Quantum Gravity from Conformal Field Theory

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PRISMA+ colloquium
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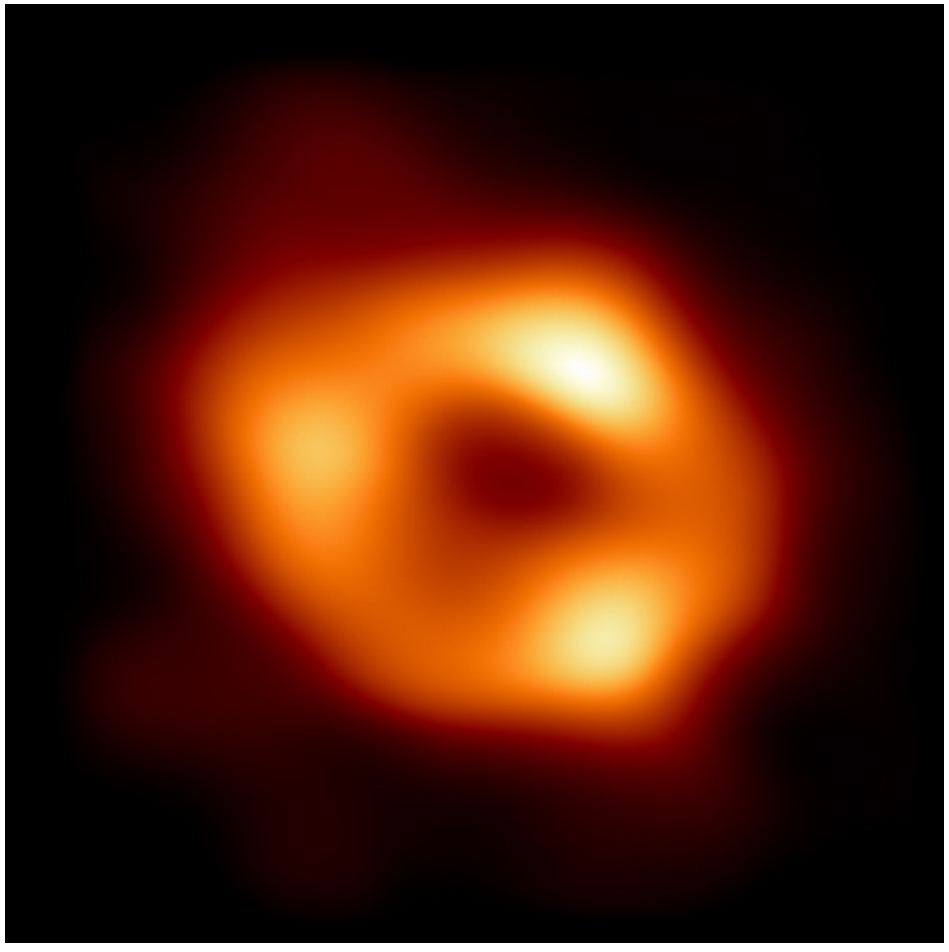
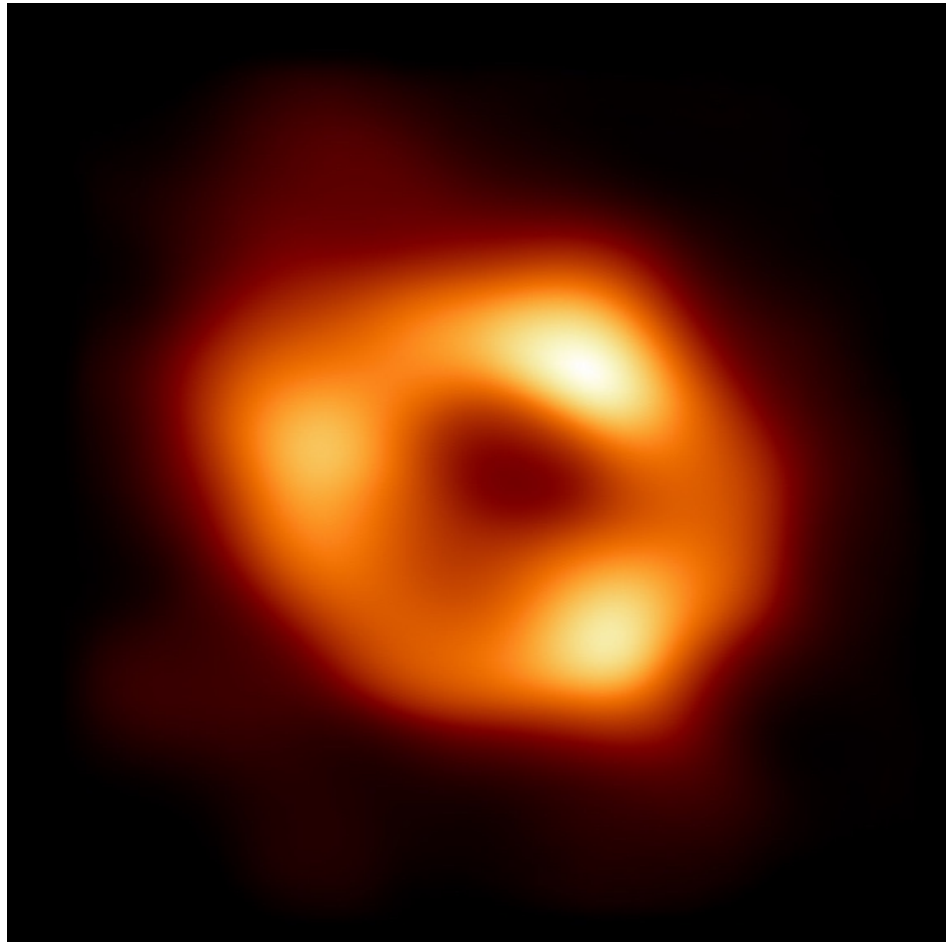
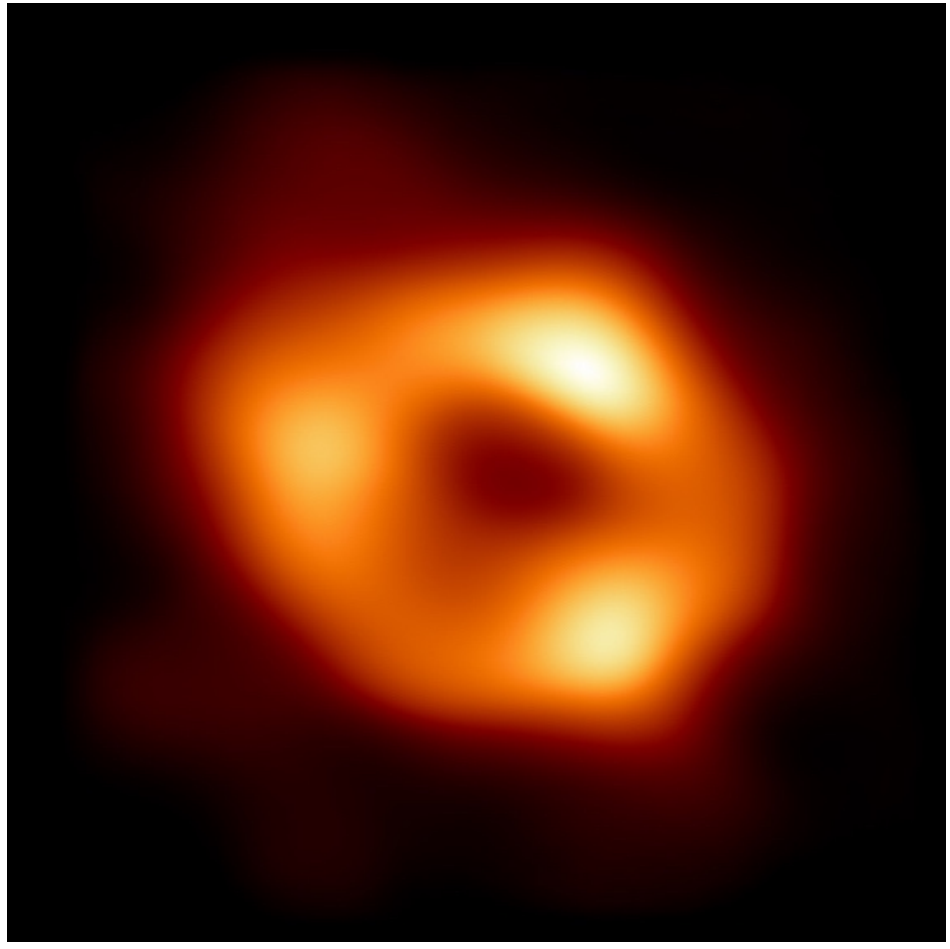


Event Horizon Telescope



Event Horizon Telescope

Black holes are the most compact objects in the Universe. $(R_{Sch} = 2\,\frac{G_N}{c^2}\,M)$



Event Horizon Telescope

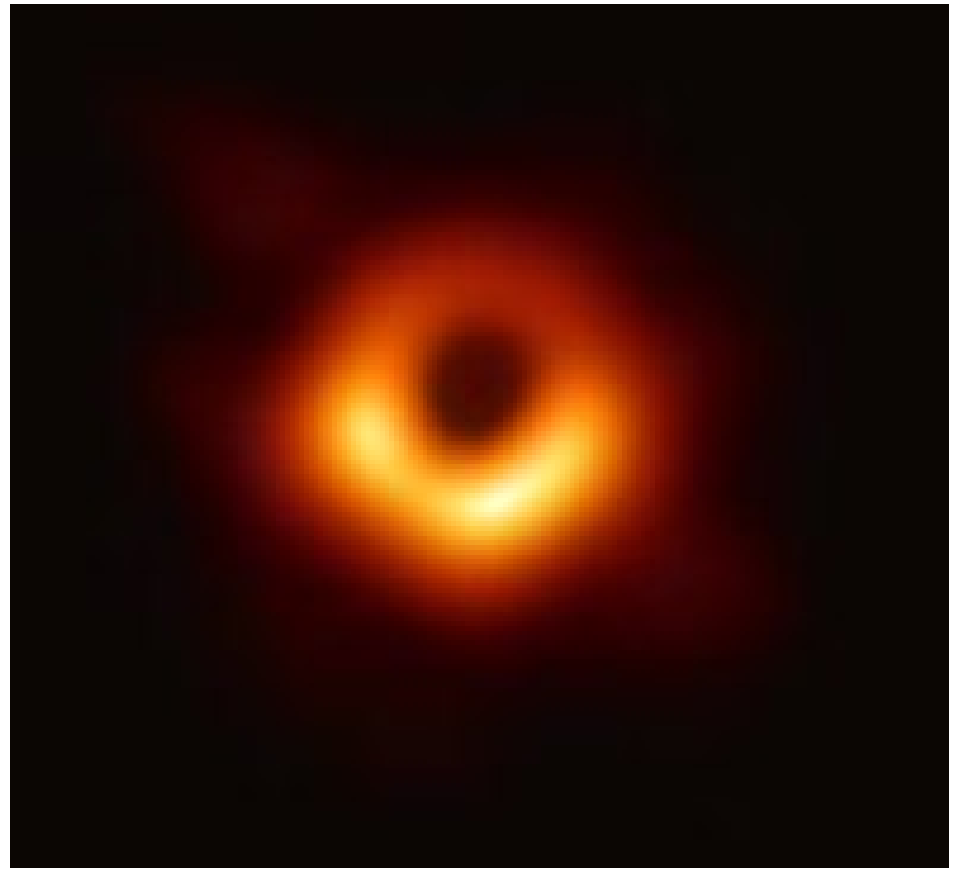
Black holes emit thermal radiation. [Hawking]

$$T \propto M^{-1}$$

Black hole entropy

$$S_{BH} = \frac{1}{4} \frac{c^3}{\hbar G_N} A$$

[Bekenstein, Hawking]



Event Horizon Telescope

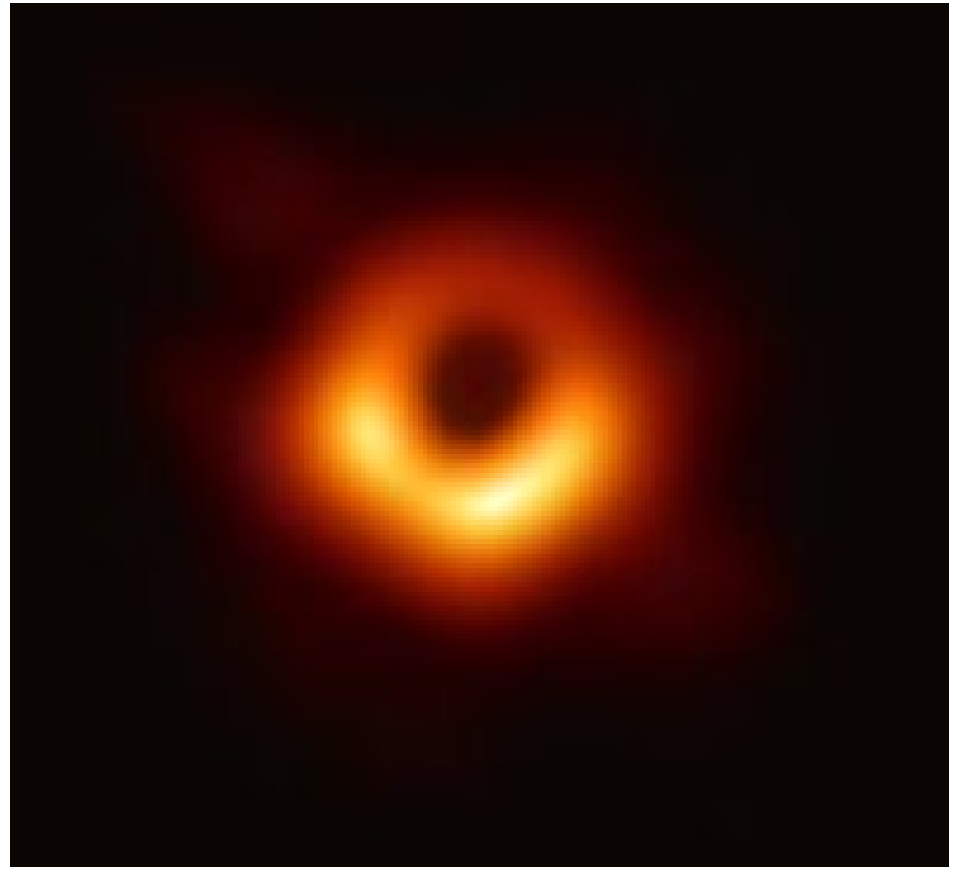
S_{BH} scales with the *area* rather than the *volume*.

Black hole entropy

$$S_{BH} = \frac{1}{4} \frac{c^3}{\hbar G_N} A$$

[Bekenstein, Hawking]

$S_{BH} \sim 10^{77}$ for $1 M_\odot$



Event Horizon Telescope

Black hole entropy

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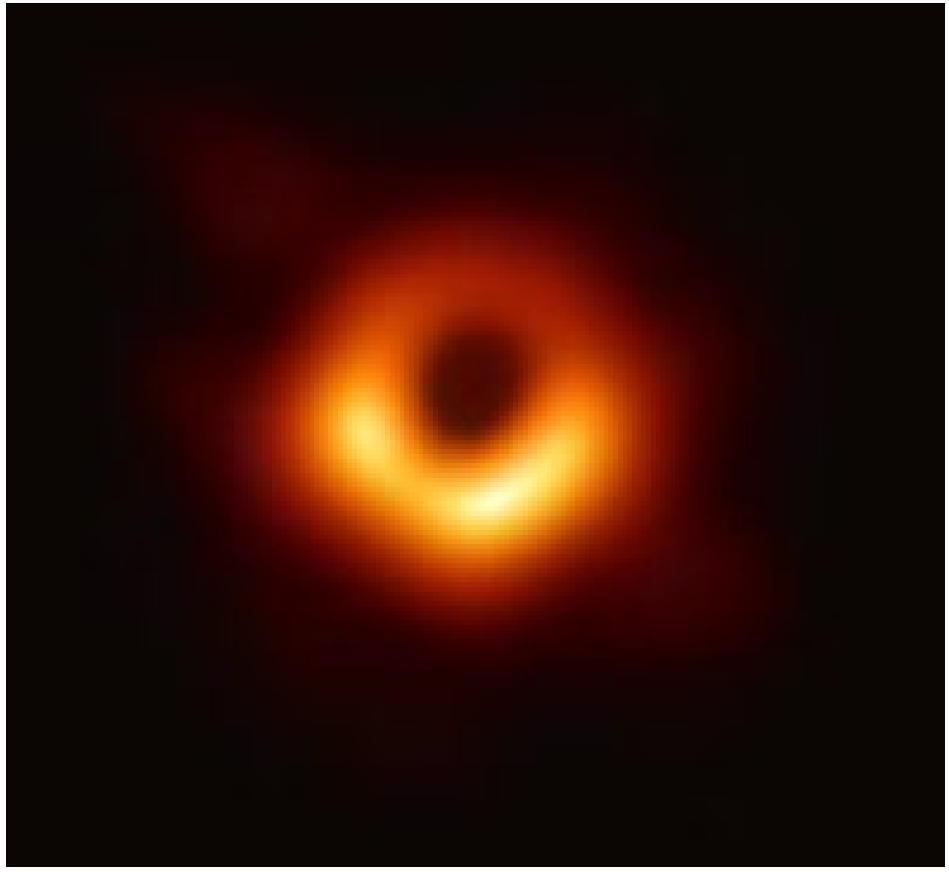
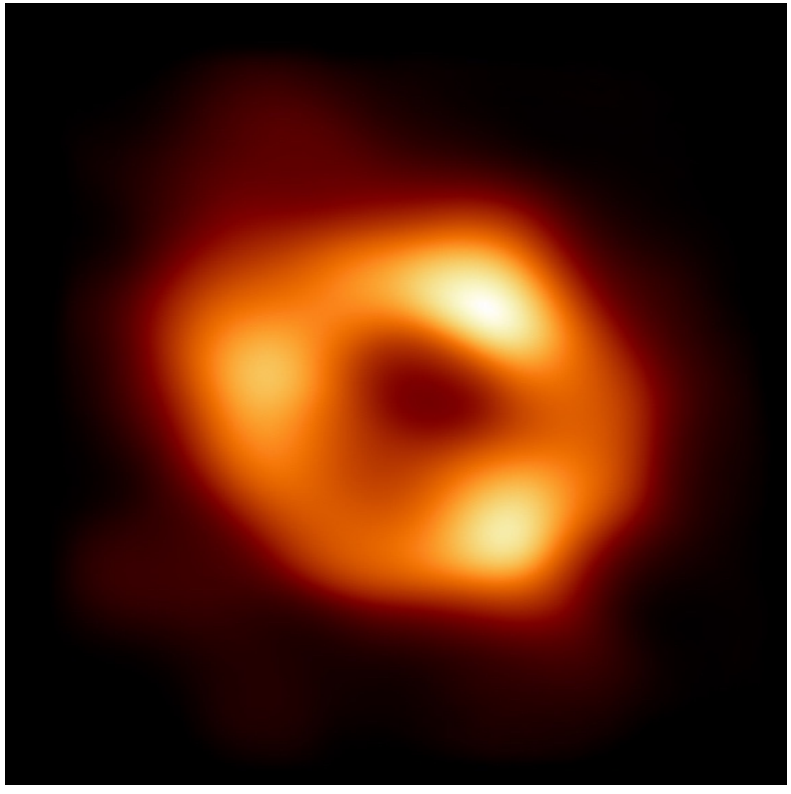


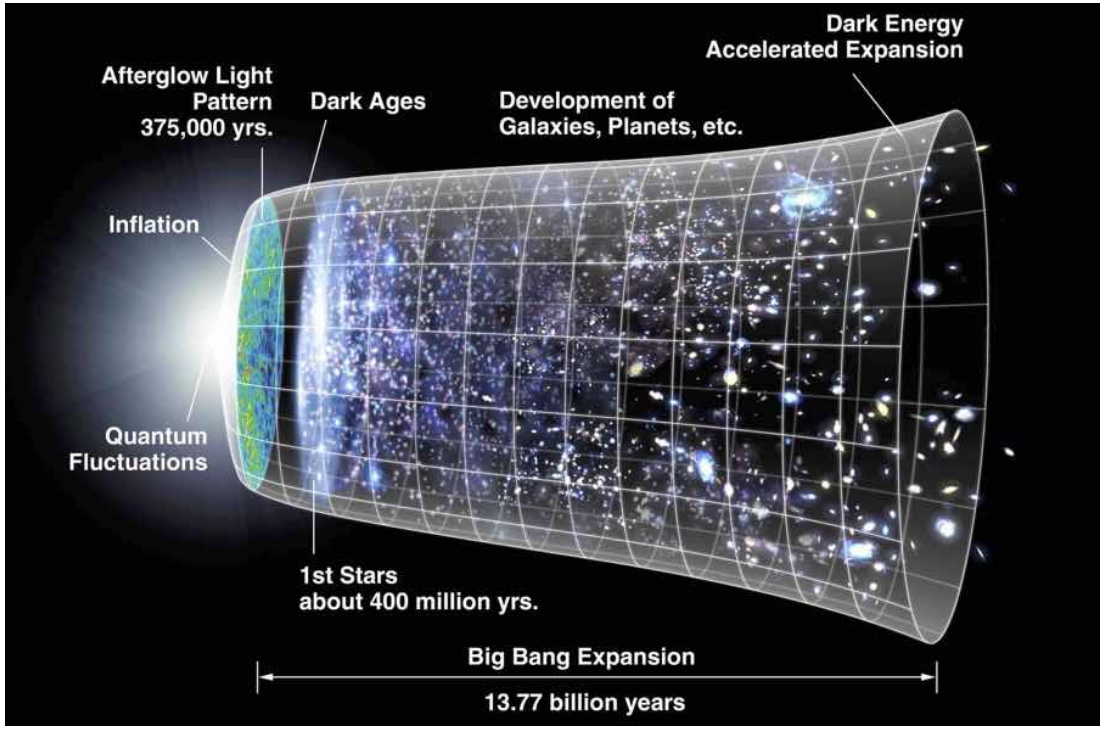
Event Horizon Telescope

What is the statistical mechanical origin of this entropy?

Why quantum gravity?



Event Horizon Telescope



Where does the enormous black hole entropy come from?

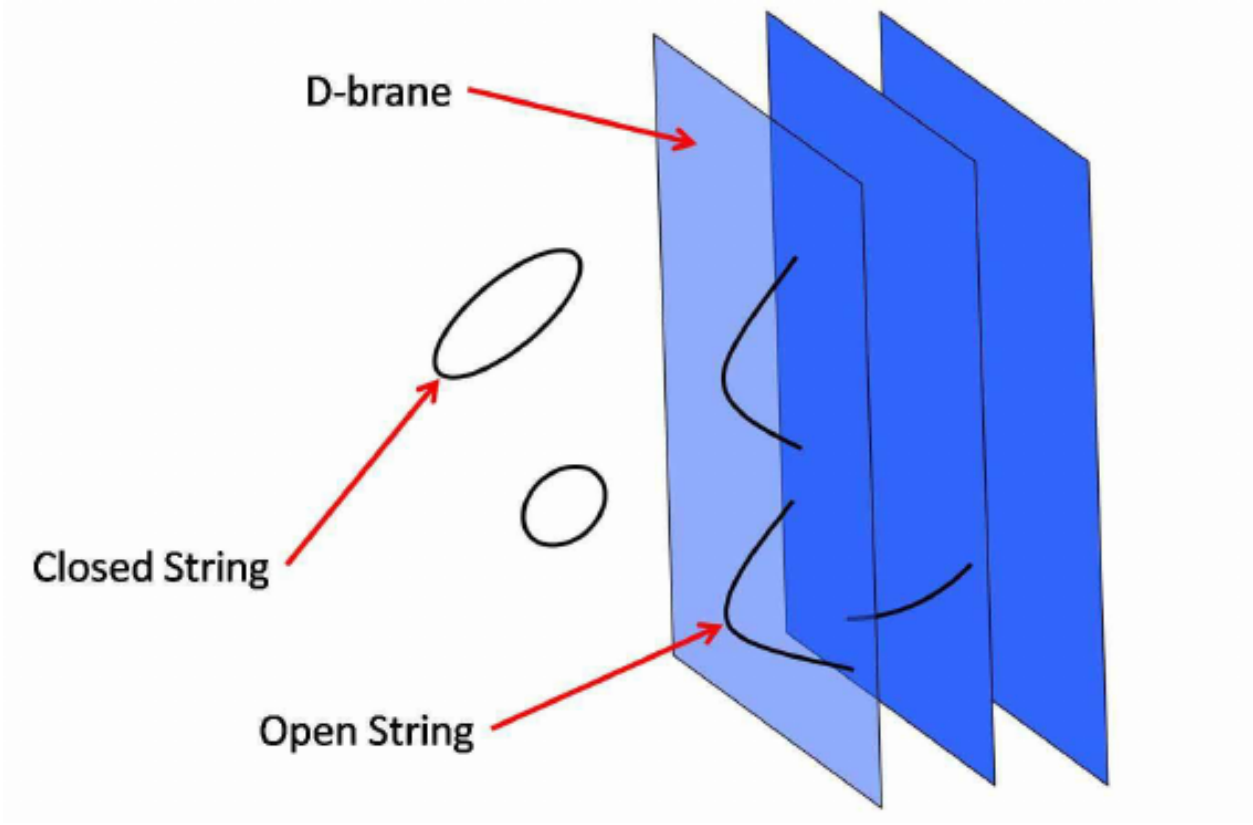
How to describe our Universe?

Where does the enormous black hole entropy come from?

How to describe our Universe?

There are different approaches to quantum gravity.

String theory is a theory of quantum gravity.



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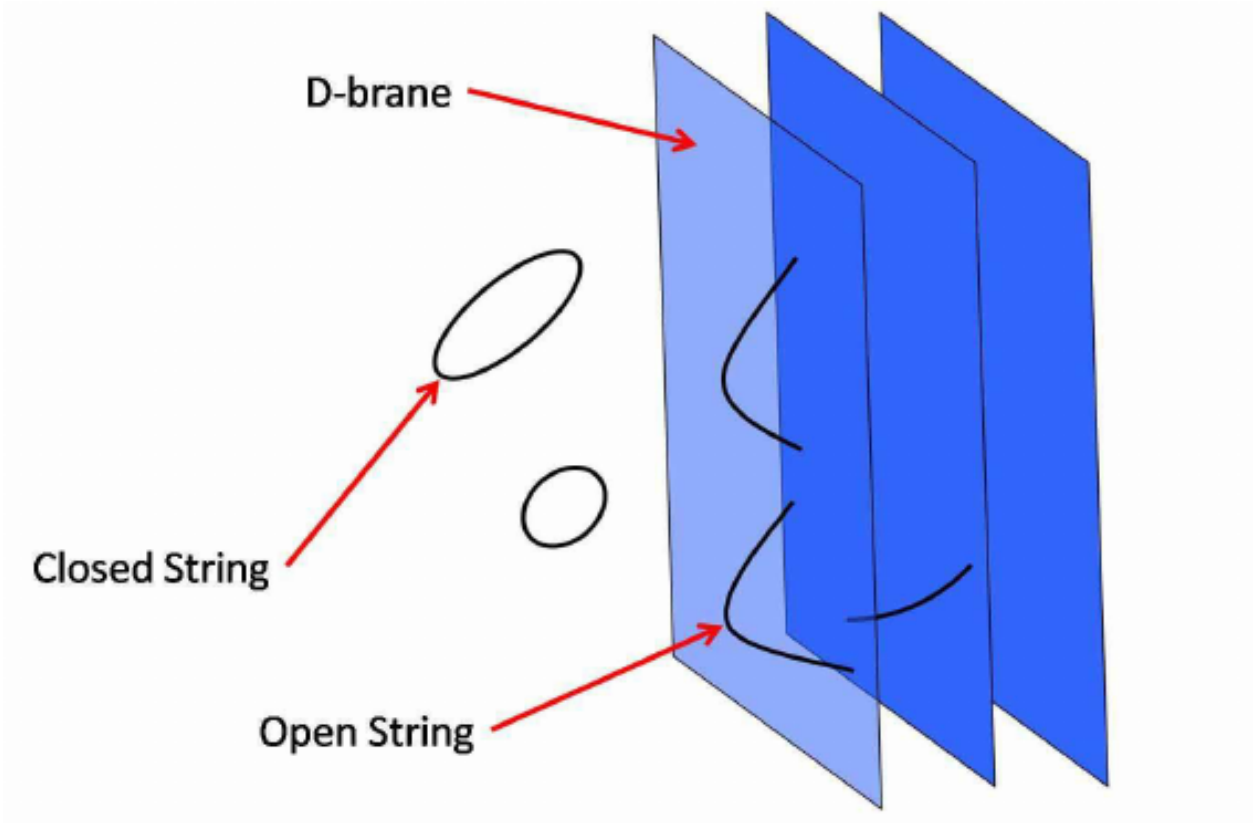
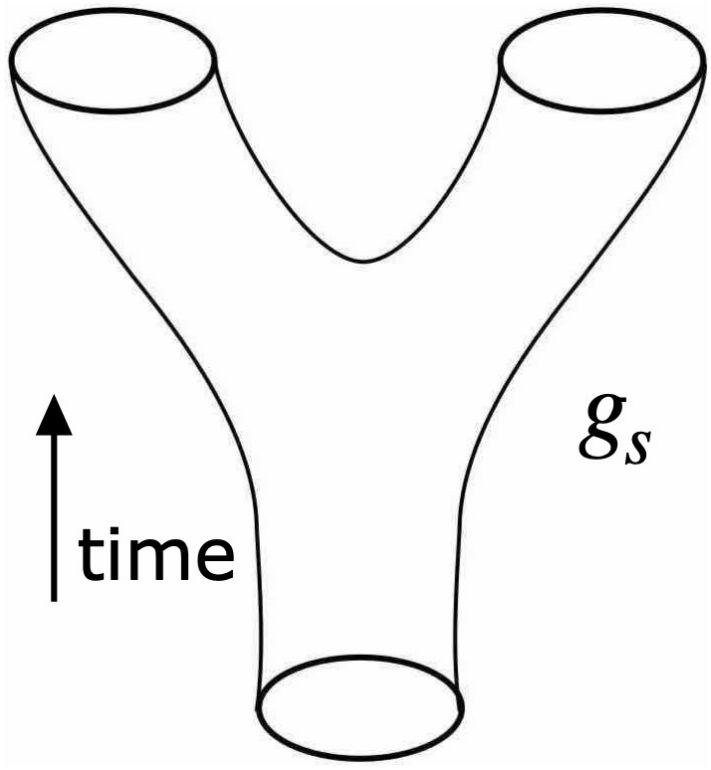


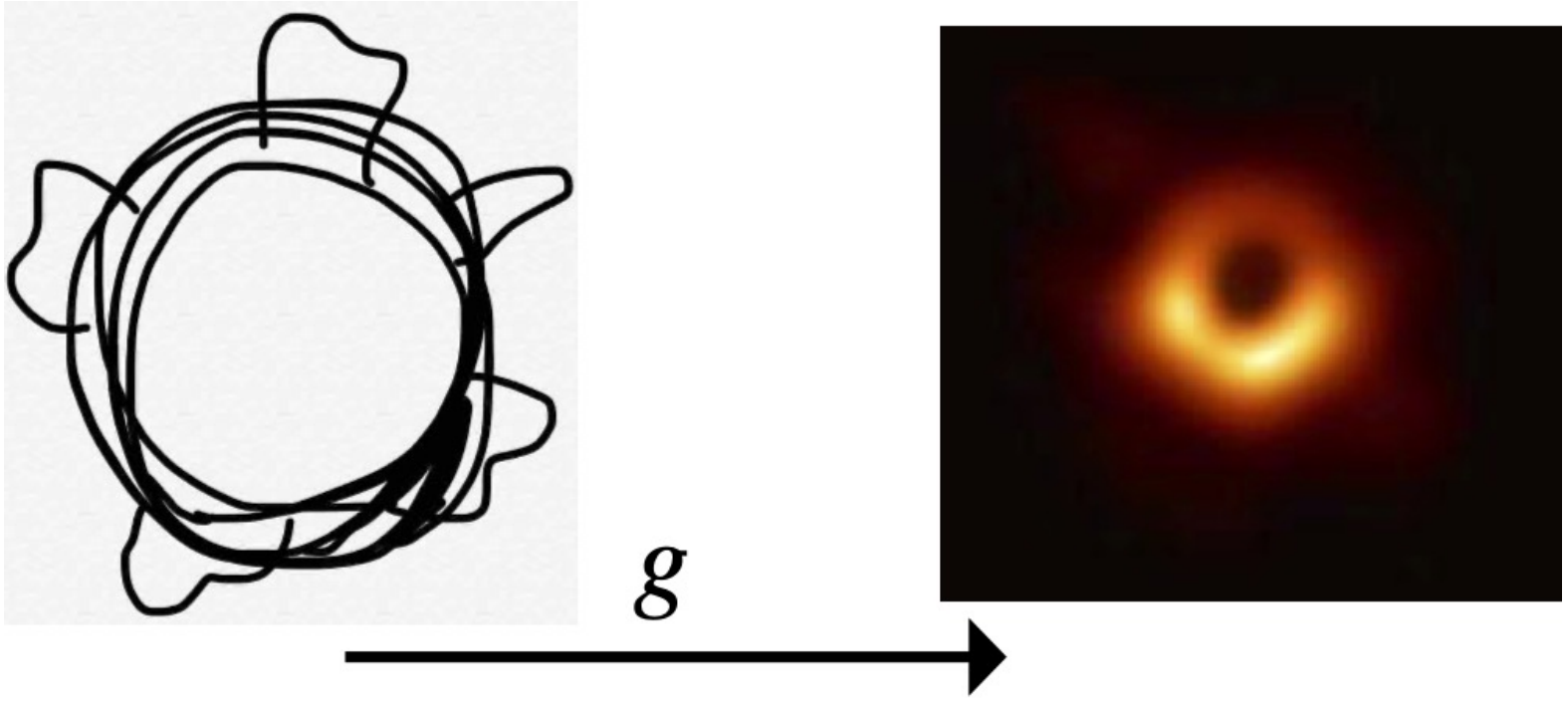
It describes all four fundamental forces of nature: strong and weak nuclear forces, electromagnetic force, *and* gravitational force.

String worldsheet



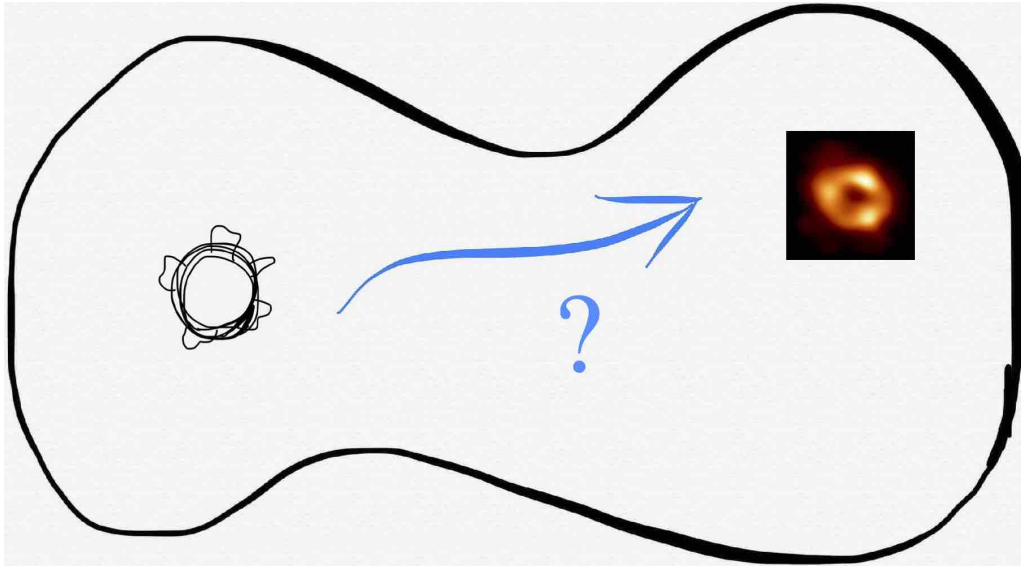
Black holes:

In string theory black holes are constructed by wrapping strings and branes on compact spaces.

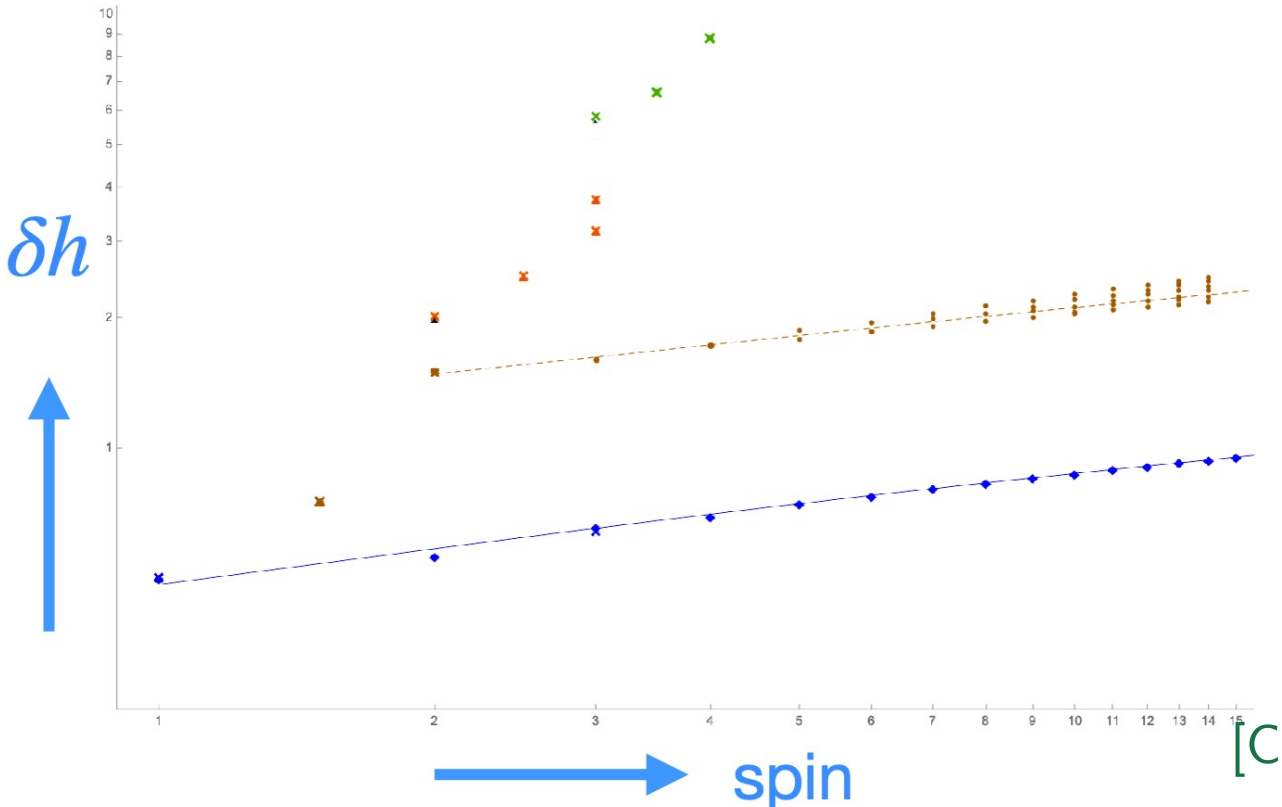


Microscopic entropy of string+brane system = BH entropy.

[Sen; Strominger, Vafa]



[IZ thesis; Cheng, Gaberdiel, IZ; Benjamin, Keller, IZ]

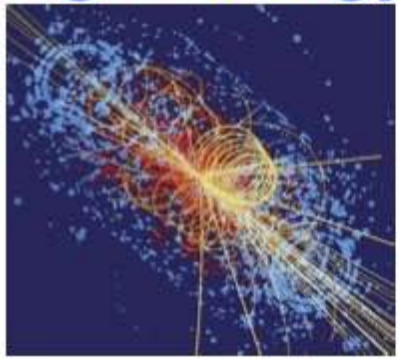


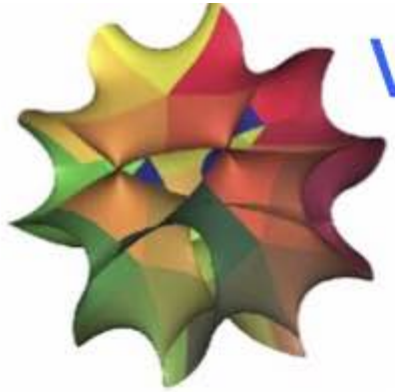
[Cheng, Gaberdiel, IZ]

- Conformal Field Theories
- Conformal Field Theories of QG
- Future

Wide applications in physics & mathematics:

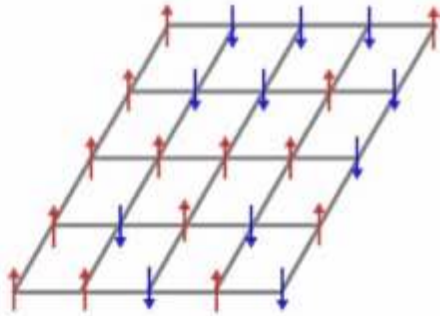
High Energy



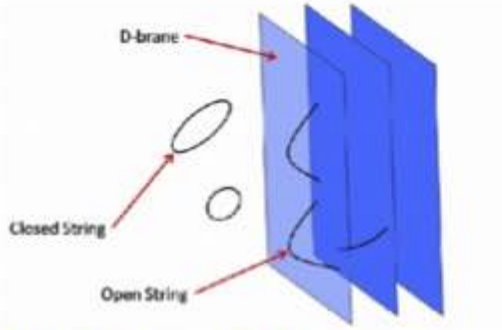


Vertex Operator Algebra

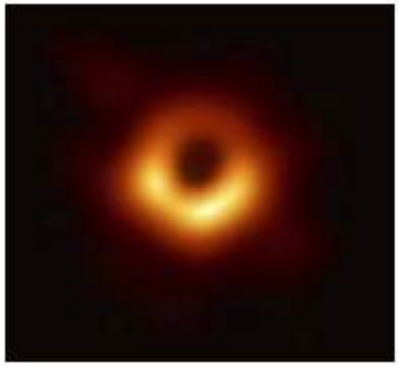
Conformal Field Theory



Statistical Mechanics



String Theory



Black Holes

Conformal symmetry: a physical system or law is invariant under changes in its length or energy scale.

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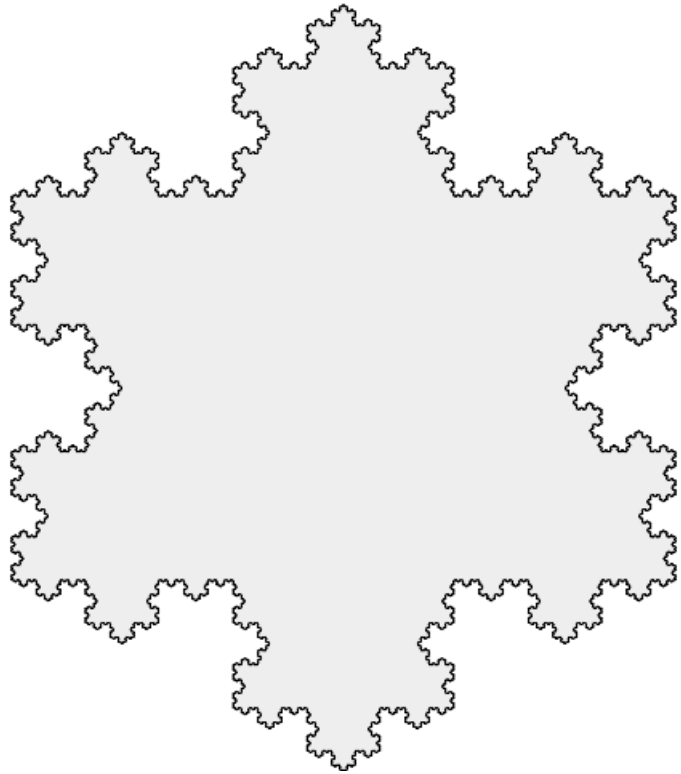
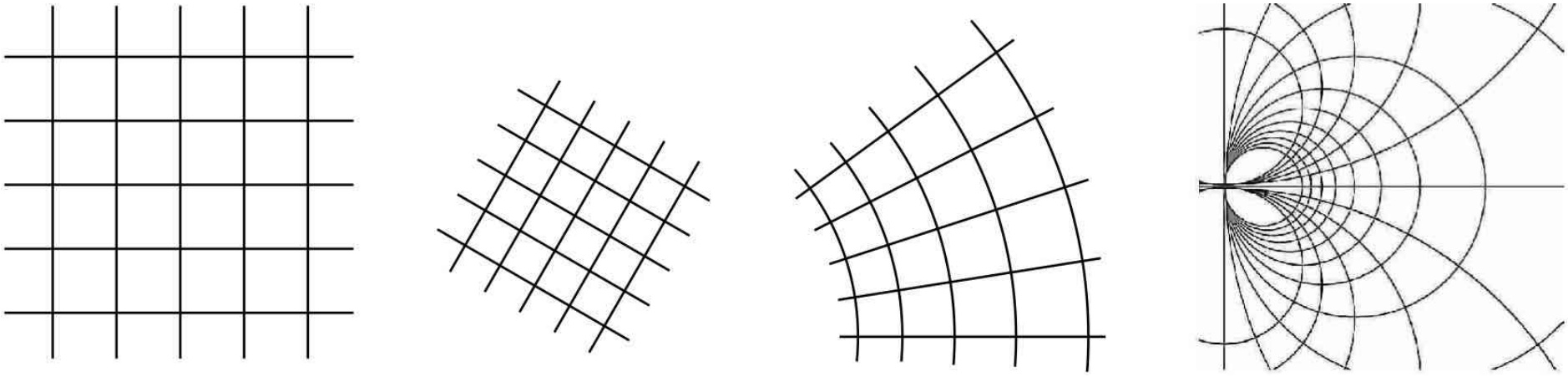


Smaller segments have the same shape and physical properties as the whole object.

Conformal field theories are interacting quantum field theories invariant under conformal transformations.

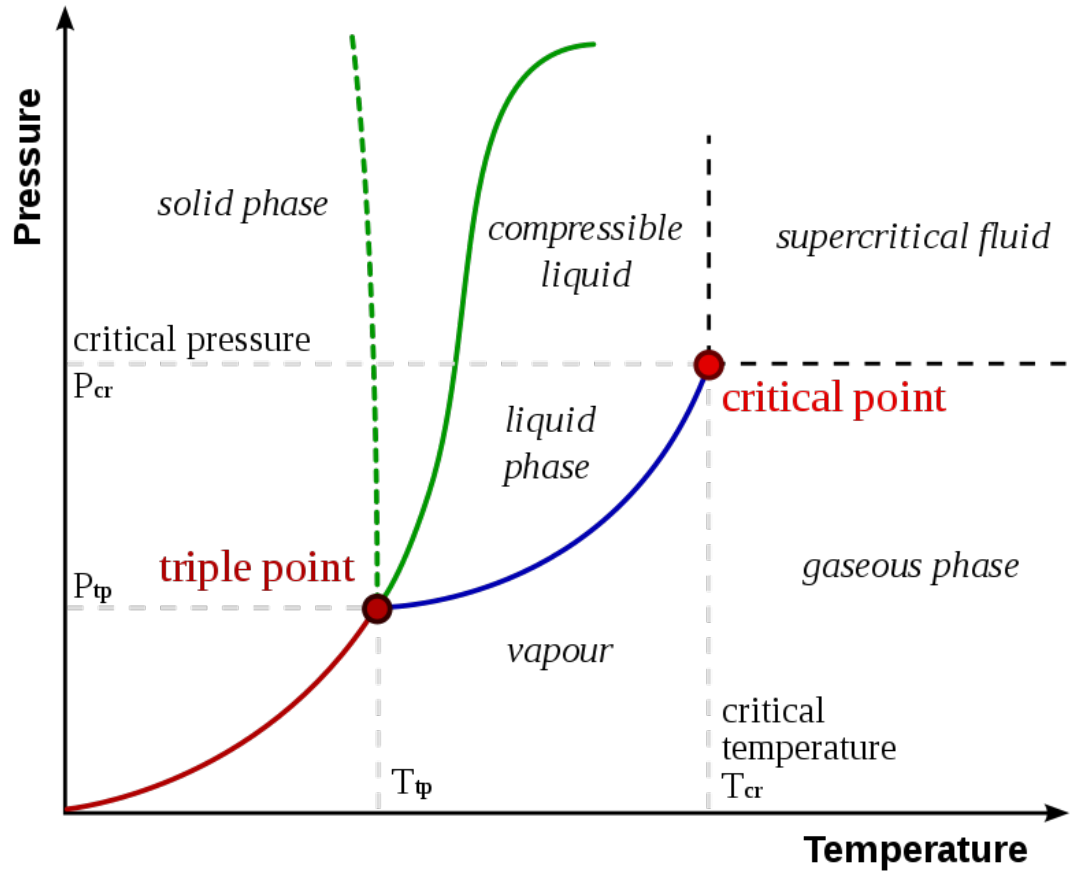


Conformal transformations preserve angles.

In d=2, symmetry algebra is ∞-dimensional (Virasoro algebra).

Rational CFTs have enhanced symmetry and are exactly solvable (*e.g.* minimal models).

- Conformal field theories describe critical points of phase transitions.



• Phase transition in ferromagnetic order: 2d Ising model at critical point is a minimal model.

Correlation function:

$$\langle \sigma_i \ \ \sigma_j \rangle \sim \frac{1}{|i-j|^{\alpha}} e^{-\frac{|i-j|}{\xi(T)}}, \quad \xi : \textit{correlation length}$$

As $T \to T_c$, $\xi(T \to T_c) \to \infty$

$$\langle \sigma_i \ \ \sigma_j \rangle \sim \frac{1}{|i-j|^{\eta}}$$

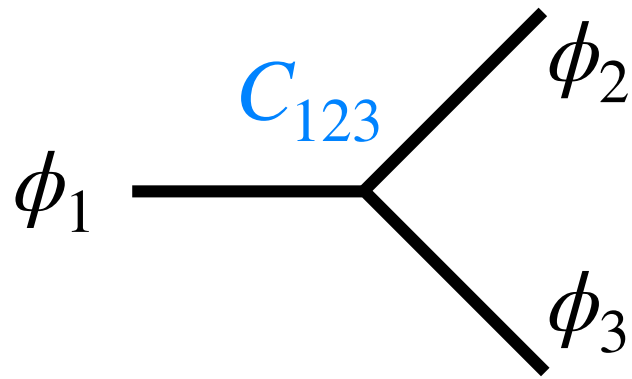
Conformal field theory data

2-point function:

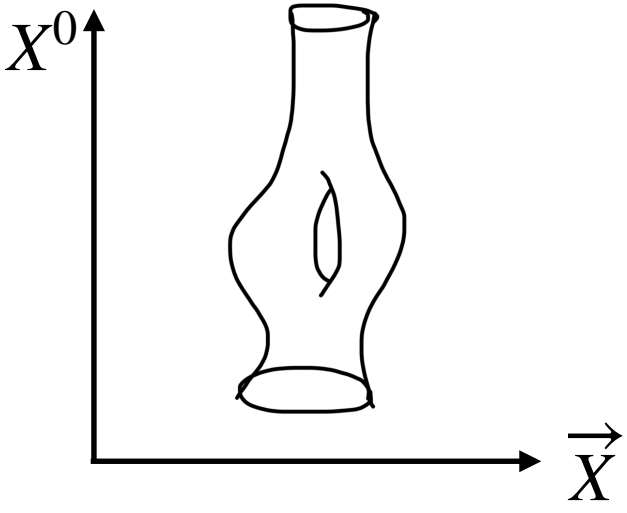
$$\langle \phi(z_1, \bar{z}_1)\, \phi(z_2, \bar{z}_2) \rangle = \frac{1}{(z_1 - z_2)^{2h}\, (\bar{z}_1 - \bar{z}_2)^{2\bar{h}}}$$

3-point function:

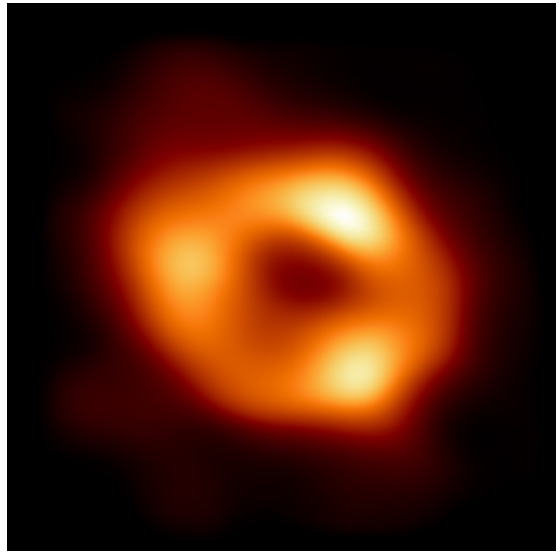
$$\langle \phi_1(z_1)\, \phi_2(z_2)\, \phi_3(z_3) \rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\, z_{23}^{h_2+h_3-h_1}\, z_{13}^{h_1+h_3-h_2}} \times c.c.$$



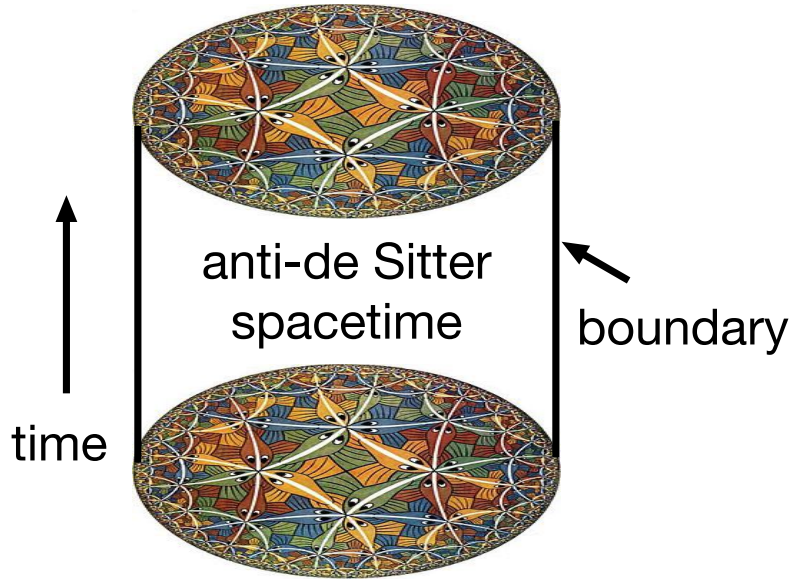
Conformal field theories provide a framework to describe quantum gravity.



Worldsheet CFT



Black Holes



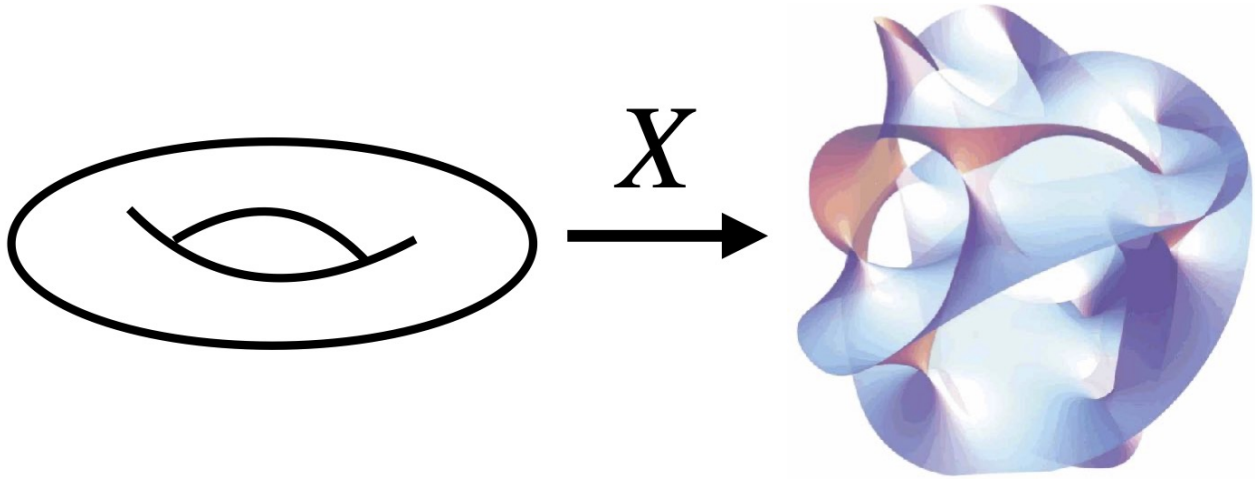
Holography

AdS_{d+1}/CFT_d

- Conformal Field Theories
- Conformal Field Theories of QG
- Future

Which CFTs appear in QG?

2d CFTs defined on a Riemann surface and with target space Σ.



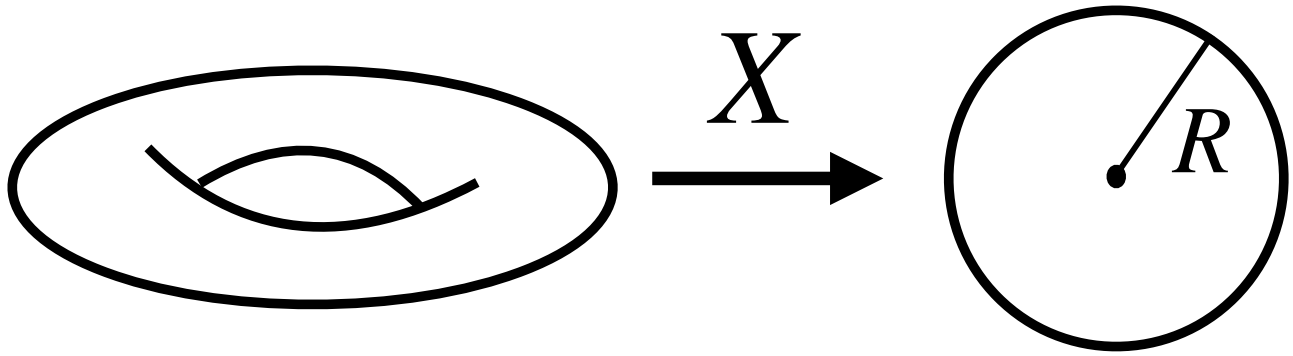
CFT of a single compact free boson

$$S = \frac{R^2}{4\pi\alpha'} \int d^2z \, \partial X \, \bar{\partial} X$$

$$Z_{S^1}(\tau, \bar{\tau}) = \mathrm{tr}_{\mathcal{H}}(q^{L_0} \, \bar{q}^{\bar{L}_0}) \ ,$$

$$= q^{h_0} \bar{q}^{\bar{h}_0} + n_1 q^{h_1} \bar{q}^{\bar{h}_1} + \cdots$$

$$q := e^{2\pi i \tau}$$



σ-model on S^1

Moduli space: $\mathcal{M}_{S^1} = \mathbb{R} \in [1, \infty)$

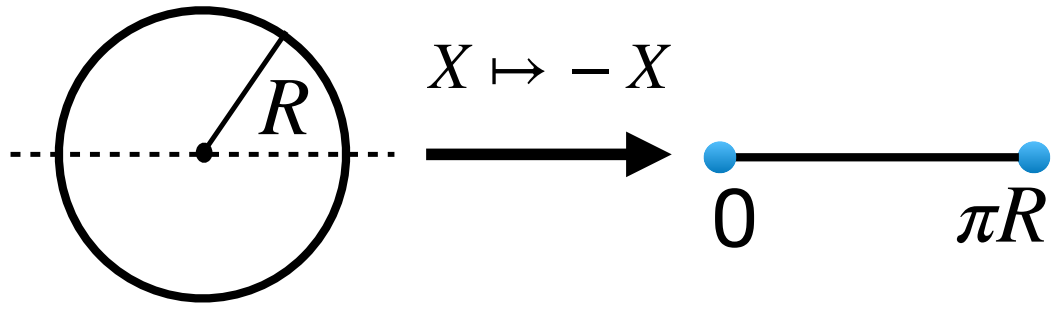
$$S = \frac{R^2}{4\pi\alpha'} \int d^2z \; \partial X \, \bar{\partial} X + \frac{2R\delta R}{4\pi\alpha'} \int d^2z \; \partial X \, \bar{\partial} X + \cdots$$

Modulus: $\partial X \bar{\partial} X$

σ-model on S^1 is a RCFT for $R^2 \in \mathbb{Q}$.

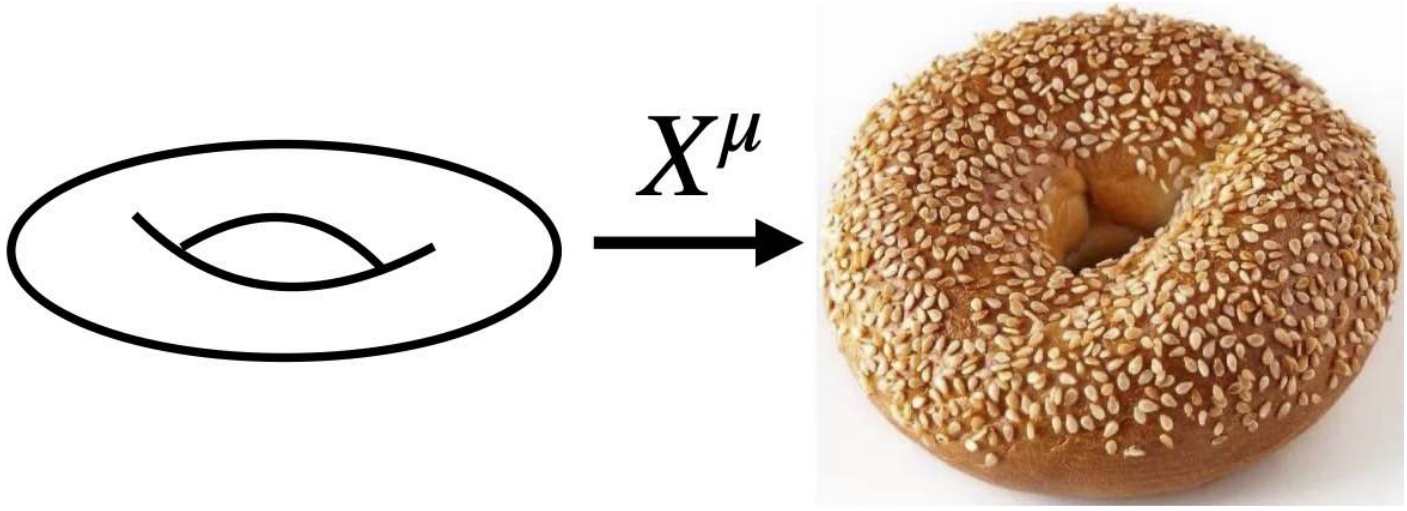
CFT on $S^1/\mathbb{Z}_2$

Action of $\mathbb{Z}_2$:

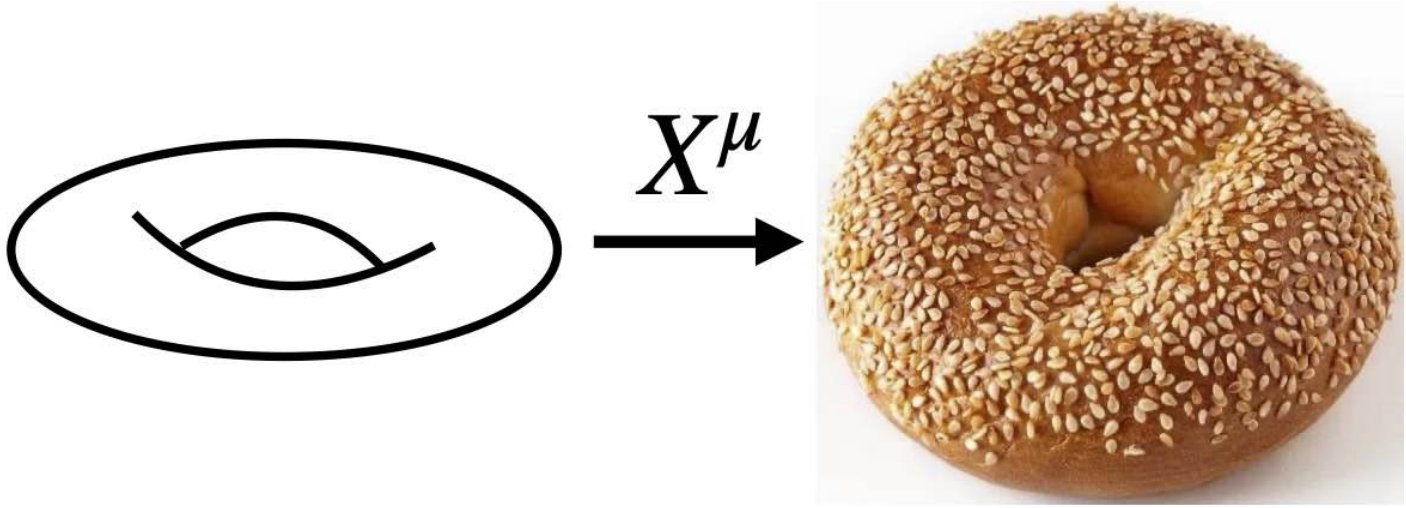


$$Z_{S^1/\mathbb{Z}_2}(\tau,\bar{\tau}) = Z_{S^1}(\tau,\bar{\tau}) + \cdots$$

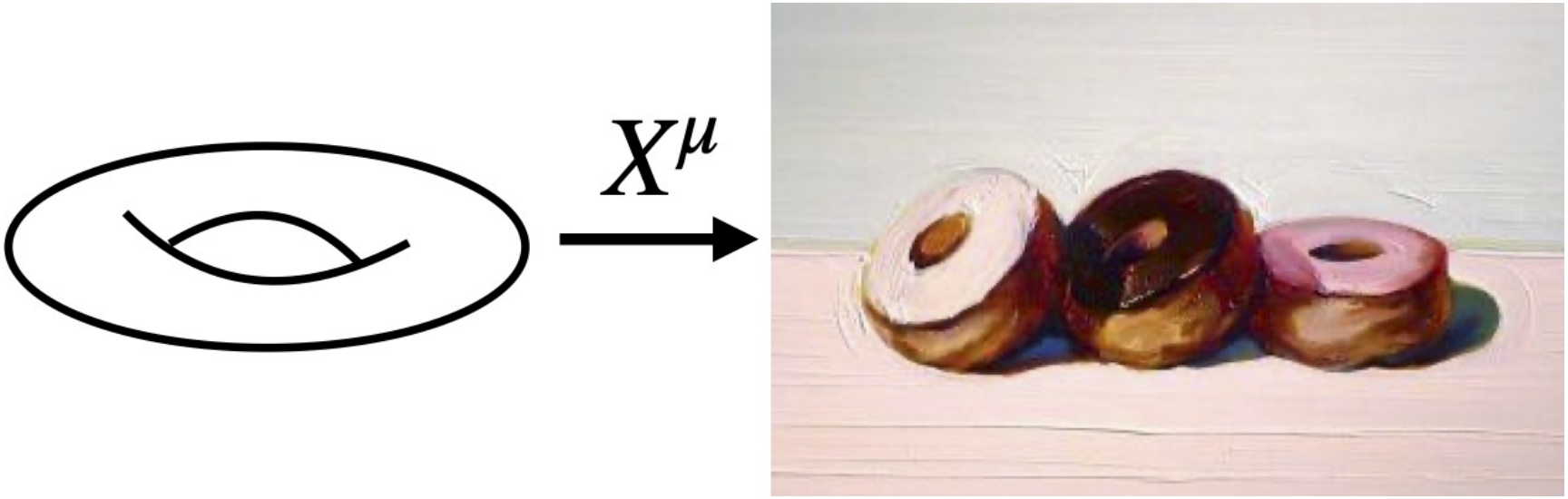
CFT on T^2



CFT on T^2



CFT on T^6

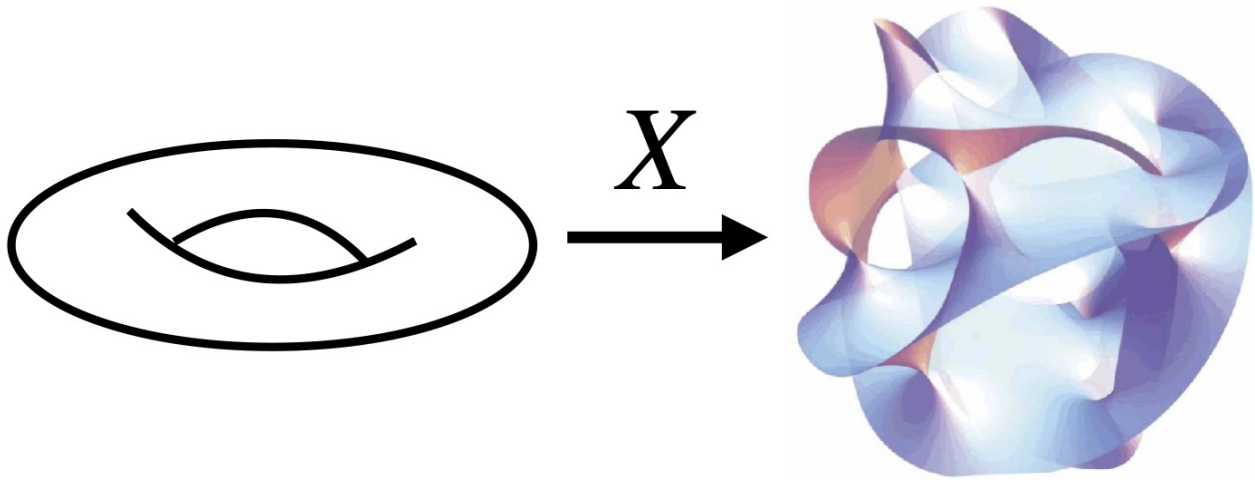


W. Thiebaud

Which CFTs appear in QG?

σ-models on torus T^d: *free* CFT and can be solved exactly.

σ-models Calabi-Yau manifolds: in general *fully interacting* CFTs.

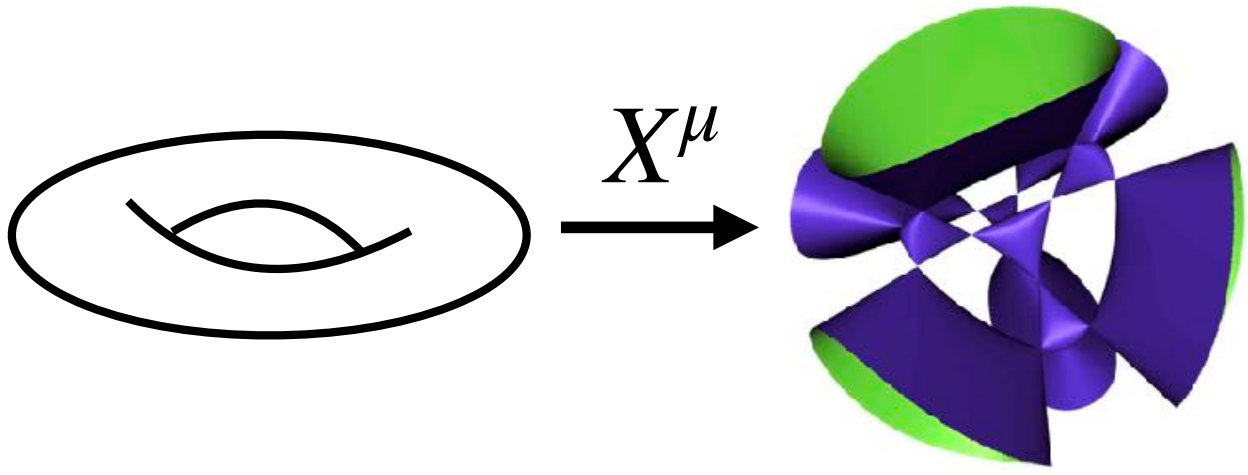


K3 Surface

In 4 real dimensions there are T^4 and K3 surfaces.

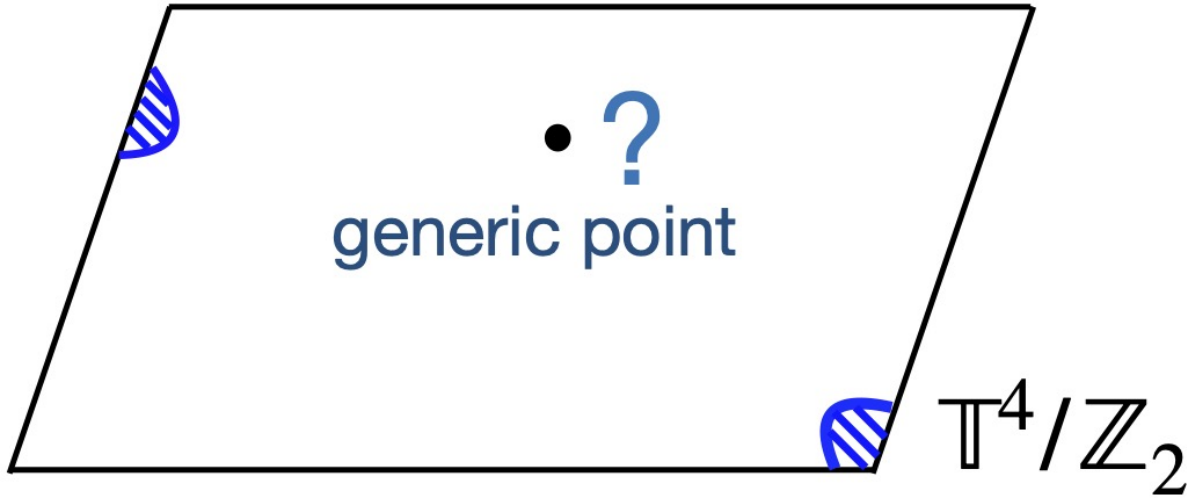
K3 Surface: the only known simply-connected compact manifold in 4d admitting Ricci-flat metrics. (Solves Einstein equations).

σ-model with target space K3



K3 CFT has an 80-d moduli space!

K3 Moduli Space



Astonishingly, apart from special regions, σ-models at generic points in K3 moduli space are *completely unknown*.

Q: What is K3 CFT at a generic point in $\mathcal{M}_{\text{K3}}$?

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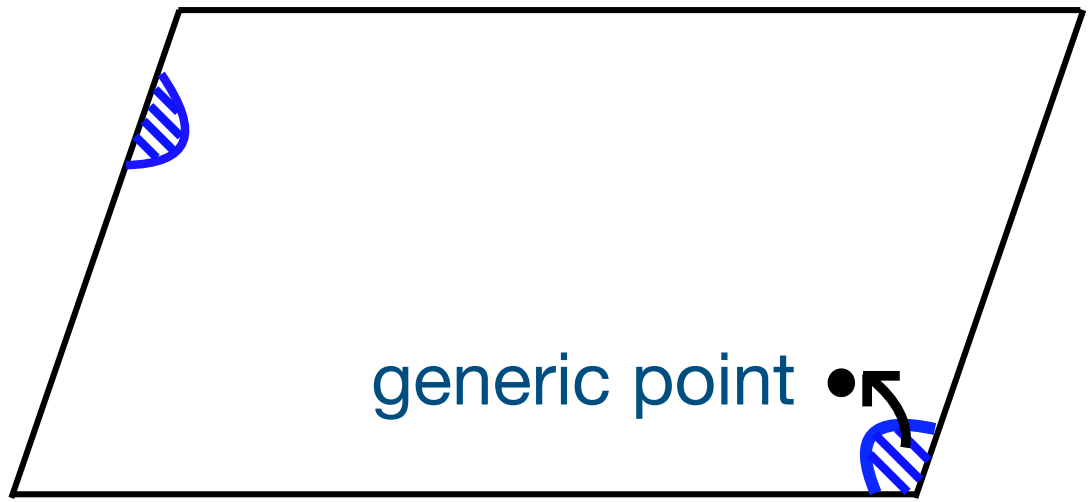
Methods: Conformal Perturbation Theory

Moduli space: averaged quantities

Worldsheet CFT, lattices

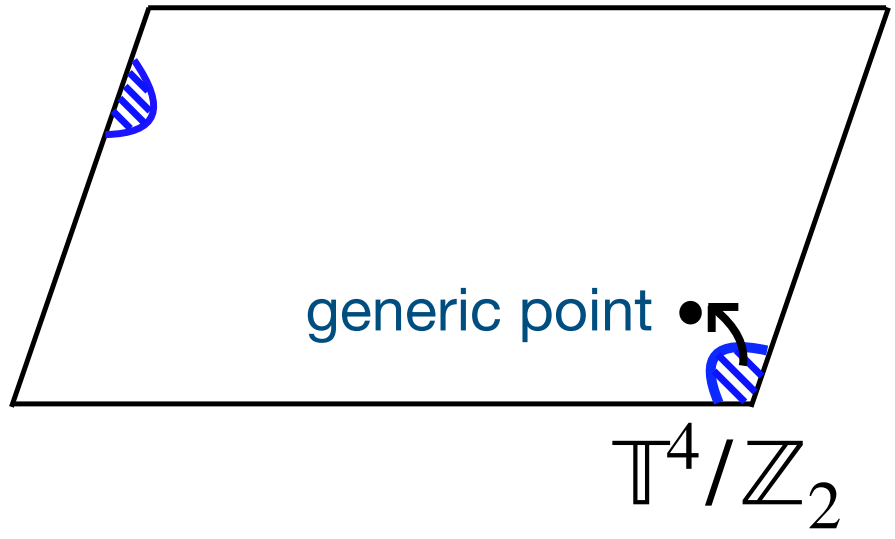
Conformal Perturbation Theory

The idea is to start from the locus with known CFT and move away from it to a generic point.



K3 CFTs

When K3 is a torus orbifold (*i.e.* $T^4/\mathbb{Z}_n$), CFT is known. Example: Kummer surface $T^4/\mathbb{Z}_2$.



Approach: deform $T^4/\mathbb{Z}_2$ CFT away from orbifold locus. [Keller, IZ]

$$S = S_0 + \lambda^a \int d^2z\, \mathcal{O}_a$$

$\mathcal{O}$: Exactly marginal operators

Shift in the spectrum:

$$\langle \varphi(w_1)\varphi(w_2)\rangle_\lambda = \left\langle \varphi(w_1)\varphi(w_2) e^{\lambda^a \int d^2z \mathcal{O}_a(z)} \right\rangle$$
$$= \frac{1}{(w_1 - w_2)^{2h(\lambda)} (\bar{w}_1 - \bar{w}_2)^{2\bar{h}(\lambda)}}$$

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$$= \frac{1}{(w_1 - w_2)^{2h(\lambda)} (\bar{w}_1 - \bar{w}_2)^{2\bar{h}(\lambda)}}$$

$$\boxed{h(\lambda) = h^{(0)} + \lambda h^{(1)} + \lambda^2 h^{(2)} + \dots}$$

$$\lambda^n h^{(n)} : \frac{\lambda^n}{n!} \int d^2z_1 \dots d^2z_n \left\langle \varphi(w_1)\varphi(w_2)\mathcal{O}(z_1)\dots\mathcal{O}(z_n) \right\rangle$$

Application: Rational Points

Q: Are RCFTs dense in $\mathcal{M}_{\mathrm{K3}}$?

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σ-model on S^1 is a RCFT for $R^2 \in \mathbb{Q}$.

$\mathcal{M}_{\mathrm{S}^1} = \mathbb{R}$ and RCFTs $\mathbb{Q} \subset \mathbb{R}$ are dense.

Similarly for σ-models on $\mathbb{T}^D$, $\mathbb{T}^D/\mathbb{Z}_N$.

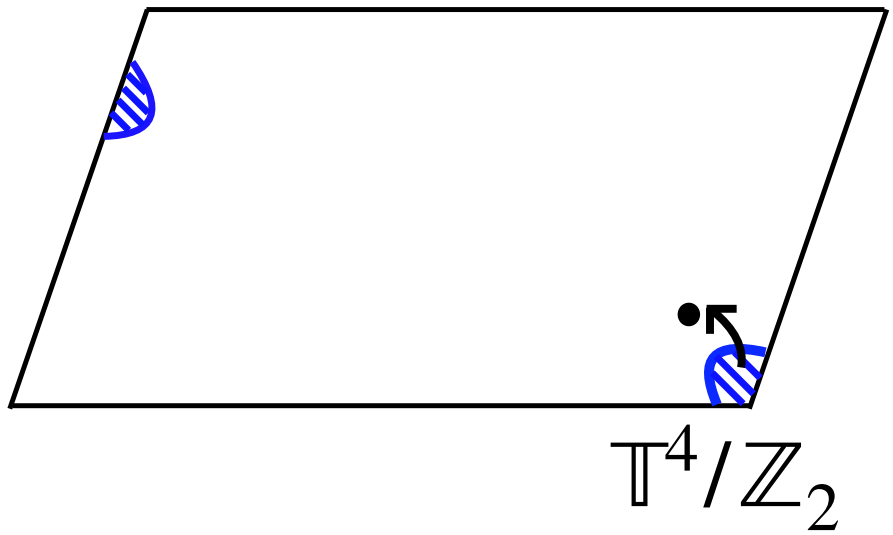
Conjecture: RCFTs on K3 are dense.

[Gukov, Vafa]

Application: Rational Points

What is the distribution of points with enhanced symmetry? [Benjamin, Keller, Ooguri, IZ]

Start from a non-rational CFT in $\mathbb{T}^4/\mathbb{Z}_2$ locus. Explore its neighbourhood.



Enhanced symmetry points can emerge, in support of denseness!

- Conformal Field Theories
- Conformal Field Theories of QG
- Future

Outlook I: K3 σ-models

- K3 CFTs at generic points?
- Are RCFTs dense on K3 moduli space?
- Denseness in higher dimensional CY CFTs.

Outlook II: Holography & Strings

- Which states count black hole entropy?

- Emergence of ensemble averages in string theory AdS_3/CFT_2 (D1-D5 brane system)?

- Find new non-supersymmetric string (M)-theory/heterotic dualities.

- Connections to automorphisms of even self-dual lattices.

Thank You!