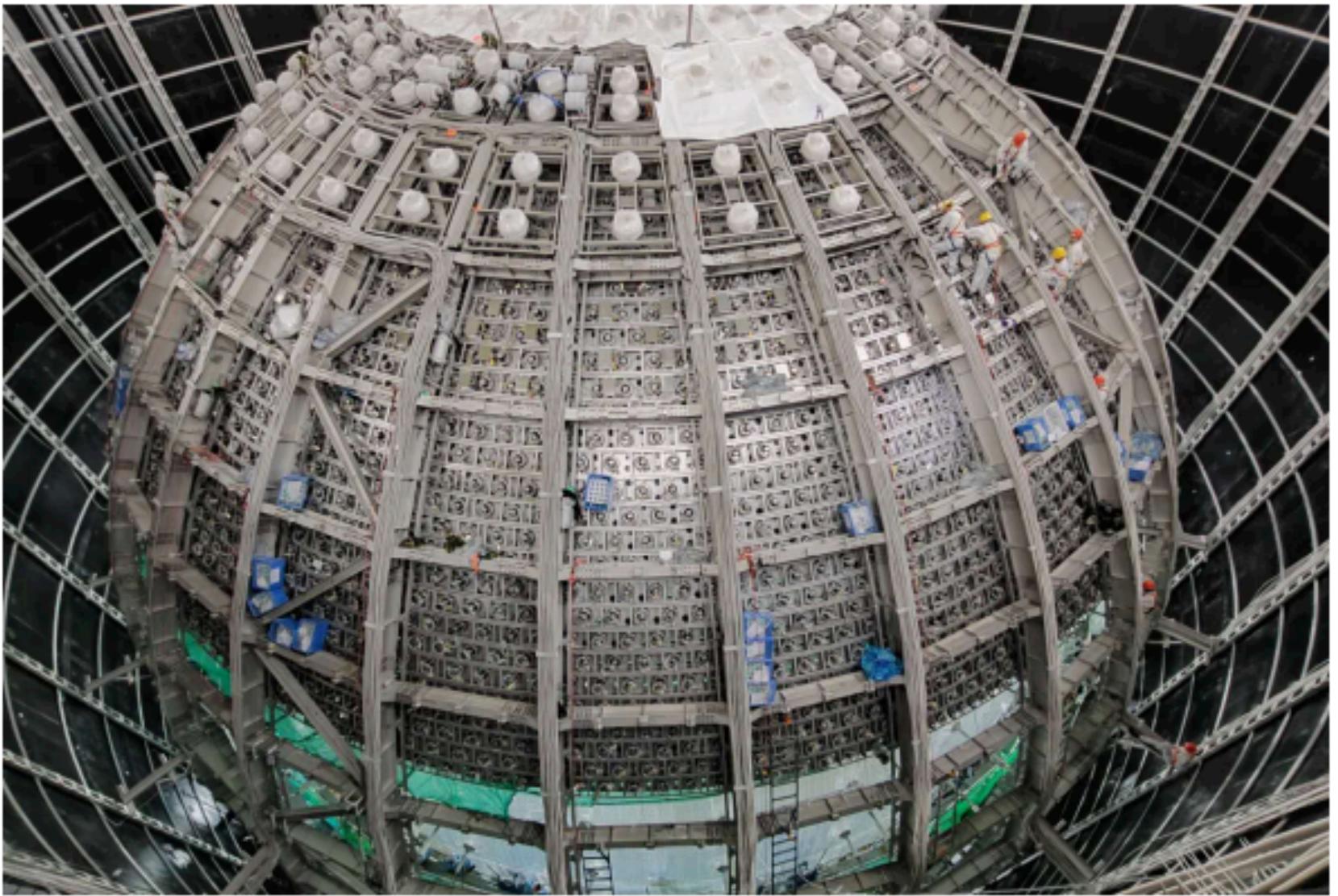


Leading Limits on Proton Decay

- Kamiokande was designed to search for proton decay (1983)
- 3 kt water $\sim 10^{32}$ nucleons, no proton decay observed
 $\Rightarrow \tau(p \rightarrow \pi^0 e^+) > 10^{32}$ years ruled out minimal $SU(5)$ GUT
- Upgrade to Super-Kamiokande (50 kt) in 1996
 $\tau(p \rightarrow \pi^0 e^+) > 2.4 \times 10^{34}$ years
- Upgrade to Hyper-Kamiokande will push the proton lifetime limit to $\tau(p \rightarrow \pi^0 e^+) > 10^{35}$ years



Reason 1: Proton Decay from GUTs

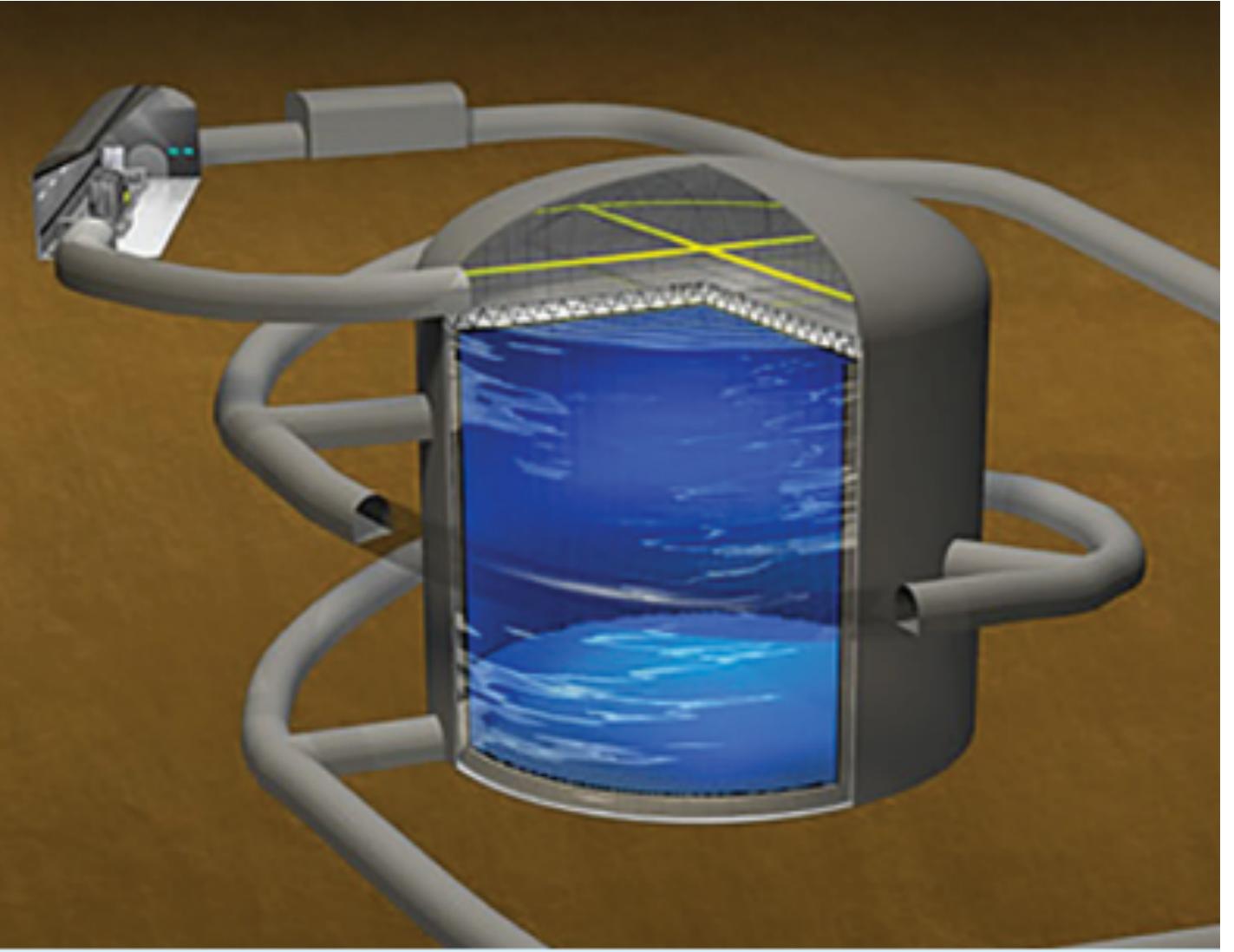


JUNO, data taking end this year

20 kiloton $\sim 7 \times 10^{33}$ protons

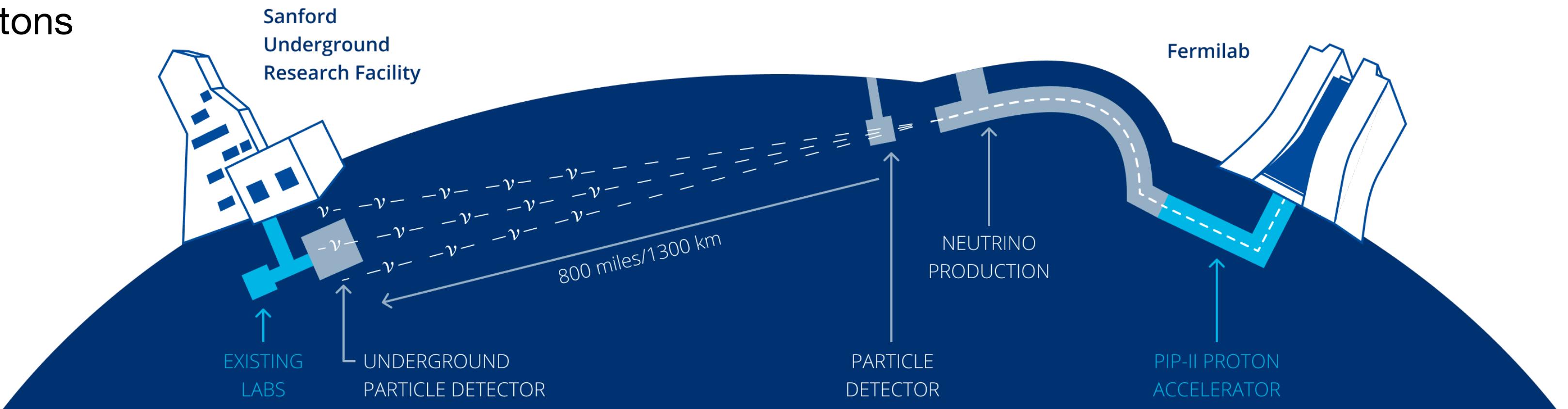
DUNE
2030(ish) expected data taking

40 kiloton $\sim 10^{34}$ protons

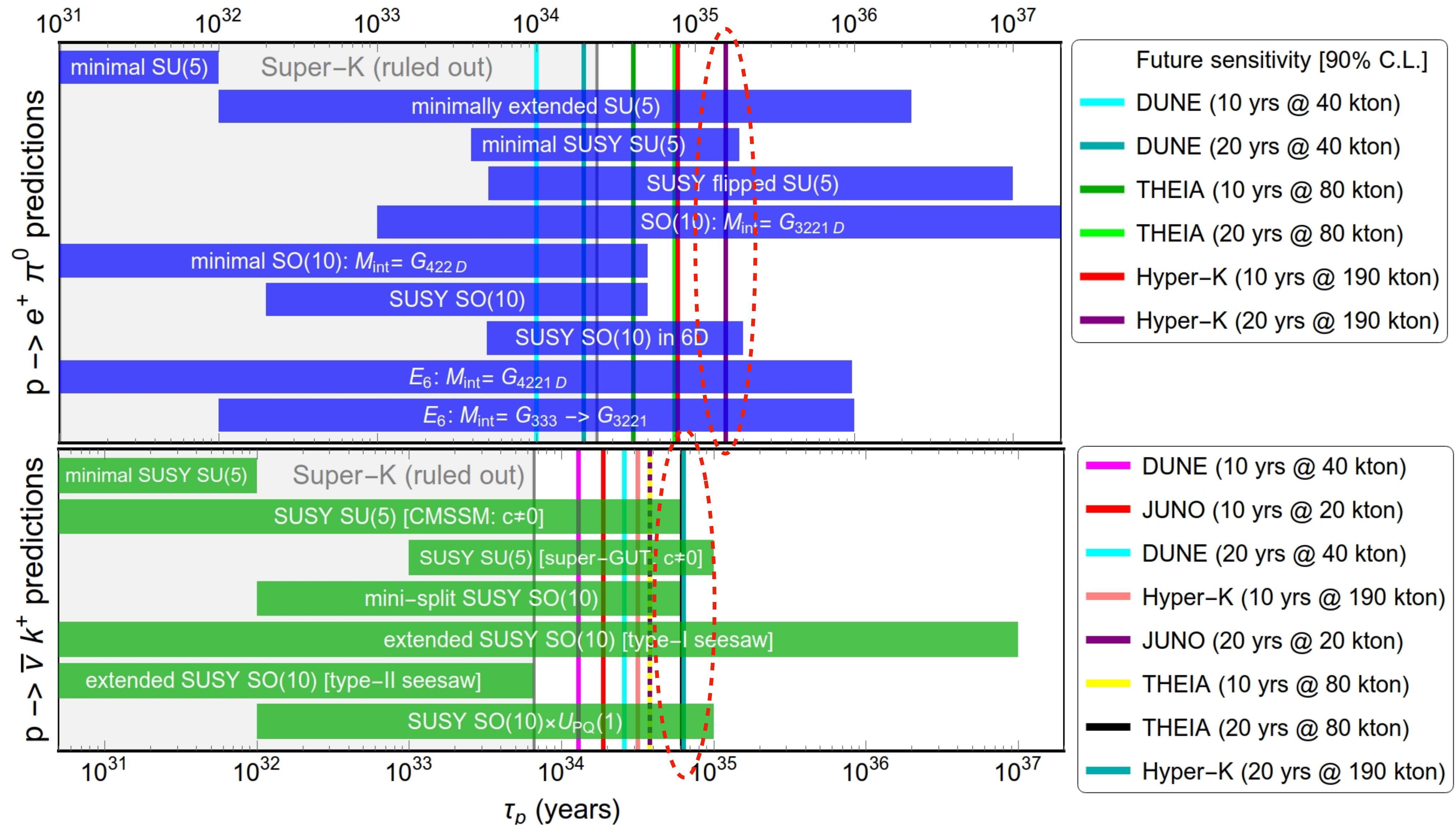


Hyper-Kamiokande
2027 expected data taking

188 kiloton $\sim 7 \times 10^{34}$ protons

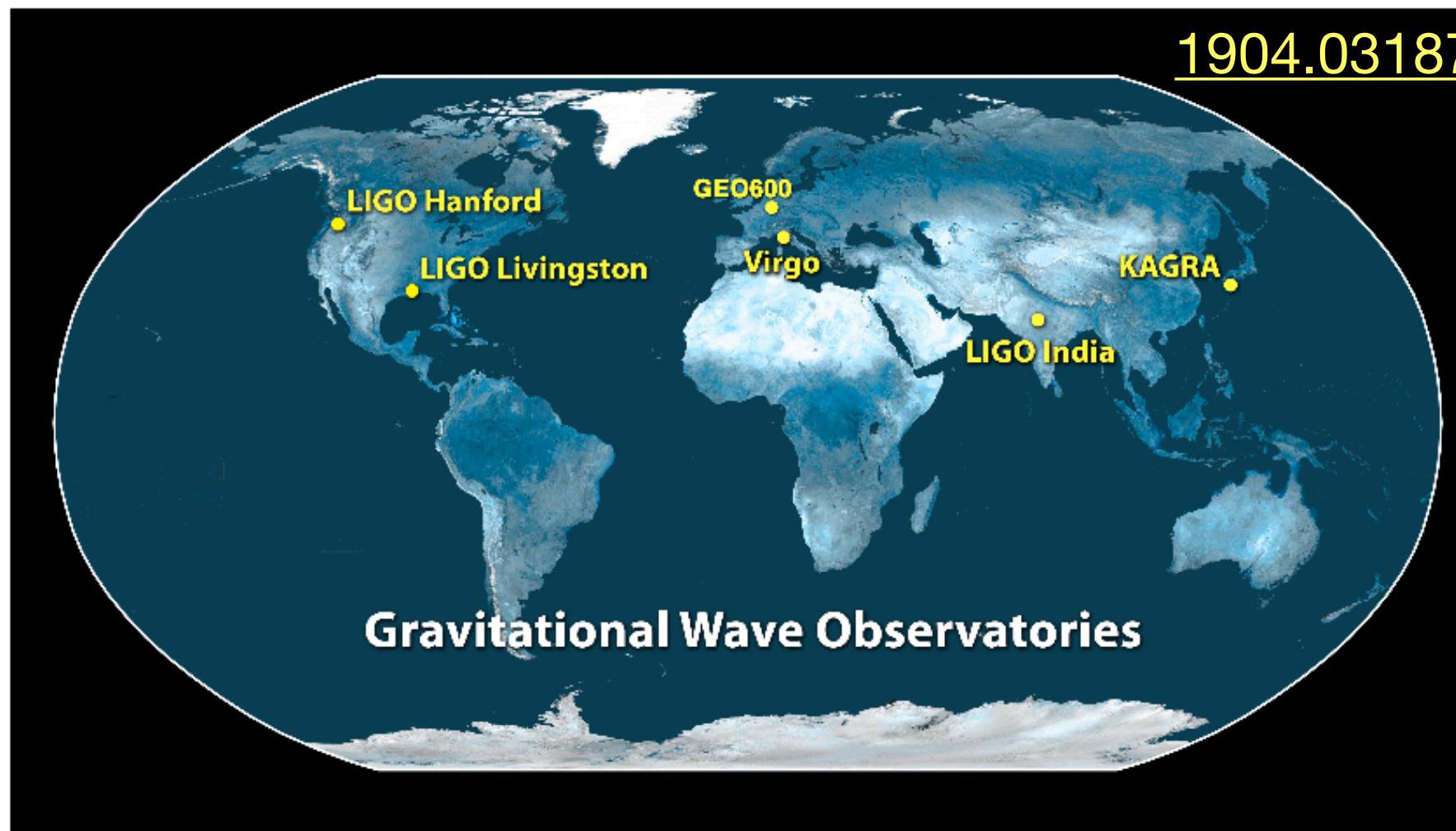


Searches for Baryon Number Violation in Neutrino Experiments: A White Paper



Reason 2: Gravitational waves from GUTs

Ground Based Interferometers

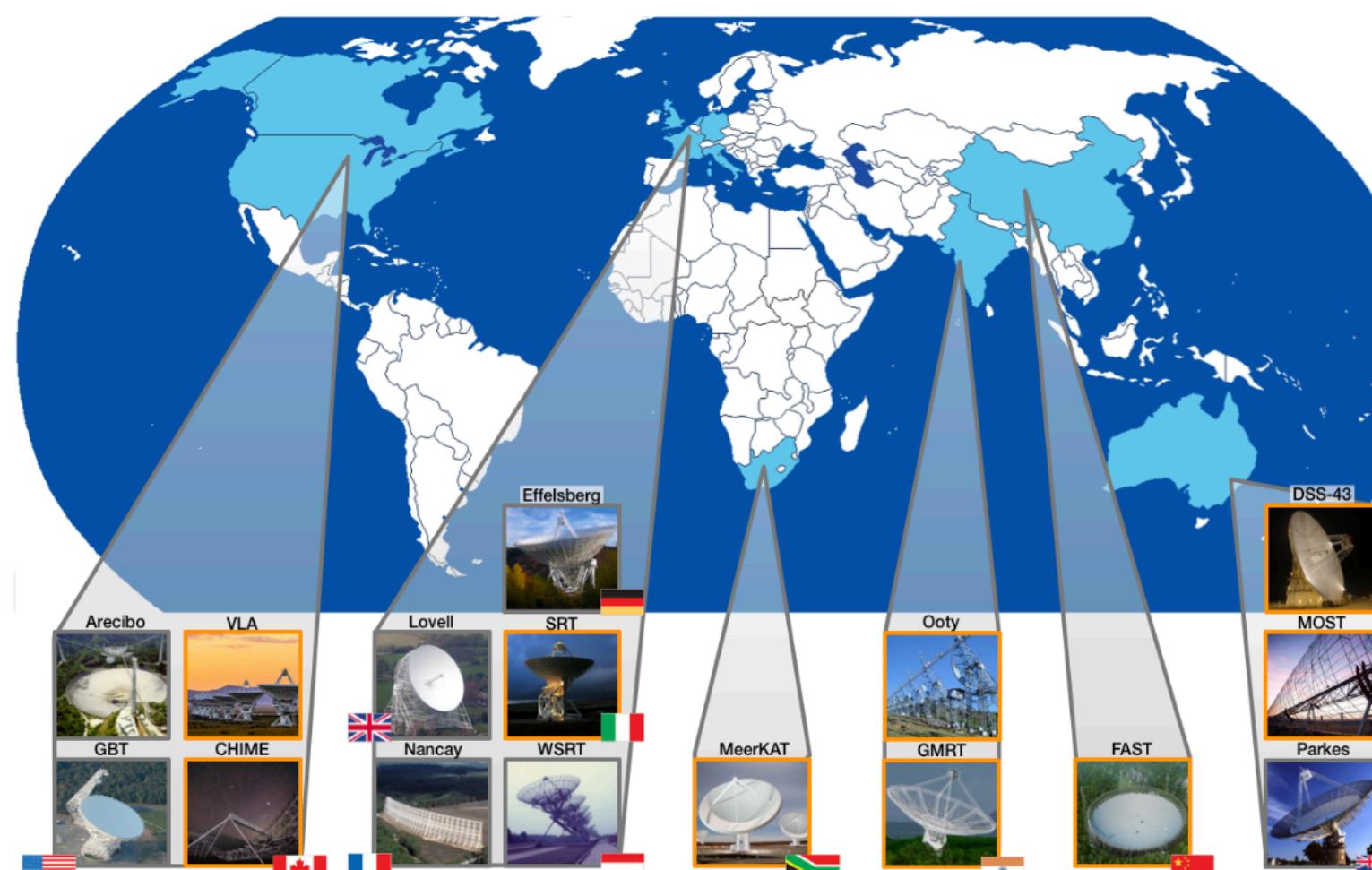


$$\mathcal{O}(1 - 10^3) \text{ Hz}$$

June 2023

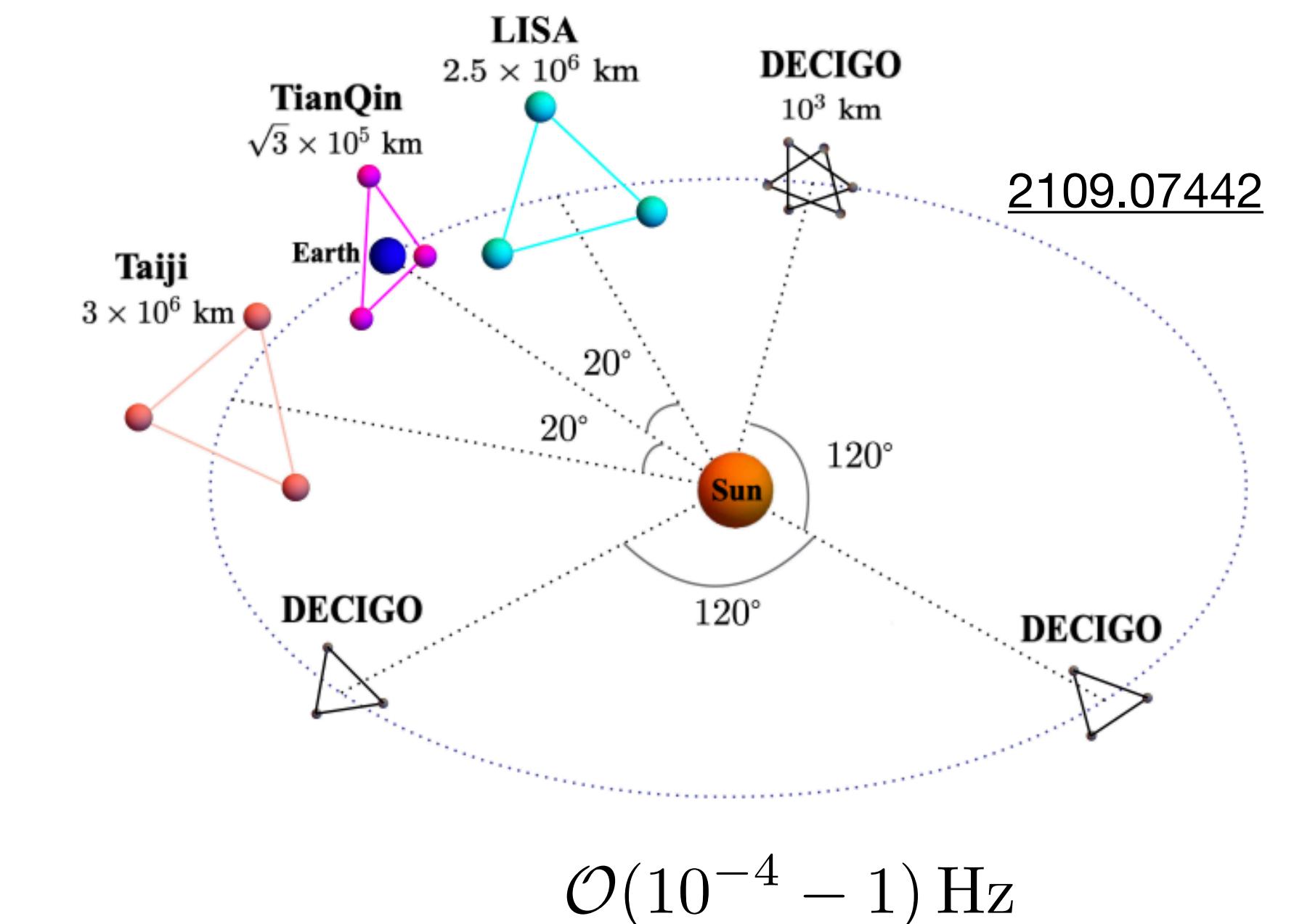
NANOGrav 15 year dataset, European Pulsar Timing Array
Parkes Pulsar Timing Array, Chinese Pulsar Timing Array
has evidence of gravitational wave in nanoHertz regime

Pulsar Timing Arrays



$$\mathcal{O}(10^{-9} - 10^{-6}) \text{ Hz}$$

Space Based Interferometers

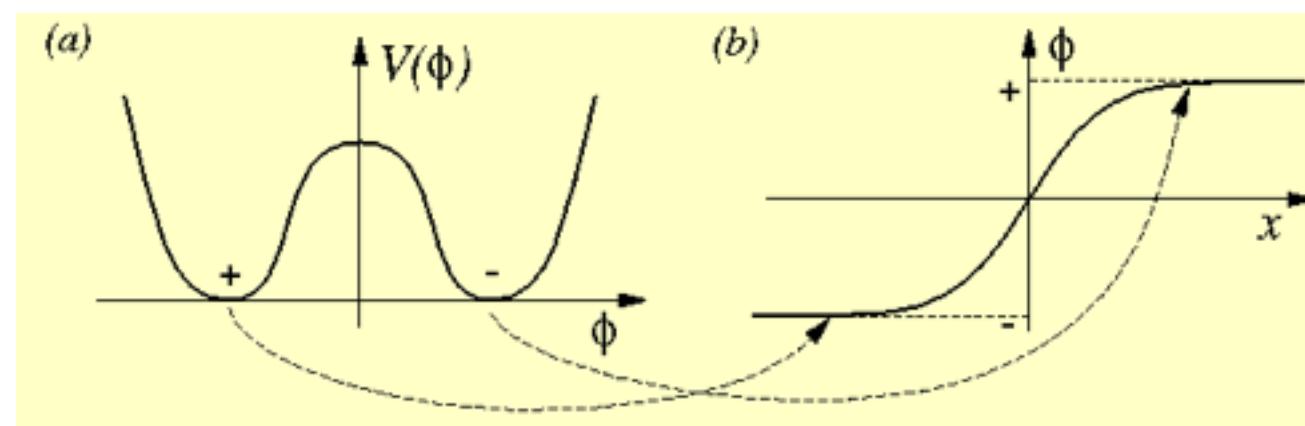


GUTs prediction: topological defects

During SSB from $G_{GUT} \rightarrow \dots \rightarrow G_{SM}$ topological defects may form.

Monopoles

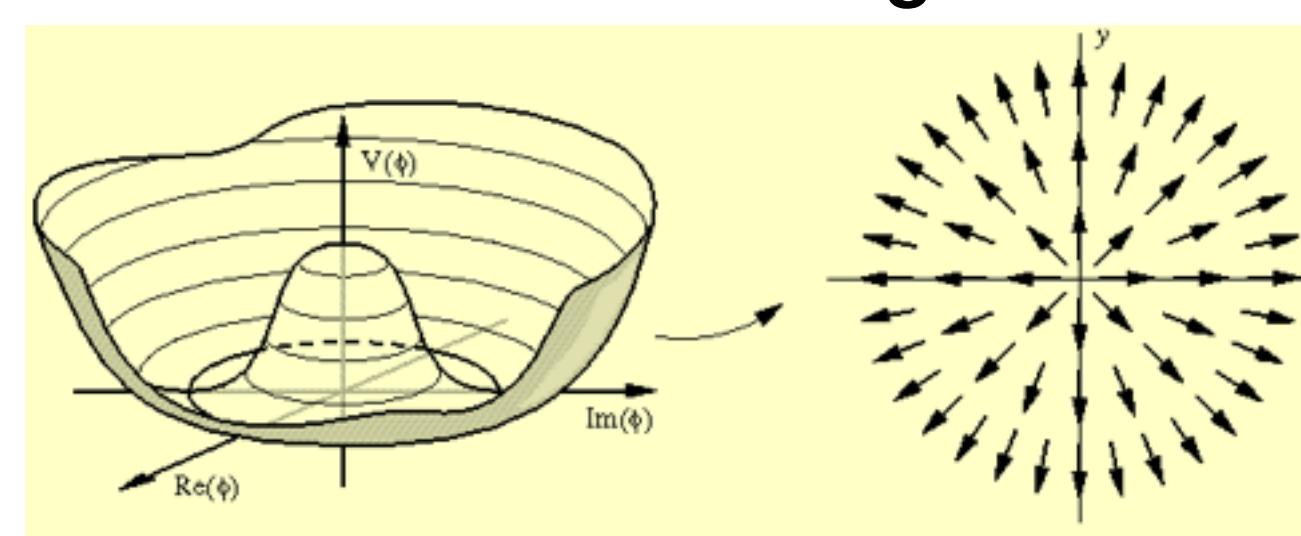
Domain wall



$$\pi_0(G/H) \neq 0$$

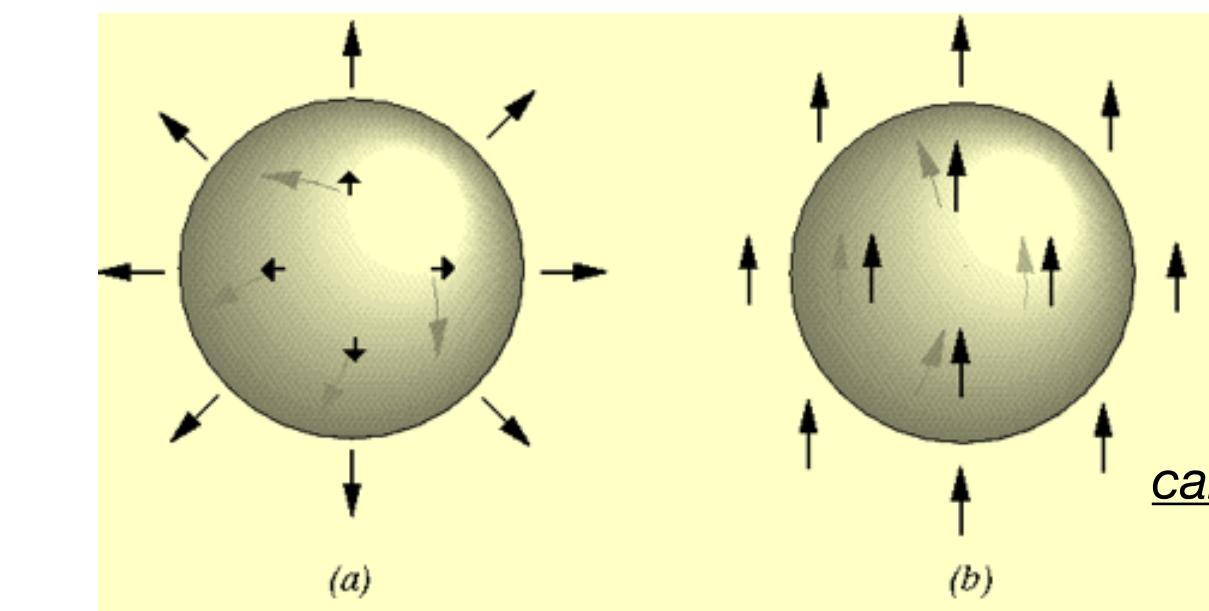
Disconnected

Cosmic strings



$$\pi_1(G/H) \neq 0$$

Non-contractible loop

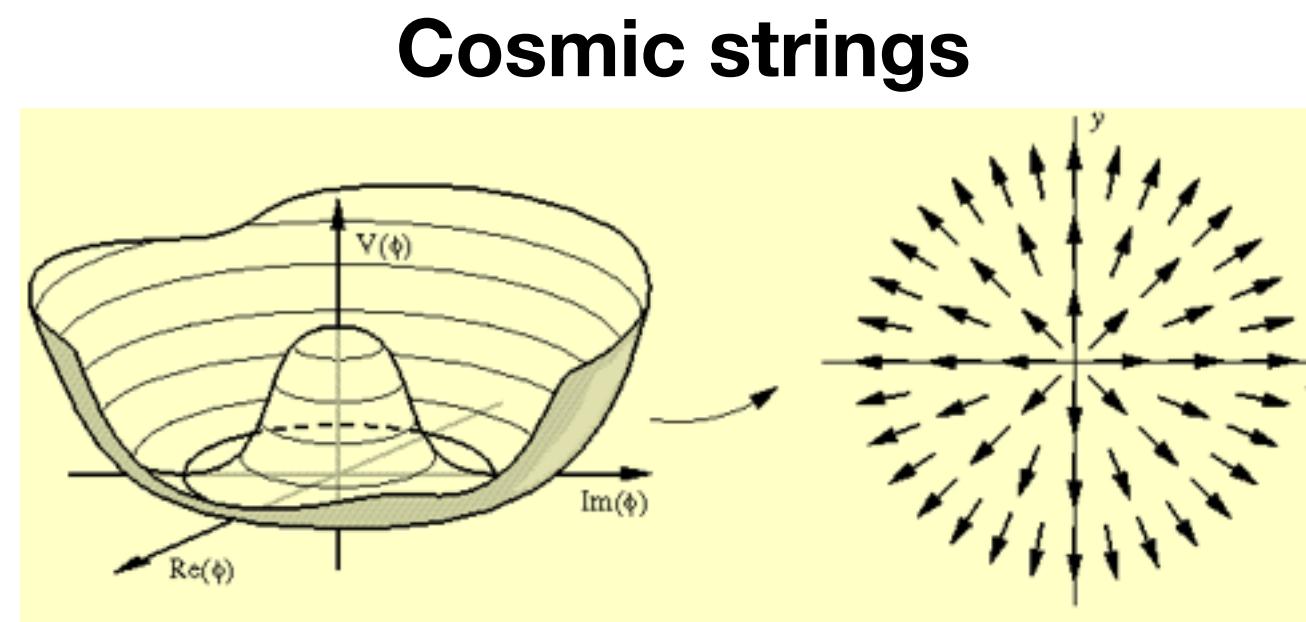
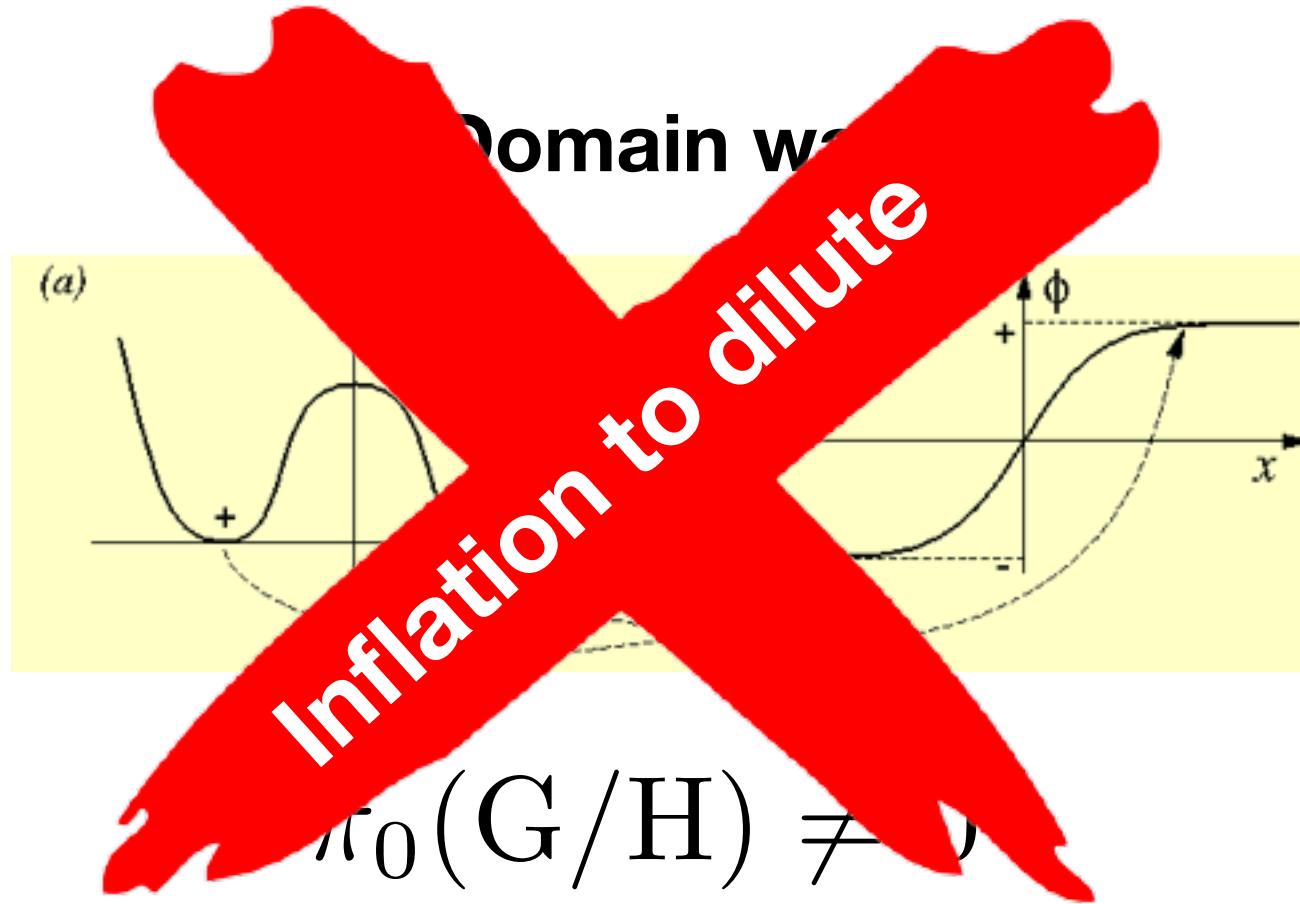


$$\pi_2(G/H) \neq 0$$

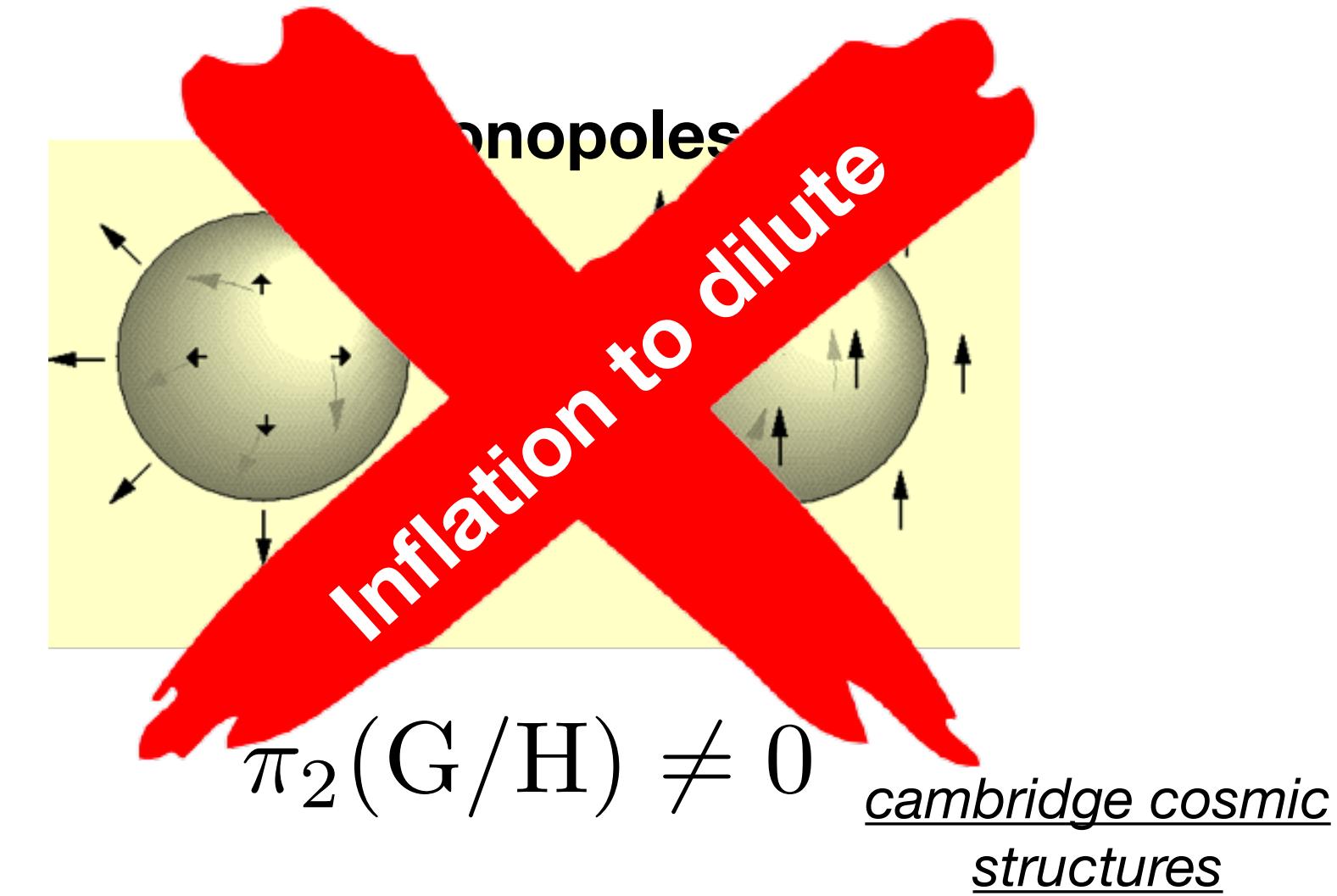
Two-sphere

GUTs prediction: topological defects

During SSB from $G_{GUT} \rightarrow \dots \rightarrow G_{SM}$ topological defects may form.



$$\pi_1(G/H) \neq 0$$

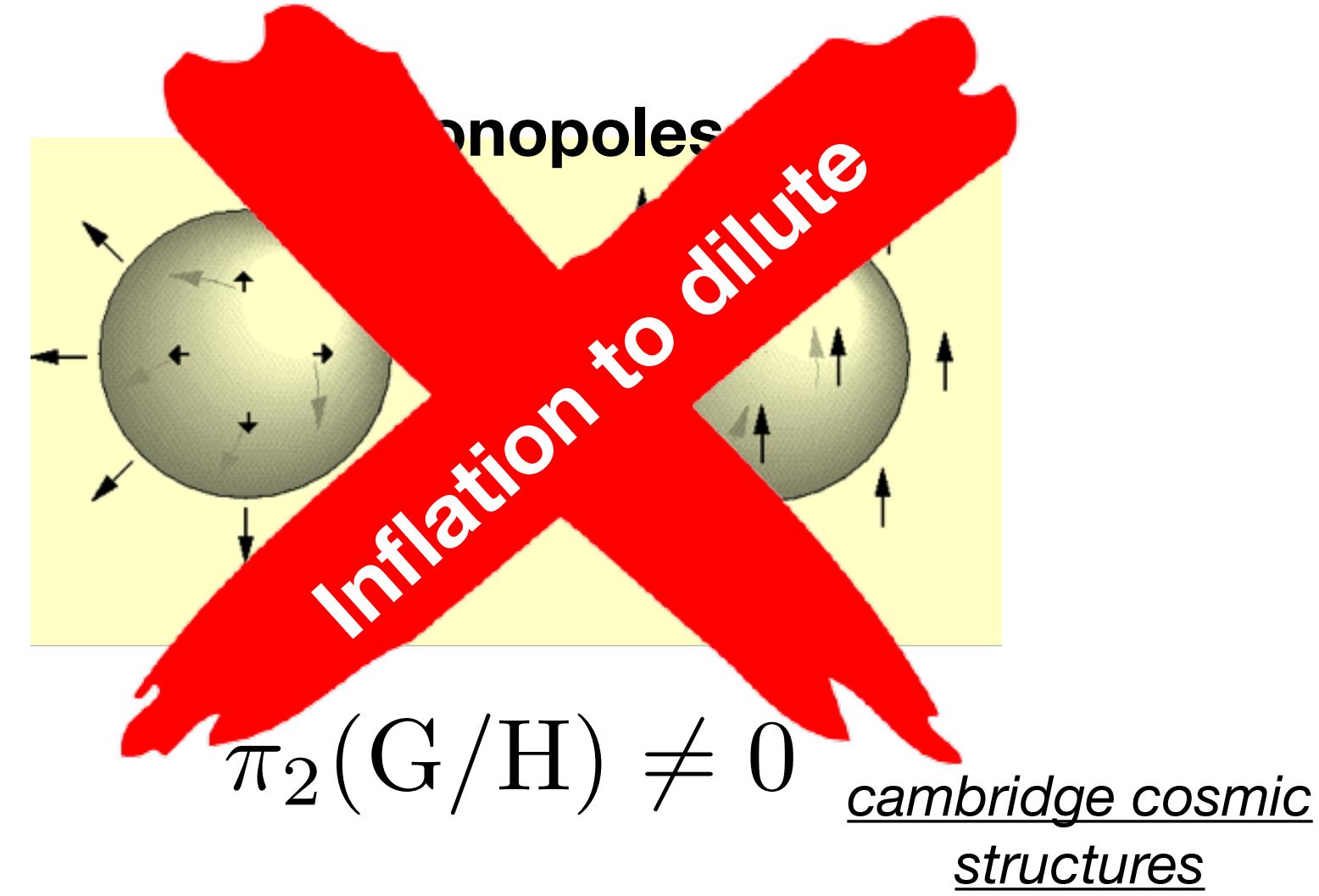
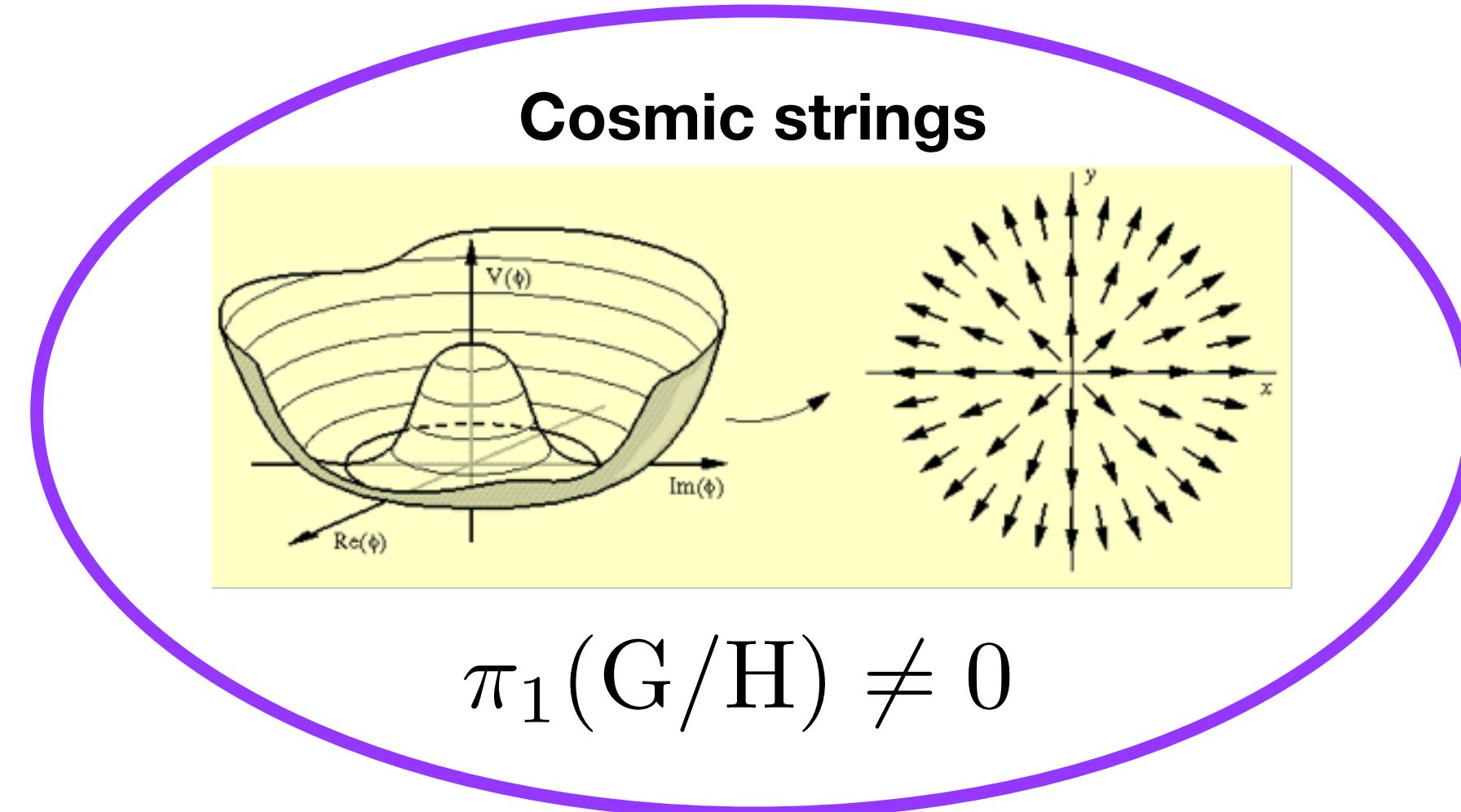
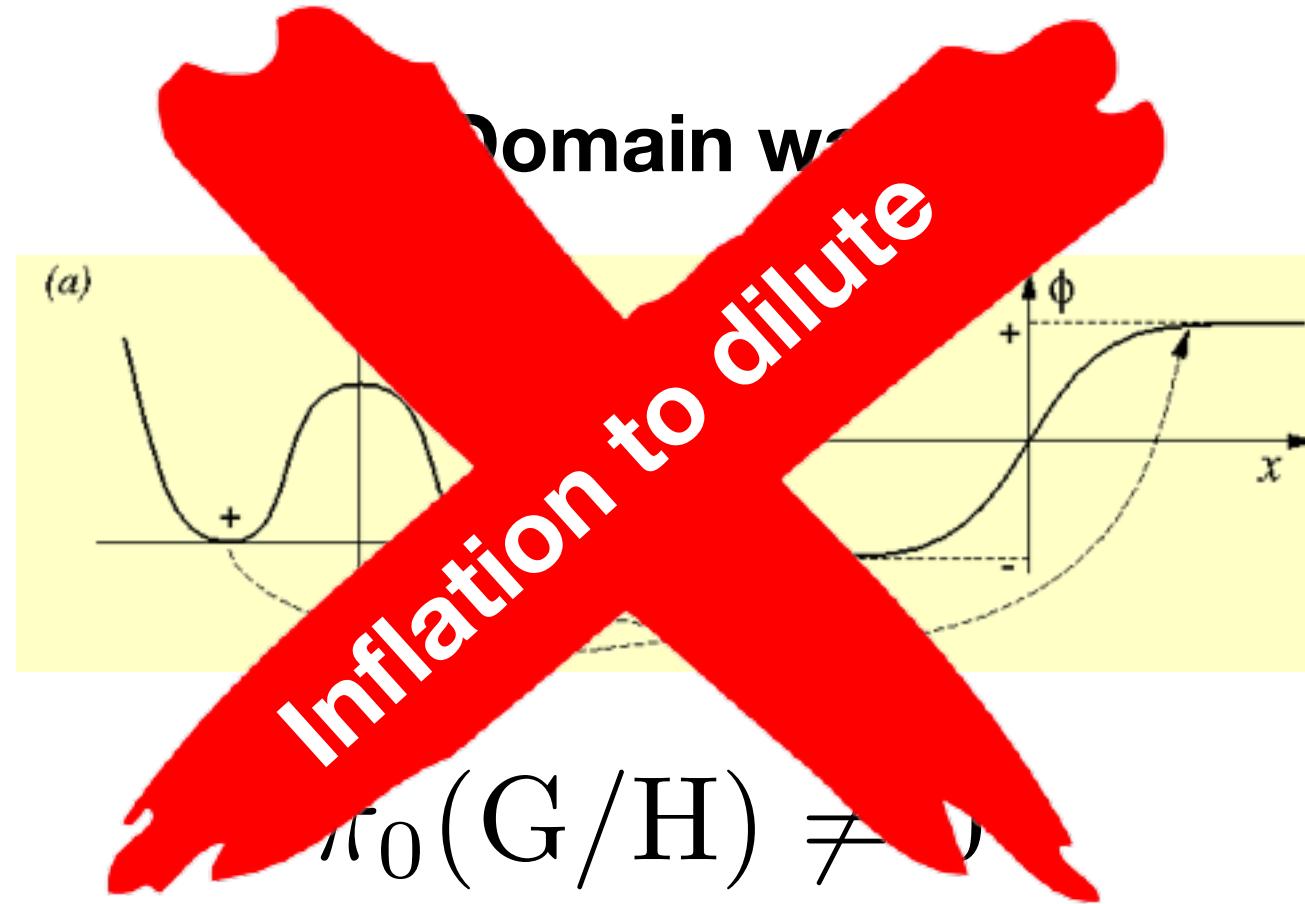


$$\pi_2(G/H) \neq 0$$

cambridge cosmic structures

GUTs prediction: topological defects

During SSB from $G_{GUT} \rightarrow \dots \rightarrow G_{SM}$ topological defects may form.



Cosmic strings induced via U(1) breaking are ubiquitously as GUT breaks to SM

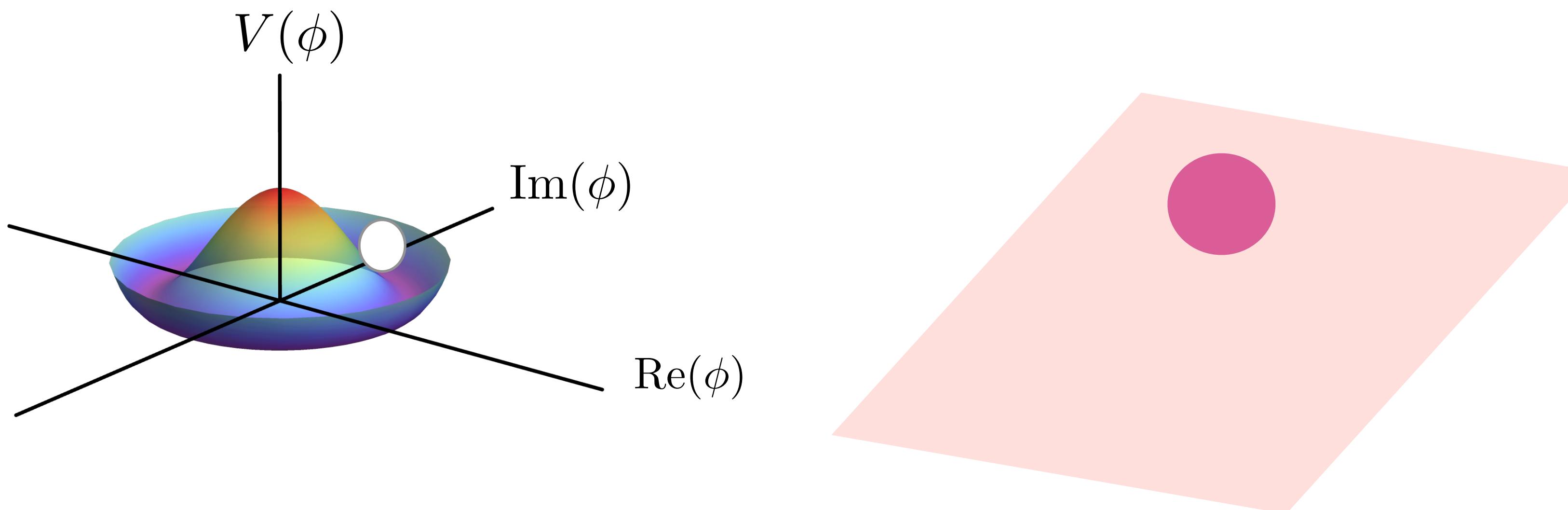
GUTs prediction: topological defects

Abelian Higgs Model

Kibble Mechanism

$$S_{U(1)} = \int d^4x [\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)]$$

$$V(\phi) = \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2$$



Fields at thermally independent regions of the Universe choose different θ

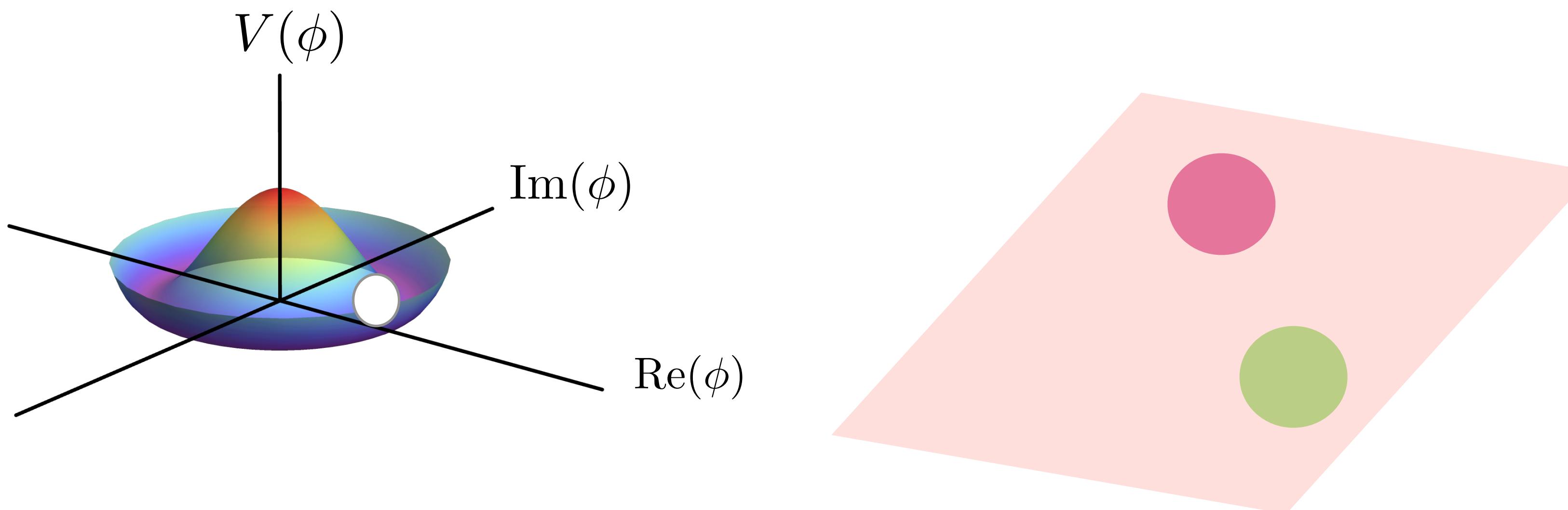
GUTs prediction: topological defects

Abelian Higgs Model

Kibble Mechanism

$$S_{U(1)} = \int d^4x [\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)]$$

$$V(\phi) = \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2$$



Fields at thermally independent regions of the Universe choose different θ

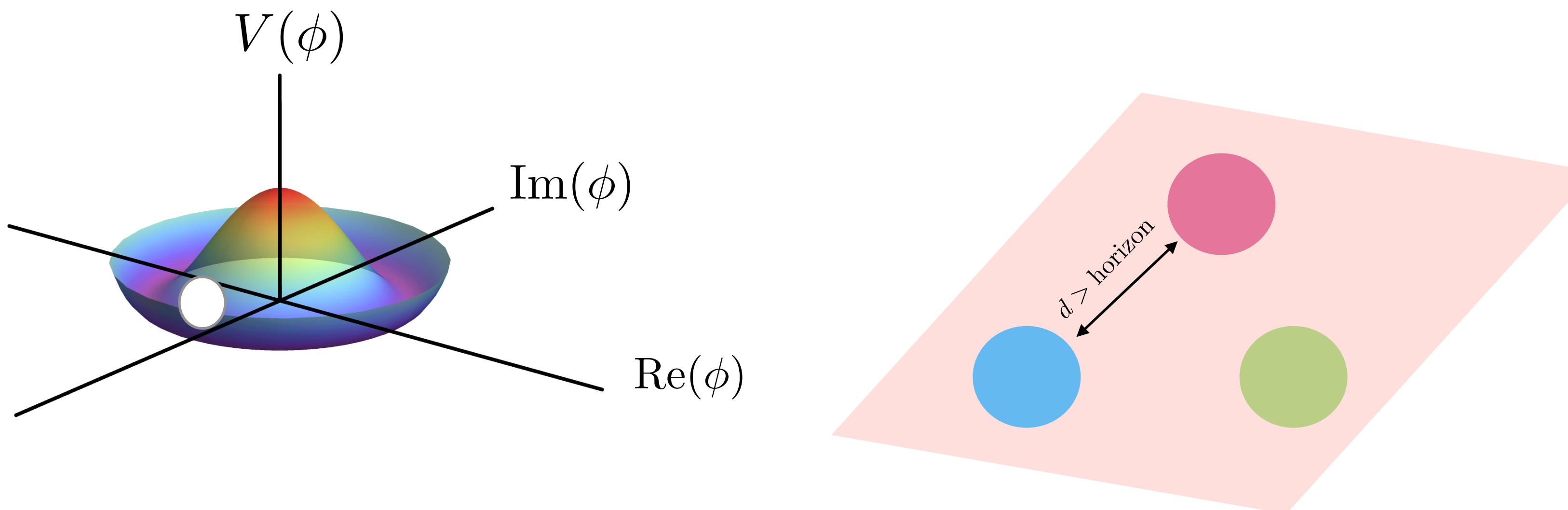
GUTs prediction: topological defects

Abelian Higgs Model

Kibble Mechanism

$$S_{U(1)} = \int d^4x [\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)]$$

$$V(\phi) = \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2$$



Fields at thermally independent regions of the Universe choose different θ

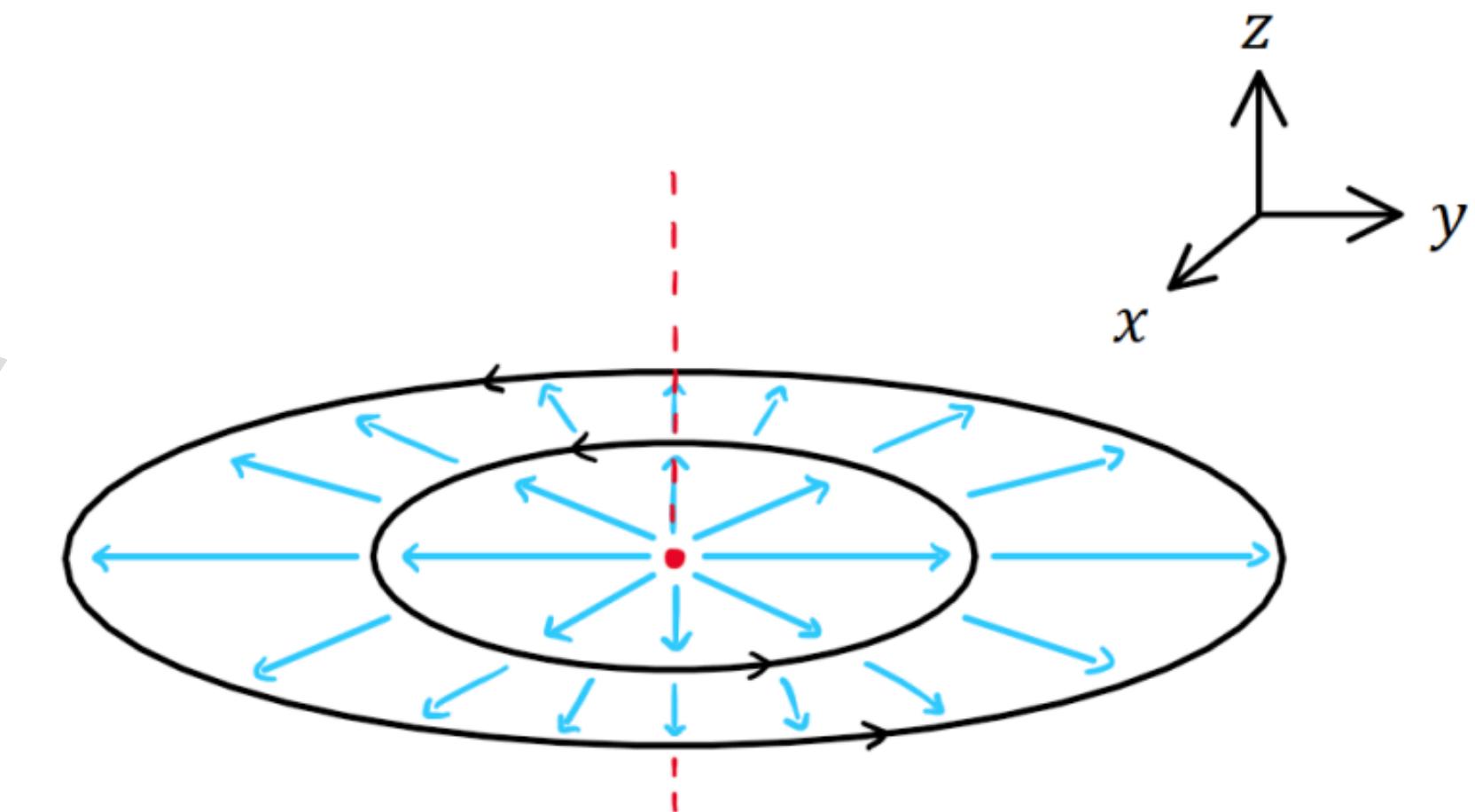
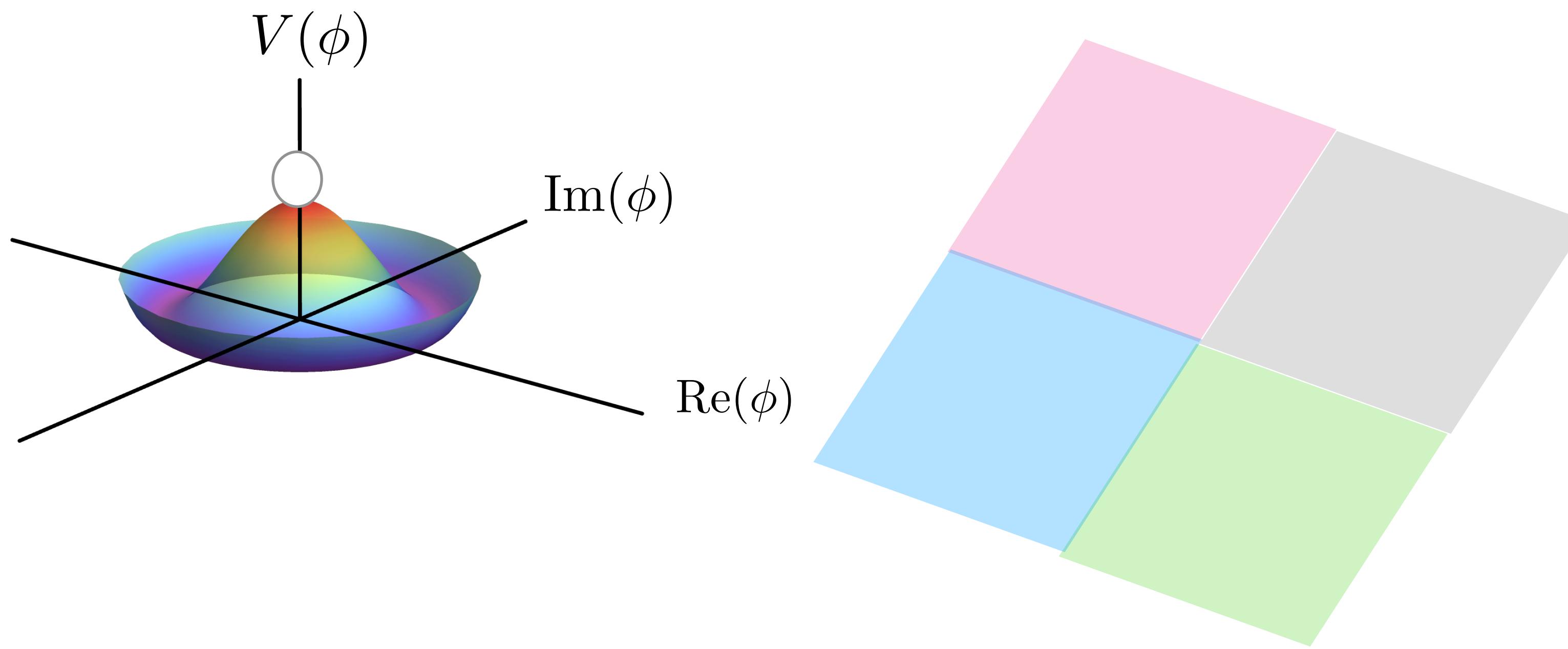
GUTs prediction: topological defects

Abelian Higgs Model

Kibble Mechanism

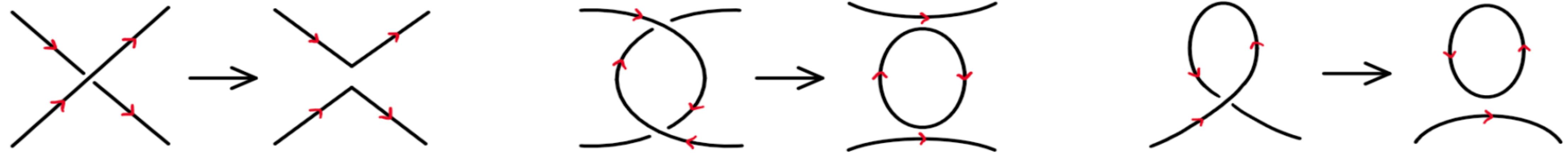
$$S_{U(1)} = \int d^4x [\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)]$$

$$V(\phi) = \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2$$



GUTs prediction: topological defects

- Assume Nambu-Goto strings, only coupling to massless mode is gravity
- String properties controlled by symmetry breaking scale, η
- $\eta = 10^{16} \text{ GeV} \implies \delta = 10^{-30} \text{ cm}$ and $\mu = 10^{22} \text{ gm/cm}$ (string parameter)
- String intercommute, can swap partners and create loops



- Gravitational effect of GUT scale string $G\mu = \left(\frac{\eta}{M_{Pl}}\right)^2 \sim 10^{-6}$

- Emission of gravitational radiation by loops:

$$\frac{dE_{GW}^{(k)}}{dt} = -\Gamma^{(k)} G\mu^2$$

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](#)

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](#)

$$\Omega_{\text{GW}}(f) = \frac{G\mu^2}{\rho_{\text{crit}}} \sum_{k=1}^{\infty} C_k(f) P_k$$

Pillado, Plum, Shlaer

Cui, Lewicki, Morrissey, Wells

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](#)

$$\Omega_{\text{GW}}(f) = \frac{G\mu^2}{\rho_{\text{crit}}} \sum_{k=1}^{\infty} C_k(f) P_k$$

Pillado, Plum, Shlaer

- GW power emitted by single loop oscillating at $f = 2k/l$

Cui, Lewicki, Morrissey, Wells

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](#)

$$\Omega_{\text{GW}}(f) = \frac{G\mu^2}{\rho_{\text{crit}}} \sum_{k=1}^{\infty} C_k(f) P_k$$

Pillado, Plum, Shlaer

- GW power emitted by single loop oscillating at $f = 2k/l$
- Number of loops that emit GW at frequency f today

Cui, Lewicki, Morrissey, Wells

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](#)

$$\Omega_{\text{GW}}(f) = \frac{G\mu^2}{\rho_{\text{crit}}} \sum_{k=1}^{\infty} C_k(f) P_k$$

Pillado, Plum, Shlaer

- **GW power emitted by single loop oscillating at $f = 2k/l$** Cui, Lewicki, Morrissey, Wells
- **Number of loops that emit GW at frequency f today**
- **GW signal is superposition of GW emission from all oscillation modes, need to sum All modes for reliable result**

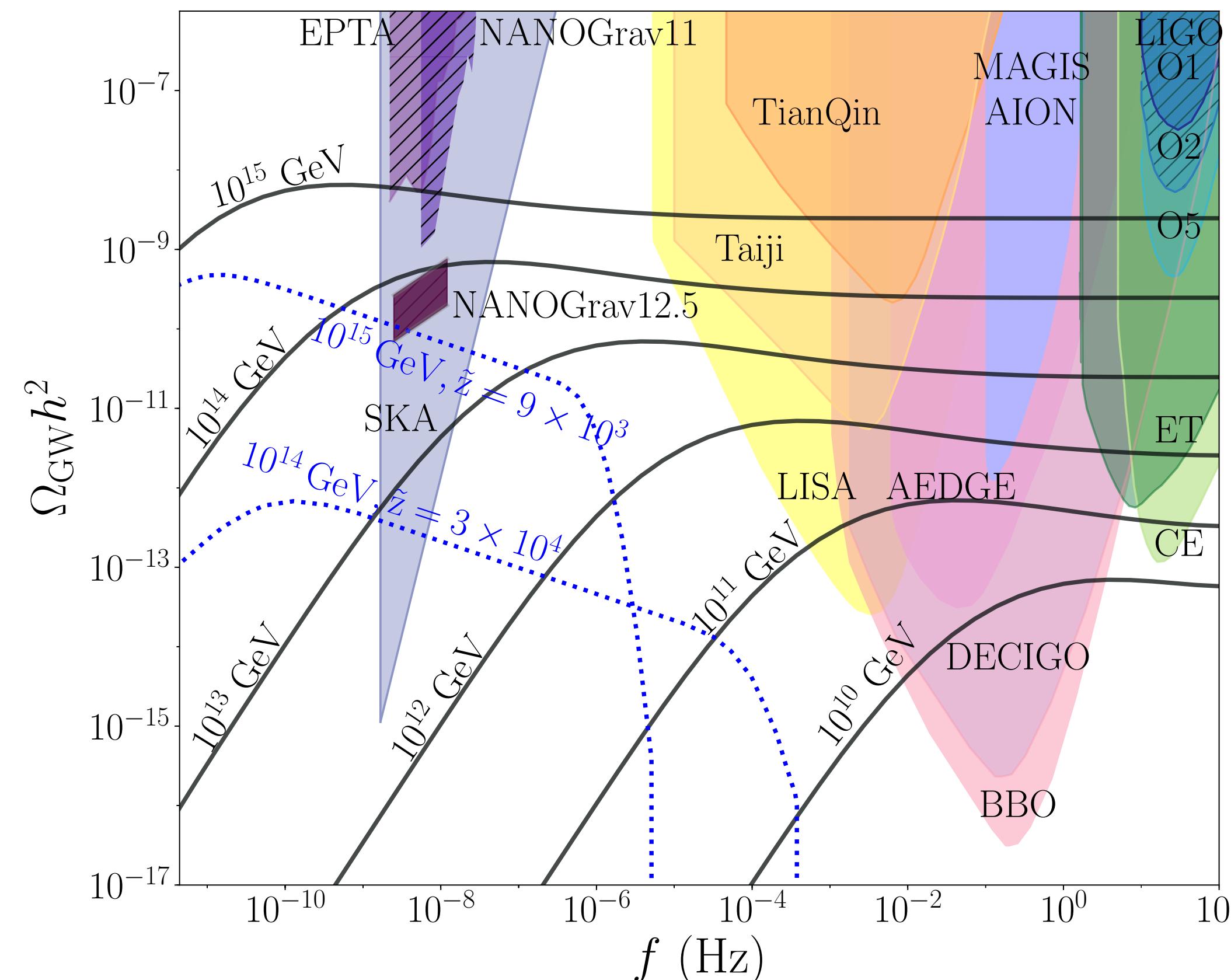
- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](https://arxiv.org/abs/1912.08832)

$$\Omega_{\text{GW}}(f) = \frac{G\mu^2}{\rho_{\text{crit}}} \sum_{k=1}^{\infty} C_k(f) P_k$$

Pillado, Plum, Shlaer

- **GW power emitted by single loop oscillating at $f = 2k/l$** Cui, Lewicki, Morrissey, Wells
- **Number of loops that emit GW at frequency f today**
- **GW signal is superposition of GW emission from all oscillation modes, need to sum All modes for reliable result**
- **All non-trivial physics contained in loop density function within $C_k(f)$**

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) [1912.08832](https://arxiv.org/abs/1912.08832)



SO(10) phenomenological predictions

- *SO(10)* provides unification without SUSY and can explain neutrino masses

SO(10) phenomenological predictions

- $SO(10)$ provides unification without SUSY and can explain neutrino masses
- 51 “breaking chains” of $SO(10)$ to SM! *Jeannerot, 0308134*

SO(10) phenomenological predictions

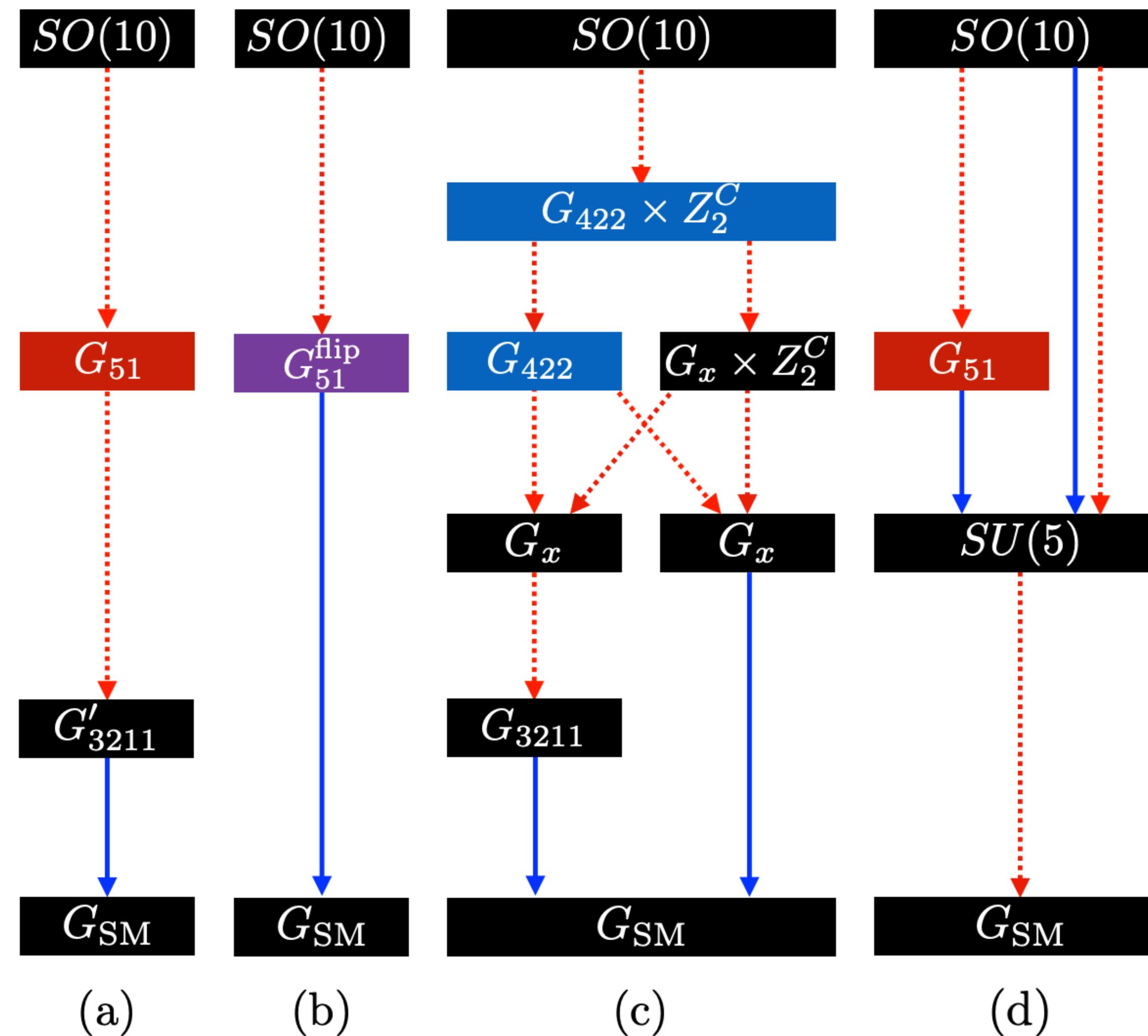
- $SO(10)$ provides unification without SUSY and can explain neutrino masses
- 51 “breaking chains” of $SO(10)$ to SM! *Jeannerot, 0308134*

$$\underbrace{SO(10)}_{G_X} \xrightarrow{M_X} \underbrace{G_{422}}_{G_2} \xrightarrow{M_2} \underbrace{G_{3221}}_{G_1} \xrightarrow{M_1} G_{SM}$$

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

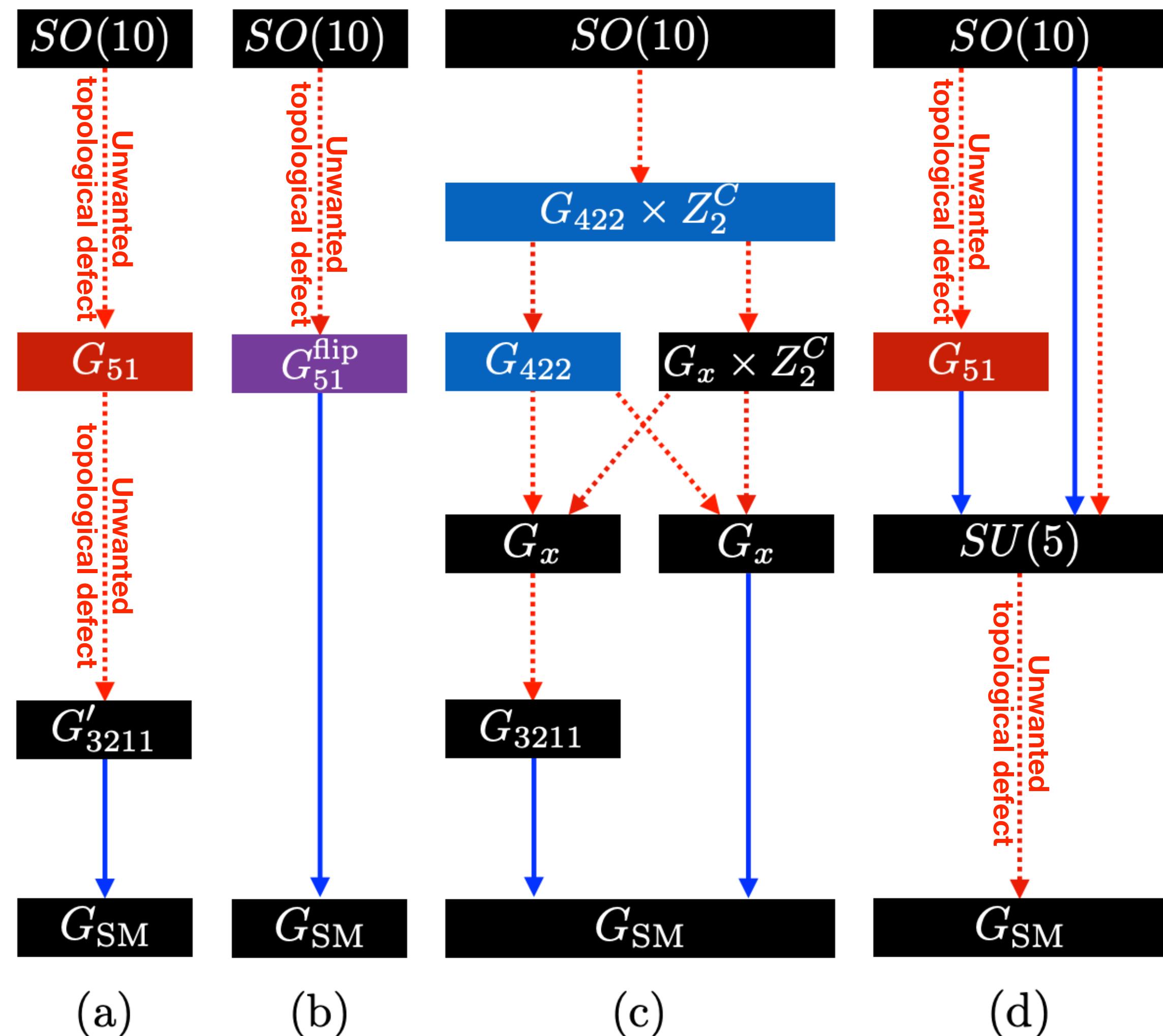
SO(10) phenomenological predictions



$$\begin{aligned}
 X &= \sqrt{\frac{3}{4}}B - L \\
 G_{51} &= SU(5) \times U(1)_X \\
 G_{51}^{\text{flip}} &= SU(5)_{\text{flip}} \times U(1)_{\text{flip}} \\
 G_{3221} &= SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
 G_{3211} &= SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\
 G'_{3211} &= SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \\
 G_{421} &= SU(4)_C \times SU(2)_L \times U(1)_Y \\
 G_{422} &= SU(4)_C \times SU(2)_L \times SU(2)_R .
 \end{aligned}$$

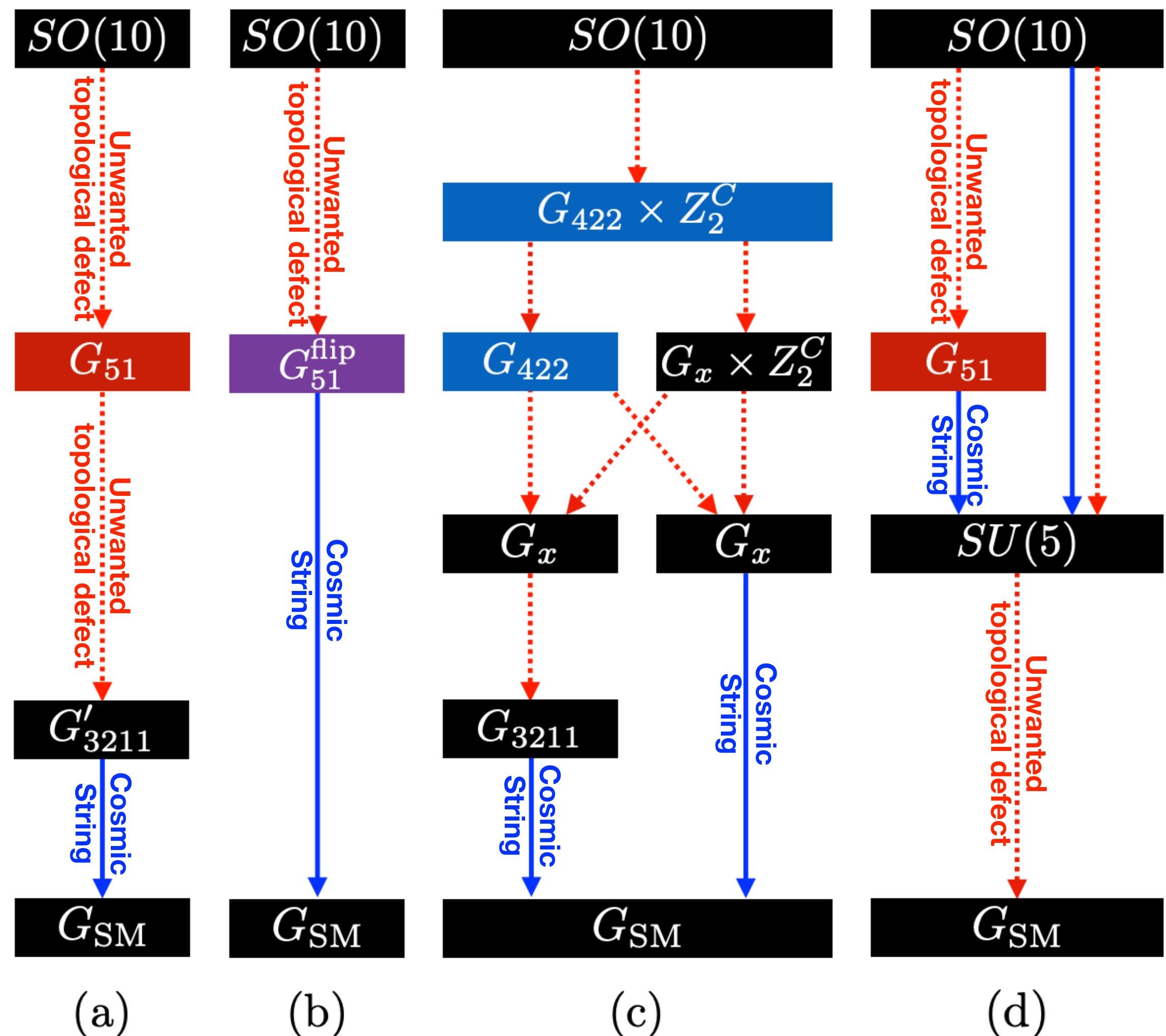
[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

$SO(10)$ phenomenological predictions



[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

$SO(10)$ phenomenological predictions

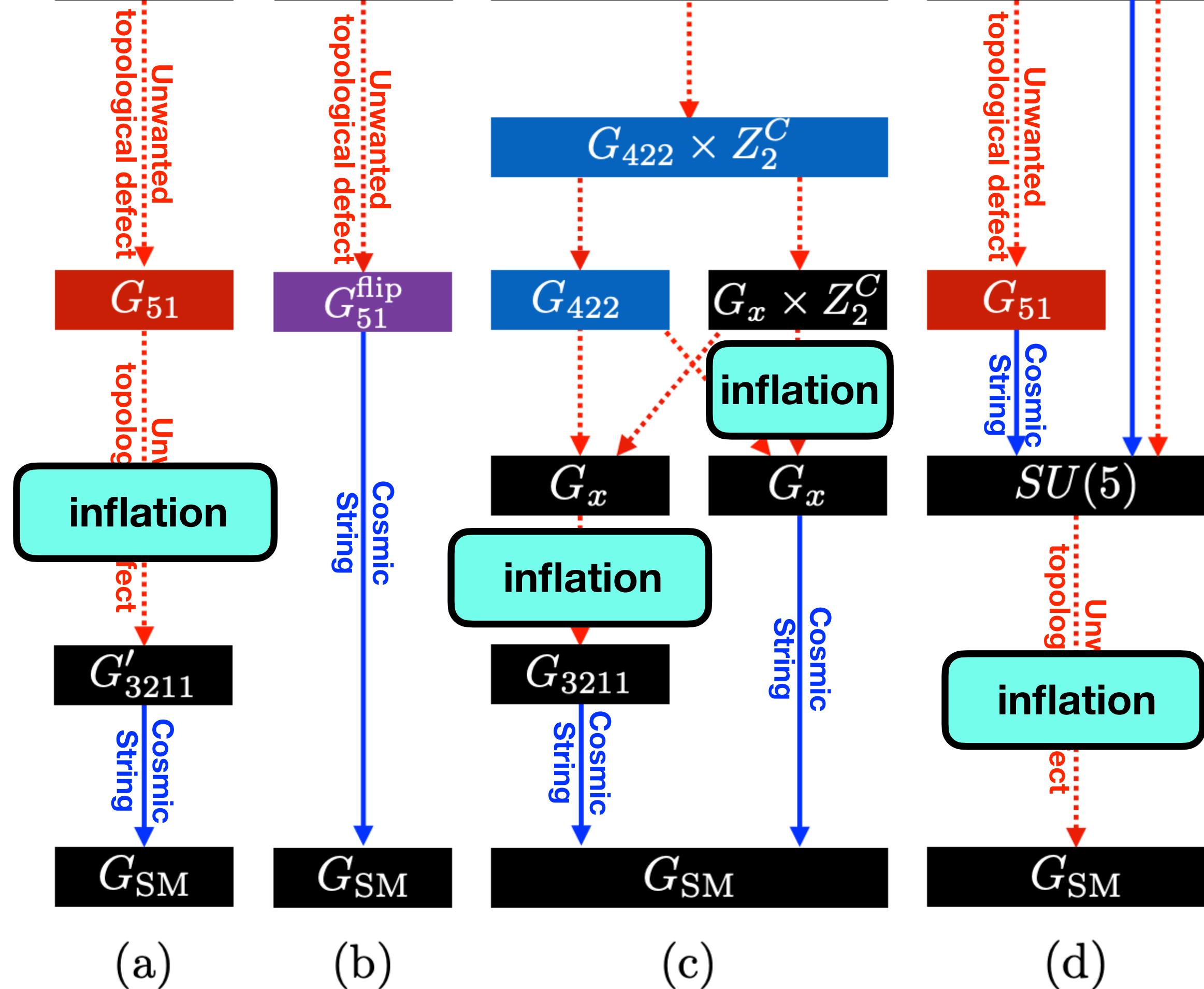


[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

$SO(10)$ phenomenological predictions

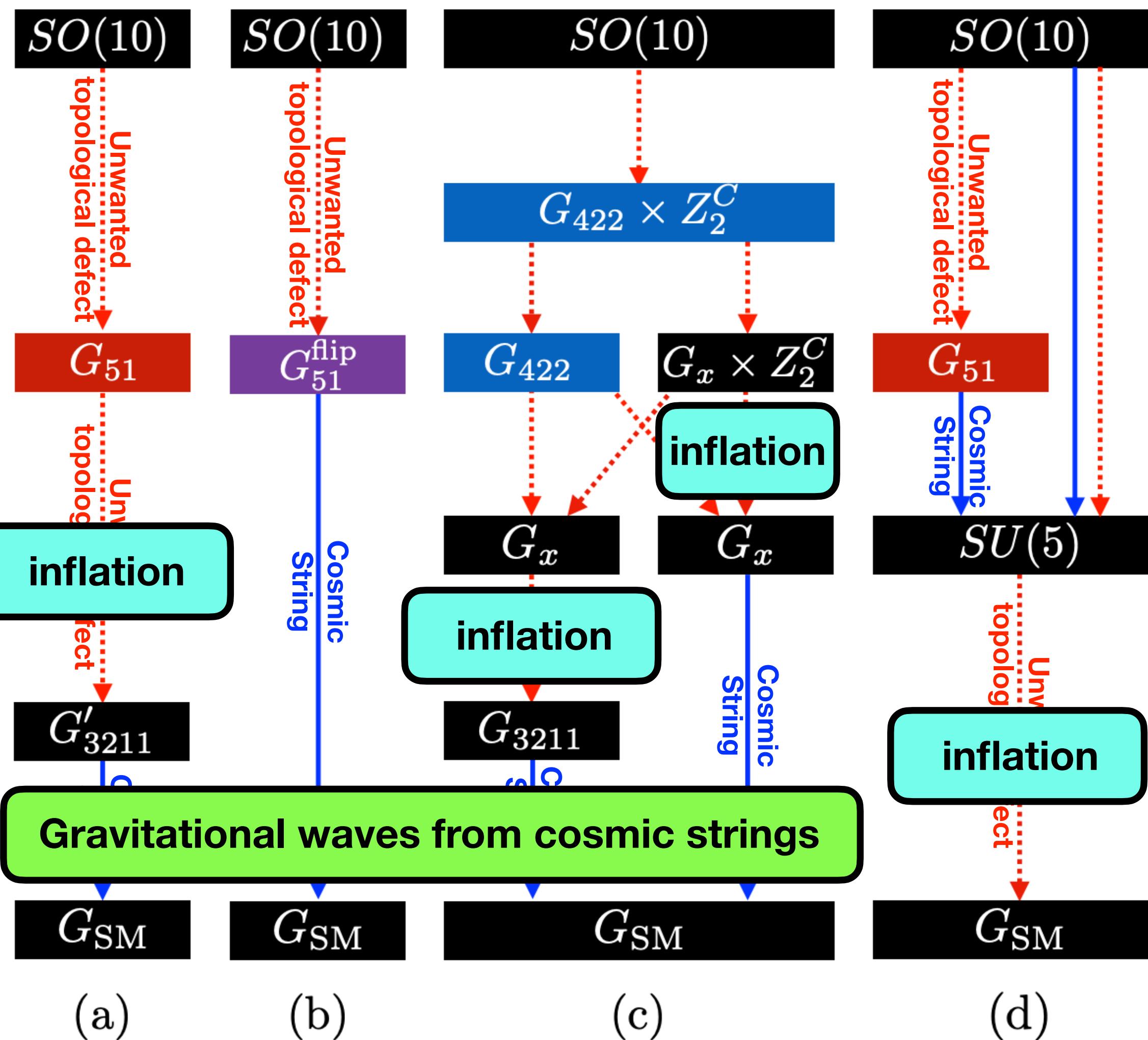
$SO(10)$ $SO(10)$ $SO(10)$ $SO(10)$

CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16} \text{ GeV}$



[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

SO(10) phenomenological predictions

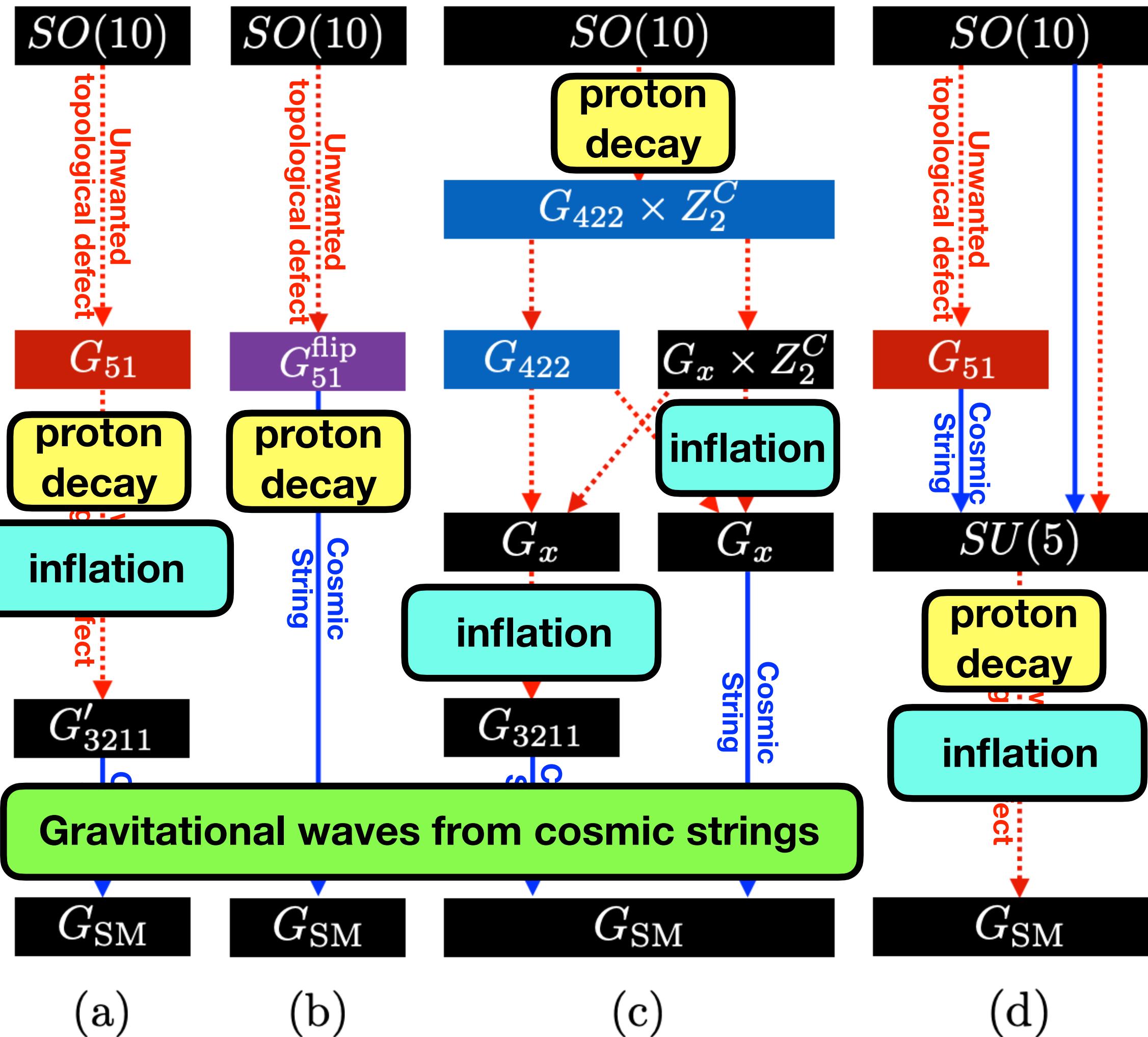


CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16} \text{ GeV}$

Non-observation GW: $\Lambda_{\text{cs}} \lesssim 5 \times 10^{14} \text{ GeV}$

[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

SO(10) phenomenological predictions



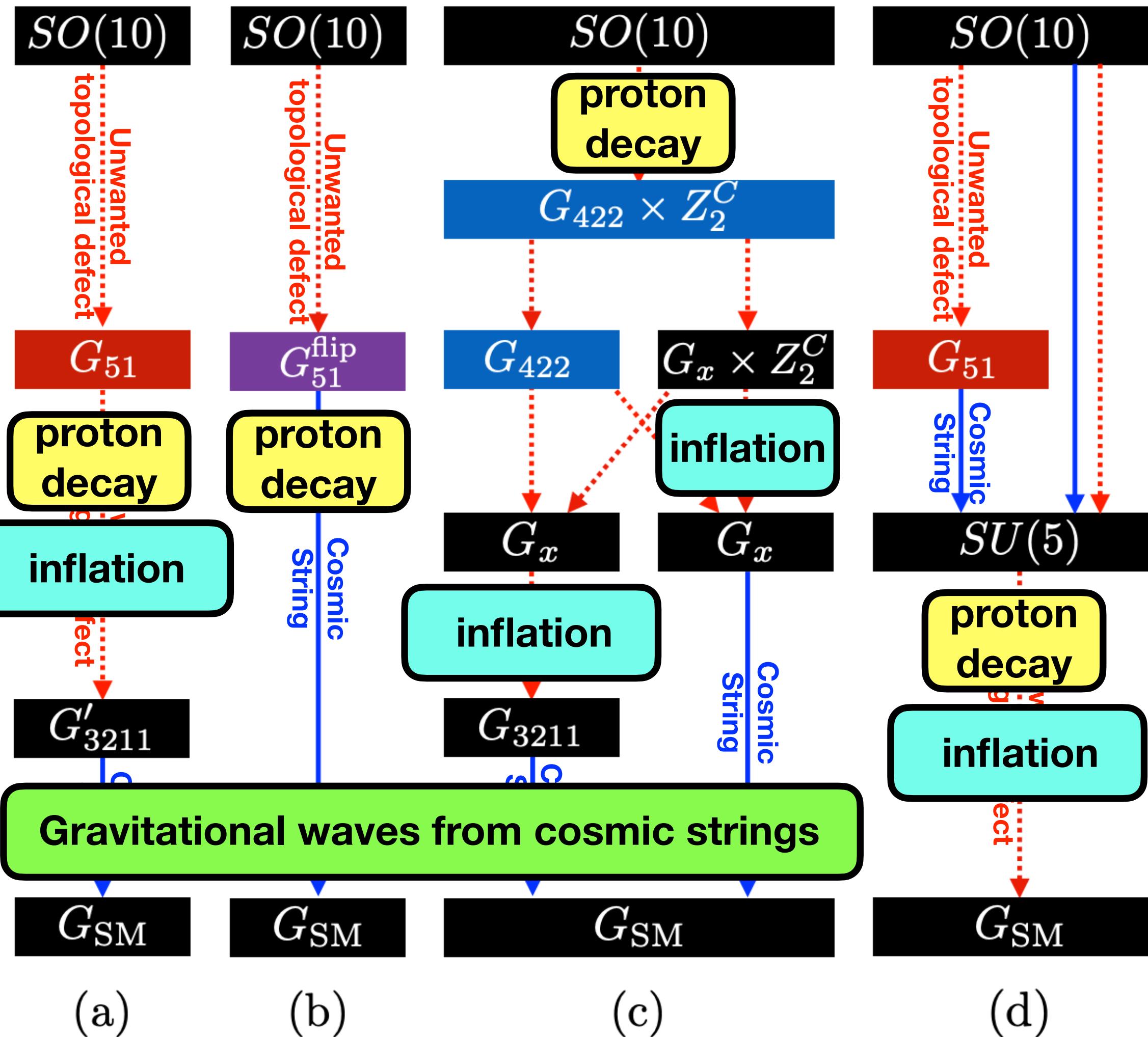
CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16} \text{ GeV}$

Non-observation GW: $\Lambda_{\text{cs}} \lesssim 5 \times 10^{14} \text{ GeV}$

Non-observation proton decay: $\Lambda_{\text{pd}} \gtrsim 10^{15} \text{ GeV}$

[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

SO(10) phenomenological predictions



CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16} \text{ GeV}$

Non-observation GW: $\Lambda_{\text{cs}} \lesssim 5 \times 10^{14} \text{ GeV}$

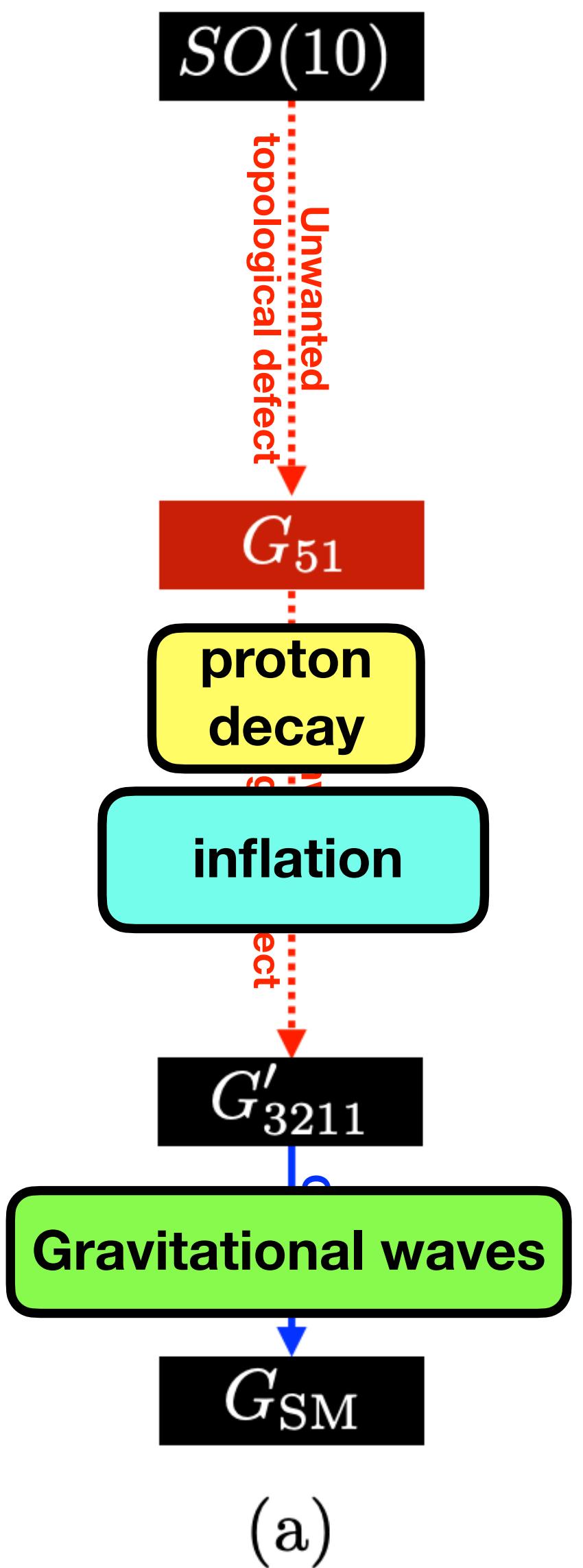
Non-observation proton decay: $\Lambda_{\text{pd}} \gtrsim 10^{15} \text{ GeV}$

Certain scale ordering excluded e.g

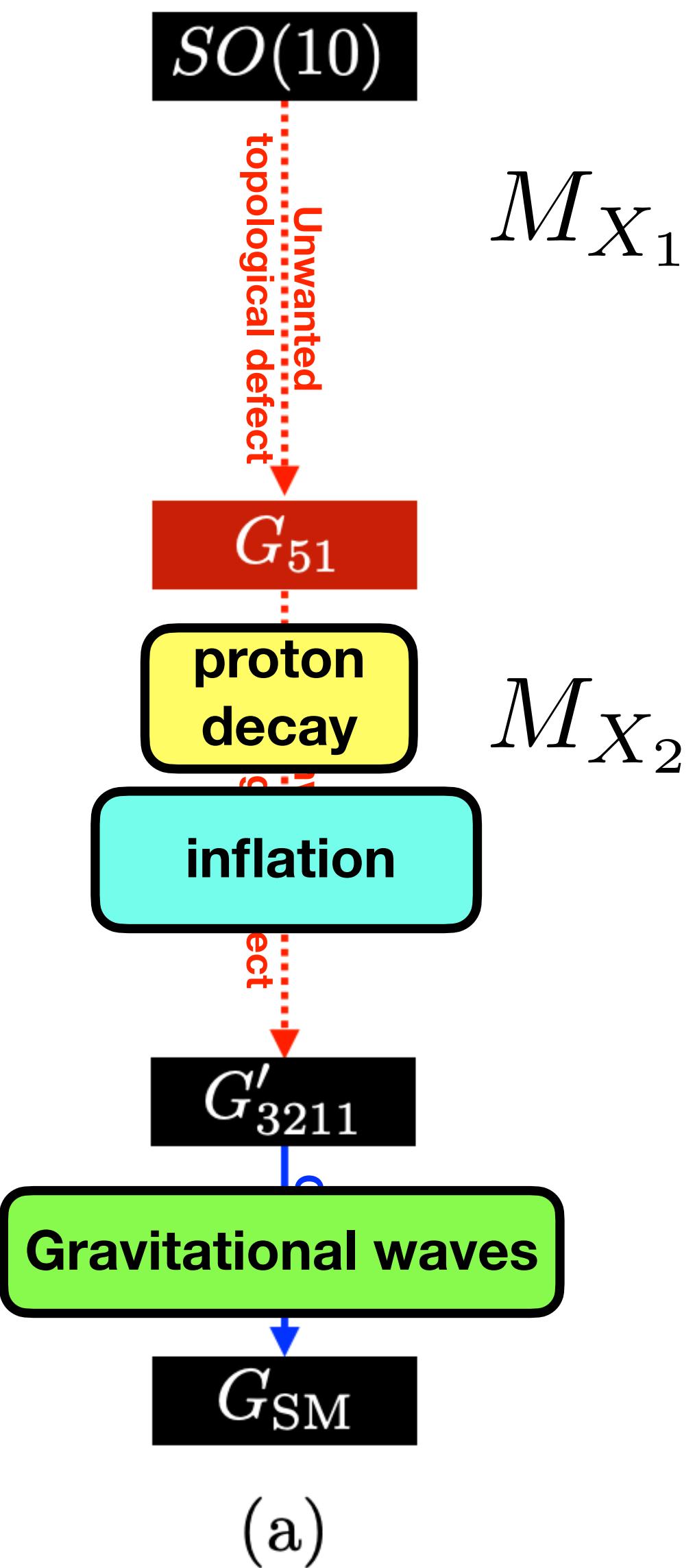
$$\Lambda_{\text{inf}} \gg \Lambda_{\text{cs}} \gg \Lambda_{\text{pd}}$$

[2005.13549 King, Pascoli, JT, Zhou](https://arxiv.org/abs/0513549)

SO(10) phenomenological predictions

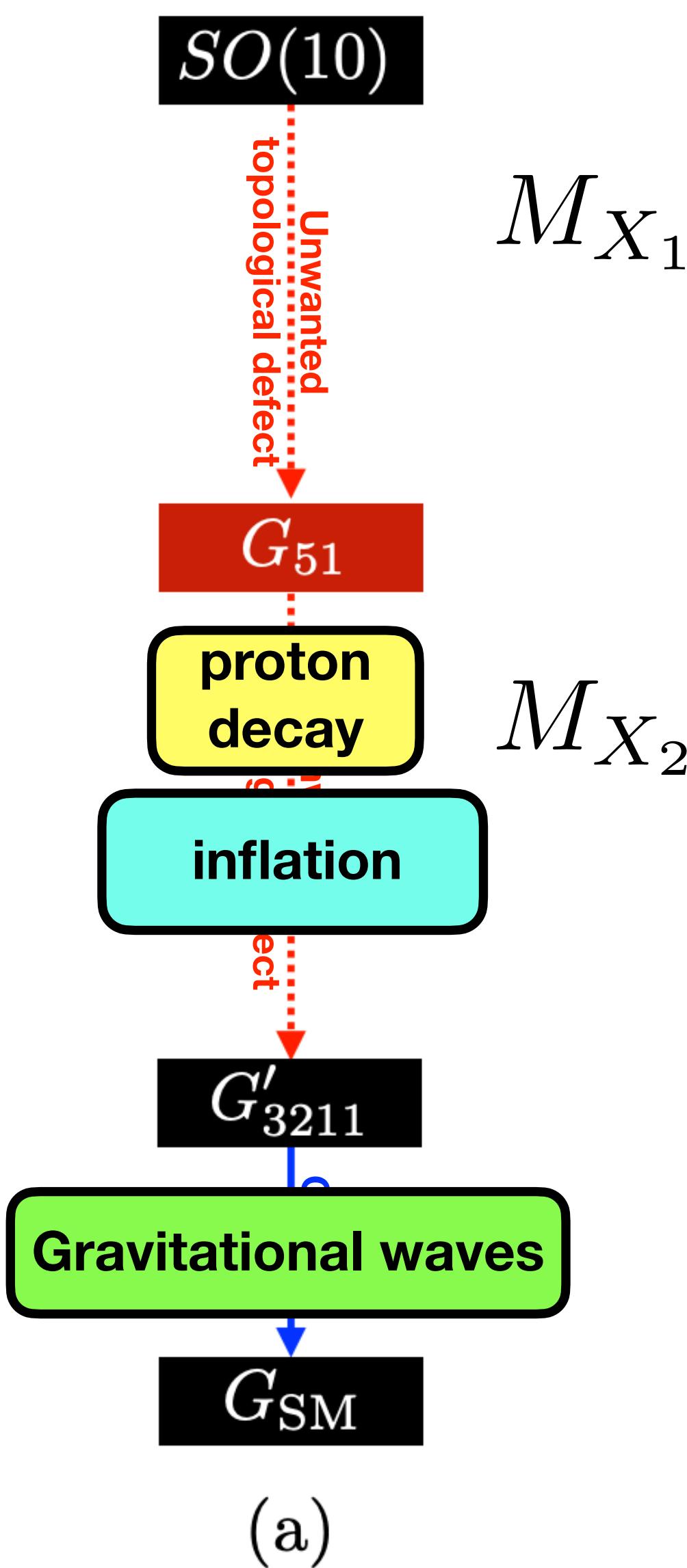


SO(10) phenomenological predictions



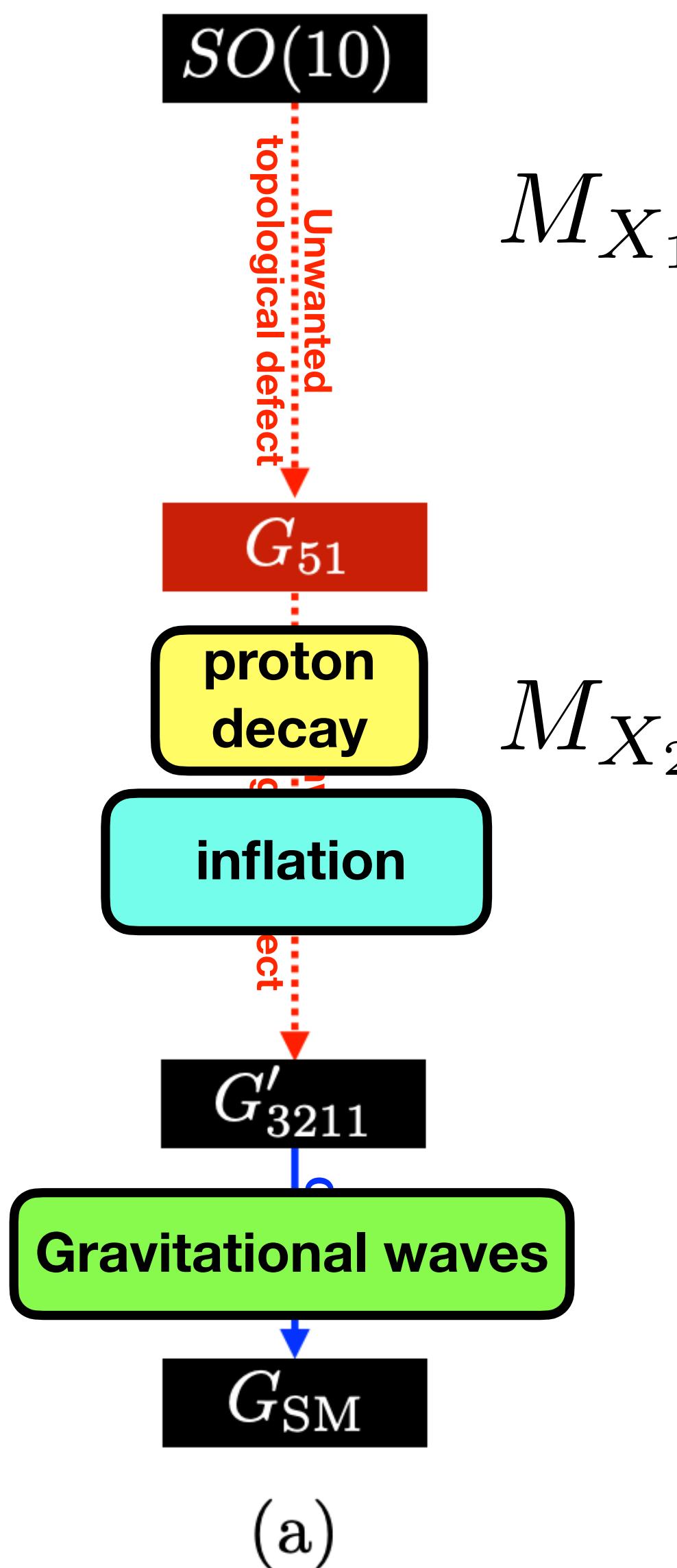
- Proton decay operators induced at M_{X_1} and M_{X_2}

SO(10) phenomenological predictions



- Proton decay operators induced at M_{X_1} and M_{X_2}
- $M_{X_2} < M_{X_1} \implies$ main proton decay channel: $p \rightarrow e^+ \pi^0$ at scale $\Lambda_{\text{pd}} = M_{X_2}$

SO(10) phenomenological predictions



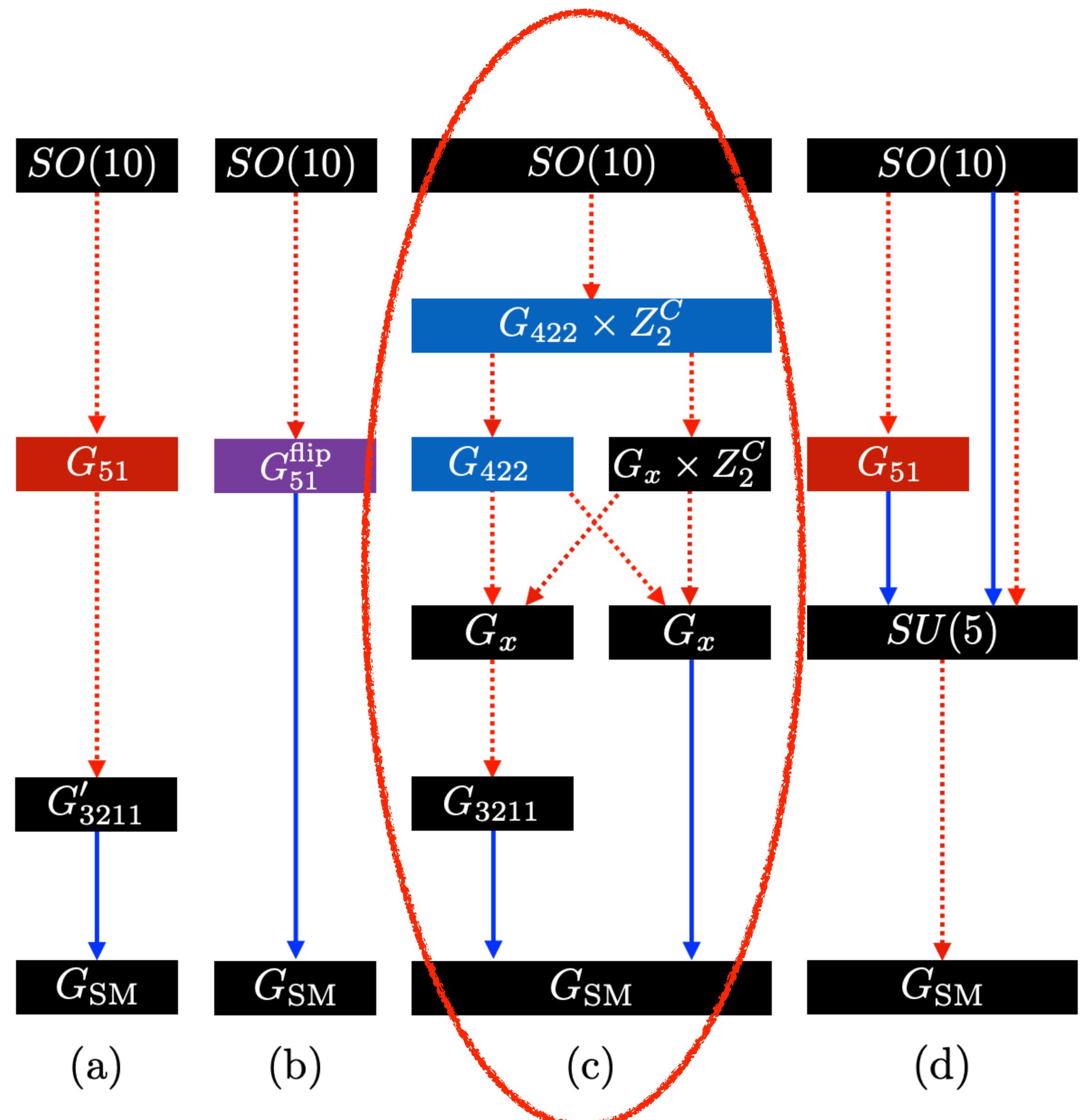
- Proton decay operators induced at M_{X_1} and M_{X_2}
- $M_{X_2} < M_{X_1} \implies$ main proton decay channel: $p \rightarrow e^+ \pi^0$ at scale $\Lambda_{\text{pd}} = M_{X_2}$
- 1. $\Lambda_{\text{pd}} > \Lambda_{\text{inf}} > \Lambda_{\text{cs}}$: PD + undiluted GW observed (ideal case)
2. $\Lambda_{\text{pd}} > \Lambda_{\text{inf}} \sim \Lambda_{\text{cs}}$: PD + diluted GW observed
3. $\Lambda_{\text{pd}} > \Lambda_{\text{cs}} > \Lambda_{\text{inf}}$: PD + no associated GW

Proton decay and GWs as complementary windows

- Type (a): $\Lambda_{pd} > \Lambda_{cs}$
- Type (b): $\Lambda_{pd} \sim \Lambda_{cs}$
- Type (c): $\Lambda_{pd} > \Lambda_{cs}$
- Type (d): no GWs

Observables		Proton decays
GWs	Observed	$p \rightarrow \pi^0 e^+$ observed \Rightarrow non-SUSY contribution indicated
	Marginal	<ul style="list-style-type: none">• types (a) and (c) favoured• types (b) and (d) excluded <ul style="list-style-type: none">• types (a) and (c) favoured• type (d) excluded• type (b) allowed if $p \rightarrow K^+ \bar{\nu}$ not observed and $\Lambda_{pd} \sim \Lambda_{cs}$

- Distinguish (a) & (c) requires model dependent study



- (d) cannot be tested with GWs since Unwanted defects formed in last SSB step
- Gauge unification not possible in (a) & (b) without SUSY
- Study (c) in more detail in [**2106.15634**](#)
31 breaking chains

Proton decay and GWs as complementary windows

$SO(10)$	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
I1:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
I2:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{s,w}} \overline{\mathbf{126}}$		✗
I3:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
I4:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \overline{\mathbf{126}, \mathbf{45}}$		✗
I5:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \overline{\mathbf{126}, \mathbf{45}}$		✗
I6:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓

Need to compute
RGEs for all 31 chains

$SO(10)$	defect Higgs	G_3	defect Higgs	G_2	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
III1:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III2:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III3:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III4:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III5:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III6:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III7:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III8:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III9:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
III10:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓

$SO(10)$	defect Higgs	G_2	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
II1:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II2:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{s,w}} \overline{\mathbf{126}}$		✗
II3:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II4:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II5:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II6:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II7:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \overline{\mathbf{126}, \mathbf{45}}$		✗
II8:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II9:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II10:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II11:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
II12:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓

$SO(10)$	defect Higgs	G_4	defect Higgs	G_3	defect Higgs	G_2	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
IV1:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
IV2:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓
IV3:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \overline{\mathbf{126}}$		✓

Proton decay and GWs as complementary windows

$SO(10)$	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
I1:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
I2:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{s,w}} \mathbf{\overline{126}}$		✗
I3:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
I4:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{\overline{126},45}$		✗
I5:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \mathbf{\overline{126},45}$		✗
I6:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓

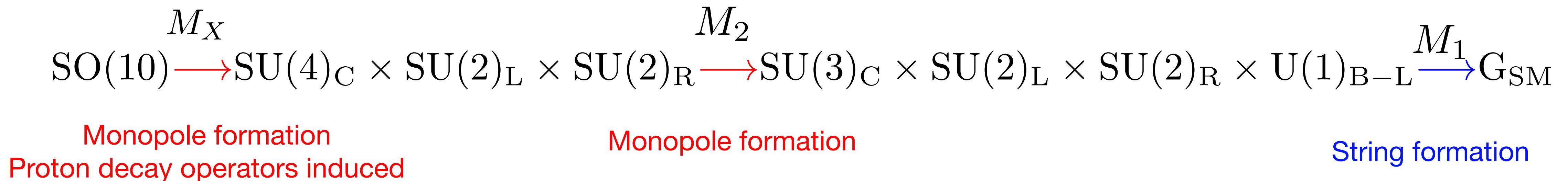
Need to compute
RGEs for all 31 chains

$SO(10)$	defect Higgs	G_3	defect Higgs	G_2	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
III1:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III2:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III3:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III4:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III5:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III6:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III7:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III8:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III9:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
III10:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓

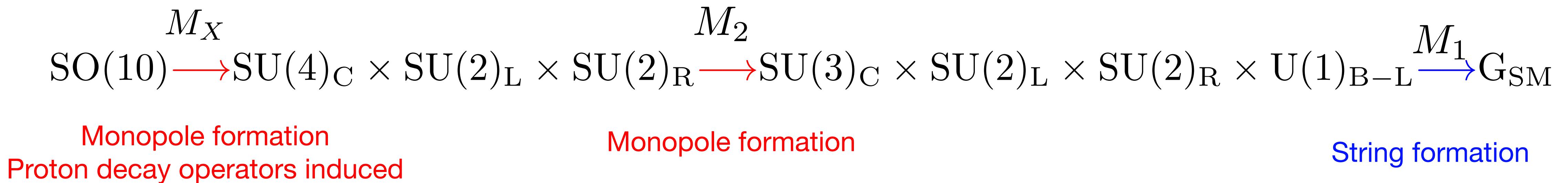
$SO(10)$	defect Higgs	G_2	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
II1:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II2:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{s,w}} \mathbf{\overline{126}}$		✗
II3:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II4:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II5:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II6:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II7:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{\overline{126},45}$		✗
II8:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II9:	$\xrightarrow{\text{m,s}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{m,w}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II10:	$\xrightarrow{\text{m}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II11:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m,w}} \mathbf{210}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
II12:	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓

$SO(10)$	defect Higgs	G_4	defect Higgs	G_3	defect Higgs	G_2	defect Higgs	G_1	defect Higgs	G_{SM}	Observable strings?
IV1:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{m}} \mathbf{210}$	G_{3221}^C	$\xrightarrow{\text{w}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
IV2:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3221}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓
IV3:	$\xrightarrow{\text{m,s}} \mathbf{54}$	G_{422}^C	$\xrightarrow{\text{w}} \mathbf{210}$	G_{422}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{421}	$\xrightarrow{\text{m}} \mathbf{45}$	G_{3211}	$\xrightarrow{\text{s}} \mathbf{\overline{126}}$		✓

Proton decay and GWs as complementary windows

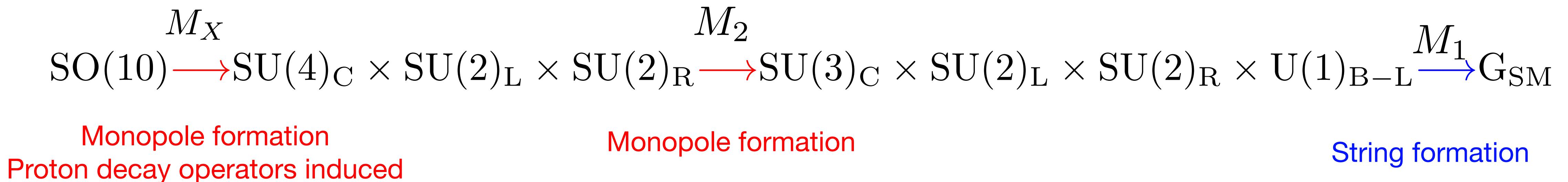


Proton decay and GWs as complementary windows



Assumptions

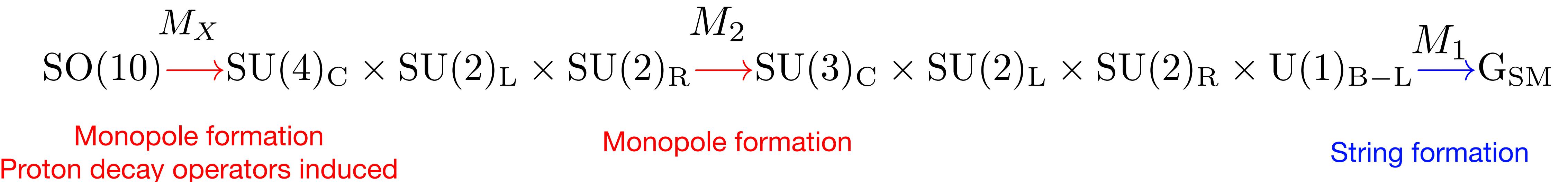
Proton decay and GWs as complementary windows



Assumptions

- Inflation after monopole formation & before cosmic string formation \implies observable GW

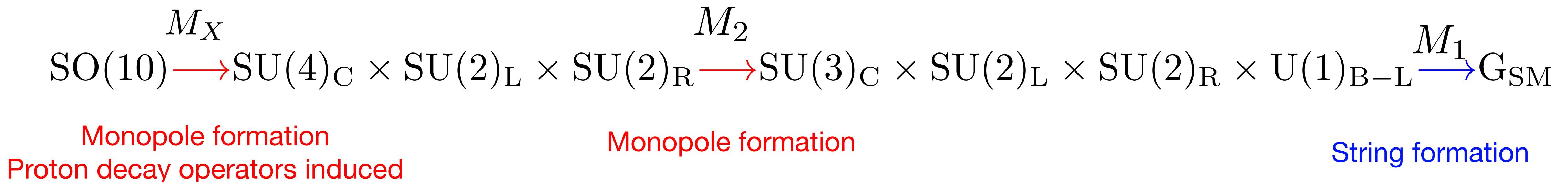
Proton decay and GWs as complementary windows



Assumptions

- Inflation after monopole formation & before cosmic string formation \implies observable GW
- **Minimal particle content:** SM, RH neutrinos and Higgs multiplet required for SSB

Proton decay and GWs as complementary windows



Assumptions

- Inflation after monopole formation & before cosmic string formation \implies observable GW
- **Minimal particle content:** SM, RH neutrinos and Higgs multiplet required for SSB
- **Assume gauge unification** at scale M_X . For each chain perform 2-loop RGE analysis to determine couplings & M_X, M_2, M_1

Proton Decay

- From RGE, α_X and M_X determined

$$\begin{aligned}\Gamma(p \rightarrow \pi^0 e^+) = & \frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL} \Lambda_1^{-2} \left(1 + |V_{ud}|^2\right) \left| \langle \pi^0 | (ud)_R u_L | p \rangle \right|^2 \right. \\ & \left. + A_{SR} \left(\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}\right) \left| \langle \pi^0 | (ud)_L u_L | p \rangle \right|^2 \right]\end{aligned}$$

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma(p \rightarrow \pi^0 e^+) = \frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL} \Lambda_1^{-2} \left(1 + |V_{ud}|^2\right) |\langle \pi^0 | (ud)_R u_L | p \rangle|^2 \right. \\ \left. + A_{SR} \left(\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}\right) |\langle \pi^0 | (ud)_L u_L | p \rangle|^2 \right]$$

**Hadronic matrix element
from lattice**

[1705.01338](#)

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma(p \rightarrow \pi^0 e^+) = \frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL} \Lambda_1^{-2} \left(1 + |V_{ud}|^2\right) |\langle \pi^0 | (ud)_R u_L | p \rangle|^2 \right. \\ \left. + A_{SR} \left(\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}\right) |\langle \pi^0 | (ud)_L u_L | p \rangle|^2 \right]$$

$$\Lambda_1 \simeq \Lambda_2 = \frac{g_X M_X}{2}$$

**Hadronic matrix element
from lattice**
1705.01338

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma(p \rightarrow \pi^0 e^+) = \frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times [A_{SL} \Lambda_1^{-2} (1 + |V_{ud}|^2) |\langle \pi^0 | (ud)_R u_L | p \rangle|^2 + A_{SR} (\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}) |\langle \pi^0 | (ud)_L u_L | p \rangle|^2]$$

Long & short
range effects
from renormalisation

$$\Lambda_1 \simeq \Lambda_2 = \frac{g_X M_X}{2}$$

Hadronic matrix element
from lattice
[1705.01338](#)

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string \implies gravitational radiation primary emission

$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string \implies gravitational radiation primary emission

$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

$$M_1^2 \sim g_1^2 v^2 \implies G\mu = \frac{1}{M_{\text{Pl}}^2} \frac{2\pi M_1^2}{g_1^2} = \frac{M_1^2}{2\alpha_1 M_{\text{Pl}}^2}$$

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string \implies gravitational radiation primary emission

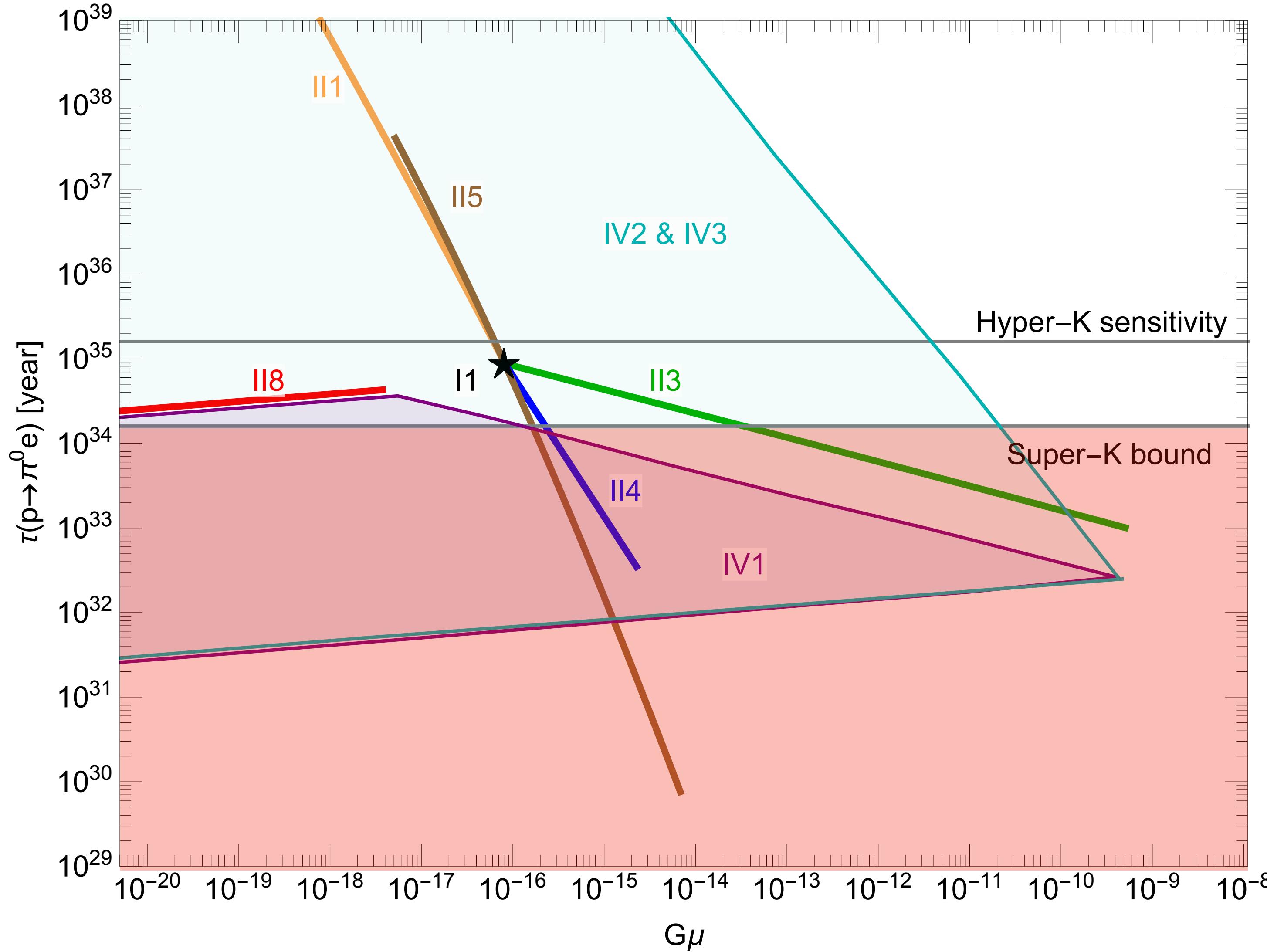
$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

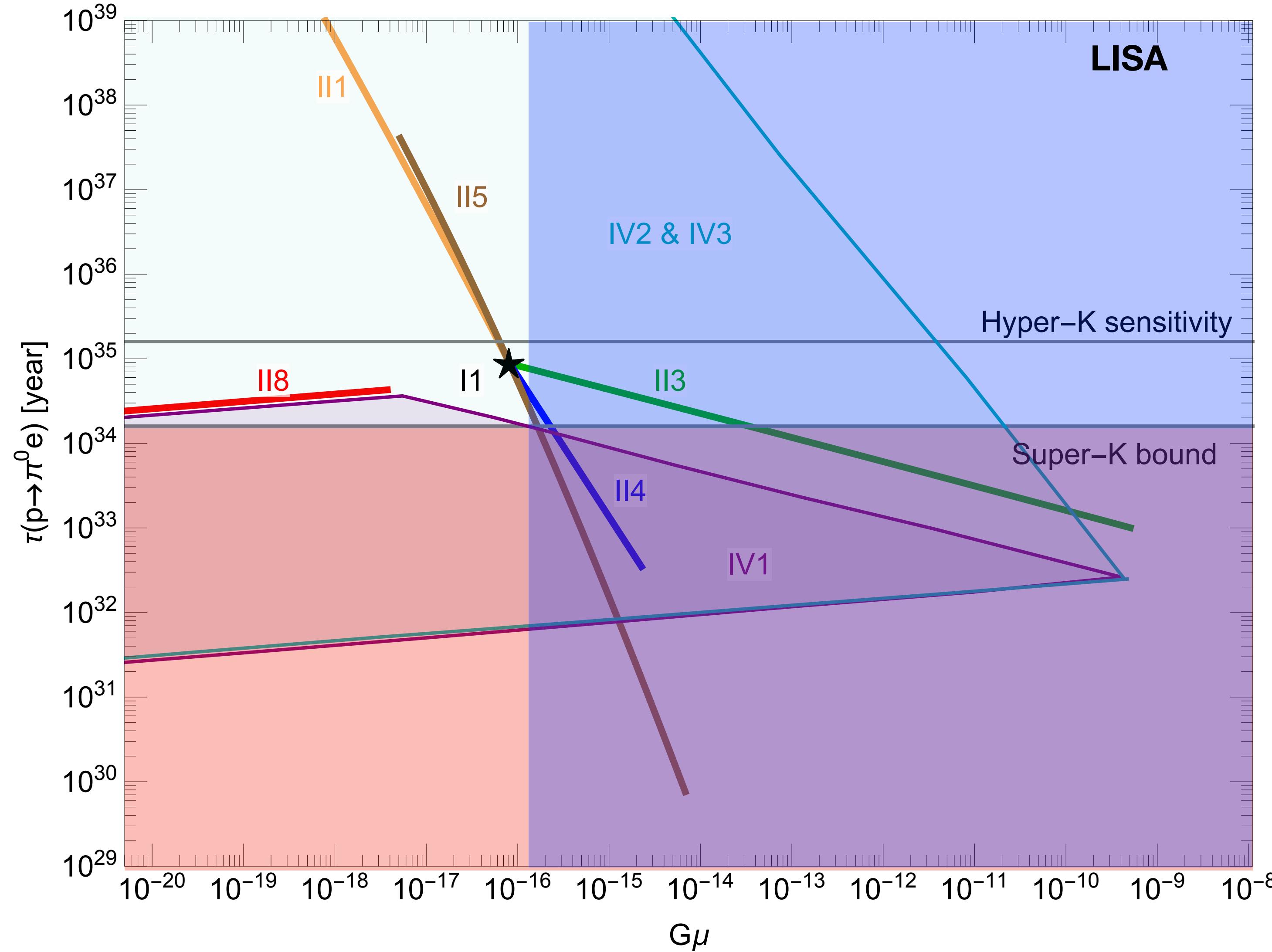
$$M_1^2 \sim g_1^2 v^2 \implies G\mu = \frac{1}{M_{\text{Pl}}^2} \frac{2\pi M_1^2}{g_1^2} = \frac{\cancel{M_1^2}}{2\cancel{\alpha_1} M_{\text{Pl}}^2}$$

Determined from RGE

Correlation of GW and PD signals



Correlation of GW and PD signals



Correlation of GW and PD signals

- non-SUSY SO(10) Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18} - 6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18} - 1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

- If HyperK **does not observe PD** \Rightarrow 9 chains excluded
- **8 survivors!** If we observe GW signal **larger than upper bounds** \Rightarrow exclude those breaking chains
- If we observe PD $\Rightarrow M_1$ determined so is GW signal. Correlations between observables matter and need to be compared on case by case basis.

**Testable by LIGO,
DECIGO, AEDGE,
C, ET, MAGIS..**

Correlation of GW and PD signals

- non-SUSY SO(10) Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K lower bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18} - 6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18} - 1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Study specific breaking chain 2209.00021

Why? **Can be tested by Hyper-K & has an associated GW signal**

SO(10) Model confronting data

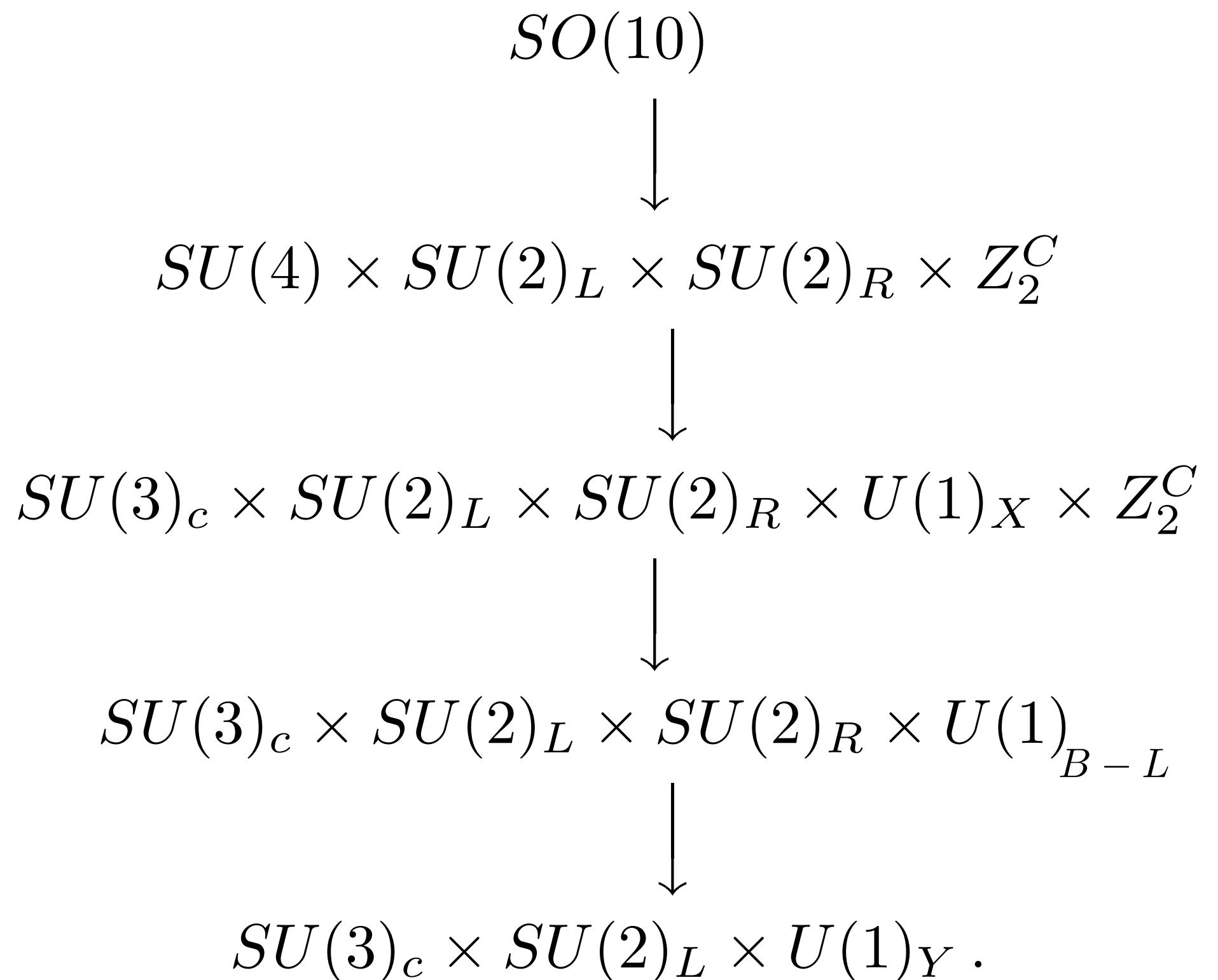
- Treatment has been **model-independent**

SO(10) Model confronting data

- Treatment has been **model-independent**
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** [2209.00021](#)

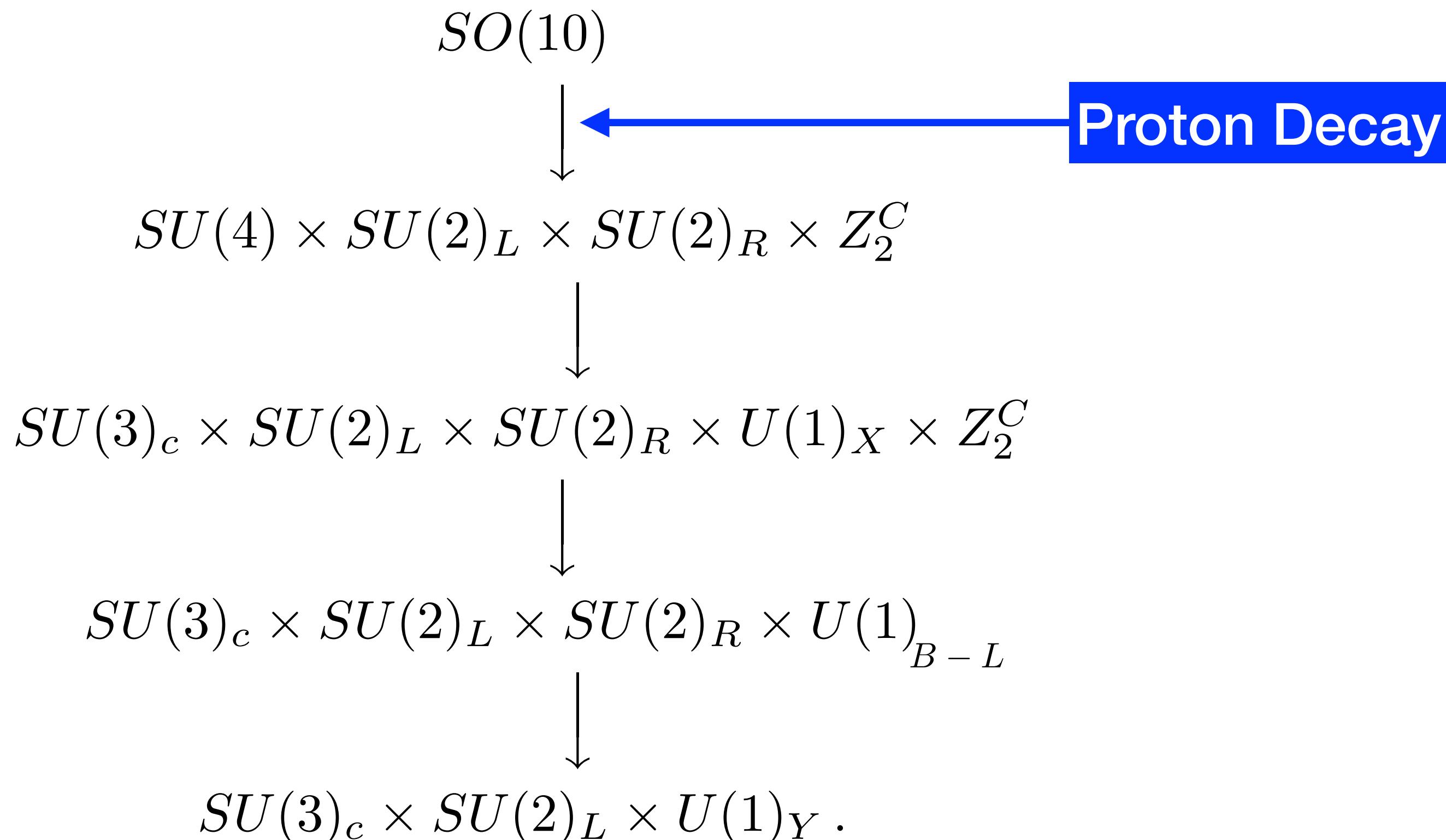
$SO(10)$ Model confronting data

- Treatment has been **model-independent**
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** [2209.00021](#)



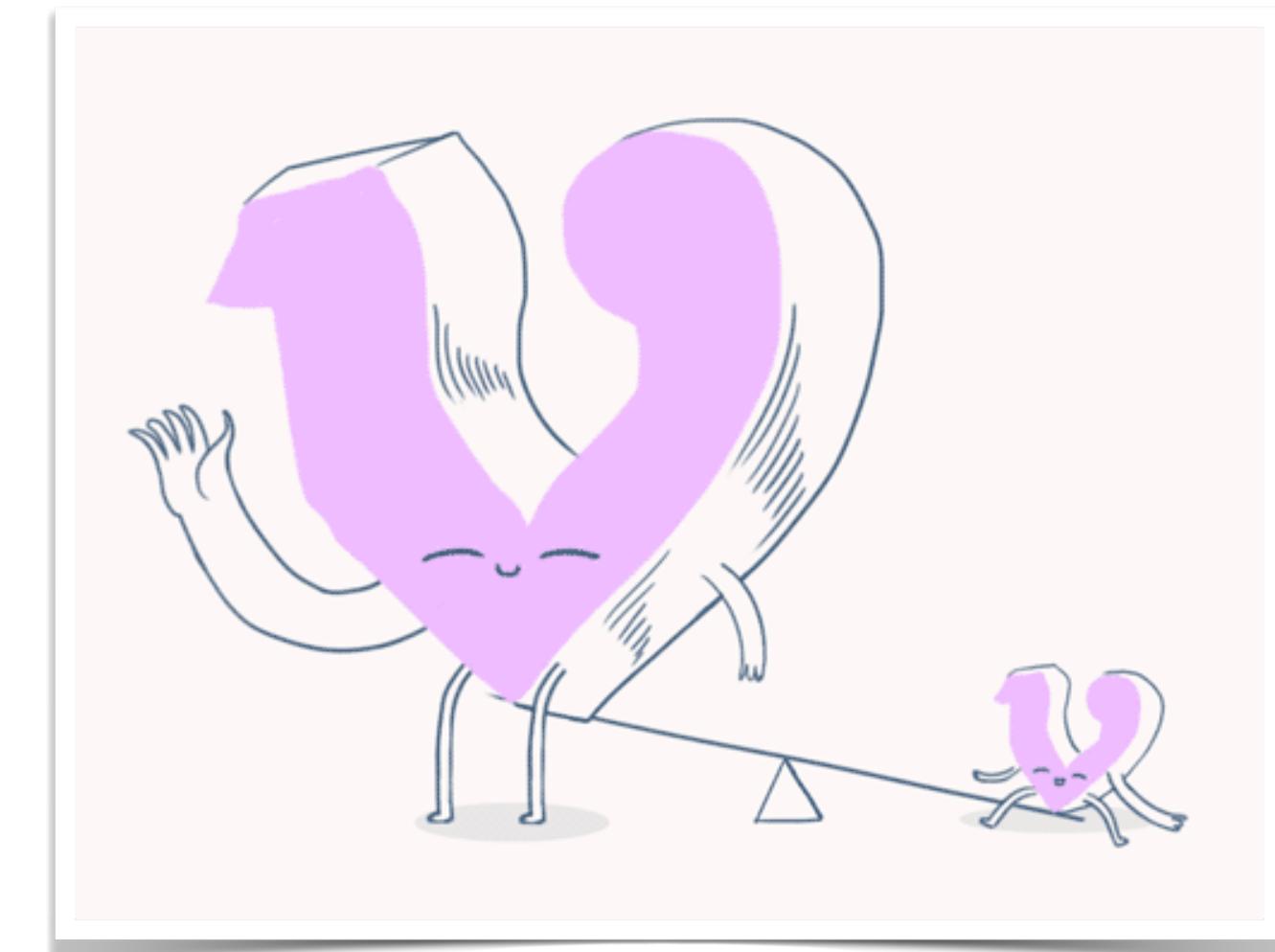
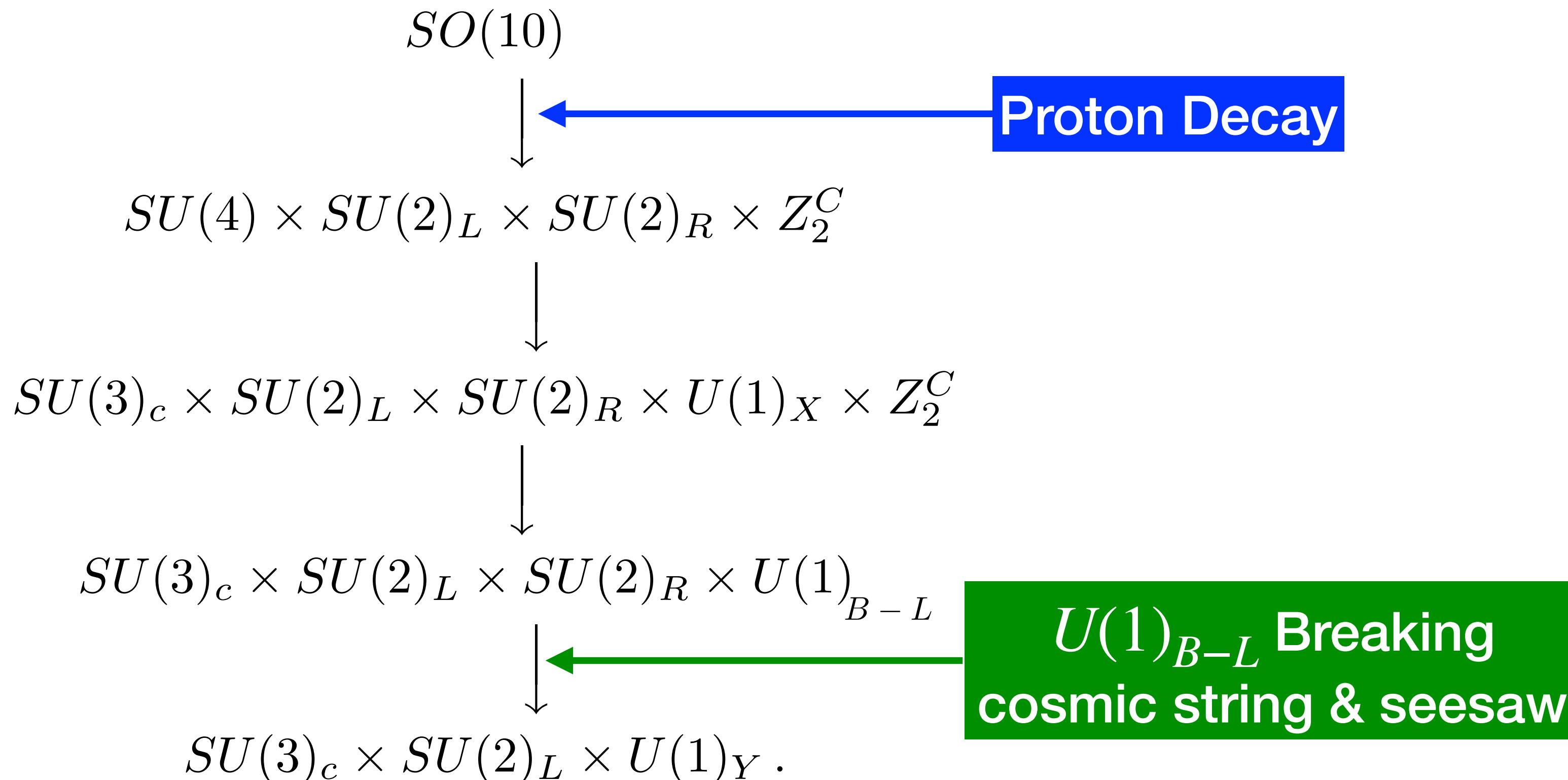
SO(10) Model confronting data

- Treatment has been **model-independent**
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** [2209.00021](#)



$SO(10)$ Model confronting data

- Treatment has been **model-independent**
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou 2209.00021**



SO(10) Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

SO(10) Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{\mathbf{126}}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{\mathbf{120}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

- After breaking to SM:

$$Y_u \bar{Q} \tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L} \tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{h.c.}$$

SO(10) Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

- After breaking to SM:

$$Y_u \bar{Q} \tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L} \tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{h.c.}$$

$$Y_u = Y_{10} V_{11}^* + \frac{1}{\sqrt{3}} Y_{\overline{126}} V_{12}^* + Y_{120} \left(V_{13}^* + \frac{1}{\sqrt{3}} V_{14}^* \right)$$

SM up Yukawa



$SO(10)$ Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

- After breaking to SM:

$$Y_u \bar{Q} \tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L} \tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{h.c.}$$

$$Y_u = Y_{10} V_{11}^* + \frac{1}{\sqrt{3}} Y_{\overline{126}} V_{12}^* + Y_{120} \left(V_{13}^* + \frac{1}{\sqrt{3}} V_{14}^* \right)$$

GUT Yukawa Parameter

SM up Yukawa

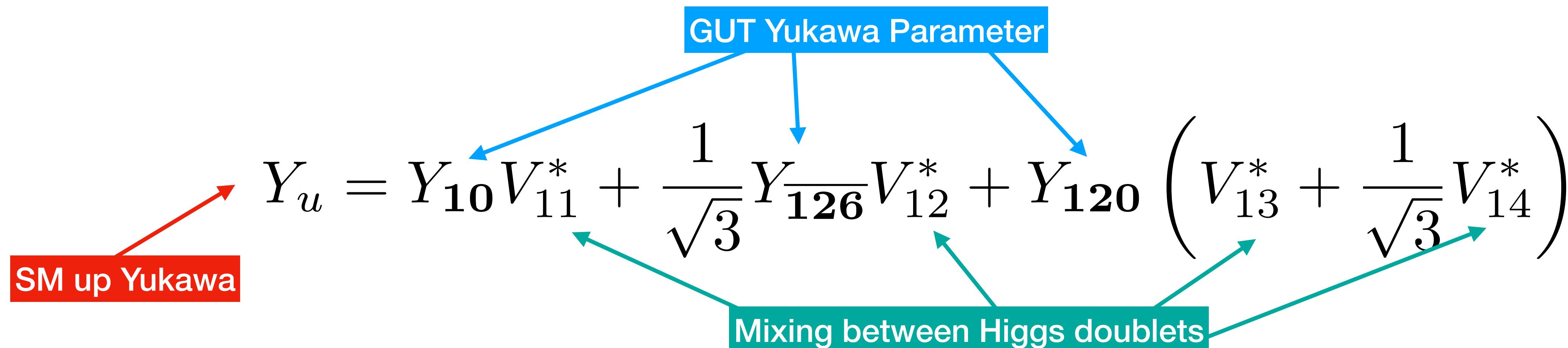
$SO(10)$ Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

- After breaking to SM:

$$Y_u \bar{Q} \tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L} \tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{h.c.}$$



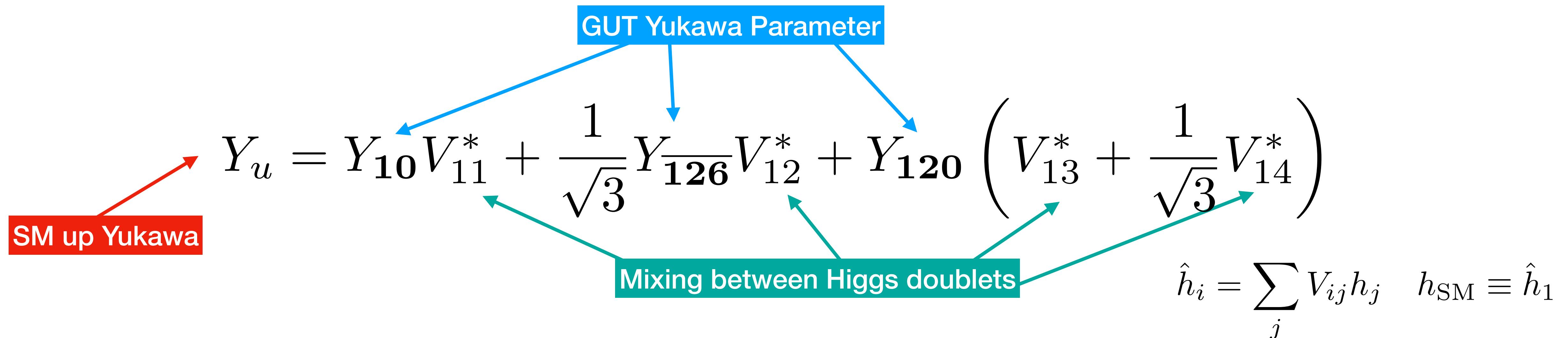
$SO(10)$ Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

- After breaking to SM:

$$Y_u \bar{Q} \tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L} \tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{h.c.}$$



$SO(10)$ Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{10}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{126}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{120}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

$$Y_u = Y_{10} V_{11}^* + \frac{1}{\sqrt{3}} Y_{\overline{126}} V_{12}^* + Y_{120} \left(V_{13}^* + \frac{1}{\sqrt{3}} V_{14}^* \right)$$

$$Y_d = Y_{10} V_{15} + \frac{1}{\sqrt{3}} Y_{\overline{126}} V_{16} + Y_{120} \left(V_{17} + \frac{1}{\sqrt{3}} V_{18} \right)$$

$$Y_\nu = Y_{10} V_{11}^* - \sqrt{3} Y_{\overline{126}} V_{12}^* + Y_{120} \left(V_{13}^* - \sqrt{3} V_{14}^* \right)$$

$$Y_e = Y_{10} V_{15} - \sqrt{3} Y_{\overline{126}} V_{16} + Y_{120} \left(V_{17} - \sqrt{3} V_{18} \right).$$

SO(10) Model confronting data

- Model of [Altarelli & Blankenburg](#)
- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{\mathbf{126}}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{\mathbf{120}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

- Right handed neutrino masses predicted & generate light neutrino masses via seesaw

$$Y_{\overline{\mathbf{126}}} \bar{\nu}_R \phi_S \nu_R^c \implies M_{\nu_R} = Y_{\overline{\mathbf{126}}} v_S \implies M_\nu = \frac{Y_\nu Y_\nu^T v_{\text{SM}}^2}{M_{\nu_R}}$$

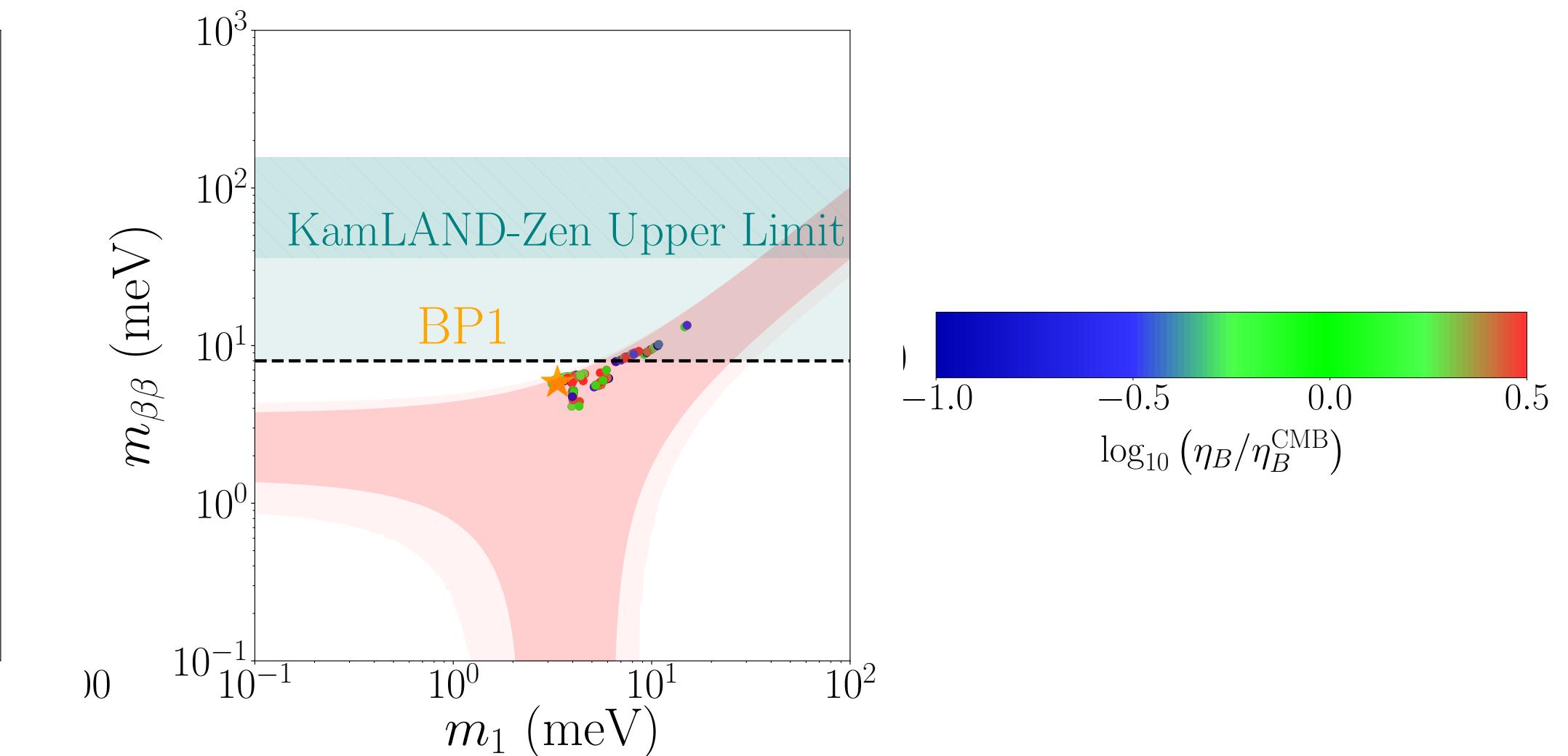
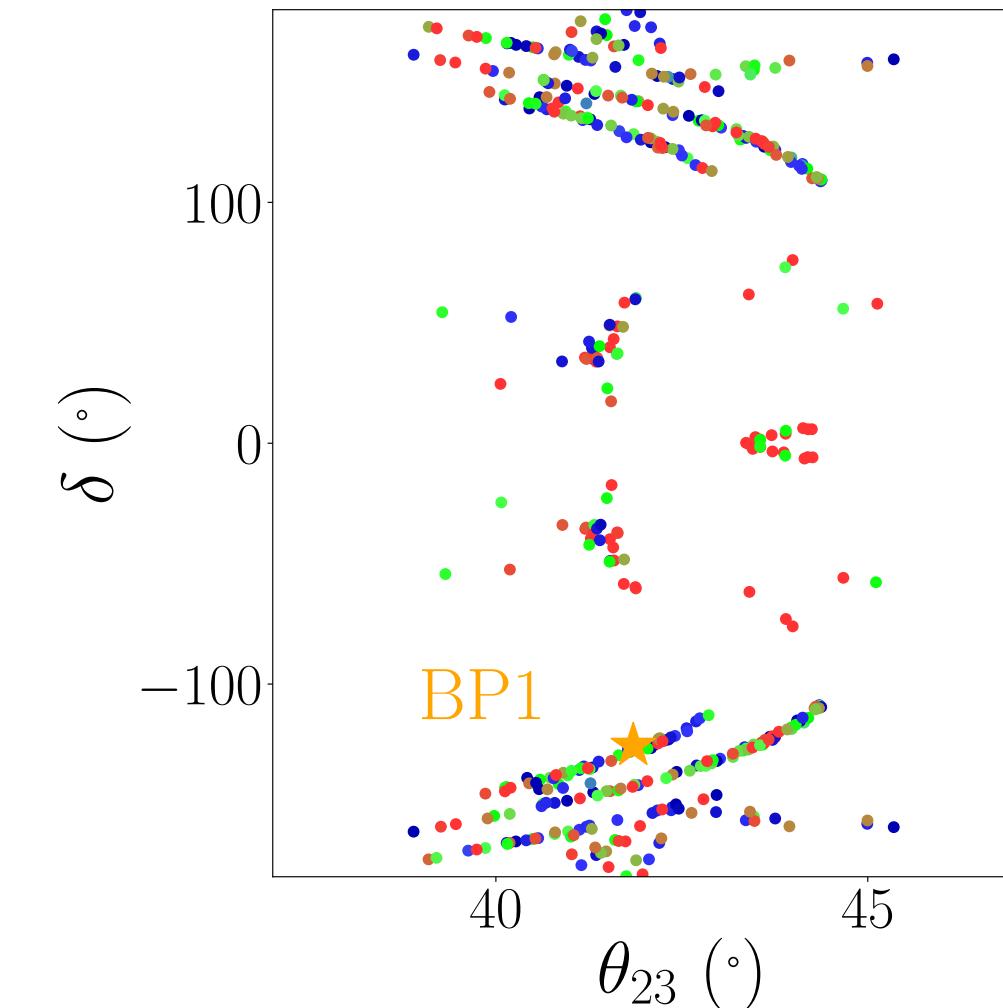
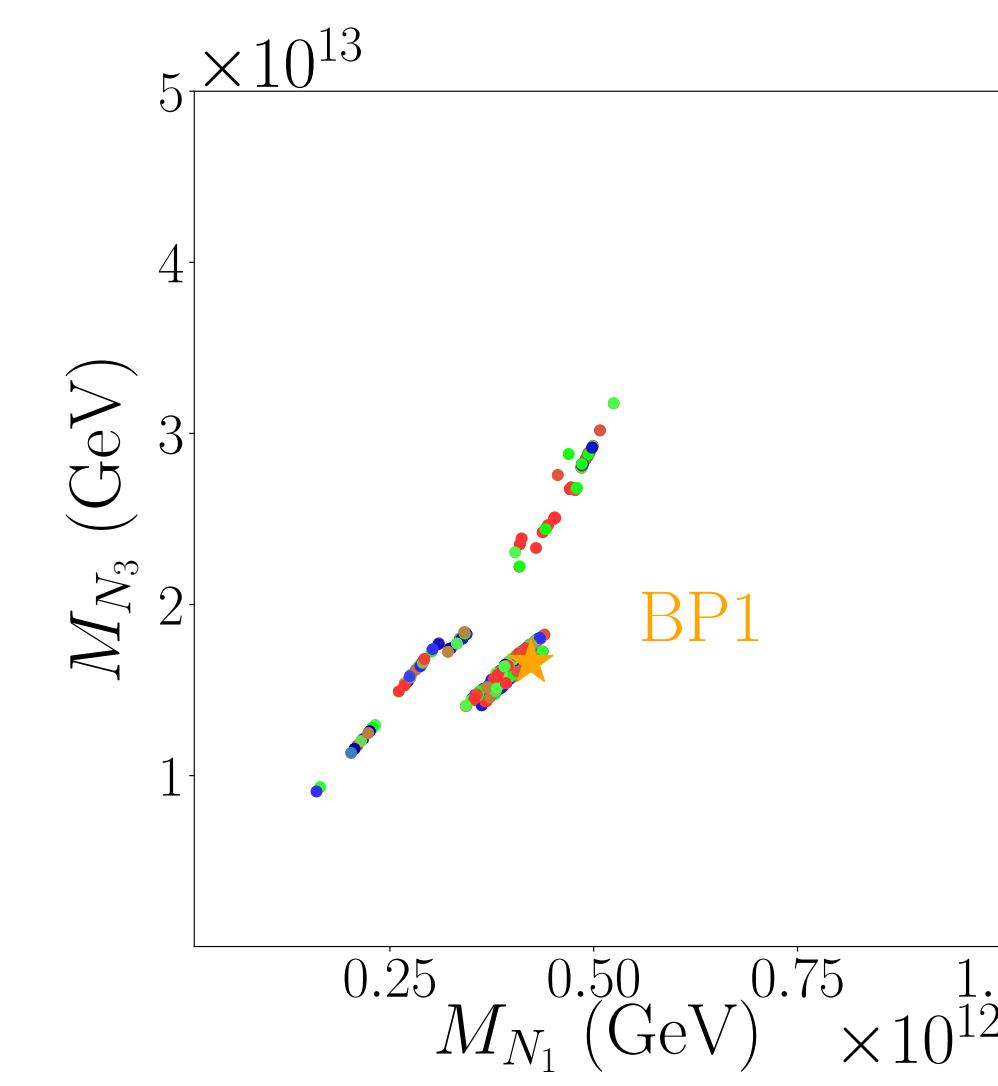
- RHN masses and Y_ν predicted \implies leptogenesis parameter space fixed and η_B predicted

SO(10) Model confronting data

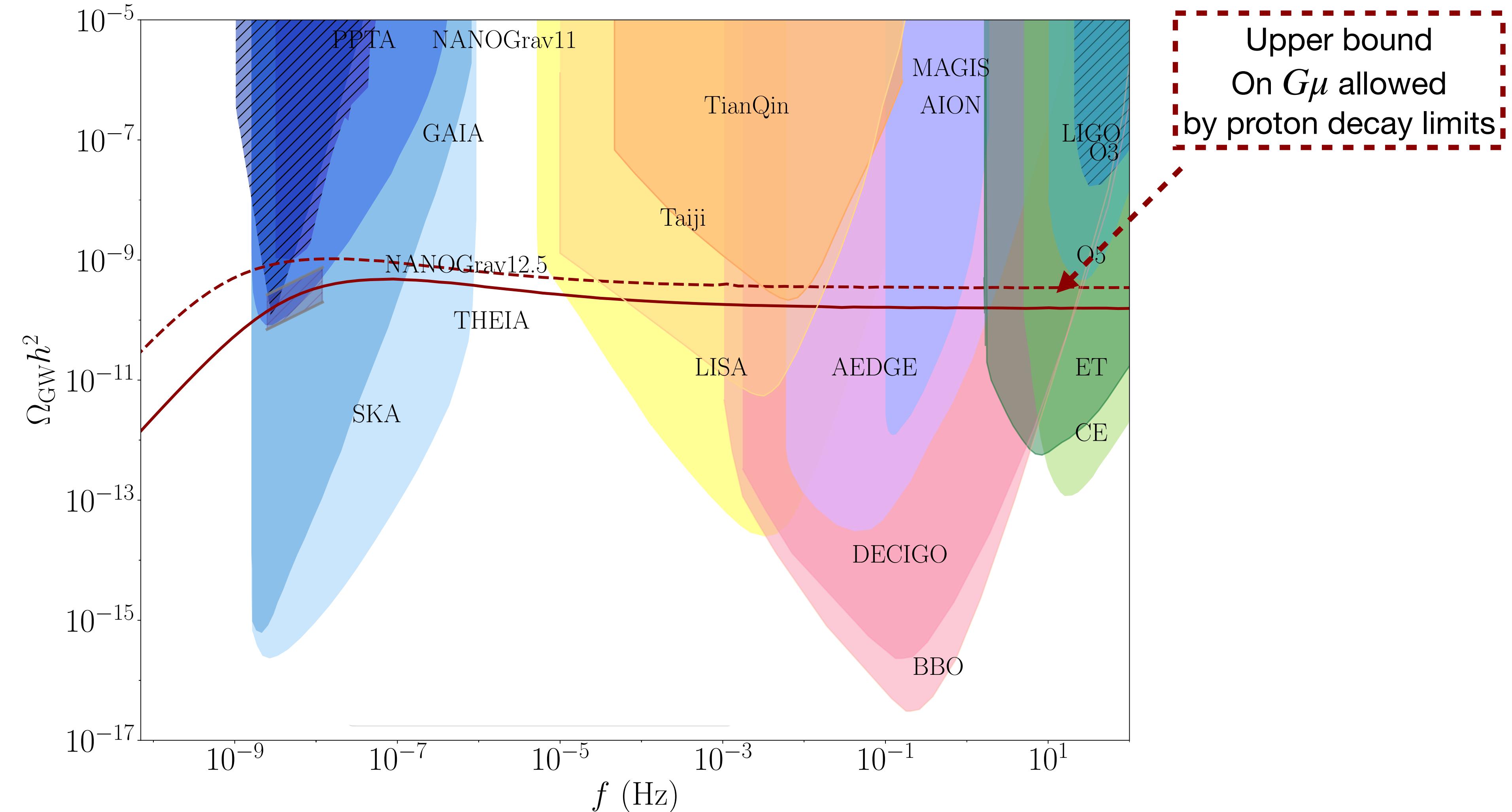
- **Input:** quark masses, mixing parameters, charged lepton Yukawa matrix
- **Theory Model parameters:** $\mathcal{P}_m \in \left\{ a_1, a_2, c_\nu, m_0, \eta_{q_{u,c,t,d,s,b}} \right\}$
- **Output:** predictions for $\mathcal{O}_m \in \left\{ \theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}, \Delta m_{21}^2, \Delta m_{31}^2 \right\}$
- **BP1** consistent with all flavour data, $\eta_B \sim \eta_{B_{cmb}}$ ($\chi^2 < 10$) & testable by HyperK. GW and PD calculated same as before

SO(10) Model confronting data

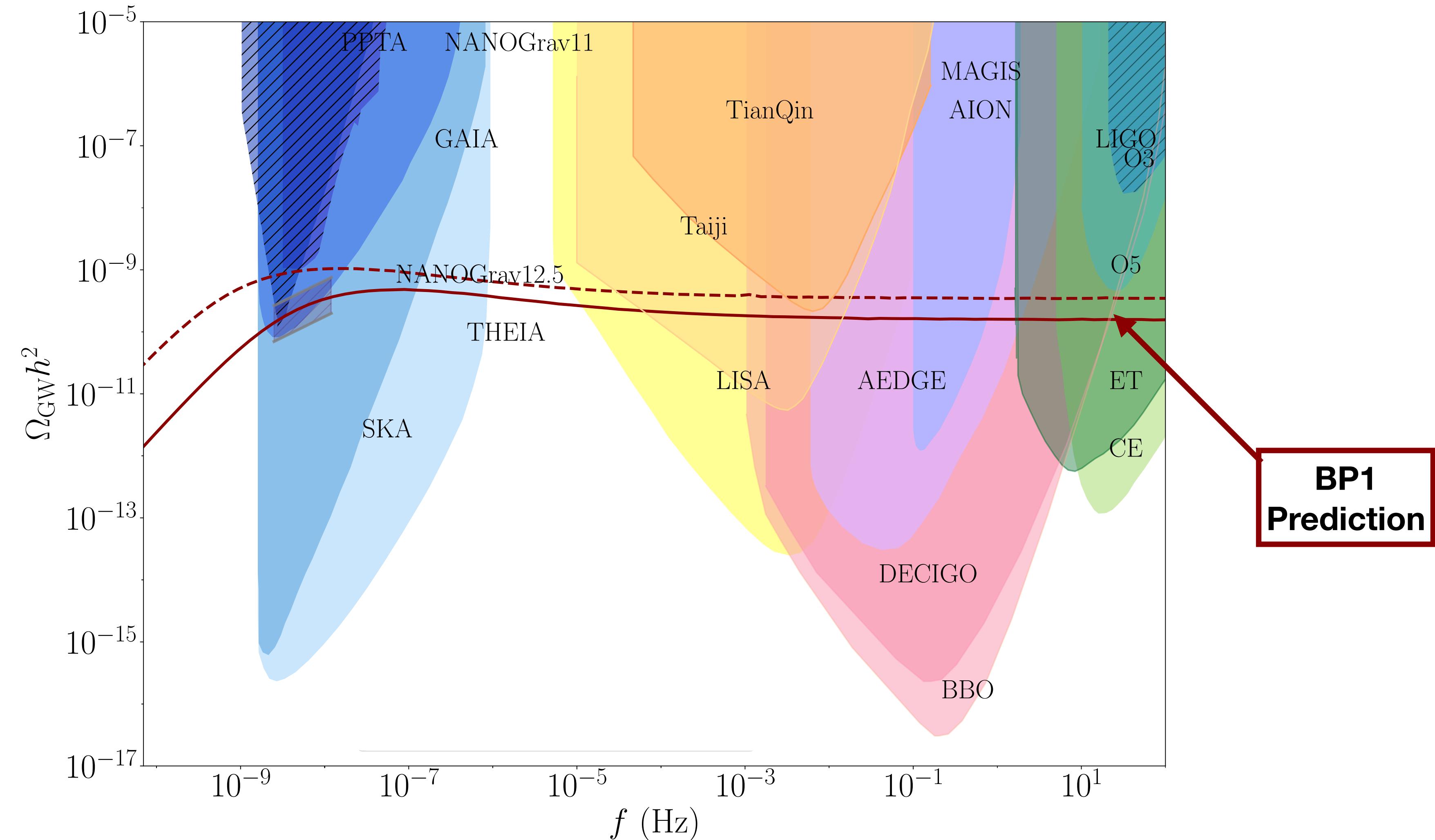
- **Input:** quark masses, mixing parameters, charged lepton Yukawa matrix
- **Theory Model parameters:** $\mathcal{P}_m \in \{a_1, a_2, c_\nu, m_0, \eta_{q_{u,c,t,d,s,b}}\}$
- **Output:** predictions for $\mathcal{O}_m \in \{\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}, \Delta m_{21}^2, \Delta m_{31}^2\}$
- **BP1** consistent with all flavour data, $\eta_B \sim \eta_{B_{cmb}}$ ($\chi^2 < 10$) & testable by HyperK. GW and PD calculated same as before



SO(10) Model confronting data

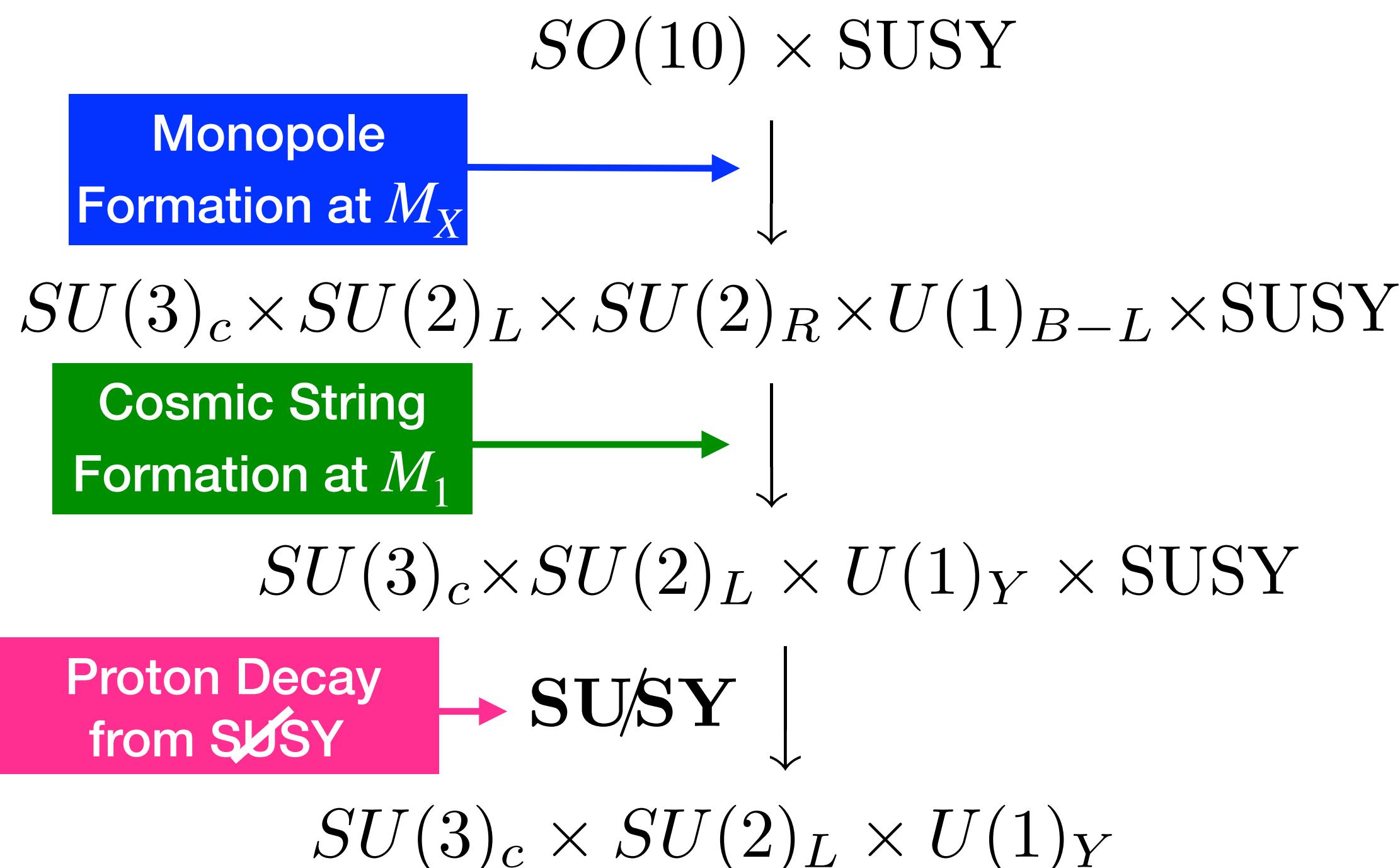


SO(10) Model confronting data



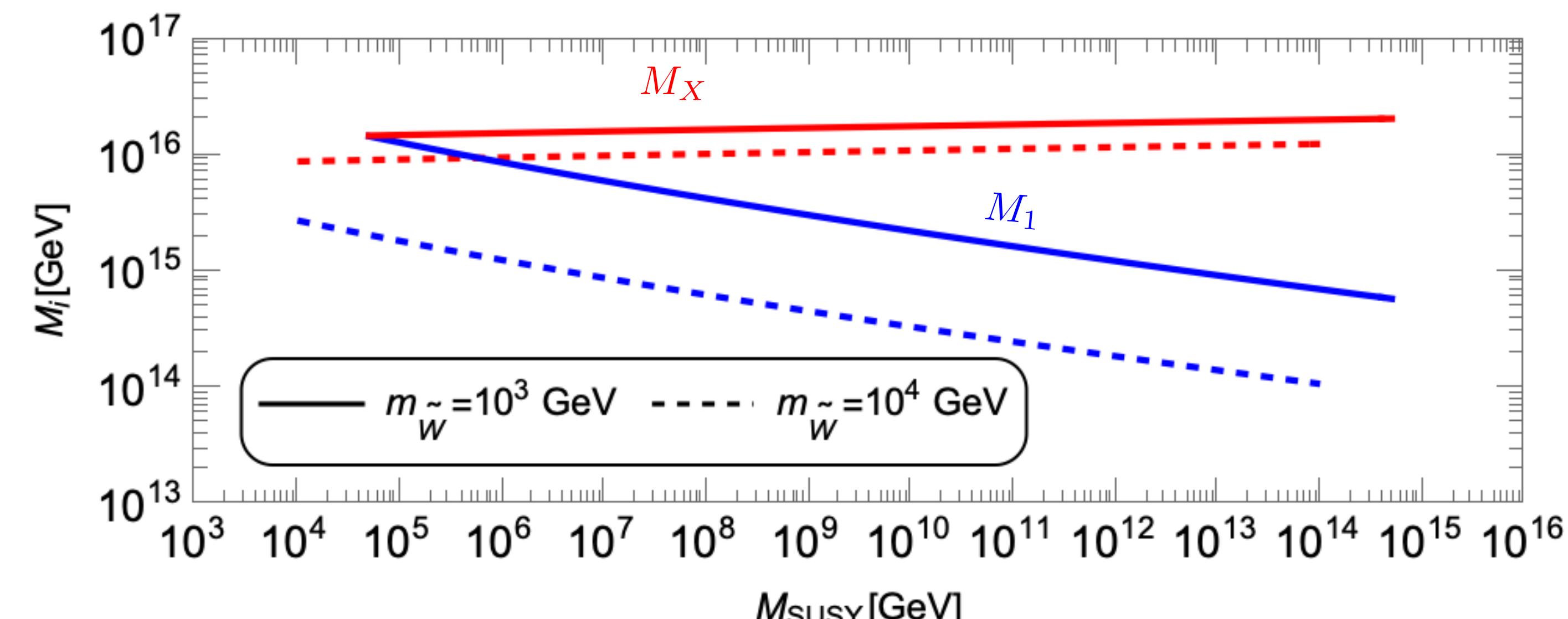
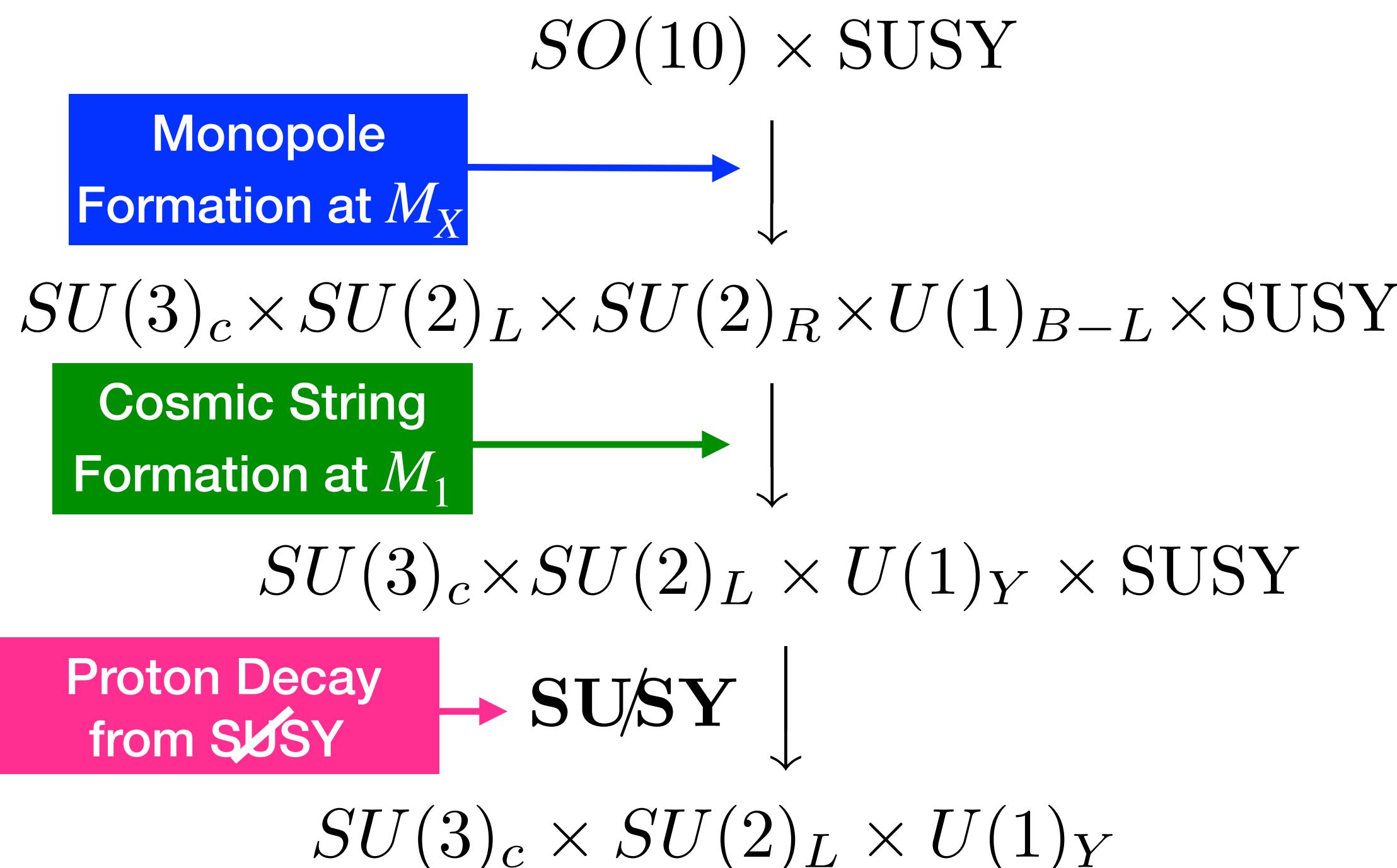
SUSY SO(10) Model confronting data

- Many GUTs are supersymmetric: gauge unification achieved in MSSM $\sim 10^{16}$ GeV
- In [2308.05799](#) we investigated a SUSY SO(10) GUT. See also [2307.04595](#)
- Calculations very similar to previous but now we used [MultiNest](#) for fit of theory to flavour data & η_B



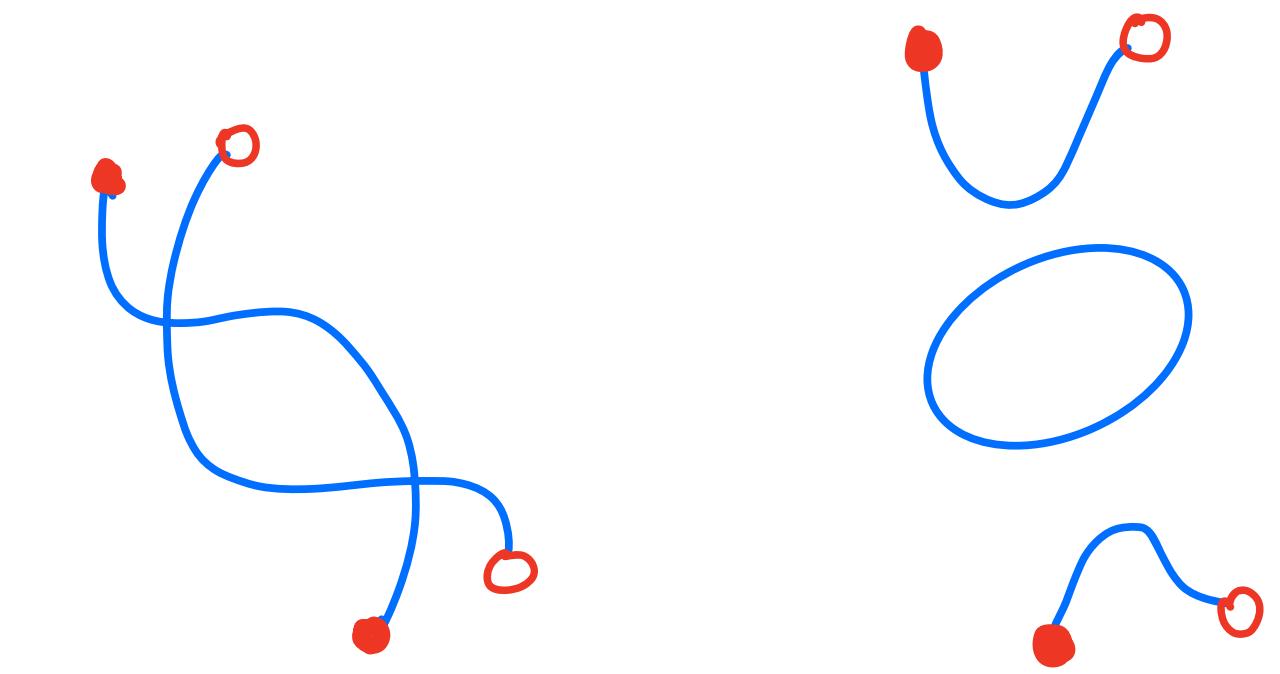
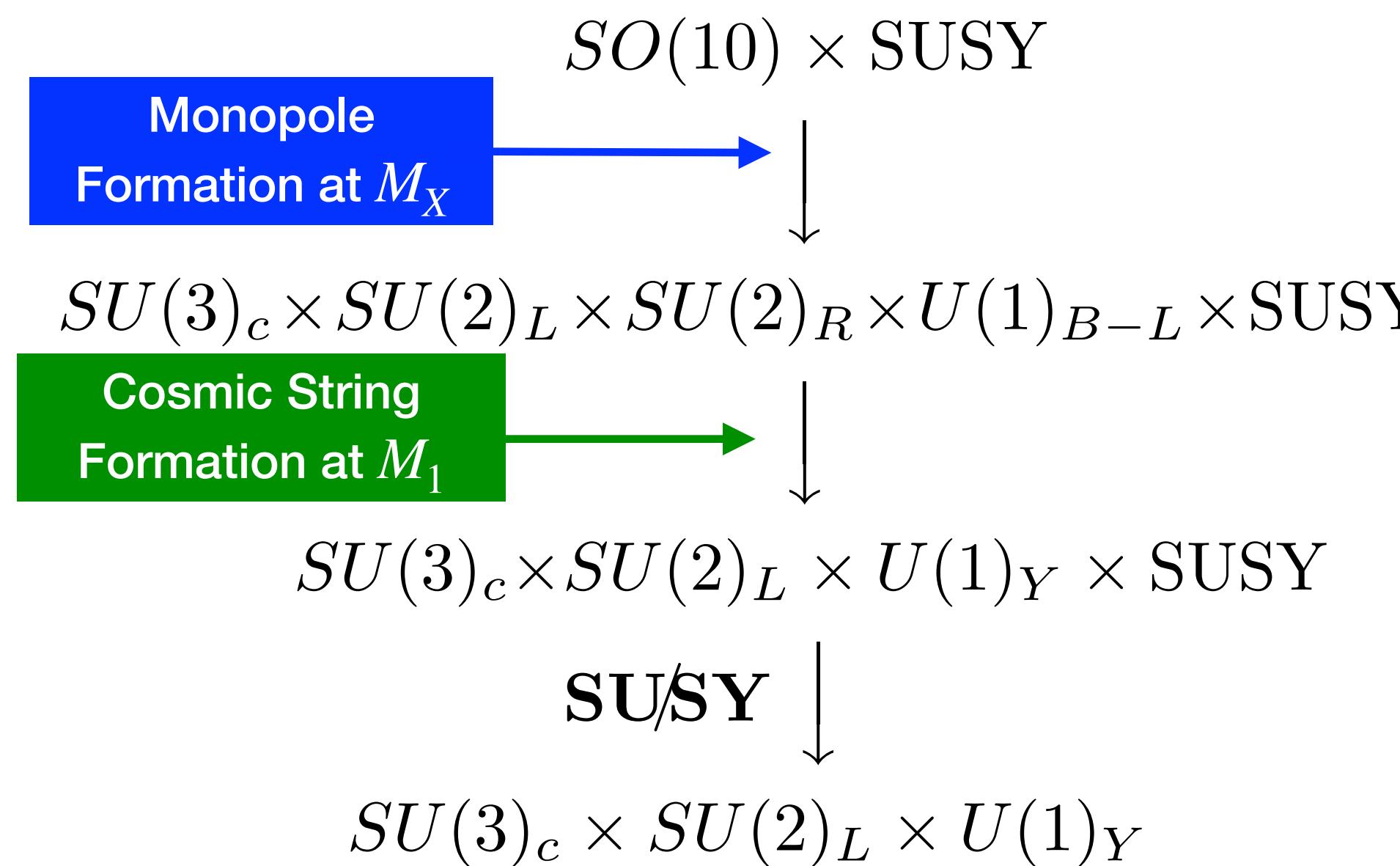
SUSY SO(10) Model confronting data

- Many GUTs are supersymmetric: gauge unification achieved in MSSM $\sim 10^{16}$ GeV
- In [2308.05799](#) we investigated a SUSY SO(10) GUT. See also [2307.04595](#)
- Calculations very similar to previous but now we used [MultiNest](#) for fit of theory to flavour data & η_B



Lower SUSY breaking \implies higher the
GUT and B-L breaking scale

SUSY SO(10) Model confronting data



$m_{\text{mono}}^2 \sim \mu \implies$ monopoles & antimonopoles can nucleate on string & annihilate \implies **metastable string**

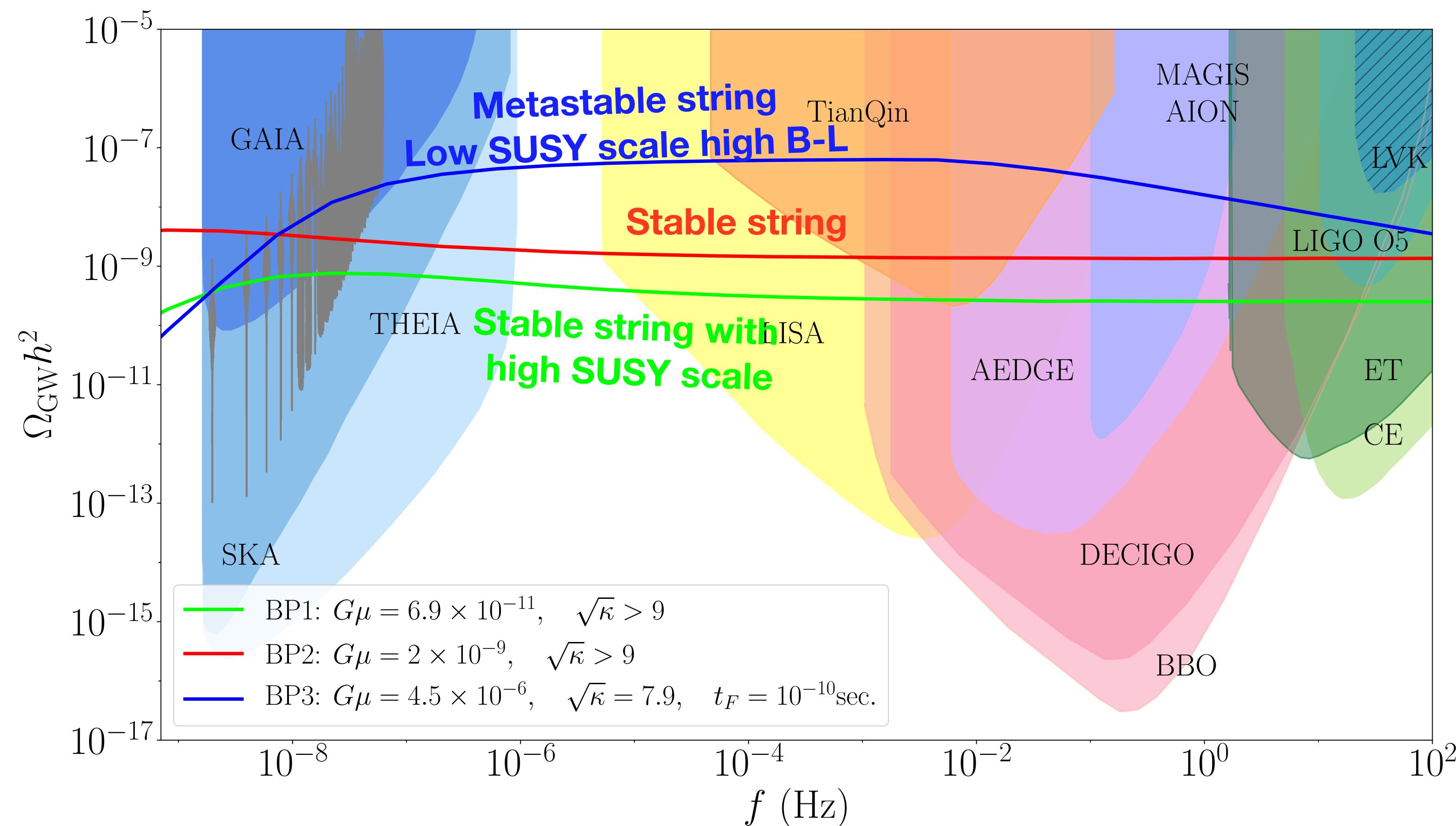
$$\Gamma_d = \frac{\mu}{2\pi} \exp(-\pi\kappa)$$

$$\kappa = \frac{m_{\text{mono}}^2}{\mu} \approx \frac{M_X^2}{\alpha_X \mu} = \frac{M_X^2}{\alpha_X M_1^2}$$

Vilenkin [1982],
Leblond, Shlaer, Siemens [2009],
Monin & Voloshin [2009],
Buchmuller, Domcke, Schmitz [2021]

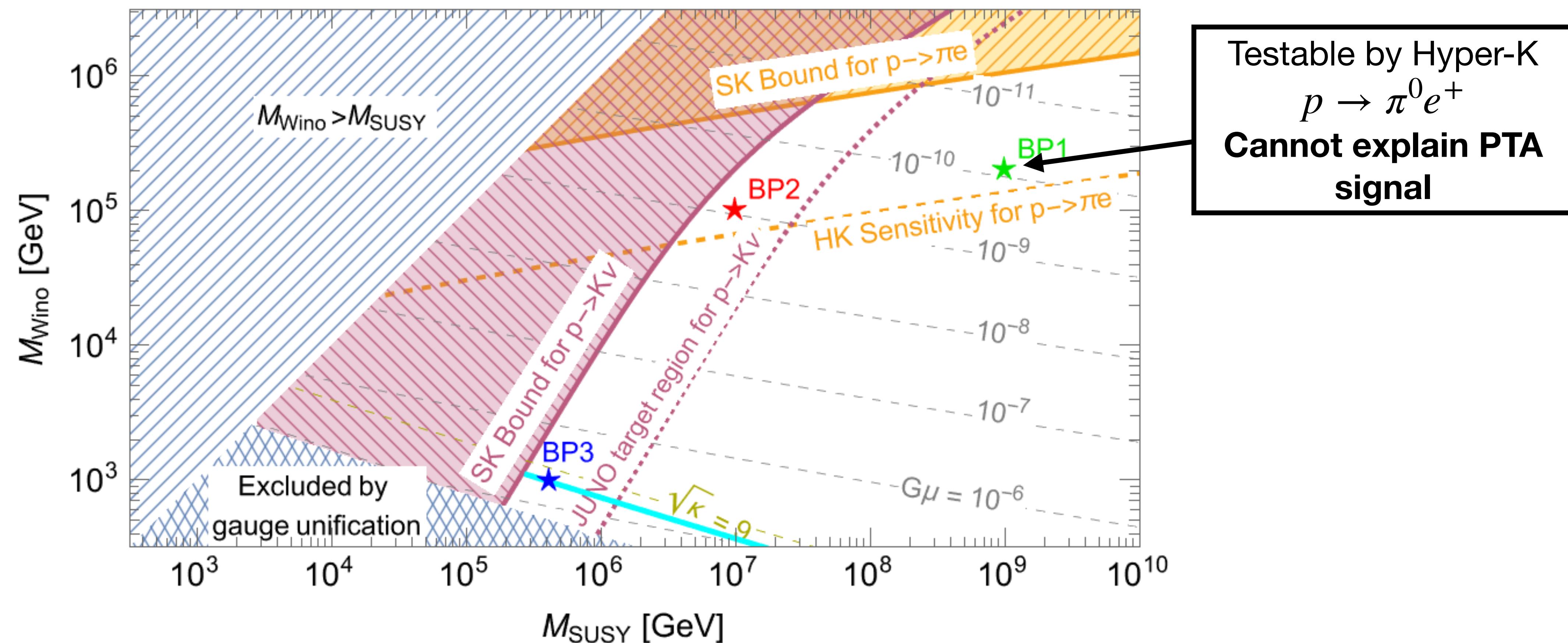
SUSY SO(10) Model confronting data

- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5 M_1$
- Achievable with SUSY but very hard in non-SUSY set up



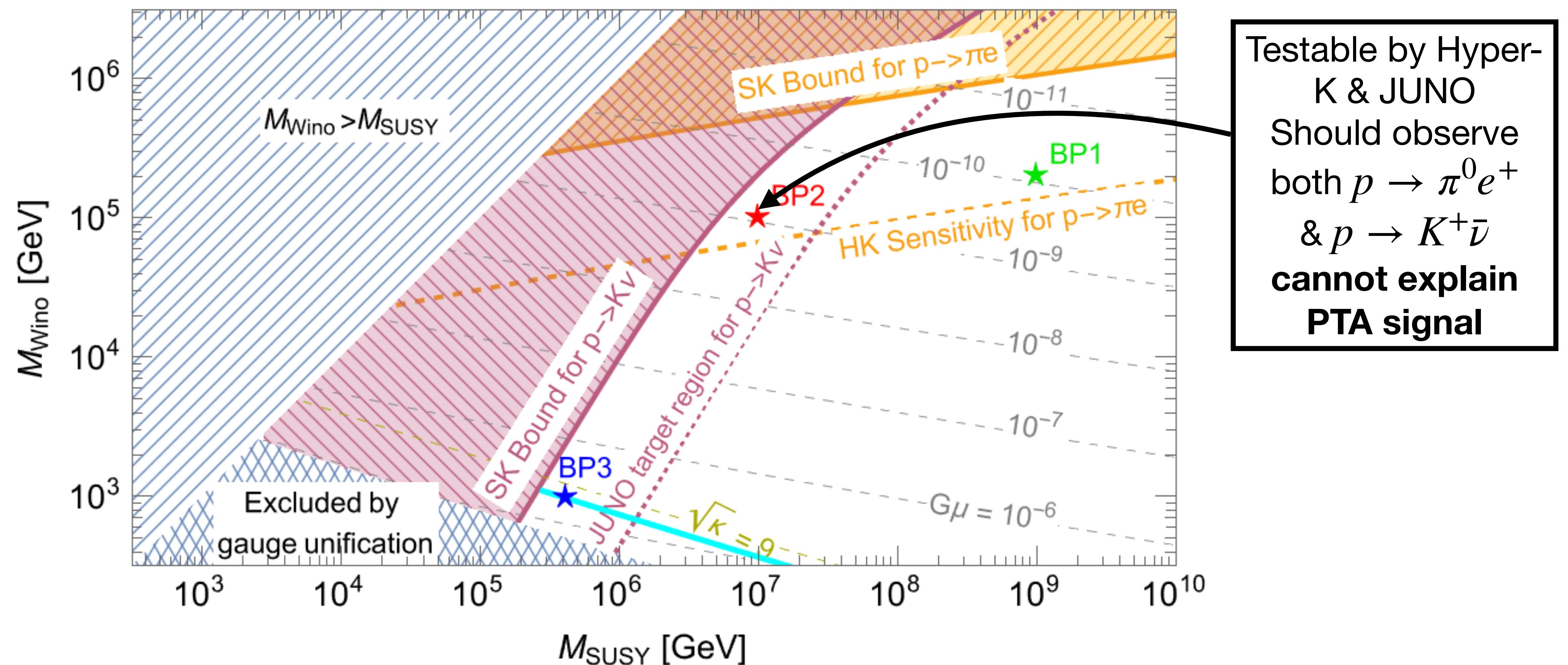
SUSY SO(10) Model confronting data

- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5 M_1$
- Achievable **with SUSY** but hard in non-SUSY set up



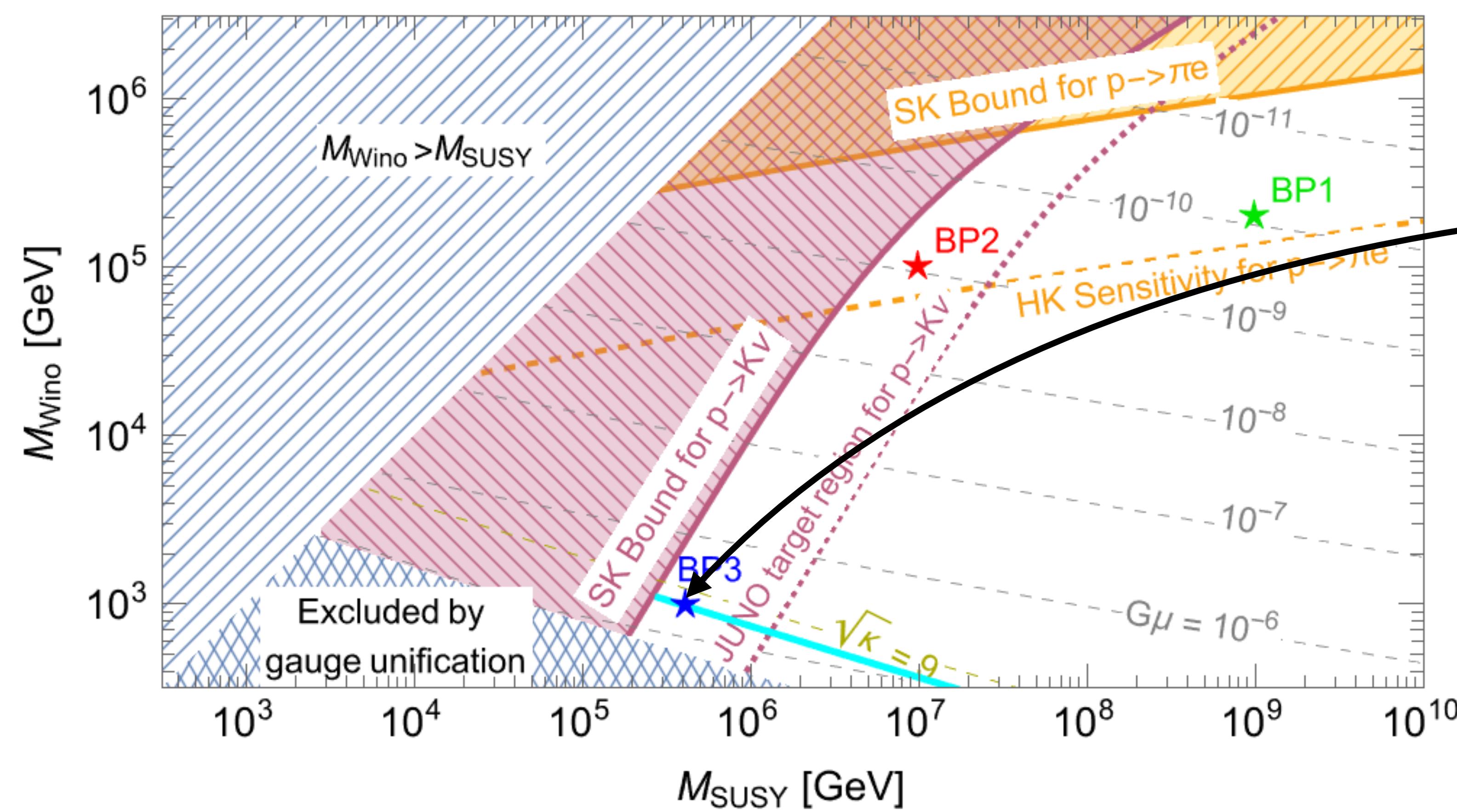
SUSY SO(10) Model confronting data

- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5 M_1$
- Achievable **with SUSY** but hard in non-SUSY set up



SUSY SO(10) Model confronting data

- Metastable strings are one possible explanation of the observed recent signal from Pulsar Timing Arrays. Signal requires $\sqrt{\kappa} \sim 7.9 \implies M_X \sim 1.5 M_1$
- Achievable **with SUSY** but hard in non-SUSY set up



Testable by Hyper-K & JUNO
Should observe both $p \rightarrow \pi^0 e^+$ & $p \rightarrow K^+ \bar{\nu}$
Can explain PTA signal

Summary

- GUTs generically predict nucleon decay and the formation of topological defects. Interplay of these observables is a powerful way of constraining GUTs.
- Coming decade is an exciting time for GUTs as neutrino and GW experiments will constrain nucleon decay, the presence of GWs and neutrinoless double beta decay ($0\nu\beta\beta$).
- Studied non-SUSY & SUSY SO(10) breaking chains which can be tested by Hyper-K, GW detectors and $0\nu\beta\beta$.
- Parameter space consistent with fermionic masses and mixing & successful leptogenesis.
- SUSY SO(10) has some parameter space available to explain PTA signal and much of the PS can be tested by JUNO. PTA signal may be a constraint on non-SUSY GUTs and constraint many of them.

“we have entered an exciting era where new observations of GWs from the heavens and proton decay experiments from under the Earth can provide complementary windows to reveal the details of the unification of matter and forces at the highest energies.”

The background image shows the historic Durham Cathedral, a large Gothic structure with multiple towers and spires, situated on a hillside overlooking the River Wear. In the foreground, there's a stone building with a red-tiled roof, likely a mill or a house from the 18th century. The riverbank is lined with trees, some of which have autumn-colored leaves. The sky is clear and blue.

Thank you for listening

Renormalisation Group Equations

Beta function coefficients 1 and 2-loop respectively

$$b_i = -\frac{11}{3}C_2(H_i) + \frac{2}{3}\sum_F T(F_i) + \frac{1}{3}\sum_S T(S_i),$$

$$b_{ij} = -\frac{34}{3}[C_2(H_i)]^2\delta_{ij} + \sum_F T(F_i)[2C_2(F_j) + \frac{10}{3}C_2(H_i)\delta_{ij}] + \sum_S T(S_i)[4C_2(S_j) + \frac{2}{3}C_2(H_i)\delta_{ij}],$$

Two-loop RGE equation [Bertolini, di Luzio, Malinsky](#)

$$\alpha_i(\mu)^{-1} = \alpha_i(\mu_0)^{-1} - \frac{b_i}{2\pi} \log \frac{\mu}{\mu_0} + \sum_j \frac{b_{ij}}{4\pi b_i} \log \left(1 - b_j \alpha_j(\mu_0) \log \frac{\mu}{\mu_0} \right),$$

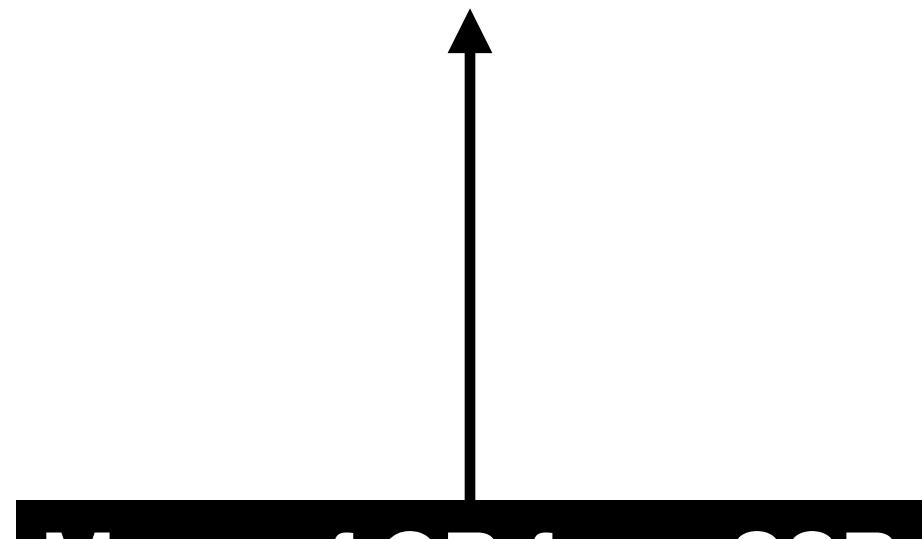
Matching condition

$$H_i \rightarrow H_j, \quad \frac{1}{\alpha_{H_i}(M_I)} - \frac{C_2(H_i)}{12\pi} = \frac{1}{\alpha_{H_j}(M_I)} - \frac{C_2(H_j)}{12\pi}.$$

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales \Rightarrow PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

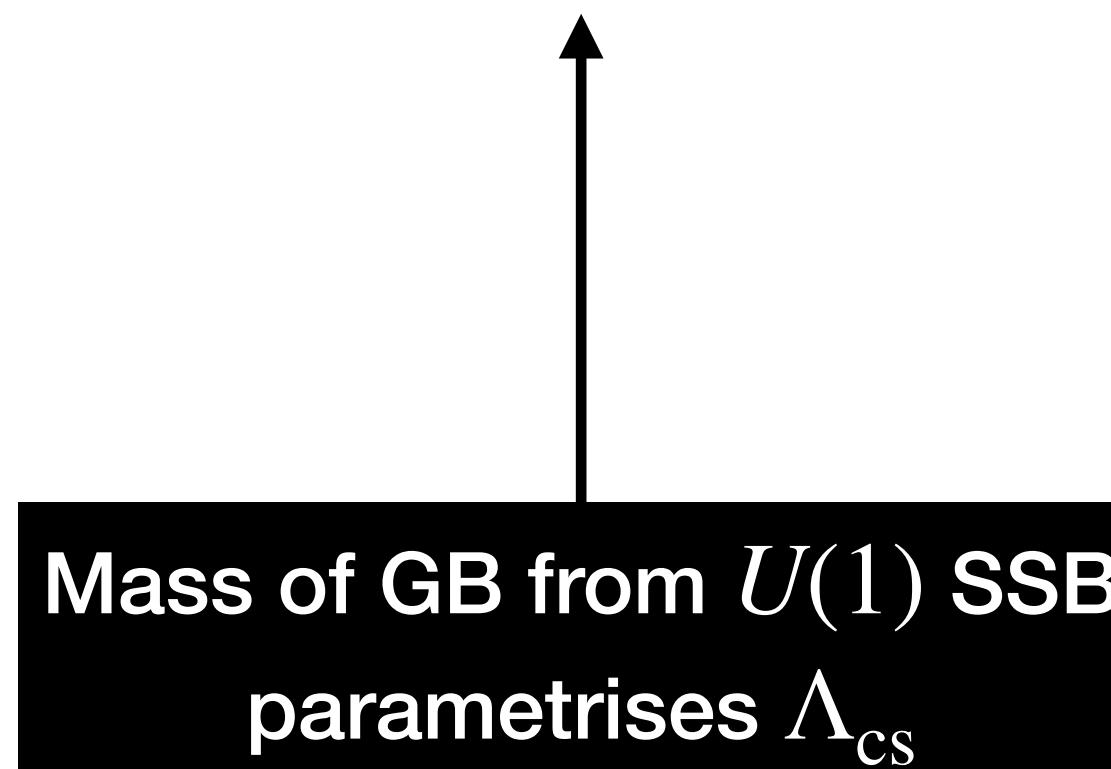
$$I1: SO(10) \xrightarrow{M_X} G_{3221} \xrightarrow{M_1} G_{SM}$$



- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales \implies PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

$$\text{I1: } SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$



- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales \implies PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

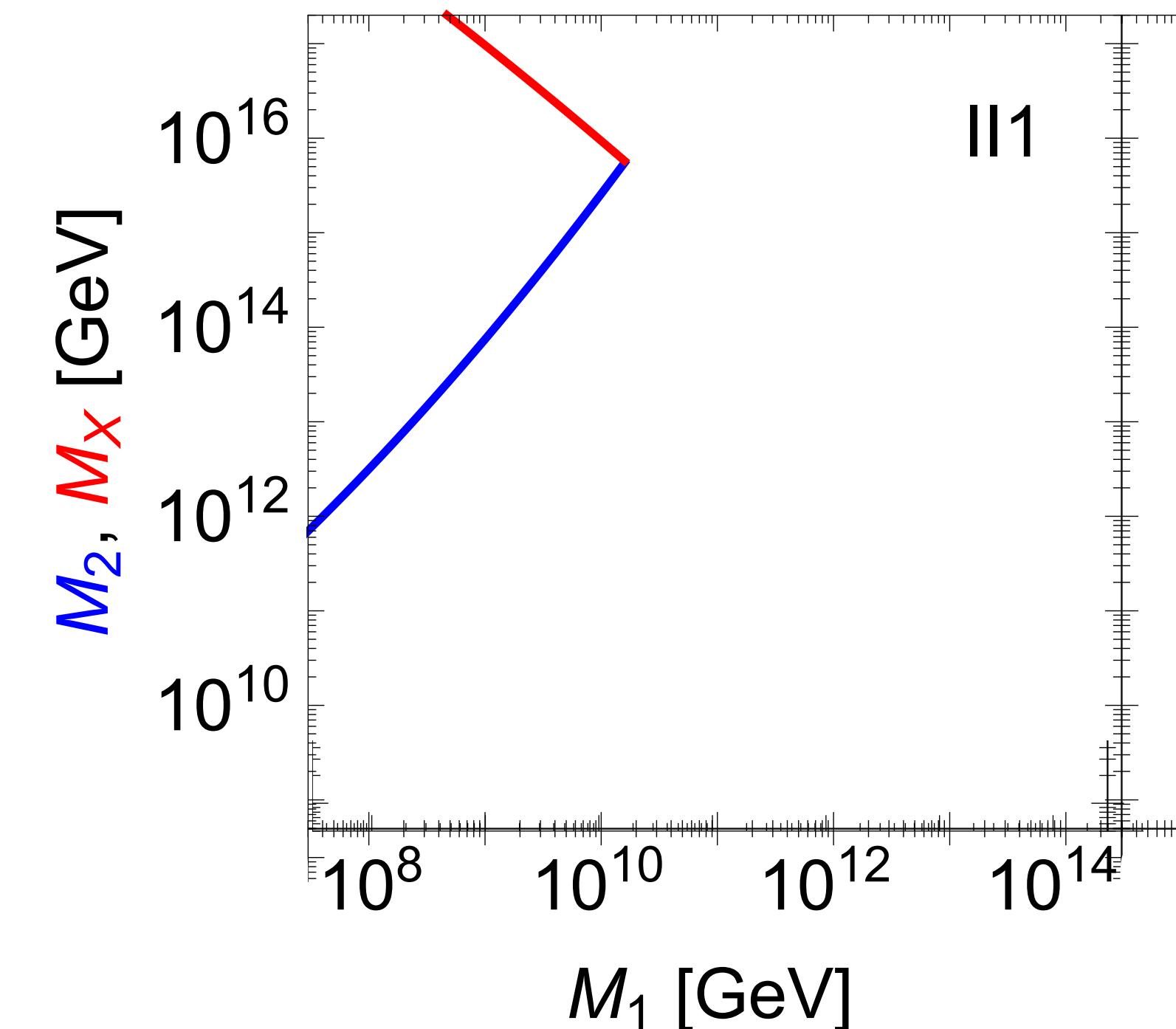
$$\text{I1: } SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$

Chains	M_X [GeV]	M_1 [GeV]
I1	5.660×10^{15}	1.617×10^{10}
I2	1.410×10^{15}	8.630×10^{10}
I3	2.902×10^{14}	1.634×10^{11}
I4	3.500×10^{16}	4.368×10^9
I5	2.722×10^{14}	1.143×10^{13}
I6	excluded	

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales \implies PD rate and GW signal

Breaking chains with **two intermediate scales** can have a range of scales

$$\text{II1 : } SO(10) \xrightarrow[M_X]{} G_{422} \xrightarrow[M_2]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$



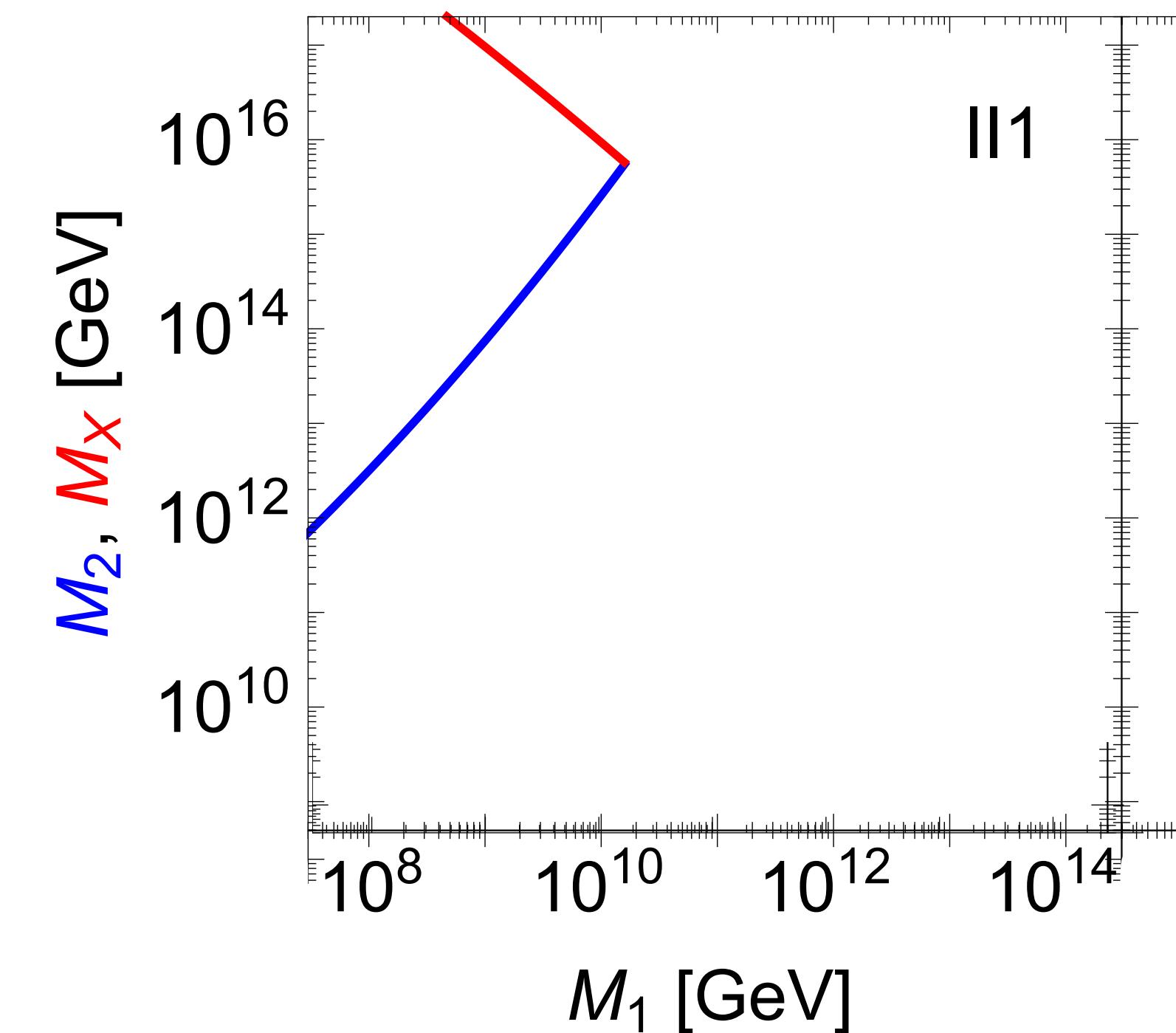
- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales \Rightarrow PD rate and GW signal

Breaking chains with **two intermediate scales** can have a range of scales

$$\text{II1 : } SO(10) \xrightarrow[M_X]{} G_{422} \xrightarrow[M_2]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$

$M_2 = M_X$ recover I1

$$SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$



- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string \Rightarrow gravitational radiation primary emission

Vilenkin & Shellard

$$\mu \approx 2\pi v^2$$

$$M_1^2 = M_{Z'}^2 \sim g^2 v^2 \implies G\mu \approx \frac{1}{M_{\text{PL}}^2} \cdot \frac{2\pi M_1^2}{g^2} = \frac{M_1^2}{2\alpha M_{\text{pl}}^2}$$

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string \Rightarrow gravitational radiation primary emission

Vilenkin & Shellard

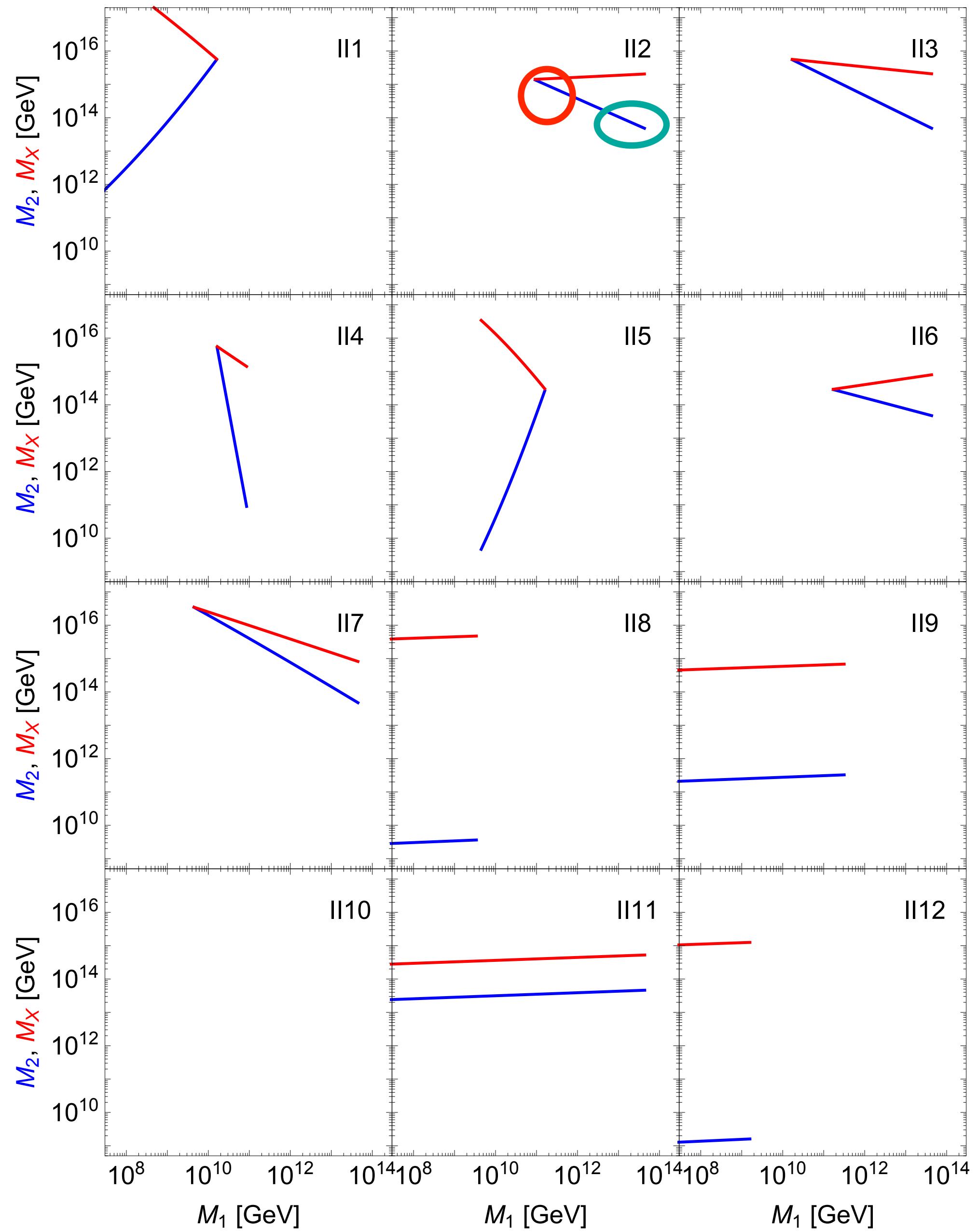
$$\mu \approx 2\pi v^2$$

$$M_1^2 = M_{Z'}^2 \sim g^2 v^2 \implies G\mu \approx \frac{1}{M_{\text{PL}}^2} \cdot \frac{2\pi M_1^2}{g^2} = \frac{M_1^2}{2\alpha M_{\text{pl}}^2}$$

Example

$$G_{3211} \rightarrow G_{SM} \quad U(1)_R \times U(1)_X \rightarrow U(1)_Y$$

$$G\mu \simeq \frac{1}{2(\alpha_{1R}(M_1) + \alpha_{1X}(M_1))} \frac{M_1^2}{M_{\text{pl}}^2}$$



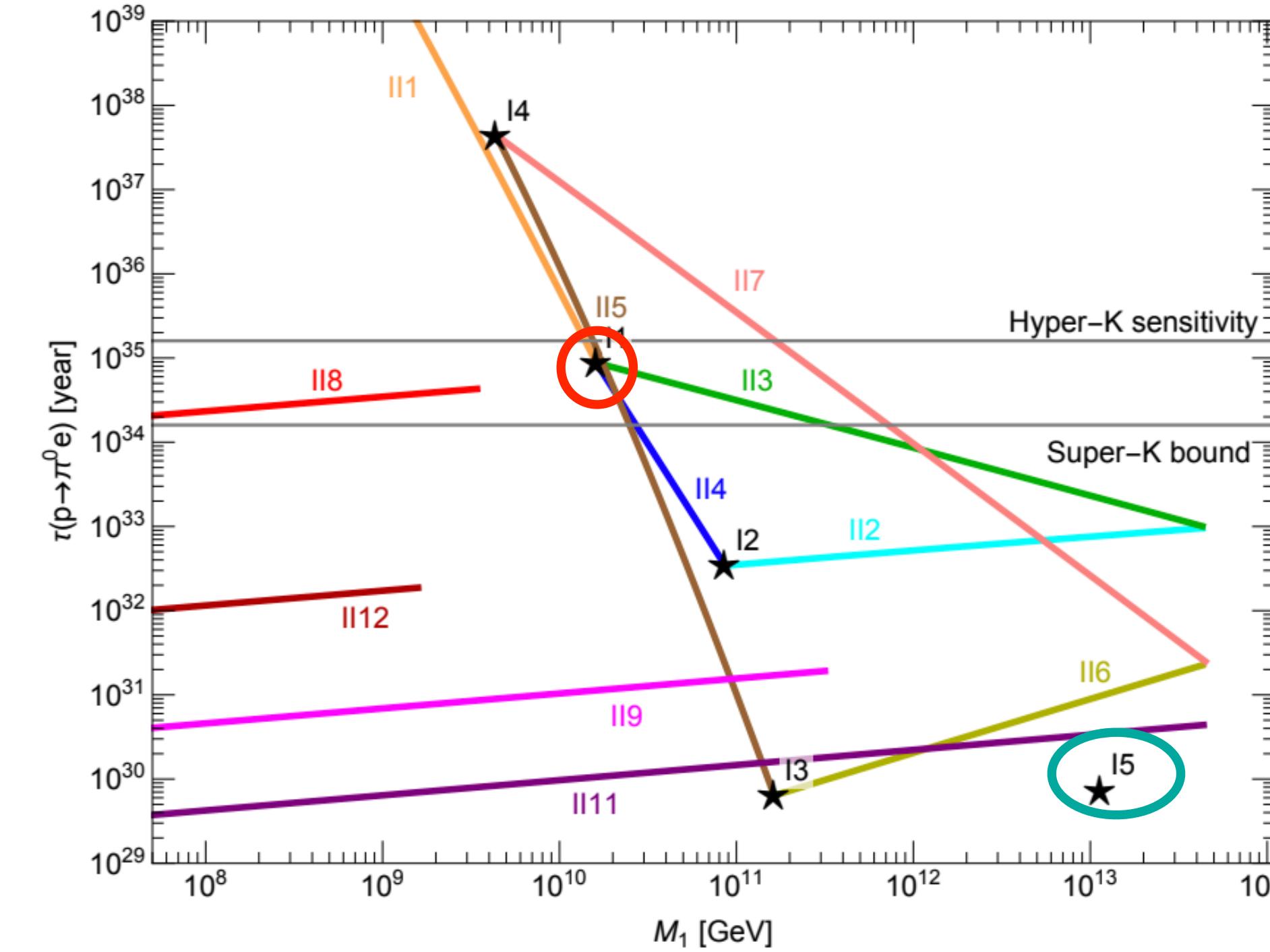
$\text{II2} : SO(10) \xrightarrow{M_X} G_{422}^C \xrightarrow{M_2} G_{3221}^C \xrightarrow{M_1} G_{\text{SM}}$

Intersection of M_2 and M_X reduces II2 to I2

$\text{I2} : SO(10) \xrightarrow{M_X} G_{3221}^C \xrightarrow{M_1} G_{\text{SM}}$ $M_X \equiv M_2$

At right side blue curve II2 becomes I5

$\text{I5} : SO(10) \xrightarrow{G_{422}^C} G_{\text{SM}}$ $M_2 \equiv M_1$



Proton Lifetime

$$\begin{aligned} & \epsilon^{ijk} \epsilon_{\alpha\beta} \left(\frac{1}{\Lambda_1^2} (\overline{u_R^{jc}} \gamma^\mu Q_\alpha^k) (\overline{d_R^{ic}} \gamma_\mu L_\beta) + \frac{1}{\Lambda_1^2} (\overline{u_R^{jc}} \gamma^\mu Q_\alpha^k) (\overline{e_R^c} \gamma_\mu Q_\beta^i) \right. \\ & \left. + \frac{1}{\Lambda_2^2} (\overline{d_R^{jc}} \gamma^\mu Q_\alpha^k) (\overline{u_R^{ic}} \gamma_\mu L_\beta) + \frac{1}{\Lambda_2^2} (\overline{d_R^{jc}} \gamma^\mu Q_\alpha^k) (\overline{\nu_R^c} \gamma_\mu Q_\beta^i) + \text{h.c.} \right) \end{aligned}$$

$$\Lambda_1 = \Lambda_2 \simeq (g_X M_X) / 2$$

$$\begin{aligned} \Gamma(p \rightarrow \pi^0 + e^+) = \frac{m_p}{32\pi} \left(1 - \frac{m_{\pi^0}^2}{m_p^2} \right)^2 A_L^2 \times & \left[A_{SL} \Lambda_1^{-2} (1 + |V_{ud}|^2) |\langle \pi^0 | (ud)_R u_L | p \rangle|^2 \right. \\ & \left. + A_{SR} (\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}) |\langle \pi^0 | (ud)_L u_L | p \rangle|^2 \right] \end{aligned}$$

$$A_{SL(R)} = \prod_A^{\substack{M_Z \leqslant M_A \leqslant M_X}} \prod_i \left[\frac{\alpha_i(M_{A+1})}{\alpha_i(M_A)} \right]^{\frac{\gamma_{iL}(R)}{b_i}}$$

Anomalous dimension

One-loop Beta coefficient

Gravitational Wave Calculation

$$l(t) = l_i - \Gamma G \mu (t - t_i) \quad l_i = \alpha t_i \text{ with } \alpha \simeq 0.1$$

Frequencies of GW released from the loops are given by $2k/l_i$ where $k = 1, 2, \dots$

Loops are found to emit energy in the form of gravitational radiation at a constant rate

$$\frac{dE}{dt} = -\Gamma G \mu^2 \quad \Gamma \sim 50$$

Assuming the fraction of the energy transfer in the form of large loops is $F_\alpha \sim 0.1$

$$\begin{aligned} \Omega_{\text{GW}}(f) &= \sum_k \Omega_{\text{GW}}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k}{f} \frac{\mathcal{F}_\alpha \Gamma^{(k)} G \mu^2}{\alpha(\alpha + \Gamma G \mu)} \\ &= \int_{t_F}^{t_0} dt \frac{C_{\text{eff}} \left(t_i^{(k)} \right)}{t_i^{(k)4}} \frac{a^2(t) a^3 \left(t_i^{(k)} \right)}{a^5(t_0)} \theta \left(t_i^{(k)} - t_F \right) \quad C_{\text{eff}} = 5.7, 0.5 \end{aligned}$$

[1101.5173](#) [1808.08968](#) [0003298](#)

GW from SSB

$$\mu = 2\pi v^2 \epsilon \quad \epsilon = \frac{m_\phi^2}{m_{Z'}^2} \quad \text{Without model information} \quad m_\phi \sim m_{Z'}$$

U(1) GB mass: $M_{Z'}^2 = 4\pi\alpha v^2 \quad M_1^2 = M_{Z'}^2 \quad G\mu \simeq \frac{GM_{Z'}^2}{(2\alpha)} = \frac{GM_1^2}{(2\alpha)}$

[1101.5173](#) [1808.08968](#) [0003298](#)

GUT Model

In the Yukawa sector, couplings above the GUT scale are given by

$$Y_{\mathbf{10}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{\mathbf{126}}}^* \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{\mathbf{120}}^* \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.},$$

After breaking to G_{SM}

$$\begin{aligned} & Y_{\mathbf{10}} \left[(\overline{Q}u_R + \overline{L}\nu_R)h_{\mathbf{10}}^u + (\overline{Q}d_R + \overline{L}e_R)h_{\mathbf{10}}^d \right] + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} \left[(\overline{Q}u_R - 3\overline{L}\nu_R)h_{\overline{\mathbf{126}}}^u + (\overline{Q}d_R - 3\overline{L}e_R)h_{\overline{\mathbf{126}}}^d \right] \\ & + Y_{\mathbf{120}} \left[(\overline{Q}u_R + \overline{L}\nu_R)h_{\mathbf{120}}^u + (\overline{Q}d_R + \overline{L}e_R)h_{\mathbf{120}}^d + \frac{1}{\sqrt{3}} (\overline{Q}u_R - 3\overline{L}\nu_R)h_{\mathbf{120}}^{u'} + (\overline{Q}d_R - 3\overline{L}e_R)h_{\mathbf{120}}^{d'} \right] + \text{h.c.} \end{aligned}$$

Rotating the Higgs fields to their mass basis, we derive Yukawa couplings to the SM Higgs

$$Y_u \bar{Q} \tilde{h}_{SM} u_R + Y_d \bar{Q} h_{SM} d_R + Y_\nu \bar{L} \tilde{h}_{SM} \nu_R + Y_e \bar{L} h_{SM} e_R + \text{h.c.}$$

$$Y_u = Y_{10} V_{11}^* + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} V_{12}^* + Y_{120} \left(V_{13}^* + \frac{1}{\sqrt{3}} V_{14}^* \right)$$

$$Y_d = Y_{10} V_{15} + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} V_{16} + Y_{120} \left(V_{17} + \frac{1}{\sqrt{3}} V_{18} \right)$$

$$Y_\nu = Y_{10} V_{11}^* - \sqrt{3} Y_{\overline{\mathbf{126}}} V_{12}^* + Y_{120} \left(V_{13}^* - \sqrt{3} V_{14}^* \right)$$

$$Y_e = Y_{10} V_{15} - \sqrt{3} Y_{\overline{\mathbf{126}}} V_{16} + Y_{120} \left(V_{17} - \sqrt{3} V_{18} \right).$$

GUT Model

$$Y_u = Y_{10}V_{11}^* + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^* + Y_{120} \left(V_{13}^* + \frac{1}{\sqrt{3}}V_{14}^* \right)$$

$$Y_d = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120} \left(V_{17} + \frac{1}{\sqrt{3}}V_{18} \right)$$

$$Y_\nu = Y_{10}V_{11}^* - \sqrt{3}Y_{\overline{126}}V_{12}^* + Y_{120} \left(V_{13}^* - \sqrt{3}V_{14}^* \right)$$

$$Y_e = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120} \left(V_{17} - \sqrt{3}V_{18} \right).$$

$$\begin{aligned} Y_u &= h + r_2 f + i r_3 h', & Y_d &= r_1 (h + f + i h'), & Y_\nu &= h - 3r_2 f + i c_\nu h' \\ Y_e &= r_1 (h - 3f + i c_e h'), & M_{\nu_R} &= f \frac{\sqrt{3}r_1}{V_{16}} v_S \end{aligned}$$

$$\begin{aligned} h &= Y_{\mathbf{10}}V_{11}, \quad f = Y_{\overline{\mathbf{126}}} \frac{V_{16}}{\sqrt{3}} \frac{V_{11}^*}{V_{15}}, \quad c_e = \frac{V_{17} - \sqrt{3}V_{18}}{V_{17} + V_{18}/\sqrt{3}}, \quad c_\nu = \frac{V_{13}^* - \sqrt{3}V_{14}^*}{V_{17} + V_{18}/\sqrt{3}} \frac{V_{15}}{V_{11}^*}, \\ r_1 &= \frac{V_{15}}{V_{11}^*}, \quad r_2 = \frac{V_{12}^*}{V_{16}} \frac{V_{15}}{V_{11}^*}, \quad r_3 = \frac{V_{13}^* + V_{14}^*/\sqrt{3}}{V_{17} + V_{18}/\sqrt{3}} \frac{V_{15}}{V_{11}^*}, \quad h' = -i Y_{\mathbf{120}} \left(V_{17} + V_{18}/\sqrt{3} \right) \frac{V_{11}^*}{V_{15}}, \end{aligned}$$

$$Y_u = h + r_2 f = \text{diag}\{\eta_u y_u, \eta_c y_c, \eta_t y_t\}$$

$$Y_d = P_a V_{\text{CKM}} \text{diag}\{\eta_d y_d, \eta_s y_s, \eta_b y_b\} V_{\text{CKM}}^\dagger P_a^* \quad P_a = \text{diag}\{e^{ia_1}, e^{ia_2}, 1\}$$

$$Y_\nu = -\frac{3r_2 + 1}{r_2 - 1} Y_u + \frac{4r_2}{r_1(r_2 - 1)} \operatorname{Re} Y_d + i \frac{c_\nu}{r_1} \operatorname{Im} Y_d$$

$$Y_e = -\frac{4r_1}{r_2 - 1} Y_u + \frac{r_2 + 3}{r_2 - 1} \operatorname{Re} Y_d + i c_e \operatorname{Im} Y_d$$

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_q} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_q} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_q} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_q} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_q} & c_{13}c_{23} \end{pmatrix},$$

$$\begin{aligned} M_\nu = & m_0 \left(\frac{8r_2(r_2 + 1)}{r_2 - 1} Y_u - \frac{16r_2^2}{r_1(r_2 - 1)} \operatorname{Re} Y_d \right. \\ & \left. + \frac{r_2 - 1}{r_1} (r_1 Y_u + i c_\nu \operatorname{Im} Y_d) (r_1 Y_u - \operatorname{Re} Y_d)^{-1} (r_1 Y_u - i c_\nu \operatorname{Im} Y_d) \right) \end{aligned}$$

RHN mass matrix obtained from inverting Seesaw Formula: i.e. we have light neutrino Yukawa, light neutrino masses

GUT Model Particle Content

	Multiplet	Role in the model
Fermions	16	Contains all SM fermions and RH neutrinos
Higgses	10	Generates fermion masses
	45	Triggers intermediate symmetry breaking
	54	Triggers GUT symmetry breaking
	120	Generates fermion masses
	126	Generates fermion masses & intermediate symmetry breaking
	210	Triggers intermediate symmetry breaking

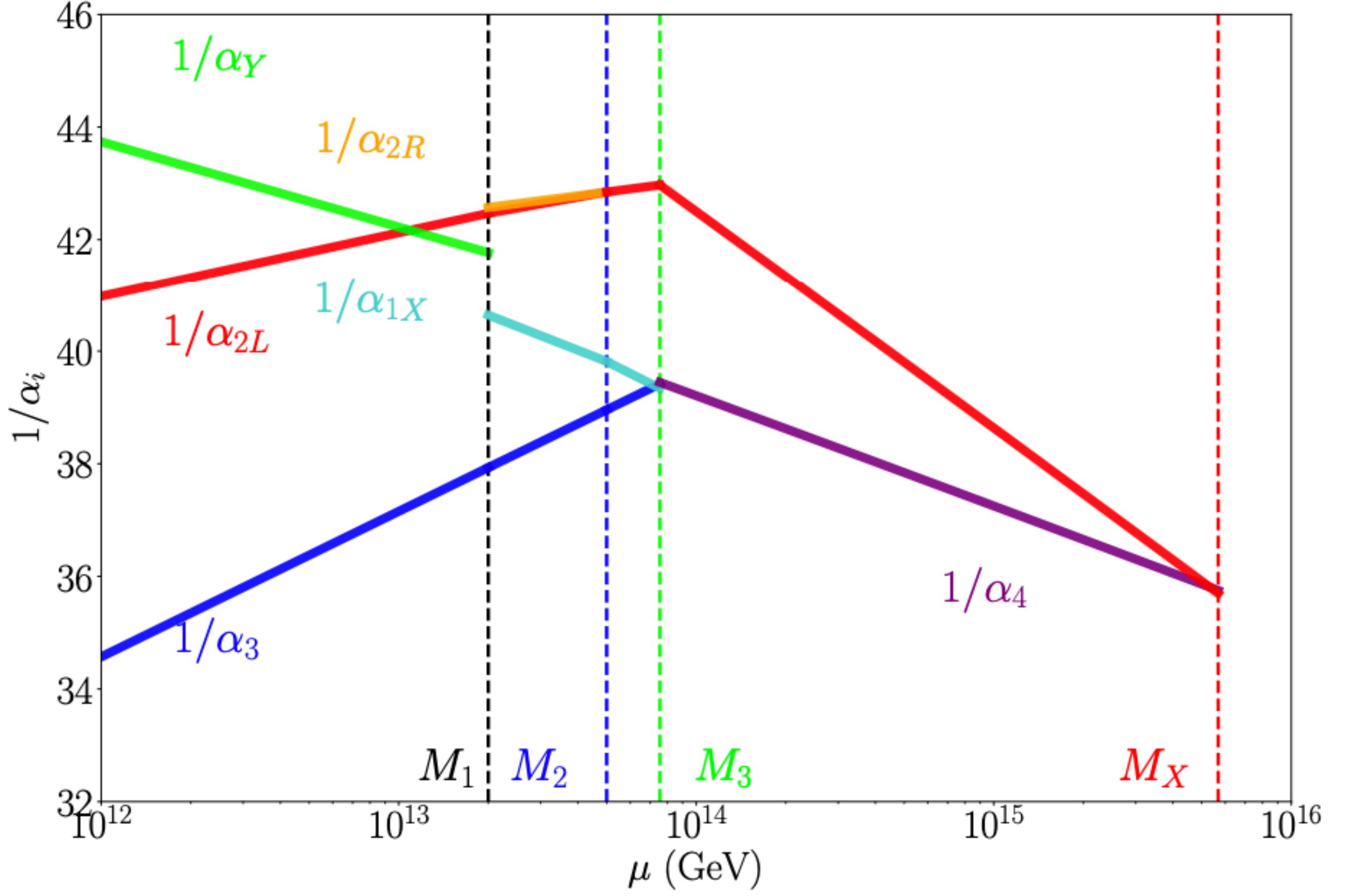
$SO(10)$	54	210	45	126
G_3	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$(\mathbf{15}, \mathbf{1}, \mathbf{1})_1$	$(\mathbf{15}, \mathbf{1}, \mathbf{1})_2$	$(\mathbf{10}, \mathbf{1}, \mathbf{3}) + (\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$
G_2	–	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{0})_1$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{0})_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{3}, -\mathbf{1}) + (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1})$
G_1	–	–	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{0})_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{3}, -\mathbf{1})$
G_{SM}	–	–	–	$(\mathbf{1}, \mathbf{1}, \mathbf{0})_S$

$SO(10)$	16
G_3	$(\mathbf{4}, \mathbf{2}, \mathbf{1})_L + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{R^c}$
G_2	$(\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/6)_{Q_L} + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -1/6)_{Q_R^c}$ + $(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1/2)_{l_L} + (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1/2)_{l_R^c}$
G_1	$(\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/6)_{Q_L} + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -1/6)_{Q_R^c}$ + $(\mathbf{1}, \mathbf{2}, \mathbf{1}, -1/2)_{l_L} + (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1/2)_{l_R^c}$
G_{SM}	$(\mathbf{3}, \mathbf{2}, 1/6)_{Q_L} + (\overline{\mathbf{3}}, \mathbf{1}, -2/3)_{u_R^c} + (\overline{\mathbf{3}}, \mathbf{1}, 1/3)_{d_R^c}$ + $(\mathbf{1}, \mathbf{2}, -1/2)_{l_L} + (\mathbf{1}, \mathbf{1}, 0)_{\nu_R^c} + (\mathbf{1}, \mathbf{1}, 1)_{e_R^c}$

Matter field decomposition

$SO(10)$	10	126	120
G_3	$(\mathbf{1}, \mathbf{2}, \mathbf{2})_1$	$(\mathbf{15}, \mathbf{2}, \mathbf{2})_1$ + $(\mathbf{10}, \mathbf{1}, \mathbf{3}) + (\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})_2 + (\mathbf{15}, \mathbf{2}, \mathbf{2})_2$
G_2	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})_1$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})_2$ + $(\mathbf{1}, \mathbf{1}, \mathbf{3}, -\mathbf{1}) + (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})_{3,4}$
G_1	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})_1$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})_2$ + $(\mathbf{1}, \mathbf{1}, \mathbf{3}, -\mathbf{1})$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0})_{3,4}$
G_{SM}	$(\mathbf{1}, \mathbf{2}, -1/2)_{h_{10}^u}$ + $(\mathbf{1}, \mathbf{2}, +1/2)_{h_{10}^d}$	$(\mathbf{1}, \mathbf{2}, -1/2)_{h_{126}^u}$ + $(\mathbf{1}, \mathbf{2}, +1/2)_{h_{126}^d}$ + $(\mathbf{1}, \mathbf{1}, 0)_S$	$(\mathbf{1}, \mathbf{2}, -1/2)_{h_{120}^u, h_{120}^{u'}}$ + $(\mathbf{1}, \mathbf{2}, +1/2)_{h_{120}^d, h_{120}^{d'}}$

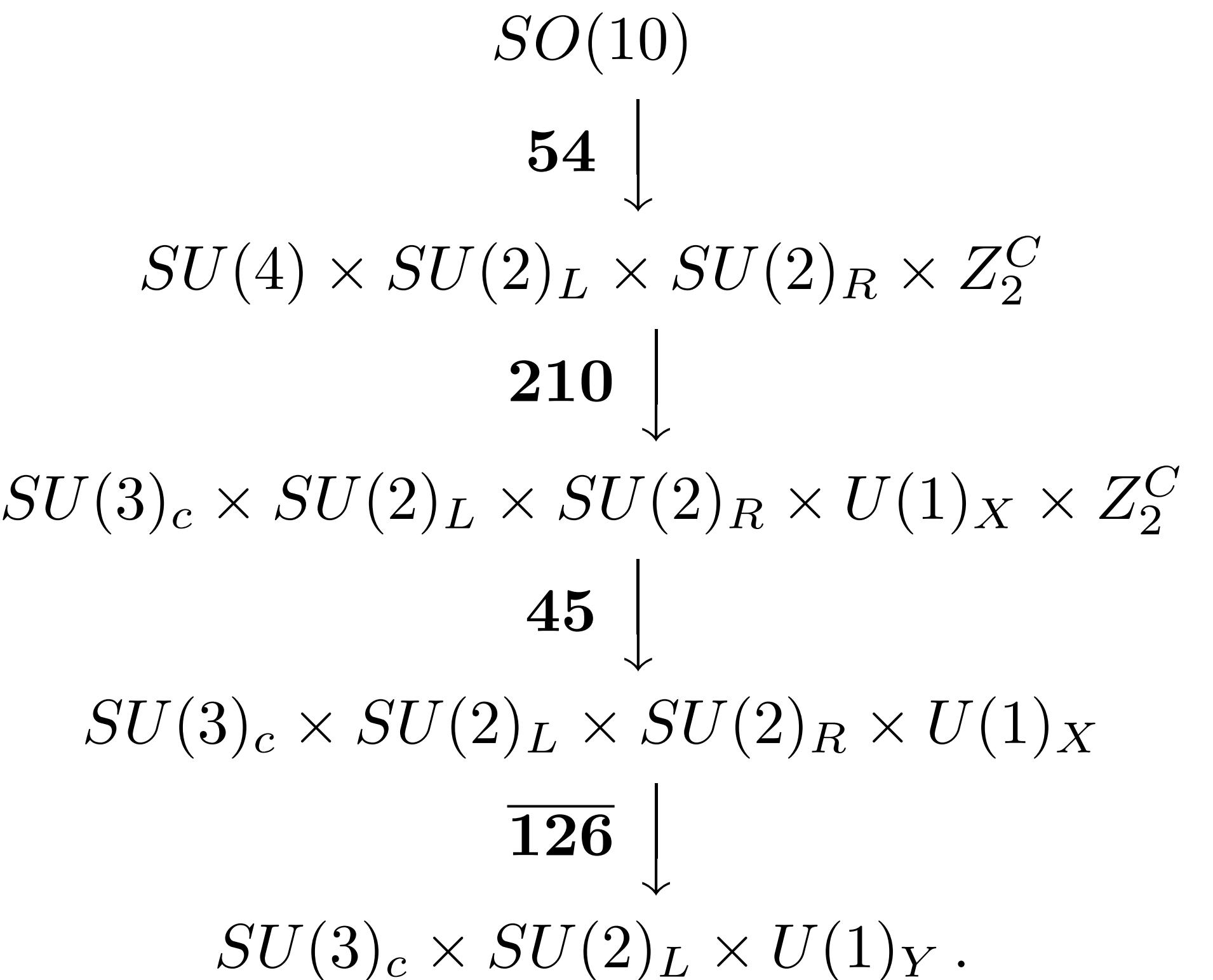
SO(10) Higgs reps for fermion mass generation



$$M_1 = 2 \times 10^{13} \text{ GeV}, \quad M_2 = 5 \times 10^{13} \text{ GeV}, \quad (1)$$

where the remaining scales and gauge coupling α_X , are then determined via the gauge unification,

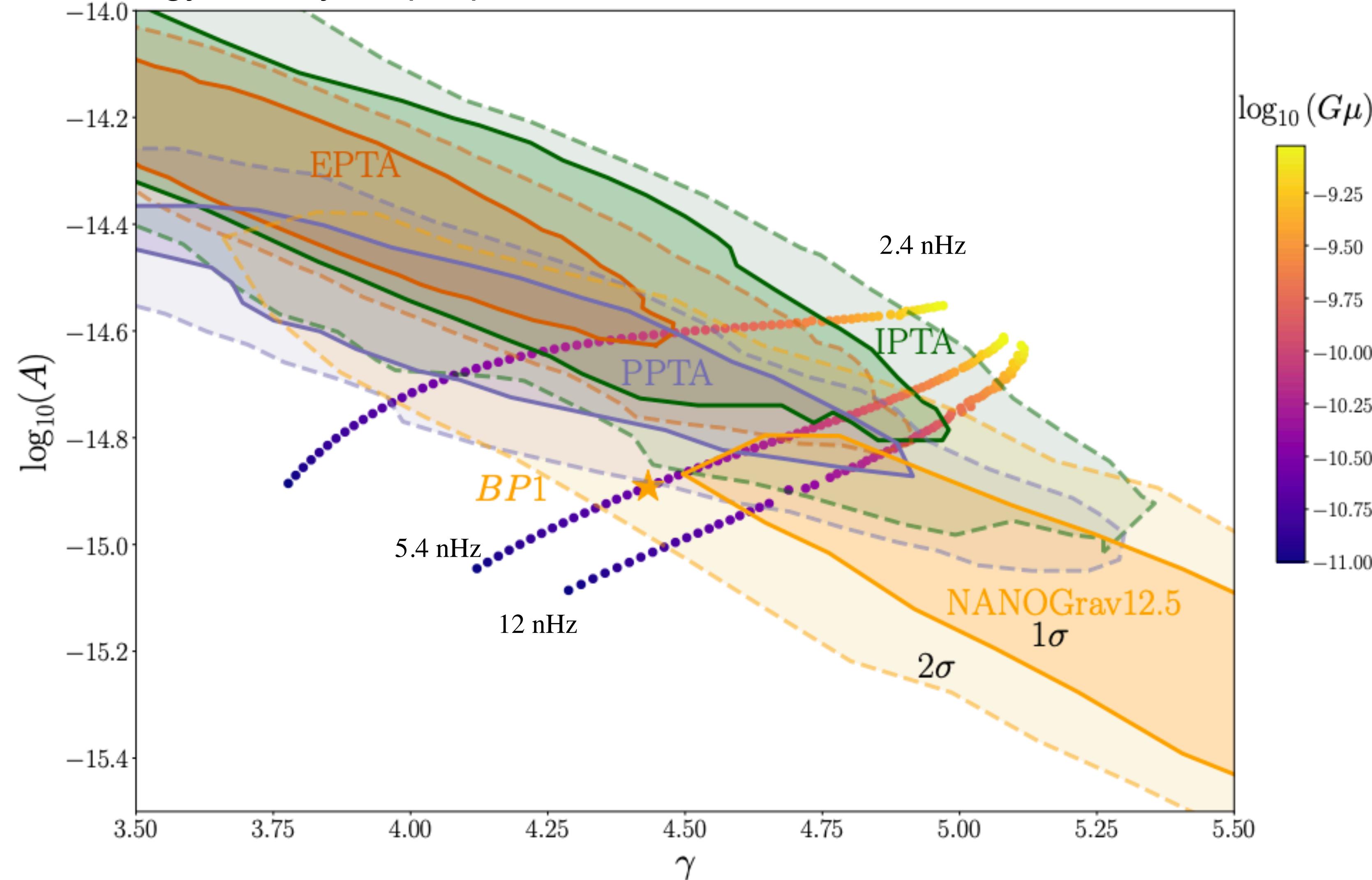
$$M_3 = 7.55 \times 10^{13} \text{ GeV}, \quad M_X = 5.68 \times 10^{15} \text{ GeV}, \quad \alpha_X = 0.0279. \quad (2)$$



Overlap with PTA experiments

$A \equiv$ amplitude parameter of correlation between pulsars.

$\gamma \equiv$ related to GW energy density freq dependence



Gravitational Wave Calculation

Use approach of Cui et al (1711.03104, 1808.08968) which is based on simulation of Pillado, Plum & Shlaer

$$l(t) = l_i - \Gamma G \mu (t - t_i) \quad l_i = \alpha t_i \text{ with } \alpha \simeq 0.1$$

Loop formation rate per unit volume per unit time

$$n(l, t) = \frac{C_{\text{eff}}(t_i)}{\alpha^2 t_i^4} \frac{a^3(t_i)}{a^3(t)}$$

Each loop radiates GW at constant rate:

$$\frac{dE}{dt} = -\Gamma G \mu^2, \quad \Gamma \approx 50$$

Gravitational Wave Calculation

Loops shrink as they radiate GWs

$$l = \alpha t_i - \Gamma G \mu (t - t_i)$$

Redshift GW emitted from loop of length l with where k is the oscillation mode

$$f = \frac{a(\tilde{t})}{a(t_0)} \frac{2k}{l}$$

Energy density of GWs today

$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df} = \sum_k \Omega_{GW}^{(k)}(f)$$

$$\Omega_{GW}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k}{f} \frac{\mathcal{F}_\alpha \Gamma^{(k)} G \mu^2}{\alpha(\alpha + \Gamma G \mu)} \int_{t_F}^{t_0} d\tilde{t} \frac{C_{\text{eff}}(t_i^{(k)})}{t_i^{(k)4}} \left[\frac{a(\tilde{t})}{a(t_0)} \right]^5 \left[\frac{a(t_i^{(k)})}{a(\tilde{t})} \right]^3 \Theta(t_i^{(k)} - t_F)$$

Gravitational Wave Calculation

More generally

$$\Omega_{\text{GW}} \equiv \frac{d\rho_{\text{GW}}}{d \ln f} \frac{1}{\rho_{\text{crit}}} \quad \frac{d\rho_{\text{GW}}(t)}{df} = \int dt' \frac{a(t')^4}{a(t)^4} \int dl \frac{dn(l, t')}{dl} \frac{dP(l, t')}{df'} \frac{df'}{df}$$

There are different models for the number density of loops

VOS model velocity-dependent one-scale model: describe long string network as a function of characteristic length & average velocity of string. Tractable analytically once scaling is reached

Blanco-Pillado-Olum-Shlaer based on simulation of network, include backreaction of intersections
On loops

Lorenz-Ringeval-Sakellariadou based on simulation of network with a focus on small-scale structure

First two approaches match better with each other than the latter. In the BOS approach large loops
Tend to dominate GW spectrum but in the LRS approach small loops tend to dominate