Precision Measurement of V_{ud} from Beta Decays

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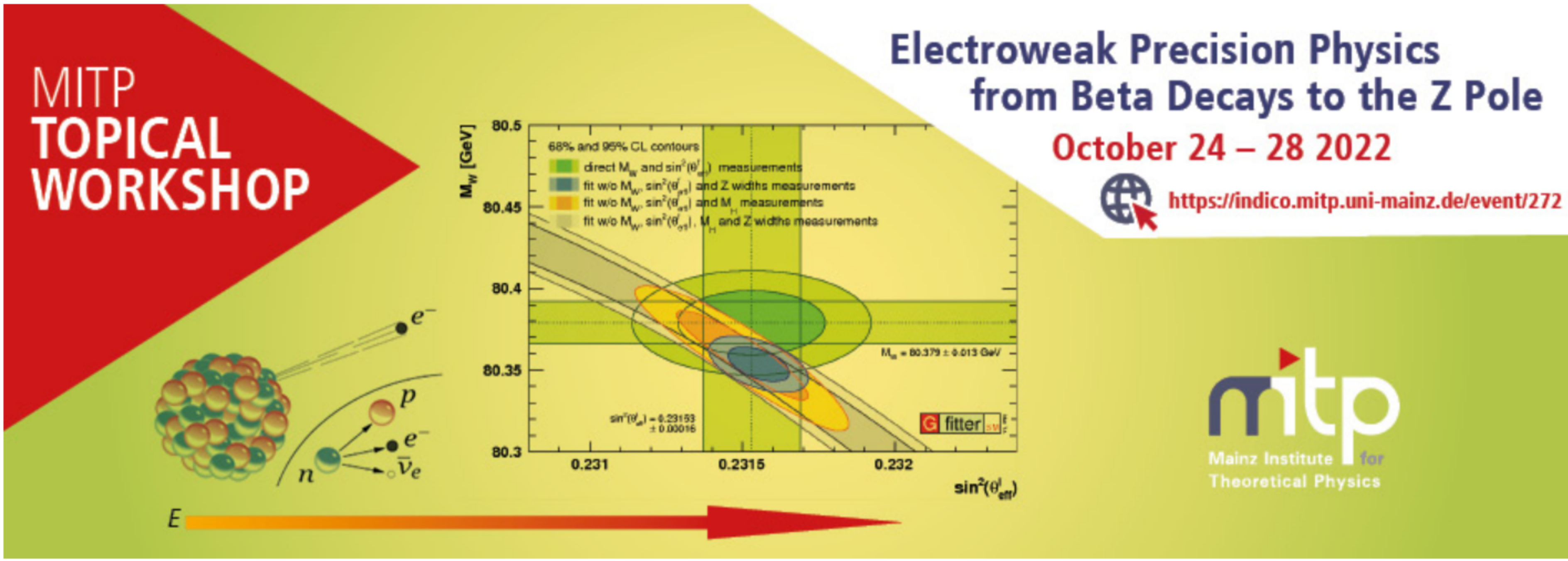
PRISMA^{+} Colloquium, Johannes Gutenberg Universität Mainz

26 October, 2022



Electroweak Precision Physics from Beta Decays to the Z Pole

My career path:

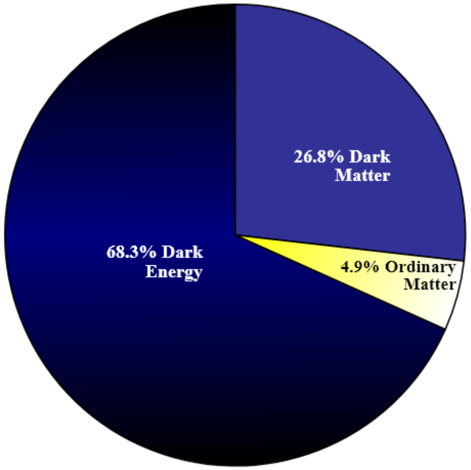
- Born in Malaysia
- 2004-2008: Bsc. in Physics, Tsinghua University
- 2008-2010: Mphil in Physics,
  Hong Kong University of Science and Technology
- 2010-2013: University of Wisconsin-Madison
- 2013-2016: PhD in Physics,
  University of Massachusetts Amherst
- 2016-2018: Postdoc, Shanghai Jiao Tong University
- 2018-2022: Postdoc (Humboldt fellow), Bonn University
- 2022-present: FRIB Theory Fellow, University of Washington

Outline

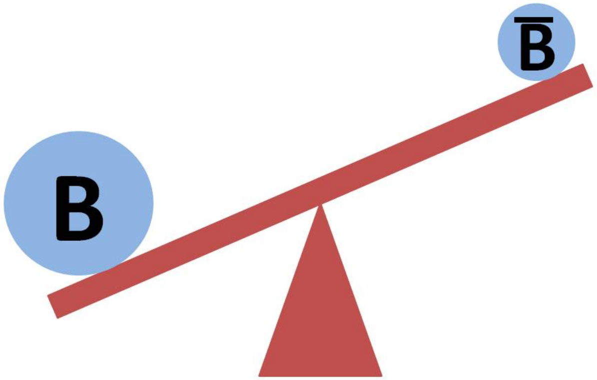
1. Beta decays and V_{ud}
2. Pion beta decay
3. Neutron beta decay
4. Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay
5. Summary

Beta decays and V_{ud}

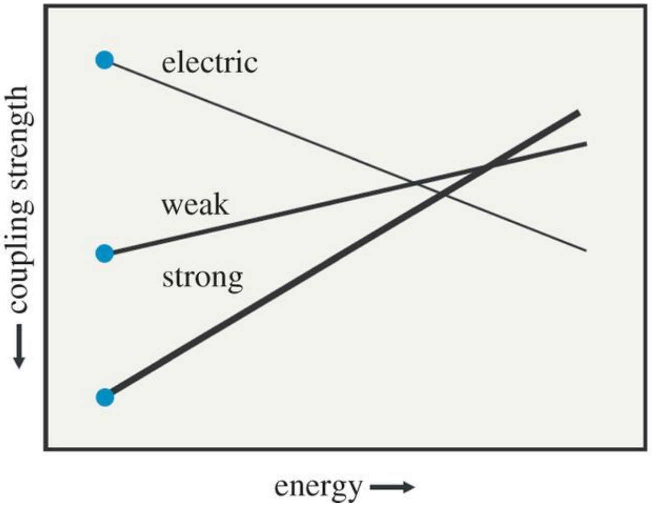
Many unresolved problems call for physics beyond the Standard Model (BSM)



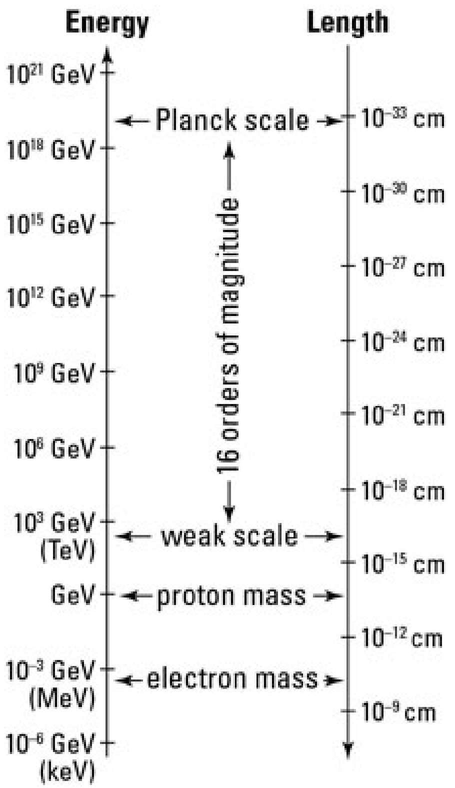
Dark energy, dark matter



Matter-antimatter asymmetry

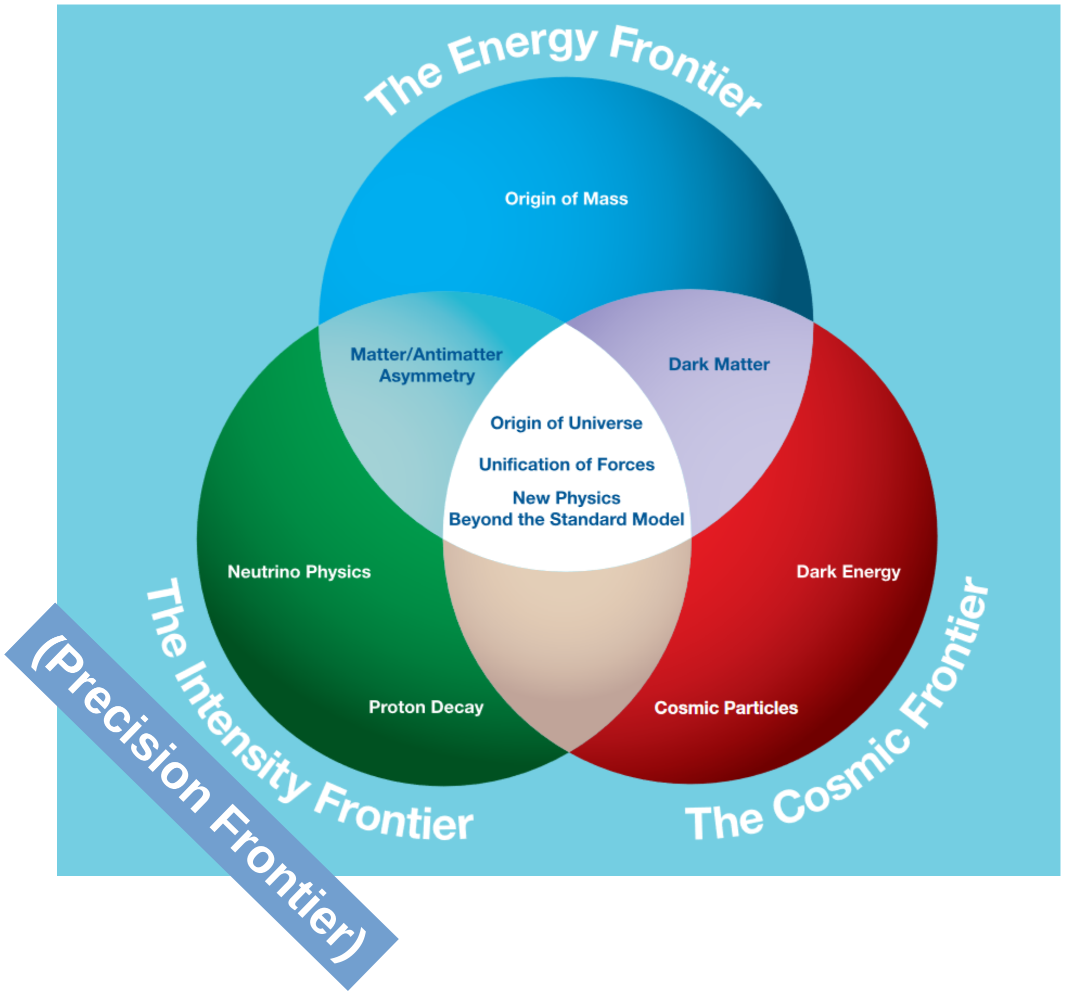


Unification of forces



Hierarchy problem

Beta decays and V_{ud}



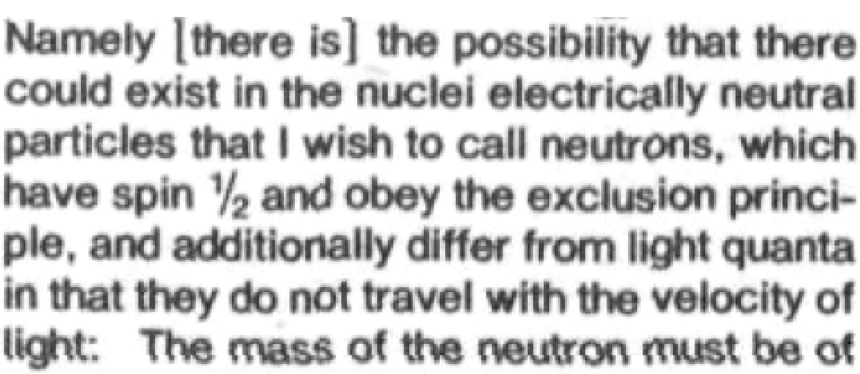
Precision Frontier: Measure things very precisely, and look for their deviations from SM prediction!

Beta decays

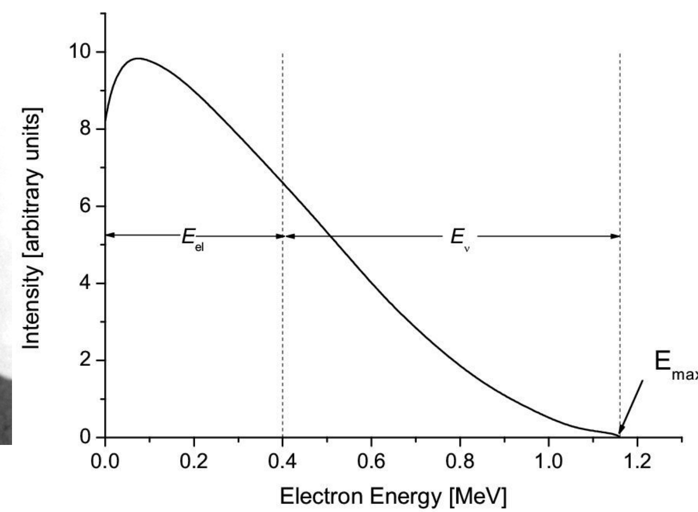
$$H_i \rightarrow H_f \beta^- \bar{\nu} \qquad\qquad H_i \rightarrow H_f \beta^+ \nu$$

had been crucial in the shaping of Standard Model!

1930: Pauli postulated the existence of “neutron” (neutrino) to explain the continuous beta decay spectrum



Namely [there is] the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin ½ and obey the exclusion principle, and additionally differ from light quanta in that they do not travel with the velocity of light: The mass of the neutron must be of



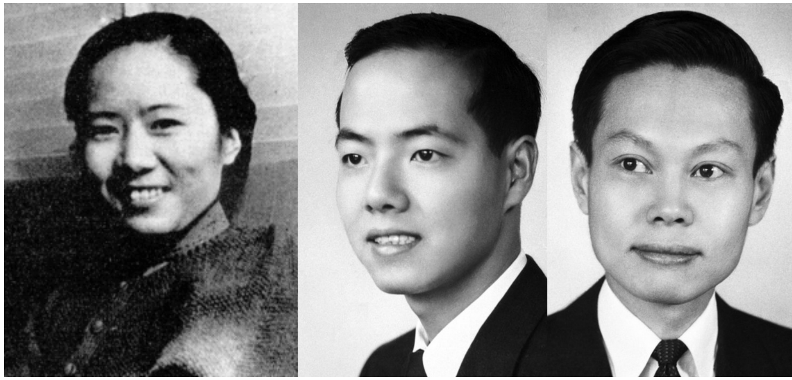
Statements in Pauli’s letter (1930), translated by *Physics Today*

Beta decays

$$H_i \to H_f \beta^- \bar{\nu} \qquad H_i \to H_f \beta^+ \nu$$

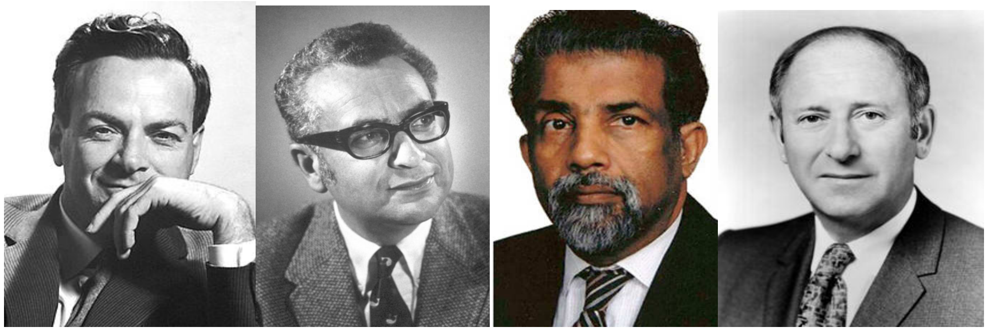
had been crucial in the shaping of Standard Model!

1956: The Wu experiment of ^{60}Co decay confirmed P-violation in weak interaction, led to the 1957 Nobel Prize by Lee and Yang.



$$^{60}_{27}\mathrm{Co} \to ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

1957: Feynman, Gell-Mann, Sudarshan and Marshak postulated the V-A structure in the charged weak interaction



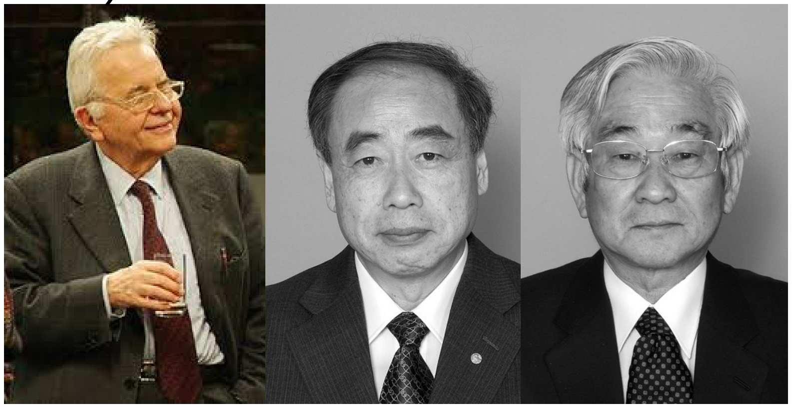
Beta decays and V_{ud}

1963: Cabibbo proposed a 2*2 matrix to mix the ΔS=0 and ΔS=1 charged weak current

1973: Kobayashi and Maskawa extended the matrix to 3*3 (**the CKM matrix**), introduced the 3rd generation quarks (Nobel Prize 2008)

$$\psi_{d,f} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_f = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

The CKM matrix

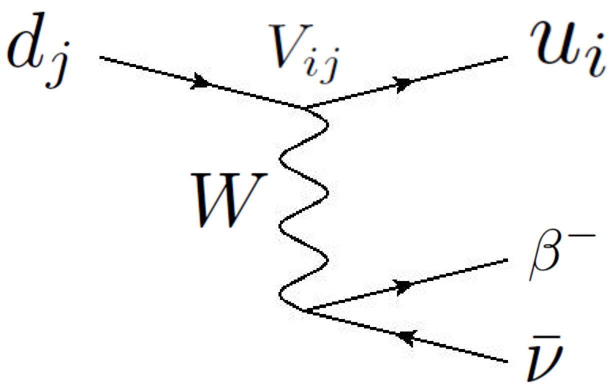


Starting from a **universal** charged weak coupling:

$$-\frac{g}{2\sqrt{2}}\bar{u}_i\gamma^\mu(1-\gamma_5)d_iW_\mu$$

$$\rightarrow \quad -\frac{g}{2\sqrt{2}}\bar{u}_iV_{ij}\gamma^\mu(1-\gamma_5)d_jW_\mu$$

$$\text{Universality} \implies VV^\dagger = 1$$



the **measured** CKM matrix must be **unitary**!

Beta decays and V_{ud}

SM prediction: **"First-row CKM unitarity"**

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 1}$$

can be tested with **0.01%** precision!

V_{ud}
- Pion beta decay
- Neutron beta decay
- Nuclear beta decay

~0.97

V_{us}
- Leptonic/semileptonic kaon decay
- Tau decay
- Hyperon decay

~0.22

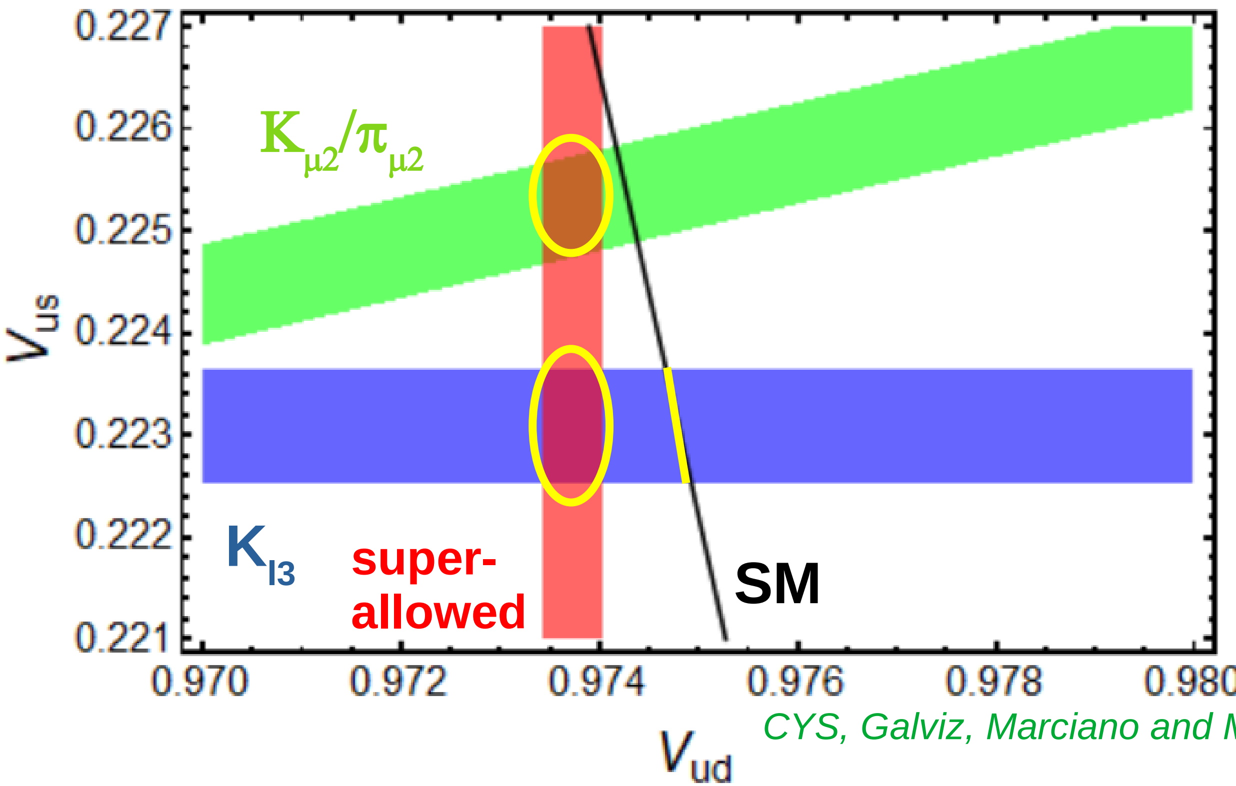
V_{ub}
- Too small!

~4x10^{-3}

"Cabibbo unitarity": $|V_{ud}| = \cos\theta_C \;,\; |V_{us}| = \sin\theta_C$

Beta decays and V_{ud}

Inconsistencies are found between different measurements of **V_{ud}**, **V_{us}** and **SM predictions**!



CYS, Galviz, Marciano and Meißner, 2022 PRD

"Cabibbo Angle Anomaly (CAA)" ~3σ

Beta decays and V_{ud}

An example:

First-row CKM unitarity with $|V_{ud}|$ from superallowed ($0^+ \rightarrow 0^+$) beta decays and $|V_{us}|$ from semileptonic kaon decays (K_{l3})

$$\boxed{|V_{ud}|^2_{0^+} + |V_{us}|^2_{K_{\ell 3}} + \cancel{|V_{ub}|^2} - 1 = -0.0021(7)}$$

$\lvert V_{ud}\rvert^2_{0^+} + \lvert V_{us}\rvert^2_{K_{\ell 3}} - 1$	-2.1×10^{-3}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **exp**	2.1×10^{-4}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **RC**	1.8×10^{-4}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **NS**	5.3×10^{-4}
$\delta\lvert V_{us}\rvert^2_{K_{\ell 3}}$, **exp+th**	1.8×10^{-4}
$\delta\lvert V_{us}\rvert^2_{K_{\ell 3}}$, **lat**	1.7×10^{-4}
Total uncertainty	6.5×10^{-4}
Significance level	3.2σ

CYS, Galviz, Marciano and Meißner, 2022 PRD

Vud inputs from Hardy and Towner, 2020 PRC

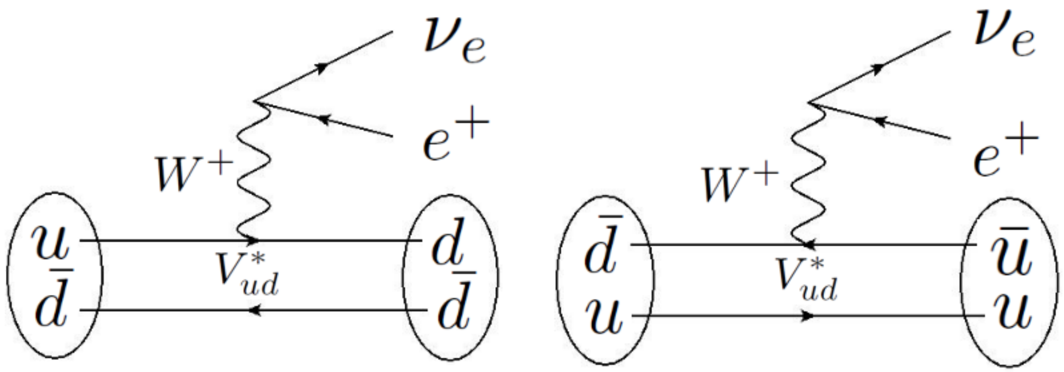
Case Study No. 1:
Pion beta decay

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$

Pion beta decay

The simplest beta decay!

$$\boxed{\pi^+ \to \pi^0 + e^+ + \nu_e} \quad (\pi_{e3})$$



Fermi constant

charged pion mass

$$\Gamma_{\pi_{e3}} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} (1+\delta)\, I_\pi$$

Partial decay rate

Fermi matrix element

Radiative Correction

Phase space integral

Pion beta decay

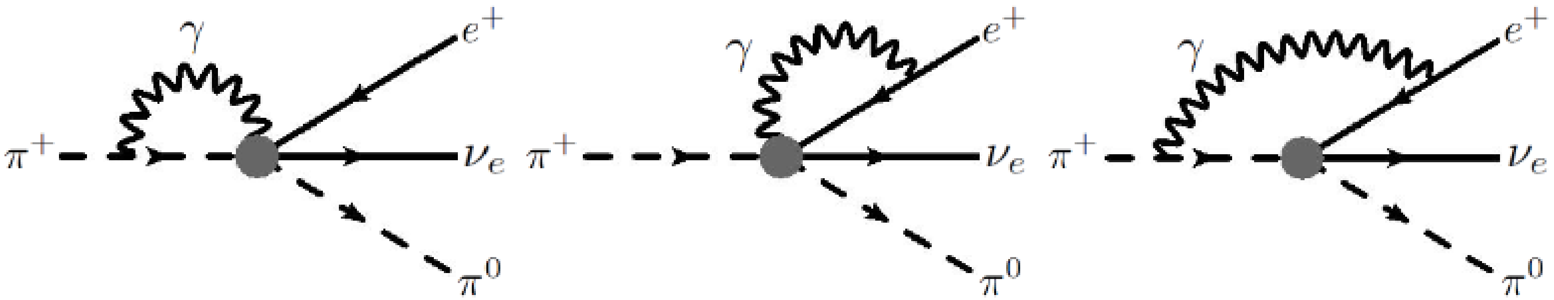
Theory Inputs:

1. **Fermi matrix element:** $|f_+^\pi(0)| = 1$ for practical purposes.

2. **Phase-space factor:** $I_\pi = 7.3764 \times 10^{-8}$

CYS, Galviz, Marciano and Meißner, 2022 PRD

3. **Electromagnetic (EM) radiative correction (RC):**



The only (highly) non-trivial theory input!

Chiral Perturbation Theory (ChPT) calculation: $\delta = 0.0334(10)$

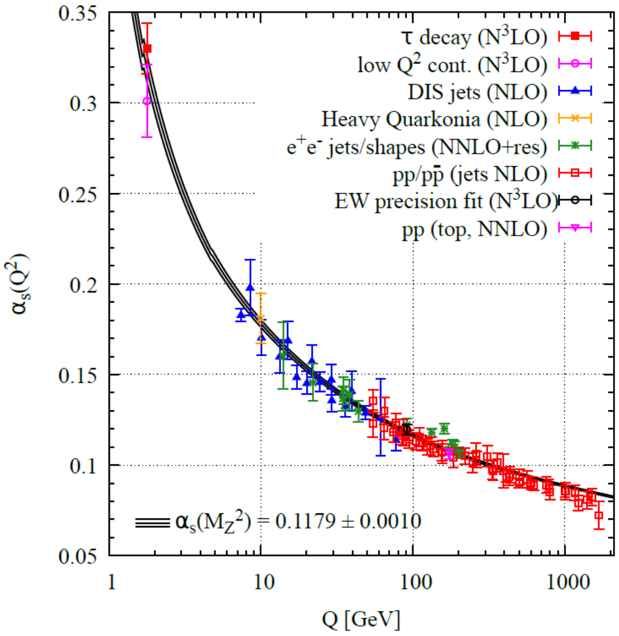
Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003

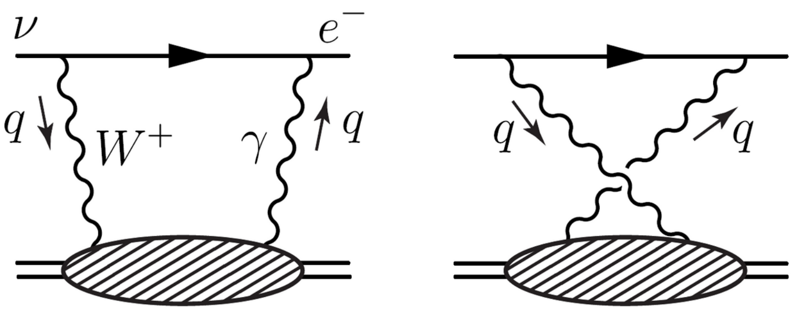
Pion beta decay

Sirlin's representation: A systematic approach to study O(α) electroweak RC

Sirlin, 1978 RMP

Important observation: The only non-trivial one-loop diagram in the electroweak RC to (Fermi) beta decays of nearly-degenerate system is the "**γW-box diagram**"





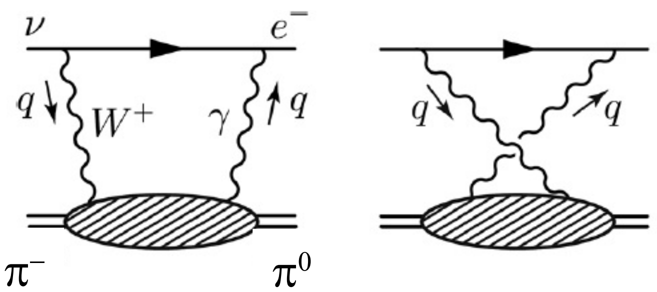
$Q^2=-q^2$

Main issue:**Strong interactions** governed by **QCD** become **non-perturbative** at $Q^2 \sim 1\ \mathrm{GeV}^2$

Pion beta decay

First-principles calculation of the **pion axial γW-box diagram**

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL



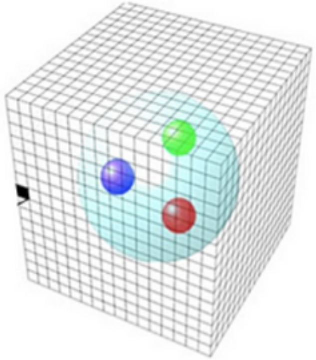
$$\square^{VA}_{\gamma W}\Big|_\pi = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_\pi(Q^2)$$

Integral sensitive to **all values of Q^2**

Large Q^2 (> 2 GeV^2): **perturbative QCD** *Baikov, Chetyrkin and Kuhn, 2010 PRL*

$$M_\pi(Q^2) = \frac{1}{12}\left[1-\tilde{C}_1\left(\frac{\alpha_S}{\pi}\right)-\tilde{C}_2\left(\frac{\alpha_S}{\pi}\right)^2-\tilde{C}_3\left(\frac{\alpha_S}{\pi}\right)^3-\tilde{C}_4\left(\frac{\alpha_S}{\pi}\right)^4-\ldots\right]$$

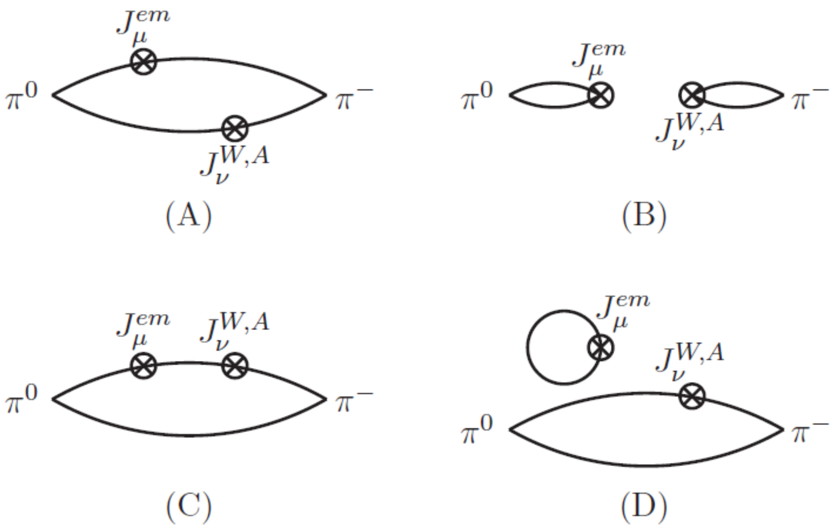
At **low Q^2** (< 2 GeV^2): **lattice QCD computation** of the **generalized Compton tensor**



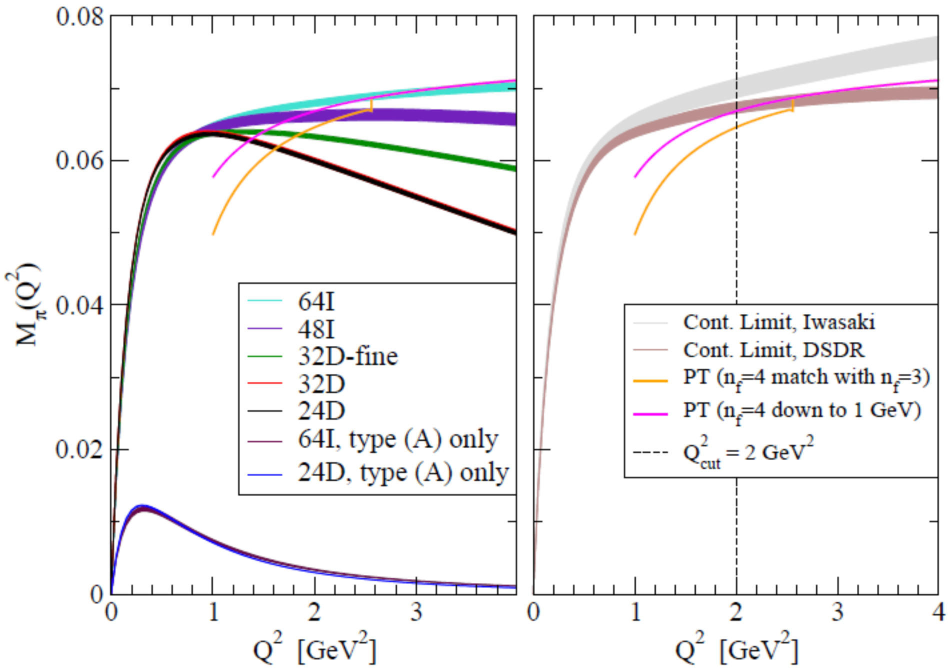
$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p)\big|\,T[J^{\text{em}}_\mu(x)J^{W,A}_\nu(0)]\,\big|\pi^-(p)\rangle$$

$$M_\pi(Q^2) = -\frac{1}{6\sqrt{2}}\frac{\sqrt{Q^2}}{m_\pi}\int d^4x\,\omega(Q,x)\epsilon_{\mu\nu\alpha 0}x_\alpha\mathcal{H}^{VA}_{\mu\nu}(x)$$

Pion beta decay



Quark contraction diagrams



(before cont. extrapolation) (after cont. extrapolation)

Significant improvement of the RC precision: $\boxed{\delta = 0.0334(10) \rightarrow 0.0332(3)}$

Pion beta decay is now the **theoretically cleanest channel** to extract V_{ud}

Pion beta decay

Experimental Inputs:

1. Fermi constant, total lifetime and masses are well-measured
2. Pion beta decay **branching ratio (BR)**:

$$\mathrm{BR}(\pi_{e3}) = 1.036(6) \times 10^{-8}$$

Pocanic et al (PIBETA), 2004 PRL

hard to achieve high precision due to its smallness

Future experiment: PIONEER at PSI

Phase I : > yr 2029
Phase II : Improve $\mathrm{BR}(\pi_{e3})$ precision by a factor 3
Phase III: Improve $\mathrm{BR}(\pi_{e3})$ precision by a factor 10

PIONEER Collaboration, 2022 Snowmass Summer Study, 2203.05505

may lead to one of the most precise determinations of V_{ud}!

Short Summary:

$$\boxed{|V_{ud}|_{\pi_{e3}} = 0.9740(28)_{\text{exp}}(1)_{\text{th}}}$$

exp: From BR(π_{e3}). May see improvements in the next decade

th: From higher-order electroweak RC. Further improvement not urgent

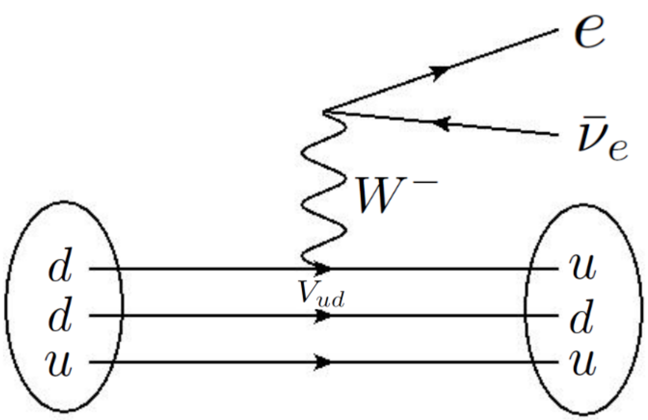
Case Study No. 2: Neutron beta decay

$$n \to p + e + \bar{\nu}_e$$

Neutron beta decay

The simplest beta decay of spinful-particles!

$$\boxed{n \to p + e + \bar{\nu}_e}$$



$$|V_{ud}|^2 = \frac{5024.7\,\mathrm{s}}{\tau_n(1+3\lambda^2)(1+\Delta_R^V)}$$

Neutron lifetime

(Renormalized) axial-to-vector coupling ratio

Inner RC to the Fermi matrix element

Neutron beta decay

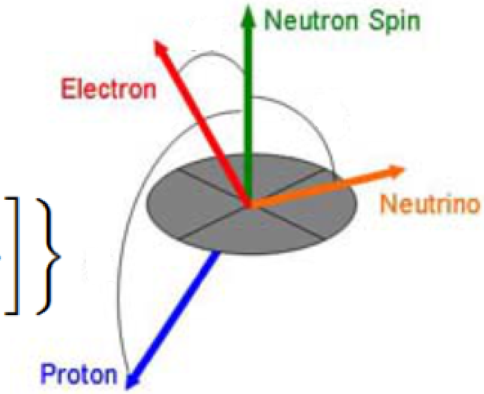
New features compare to pion:

Fermi　　Gamow-Teller (GT)

$$\boxed{\langle p|J_W^{\mu}|n\rangle = \bar{u}\gamma^{\mu}(\mathring{g}_V + \mathring{g}_A\gamma_5)u}$$

After RC: $\mathring{g}_{V,A} \to g_{V,A}$　　**Axial-to-vector ratio:** $\lambda = g_A/g_V$

λ requires separate measurement through the correlation coefficients of the differential decay rate



$$d\Gamma \propto F(E_e)\left\{1 + b\frac{m_e}{E_e} + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_eE_\nu} + \langle\vec{J}\rangle\cdot\left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + \cdots\right]\right\}$$

- The **bare GT matrix element** is calculable from lattice to sub-percent level

  PNDME 18, CalLat 18, CalLat 19...

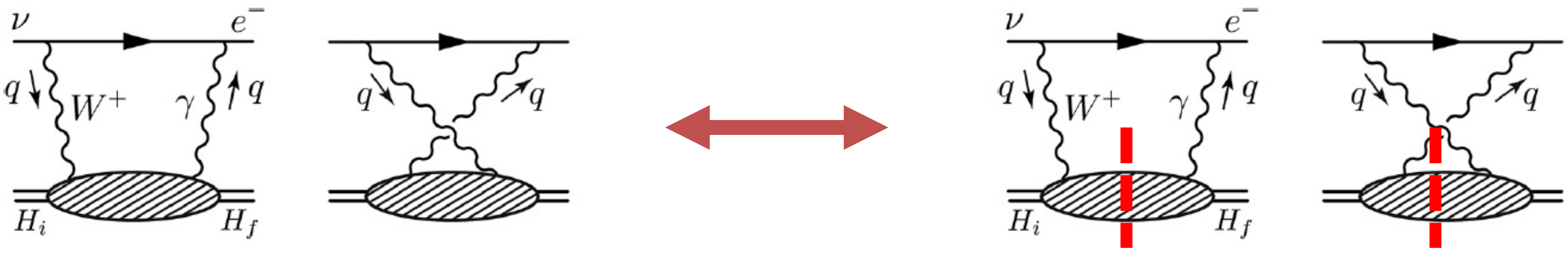
- Direct comparison between theory and experiment may serve as **a strong probe of BSM physics**

  Gonzalez-Alonso, Naviliat-Cuncic and Severijns, 2019 Prog.Part.Nucl.Phys

Theory Input 1: RC to the Fermi matrix element

Again, the only non-trivial part is the axial γW-box diagram.

Dispersion relation (DR) treatment --- relate the loop integral to experimentally-measurable structure functions *CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL*



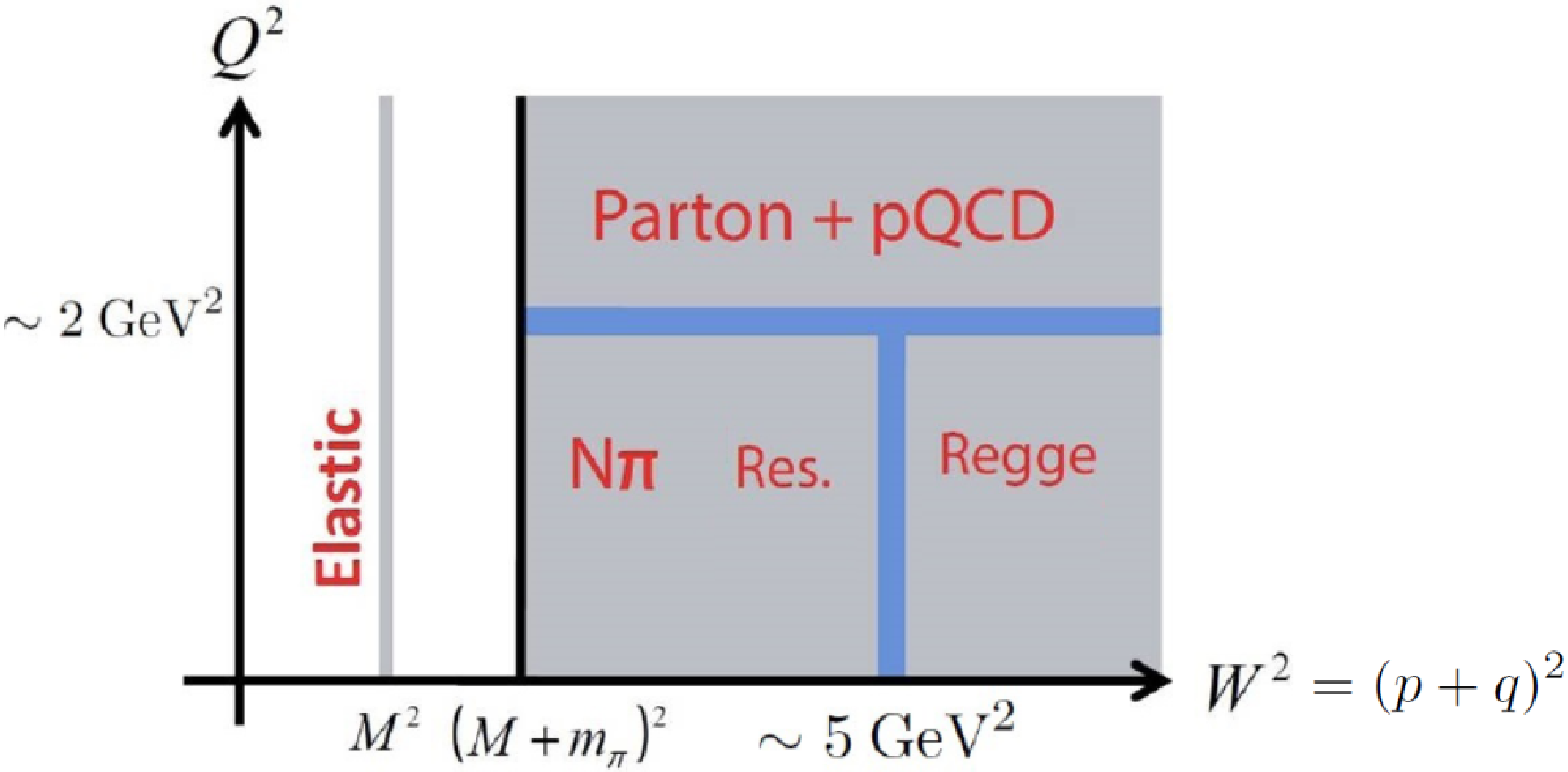
Generalized Compton tensor **On-shell hadronic tensor**

$$\frac{1}{2}\int d^4x e^{iq\cdot x}\langle H_f(p)|T[J^\mu_{\text{em}}(x)J^\nu_W(0)]|H_i(p)\rangle \longleftrightarrow \frac{1}{8\pi}\int d^4x e^{iq\cdot x}\langle H_f(p)|[J^\mu_{\text{em}}(x),J^\nu_W(0)]|H_i(p)\rangle$$

$$\square^V_{\gamma W}=\frac{\alpha_{em}}{\pi\mathring{g}_V}\int_0^\infty\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\frac{1+2r}{(1+r)^2}F_3^{(0)}(x,Q^2)$$

Neutron beta decay

Dominant intermediate state contributions at different kinematic regions:



Neutron beta decay

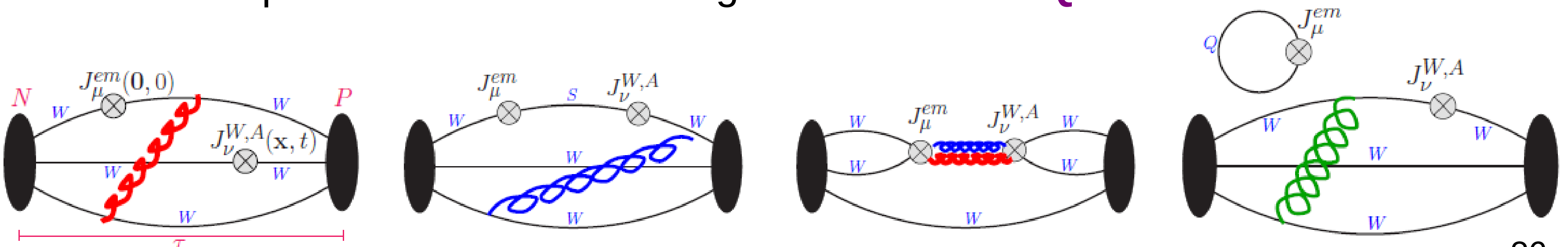
Different evaluations of the inner RC:

Method	Δ_R^V	
DR with neutrino data (1)	0.02467(22)	*CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL*
DR with neutrino data (2)	0.02471(18)	*Shiells, Blunden and Melnitchouk, 2021 PRD*
DR with indirect lattice data	0.02477(24)	*CYS, Feng, Gorchtein and Jin, 2020 PRD*
Non-DR (1)	0.02426(32)	*Czarnecki, Marciano and Sirlin, 2019 PRD*
Non-DR (2)	0.02473(27)	*Hayen, 2021 PRD*

All **systematically larger** than the pre-2018 state-of-the-art determination: $\Delta_R^V = 0.02361(38)$ *Marciano and Sirlin, 2006 PRL*

$$\Delta_R^V \uparrow \Longrightarrow |V_{ud}| \downarrow$$

Future: Compute the neutron box diagram with **lattice QCD**

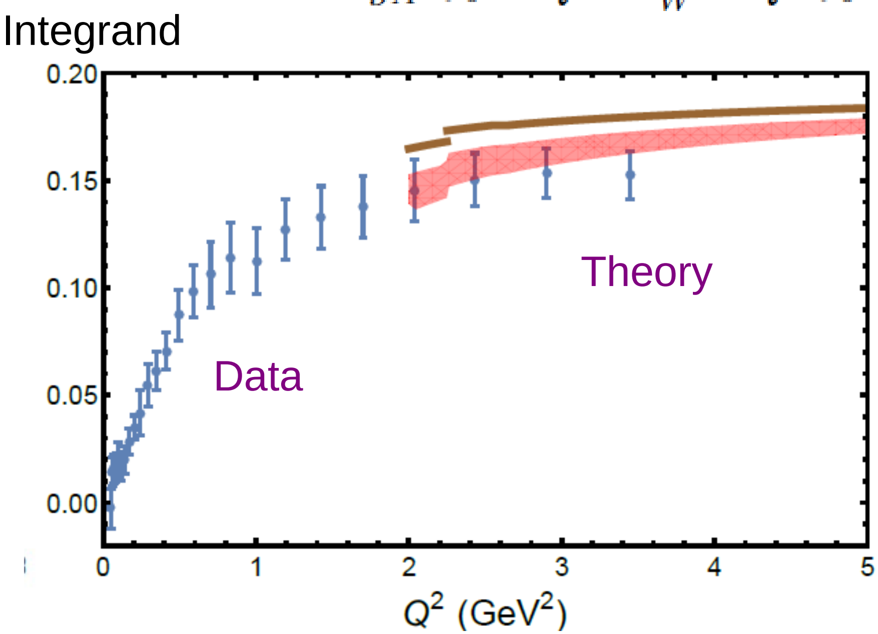


from R. Gupta, LANL

Theory Input 2: RC to the GT element

Box diagram contribution can be related to the spin-dependent structure functions g_1 and g_2 *Gorchtein and CYS, JHEP 10 (2021) 053*

$$\Box^{A}_{\gamma W} = -\frac{2\alpha_{em}}{\pi \mathring{g}_A}\int_0^{\infty}\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1\frac{dx}{(1+r)^2}\left[\frac{5+4r}{3}g_1^{(0)}(x,Q^2)-\frac{4M^2x^2}{Q^2}g_2^{(0)}(x,Q^2)\right]$$



Data obtained from **deep inelastic scattering (DIS) off proton and neutron**

CLAS Collaboration (Jefferson Lab), EG1b experiment; 2015 PRC and 2017 PRC

Method	$(\Delta^A_R - \Delta^V_R)_{\gamma W}$
DR	$0.26(25)\times 10^{-3}$
Non-DR	$0.60(5)\times 10^{-3}$

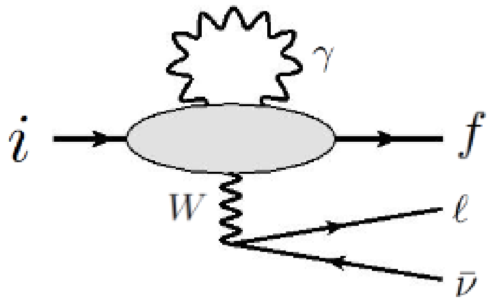
CYS and Gorchtein, 2021 JHEP

Hayen, 2021 PRD

Box diagram correction to g_V and g_A are almost identical!

Neutron beta decay

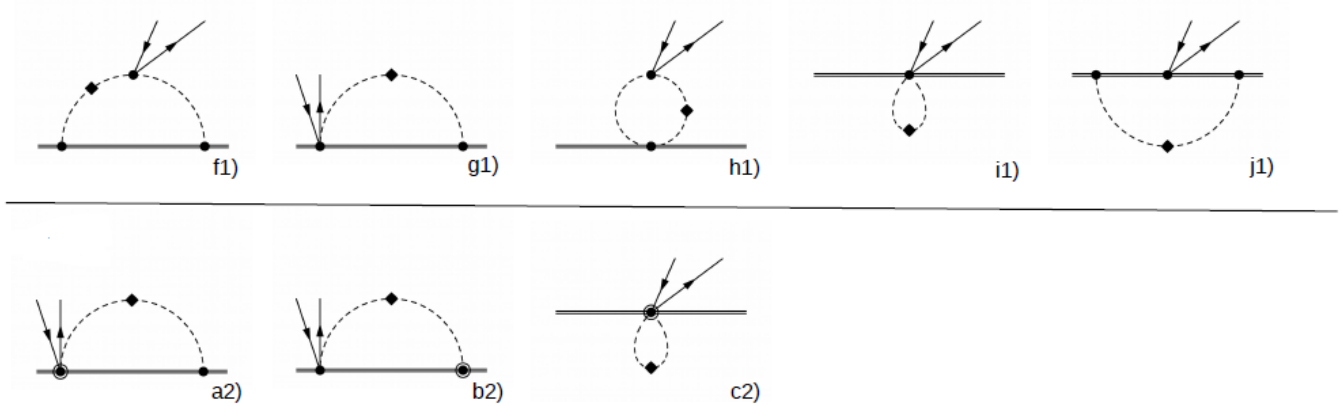
But unlike g_V, the box diagram is NOT the only non-trivial part in the g_A RC !



"three-point function"

In terms of effective field theory (EFT): **"Pion-induced RC"**

Cirigliano, de Vries, Hayen, Mereghetti and Walker-Loud, 2022 PRL



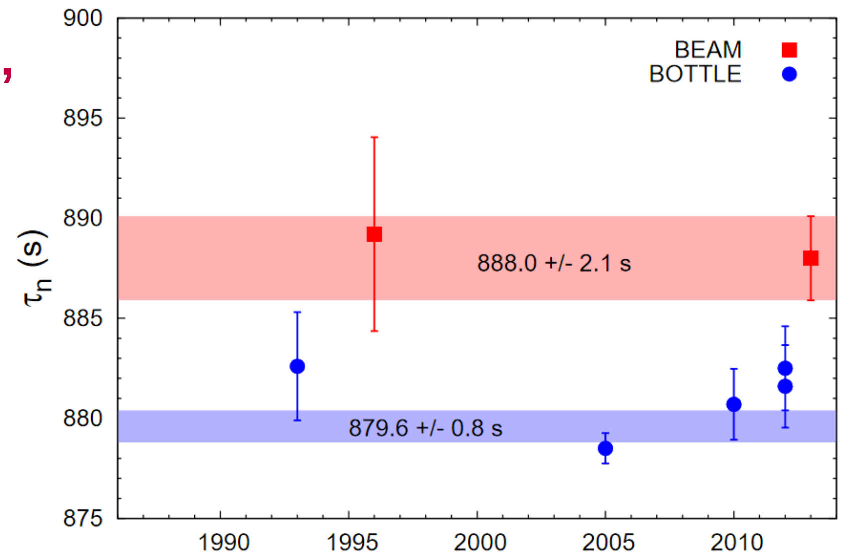
Possibly-large correction to g_A, **~10^{-2}**. Remains to be understood.
Not directly relevant to V_{ud} extraction, but affects BSM search.

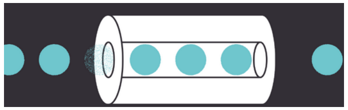
Experimental Input 1: Neutron lifetime

"Beam-bottle discrepancy" remains unresolved

"Beam" average:

$$(\tau_n)_{\mathrm{beam}} = 888.0(2.1)\ \mathrm{s}$$





ACFI workshop white paper, year 2014

Current particle data group (PDG) average from "bottle":

$$(\tau_n)_{\mathrm{bottle}} = 878.4(5)\ \mathrm{s}$$

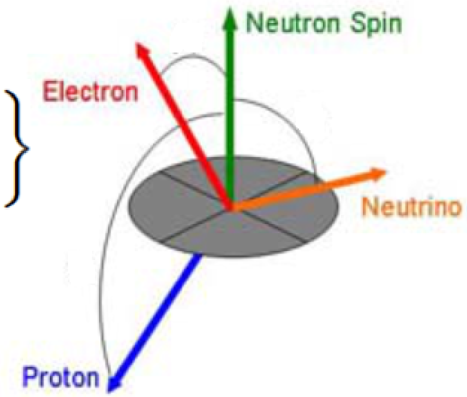
The single best measurement: *UCNτ Collaboration, 2021 PRL*

$$\tau_n = 877.75 \pm 0.28_{\mathrm{stat}} + 0.22/-0.16_{\mathrm{syst}}\ \mathrm{s}$$

Neutron beta decay

Experimental Input 2: Axial-to-vector ratio λ

$$d\Gamma \propto F(E_e)\left\{1 + b\,\frac{m_e}{E_e} + a\,\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_eE_\nu} + \langle\vec{J}\rangle\cdot\left[\,A\,\frac{\vec{p}_e}{E_e} + B\,\frac{\vec{p}_\nu}{E_\nu} + \cdots\right]\right\}$$



Polarized neutron => A,B
Proton recoil spectrum => a

The single best measurement (from A): *Märkisch et al (PERKEO III), 2019 PRL*

$$\boxed{\lambda = -1.27641(45)_{\mathrm{stat}}(33)_{\mathrm{syst}}}$$

PDG average:

$$\boxed{\lambda = -1.2754(13)}$$

Inflated error due to the discrepancy of λ determined from A and a.

Short Summary:

Adopting an averaged value of single-nucleon inner RC: $\Delta_R^V = 0.02467(27)$

Cirigliano, Crivellin, Hoferichter and Moulson, 2208.11707

We obtain: $$\boxed{|V_{ud}|_n = 0.97441(13)_{\rm th}(82)_\lambda(28)_{\tau_n}}$$

using the PDG-average of τ_n and λ, or

$$\boxed{|V_{ud}|_n = 0.97413(13)_{\rm th}(35)_\lambda(20)_{\tau_n}}$$

using the single best measurement of τ_n and λ.

th: γW-box diagram, possible future improvements using lattice QCD

τ_n and λ: Future experiments including UCNτ+ , UCNA+, Nab, PERC...

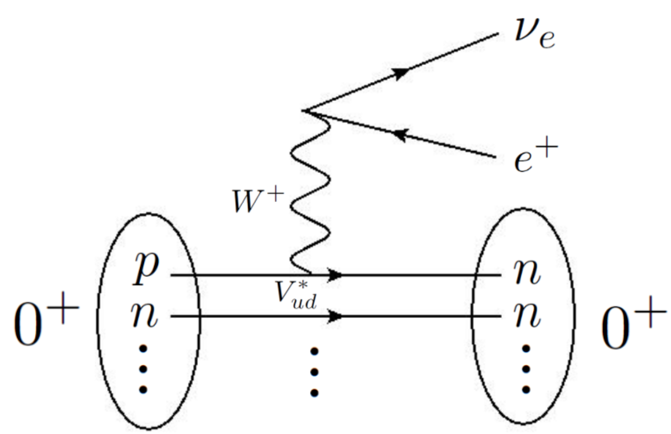
Case Study No. 3:
Superallowed 0+ → 0+ nuclear beta decay

$$i(0^+) \to f(0^+) + e^+ + \nu_e$$

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

The simplest nuclear beta decay!

$$\boxed{i(0^+) \rightarrow f(0^+) + e^+ + \nu_e}$$



experimental ft-value

$$\boxed{|V_{ud}|^2 = \frac{2984.43\ s}{\mathcal{F}t\left(1+\Delta_R^V\right)}.}$$

free-nucleon inner RC

(discussed before)

$$\mathcal{F}t = ft\left(1+\delta_R'\right)\left(1+\delta_{\rm NS}-\delta_{\rm C}\right)$$

"Outer correction"

(well under control)

Nuclear structure effects in inner RC

Isospin-breaking correction

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Superallowed nuclear beta decays of **T=1, $J^p=0^+$ nuclei** provide currently the **best** measurement of V_{ud}

1. Conserved vector current (CVC) at tree level
2. Large number of measured transitions → Huge gain in statistics
3. Price to pay: Nuclear-structure-dependent corrections

Experimental Input: ft-values

23 measured superallowed transitions, 15 among them whose lifetime precision is 0.23% or better

Hardy and Towner, 2020 PRC

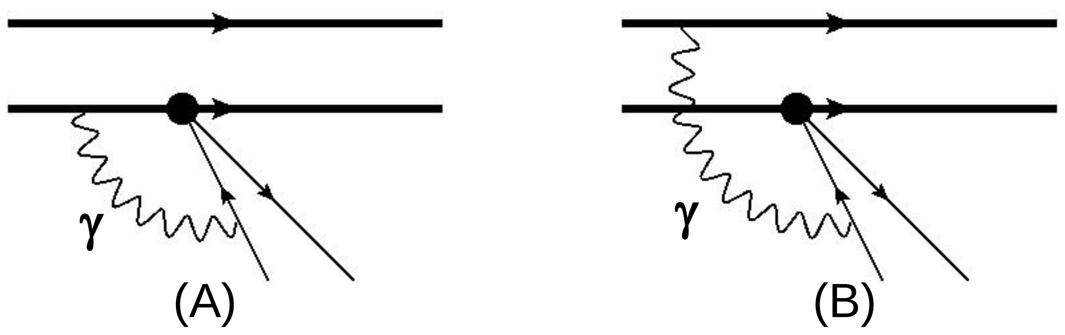
$T_z = -1$	
${}^{10}_{6}\mathrm{C} \rightarrow {}^{10}_{5}\mathrm{B}$	●
${}^{14}_{8}\mathrm{O} \rightarrow {}^{14}_{7}\mathrm{N}$	●
${}^{18}_{10}\mathrm{Ne} \rightarrow {}^{18}_{9}\mathrm{F}$	
${}^{22}_{12}\mathrm{Mg} \rightarrow {}^{22}_{11}\mathrm{Na}$	●
${}^{26}_{14}\mathrm{Si} \rightarrow {}^{26}_{13}\mathrm{Al}$	●
${}^{30}_{16}\mathrm{S} \rightarrow {}^{30}_{15}\mathrm{P}$	
${}^{34}_{18}\mathrm{Ar} \rightarrow {}^{34}_{17}\mathrm{Cl}$	●
${}^{38}_{20}\mathrm{Ca} \rightarrow {}^{38}_{19}\mathrm{K}$	●
${}^{42}_{22}\mathrm{Ti} \rightarrow {}^{42}_{21}\mathrm{Sc}$	
${}^{46}_{24}\mathrm{Cr} \rightarrow {}^{46}_{23}\mathrm{V}$	
${}^{50}_{26}\mathrm{Fe} \rightarrow {}^{50}_{25}\mathrm{Mn}$	
${}^{54}_{28}\mathrm{Ni} \rightarrow {}^{54}_{27}\mathrm{Co}$	

$T_z = 0$	
${}^{26m}_{13}\mathrm{Al} \rightarrow {}^{26}_{12}\mathrm{Mg}$	●
${}^{34}_{17}\mathrm{Cl} \rightarrow {}^{34}_{16}\mathrm{S}$	●
${}^{38m}_{19}\mathrm{K} \rightarrow {}^{38}_{18}\mathrm{Ar}$	●
${}^{42}_{21}\mathrm{Sc} \rightarrow {}^{42}_{20}\mathrm{Ca}$	●
${}^{46}_{23}\mathrm{V} \rightarrow {}^{46}_{22}\mathrm{Ti}$	●
${}^{50}_{25}\mathrm{Mn} \rightarrow {}^{50}_{24}\mathrm{Cr}$	●
${}^{54}_{27}\mathrm{Co} \rightarrow {}^{54}_{26}\mathrm{Fe}$	●
${}^{62}_{31}\mathrm{Ga} \rightarrow {}^{62}_{30}\mathrm{Zn}$	●
${}^{66}_{33}\mathrm{As} \rightarrow {}^{66}_{32}\mathrm{Ge}$	
${}^{70}_{35}\mathrm{Br} \rightarrow {}^{70}_{34}\mathrm{Se}$	
${}^{74}_{37}\mathrm{Rb} \rightarrow {}^{74}_{36}\mathrm{Kr}$	●

● : Lifetime precision better than 0.23%

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Theory Input 1: Nuclear structure effects in inner RC, δ_{NS}



Classical separation: $$\boxed{\delta_{\mathrm{NS}} = \delta_{\mathrm{NS,A}} + \delta_{\mathrm{NS,B}}}$$

Type B: Weak and EM vertices act on two different nucleons.

Jaus and Rasche, 1990 PRD; Barker et al., 1992 NPA; Towner, 1992 NPA

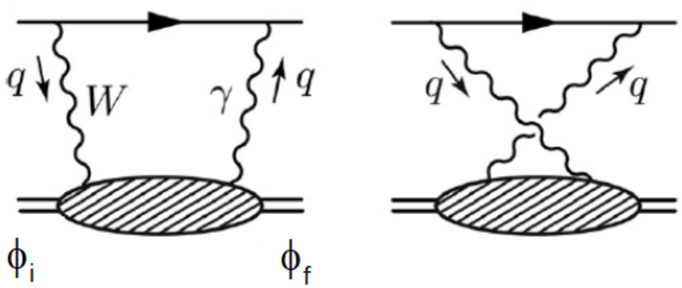
Type A: Nuclear medium effect, weak and EM vertices act on the same nucleon.
Computed with quenched coupling constants. *Towner, 1994 PLB*

All computed with non-relativistic (NR) nuclear models!

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Modern starting point: δ_{NS} in terms of **the difference between the nuclear and nucleon γW-box diagram**, both in fully-relativistic notations

CYS, Gorchtein and Ramsey-Musolf, 2019 PRD



$$\boxed{\Box^{\rm nucl.}_{\gamma W} = \Box^{n}_{\gamma W} + \underbrace{\left[\Box^{\rm nucl.}_{\gamma W} - \Box^{n}_{\gamma W}\right]}_{\sim\frac{1}{2}\,\delta_{NS}}}$$

Dispersive representation of the nuclear γW box diagram

$$\mathfrak{Re}\,\Box^{b,{\rm e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\rm thr}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

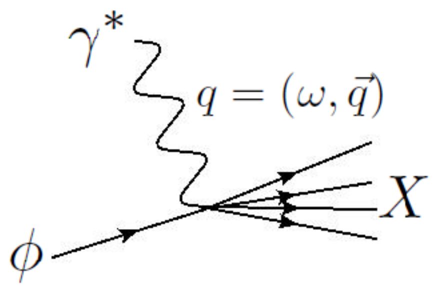
$$\mathfrak{Re}\,\Box^{b,{\rm o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\rm thr}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

Gorchtein, 2019 PRL

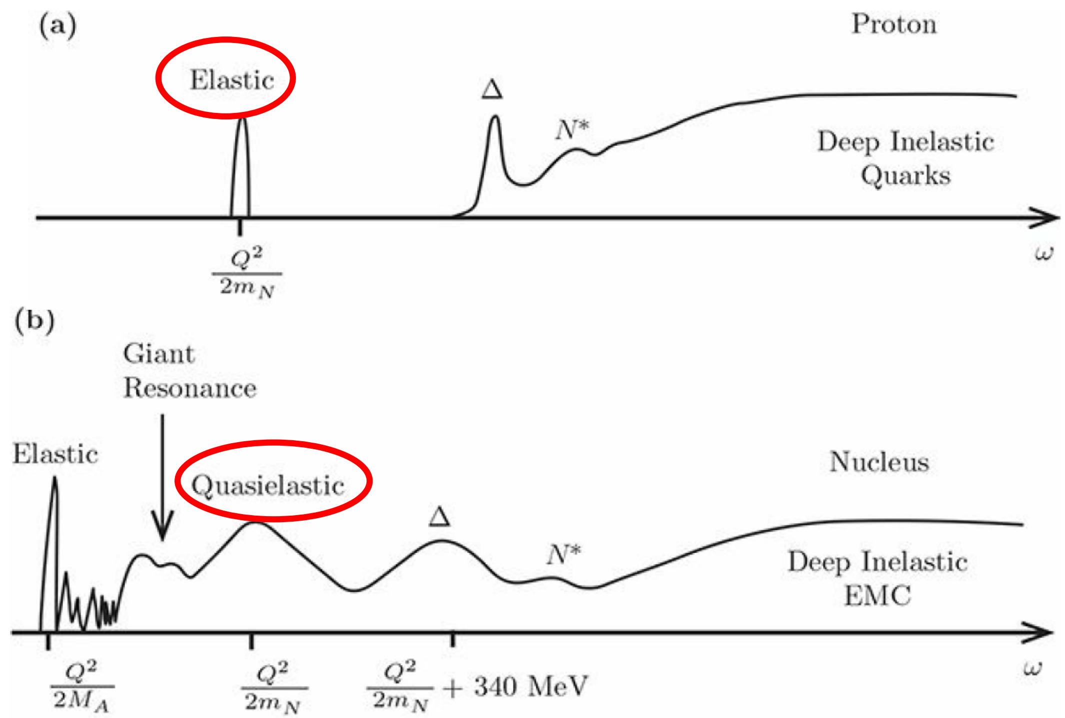
Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Nuclear structure function:

$$\frac{1}{8\pi}\sum_X (2\pi)^4\delta^{(4)}(p+q-p_X)\langle\phi_f(p)|J^\mu_{\rm em}|X\rangle\langle X|(J^{\dagger\nu}_W)_A|\phi_i(p)\rangle = -\frac{i\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{2p\cdot q}F_3(\nu,Q^2)$$



Nuclear modifications of absorption spectrum:



Donnelly, Formaggio, Holstein, Milner and Surrow, "Foundations of Nuclear and Particle Physics"

Calculations with quenched couplings did not properly account for the **quasielastic broadening effect**; adding it back results in an **inflated error in δ_{NS}**

CYS, Gorchtein and Ramsey-Musolf, 2019 PRD; Gorchtein, 2019 PRL

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Future of δ_{NS}: **Ab-initio calculations** of nuclear box diagram

(1) Nuclear forces:

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

AV18, IL7, CD-Bonn, Chiral effective field theory (ChEFT)...

(2) Solve the many-body Schrödinger equation and compute matrix elements:

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

Quantum Monte Carlo (QMC) , No-core shell model (NCSM), Coupled Cluster,
Nuclear lattice effective field theory (NLEFT)...

(3) Object of study: Nuclear response function

$$R_{xy}(q,\omega) \propto \sum_X \delta(\omega + M_\phi - E_X)\mathrm{Im}[\langle X|j_1^x(\vec{q},\omega)|\phi\rangle\langle\phi|j_2^y(\vec{q},\omega)|X\rangle^*]$$

Shen, Marcucci, Carlson, Gandolfi and Schiavilla, 2012 PRC

$^{10}C \rightarrow {}^{10}B$ transition: the first, important prototype!

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Theory Input 2: Isospin breaking (ISB) correction, δ_C

Fermi matrix element: $M_F = \langle f | \hat{\tau}_+ | i \rangle$

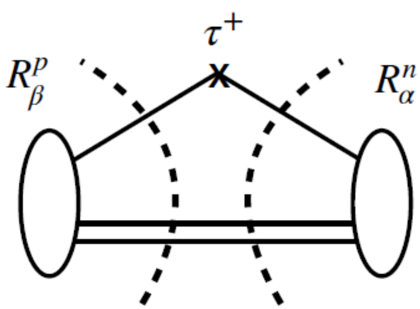
In the isospin-symmetric limit:

$$M_F^0 = \langle f_0 | \hat{\tau}_+ | i_0 \rangle = \langle T=1, T_z+1 | \hat{\tau}_+ | T=1, T_z \rangle = \sqrt{2}$$

Their **difference** defines δ_C:

$$\boxed{|M_F|^2 = |M_F^0|^2 (1 - \delta_C)}$$

ORIGIN: Slight mismatch between the initial proton and the final neutron radial wavefunction, mainly due to **Coulomb (C) repulsion between protons**



$$\int dr r^2 R_\alpha^{i,p*}(r) R_\alpha^{f,n}(r) = 1 - \delta_C/2$$

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

- Computing δ_C: Classical problem over ***6 decades***! *MacDonald, 1958 Phys.Rev*

- Current input adopted in global analysis: Shell model + Woods-Saxon (WS) potential by Hardy and Towner
- Successful in aligning Ft values of different superallowed transitions

Hardy and Towner, 2020 PRC

Transitions	δ_C (%)				
	WS	DFT	HF	RPA	Micro
$^{26m}\mathrm{Al} \rightarrow {}^{26}\mathrm{Mg}$	0.310	0.329	0.30	0.139	0.08
$^{34}\mathrm{Cl} \rightarrow {}^{34}\mathrm{S}$	0.613	0.75	0.57	0.234	0.13
$^{38m}\mathrm{K} \rightarrow {}^{38}\mathrm{Ar}$	0.628	1.7	0.59	0.278	0.15
$^{42}\mathrm{Sc} \rightarrow {}^{42}\mathrm{Ca}$	0.690	0.77	0.42	0.333	0.18
$^{46}\mathrm{V} \rightarrow {}^{46}\mathrm{Ti}$	0.620	0.563	0.38	/	0.21
$^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}$	0.660	0.476	0.35	/	0.24
$^{54}\mathrm{Co} \rightarrow {}^{54}\mathrm{Fe}$	0.770	0.586	0.44	0.319	0.28

(Selected results)

Caveats:

- Significant model dependence. Disagreement with Hartree-Fock, DFT, RPA...

- Theory inconsistencies, e.g. not using the correct isospin operator *Miller and Schwenk, 2008 PRC, 2009 PRC; Condren and Miller, 2201.10651*

- **Results solely from nuclear models, no direct experimental constraint!**

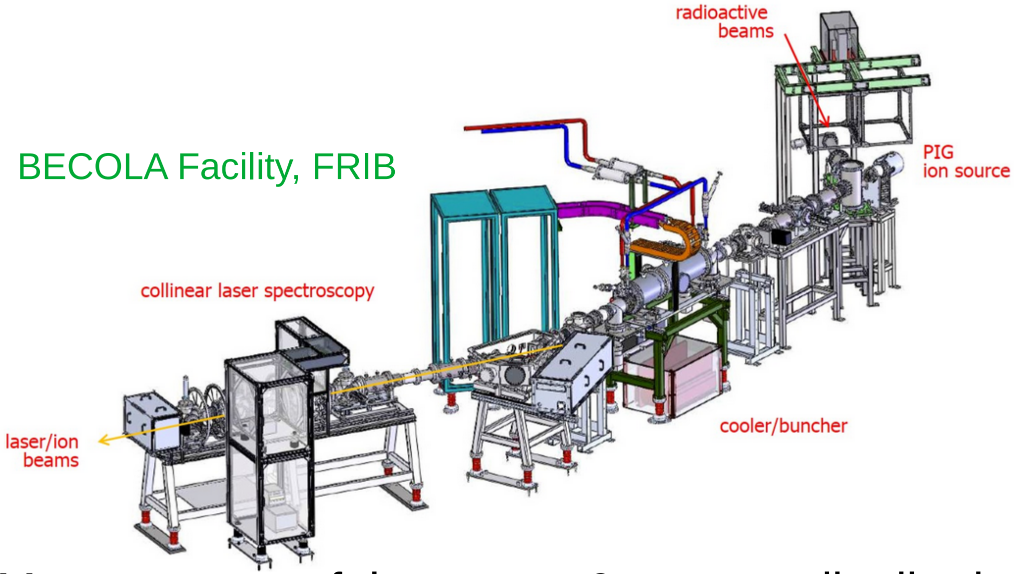
Superallowed $0^+ \to 0^+$ nuclear beta decay

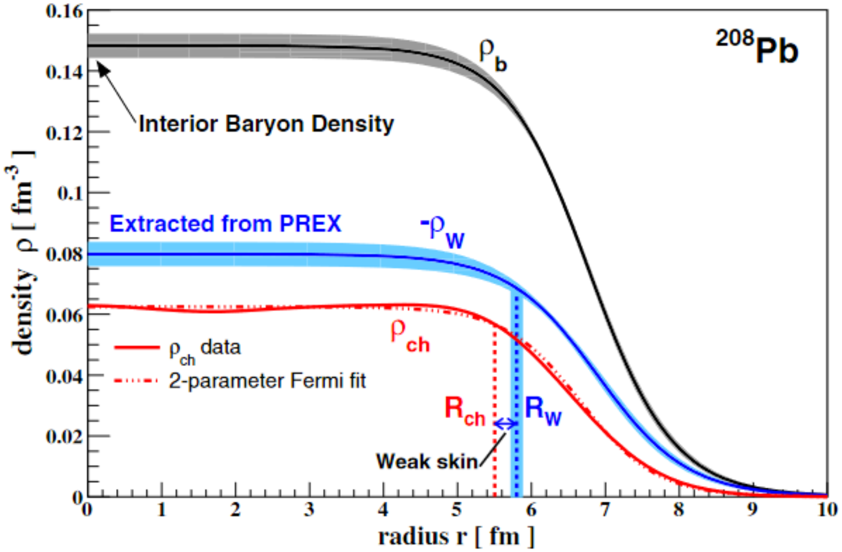
Vibrant experimental programs of the **neutron skin measurements** with **parity-violating elastic scattering (PVES)** *PREX, CREX, P2, MREX...*

$$S_n = R_n - R_p$$

Furthermore, **charge radius** for stable/unstable can be measured through **atomic spectroscopy**

BECOLA Facility, FRIB





PREX Collaboration, 2021 PRL

Measurement of the proton & neutron distribution radius in a nucleus may help to constrain δ_C model-independently

Seng and Gorchtein, 2208.03037

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Short Summary:

$$\boxed{|V_{ud}|_{0^+} = 0.97367(11)_{\mathrm{exp}}(13)_{\Delta_R^V}(27)_{\delta_{\mathrm{NS}}}}$$

Hardy and Towner, 2020 PRC;
Cirigliano, Crivellin, Hoferichter and Moulson, 2208.11707

exp: Improvements not urgent, but future half-life measurements possible at FRIB

δ_{NS}: Largest source of error at face value. Future improvements through ab-initio calculations

δ_C: Error unlisted but potentially large. Possible direct experimental constraints from electroweak nuclear radii

Summary

- I have described the extraction of V_{ud} from pion, neutron and superallowed $0^+ \rightarrow 0^+$ beta decays.

$$|V_{ud}|_{\pi_{e3}} = 0.9740(28)_{\rm exp}(1)_{\rm th}\ [28]_{\rm tot}$$
$$|V_{ud}|_{n} = 0.97441(13)_{\rm th}(82)_{\lambda}(28)_{\tau_n}\ [88]_{\rm tot}$$
$$|V_{ud}|_{0^+} = 0.97367(11)_{\rm exp}(13)_{\Delta_R^V}(27)_{\delta_{\rm NS}}\ [32]_{\rm tot}$$

- V_{ud} from pion is least precise but theoretically cleanest, thanks to recent lattice QCD inputs. Future experiment (PIONEER) may improve the π_{e3} branching ratio precision.
- V_{ud} from neutron is more precise; theory (box diagram) and experimental (τ_n,λ) precision are significantly improved.
- V_{ud} from superallowed beta decays is currently the most precise (at face value). Experimental errors are sufficiently small, but hidden systematic errors from nuclear-structure-dependent corrections are potentially large. Ab-initio calculations and future measurements of electroweak nuclear radii may help reducing them.

Backup Slides

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

The Tz=+1 state is always the most stable!

Parent daughter

Tz = -1	0	+1	
$^{10}_{6}\mathrm{C}$	$^{10}_{5}\mathrm{B}$ (ex)	$^{10}_{4}\mathrm{Be}$	10^6 yrs
$^{14}_{8}\mathrm{O}$	$^{14}_{7}\mathrm{N}$ (ex)	$^{14}_{6}\mathrm{C}$	10^3 yrs
$^{18}_{10}\mathrm{Ne}$	$^{18}_{9}\mathrm{F}$ (ex)	$^{18}_{8}\mathrm{O}$	
$^{22}_{12}\mathrm{Mg}$	$^{22}_{11}\mathrm{Na}$ (ex)	$^{22}_{10}\mathrm{Ne}$	
$^{26}_{14}\mathrm{Si}$	$^{26m}_{13}\mathrm{Al}$	$^{26}_{12}\mathrm{Mg}$	
$^{30}_{16}\mathrm{S}$	$^{30}_{15}\mathrm{P}$ (ex)	$^{30}_{14}\mathrm{Si}$	
$^{34}_{18}\mathrm{Ar}$	$^{34}_{17}\mathrm{Cl}$	$^{34}_{16}\mathrm{S}$	
$^{38}_{20}\mathrm{Ca}$	$^{38m}_{19}\mathrm{K}$	$^{38}_{18}\mathrm{Ar}$	stable
$^{42}_{22}\mathrm{Ti}$	$^{42}_{21}\mathrm{Sc}$	$^{42}_{20}\mathrm{Ca}$	
$^{46}_{24}\mathrm{Cr}$	$^{46}_{23}\mathrm{V}$	$^{46}_{22}\mathrm{Ti}$	
$^{50}_{26}\mathrm{Fe}$	$^{50}_{25}\mathrm{Mn}$	$^{50}_{24}\mathrm{Cr}$	
$^{54}_{28}\mathrm{Ni}$	$^{54}_{27}\mathrm{Co}$	$^{54}_{26}\mathrm{Fe}$	

Parent daughter

Tz = -1	0	+1	
$^{26}_{14}\mathrm{Si}$	$^{26m}_{13}\mathrm{Al}$	$^{26}_{12}\mathrm{Mg}$	
$^{34}_{18}\mathrm{Ar}$	$^{34}_{17}\mathrm{Cl}$	$^{34}_{16}\mathrm{S}$	
$^{38}_{20}\mathrm{Ca}$	$^{38m}_{19}\mathrm{K}$	$^{38}_{18}\mathrm{Ar}$	
$^{42}_{22}\mathrm{Ti}$	$^{42}_{21}\mathrm{Sc}$	$^{42}_{20}\mathrm{Ca}$	stable
$^{46}_{24}\mathrm{Cr}$	$^{46}_{23}\mathrm{V}$	$^{46}_{22}\mathrm{Ti}$	
$^{50}_{26}\mathrm{Fe}$	$^{50}_{25}\mathrm{Mn}$	$^{50}_{24}\mathrm{Cr}$	
$^{54}_{28}\mathrm{Ni}$	$^{54}_{27}\mathrm{Co}$	$^{54}_{26}\mathrm{Fe}$	
$^{62}_{32}\mathrm{Ge}$	$^{62}_{31}\mathrm{Ga}$	$^{62}_{30}\mathrm{Zn}$	9 hrs
$^{66}_{34}\mathrm{Se}$	$^{66}_{33}\mathrm{As}$	$^{66}_{32}\mathrm{Ge}$	2 hrs
$^{70}_{36}\mathrm{Kr}$	$^{70}_{35}\mathrm{Br}$	$^{70}_{34}\mathrm{Se}$	41 min
$^{74}_{38}\mathrm{Sr}$	$^{74}_{37}\mathrm{Rb}$	$^{74}_{36}\mathrm{Kr}$	11 min

However, at N≠Z, disentangling the **ISB** and **symmetry energy** contribution to the neutron skin is non-trivial

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Proposal: Measure different radii across the isotriplet!

Seng and Gorchtein, 2208.03037

"Isovector monopole operator": $$\boxed{\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)}$$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

Measurement (1): t-dependence in beta decay

Beta decay form factors:

$$\langle f(p_f)|J_W^{\lambda\dagger}(0)|i(p_i)\rangle = f_+(t)(p_i+p_f)^\lambda + f_-(t)(p_i-p_f)^\lambda$$

Recoil effects probe the t-dependence, give the off-diagonal matrix element of the isovector monopole operator:

$$\boxed{\bar{f}_+(t) = 1 - \frac{t}{6}\frac{\langle f|M_{+1}^{(1)}|i\rangle}{\sqrt{2}M_F} + \mathcal{O}(t^2)}$$

Existing recoil expt: TRIUMF, ISOLDE (CERN) etc. Future expt at FRIB?

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Proposal: Measure different radii across the isotriplet!

Seng and Gorchtein, 2208.03037

"Isovector monopole operator":

$$\boxed{\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)}$$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

Measurement (2): p/n distribution radius at $(Tz)_f$=+1

For stable daughter nucleus, fixed-target scattering can be performed to measure R_p and R_n respectively (deduced from charge and weak radii)

Can combine to get another matrix element of the isovector monopole operator:

$$\boxed{\langle f | M_0^{(1)} | f \rangle = \langle f | \sum_{i=1}^{A} r_i^2 \hat{T}_z(i) | f \rangle = \frac{N}{2} R_{n,f}^2 - \frac{Z}{2} R_{p,f}^2}$$

PREX, CREX (JLab), P2, MREX (Mainz)…

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Proposal: Measure different radii across the isotriplet!

Seng and Gorchtein, 2208.03037

"Isovector monopole operator": $\boxed{\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)}$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

Combined experimental observable

If isospin symmetry is exact, the two matrix elements are equal and opposite:

$$\langle f_0 | M_{+1}^{(1)} | i_0 \rangle = -\langle f_0 | M_0^{(1)} | f_0 \rangle$$

Therefore, the combined experimental observable:

$$\boxed{\Delta M_A^{(1)} \equiv \langle f | M_{+1}^{(1)} | i \rangle + \langle f | M_0^{(1)} | f \rangle}$$

provides a clean probe of ISB. **Deviation from zero signifies isospin mixing**

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Proposal: Measure different radii across the isotriplet!

Seng and Gorchtein, 2208.03037

"Isovector monopole operator": $\boxed{\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)}$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

Measurement (3): Charge radii across the isotriplet

Nuclear charge radii are measurable for both stable and unstable nuclei (through atomic spectroscopy)

Assuming $R_{ch} \approx R_p$, the following observable is also a clean probe of ISB:

$$\boxed{\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2}$$

Possible future measurements: BECOLA at FRIB

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay

Leading source of isospin mixing:
Isovector Coulomb potential in a uniformly charged sphere

$$\boxed{V_C^{(1)}} = \frac{Ze^2}{8\pi R_C^3}\sum_i r_i^2 \hat{T}_z(i) + \ldots$$

$$= \frac{Ze^2}{8\pi R_C^3}\boxed{M_0^{(1)}} + \ldots$$

Damgaard, 1969 Nucl.Phys.A;
Miller and Schwenk, 2008 PRC;
Auerbach, 2009 PRC

$$\Delta M_A^{(1)} \approx -\frac{8\pi R_C^3}{Ze^2}\left\{\frac{1}{3}\sum_a \frac{\boxed{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}}{E_{a,0}-E_{g,1}} + \frac{1}{2}\sum_{a\neq g}\frac{\boxed{\langle a;1||V_C^{(1)}||g;1\rangle|^2}}{E_{a,1}-E_{g,1}} + \frac{7}{6}\sum_a \frac{\boxed{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}}{E_{a,2}-E_{g,1}}\right\}$$

$$\Delta M_B^{(1)} \approx -\frac{8\pi R_C^3}{Ze^2}\left\{\frac{2}{3}\sum_a \frac{\boxed{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}}{E_{a,0}-E_{g,1}} - \sum_{a\neq g}\frac{\boxed{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}}{E_{a,1}-E_{g,1}} + \frac{1}{3}\sum_a \frac{\boxed{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}}{E_{a,2}-E_{g,1}}\right\}$$

$$\delta_C \approx \frac{1}{3}\sum_a \frac{\boxed{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}}{(E_{a,0}-E_{g,1})^2} + \frac{1}{2}\sum_{a\neq g}\frac{\boxed{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}}{(E_{a,1}-E_{g,1})^2} - \frac{5}{6}\sum_a \frac{\boxed{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}}{(E_{a,2}-E_{g,1})^2}$$

They share **identical reduced matrix elements** in the **T=0,1,2** channels!

Benefits to theorists: Methodologies capable to compute δ_C can also compute $\Delta M^{(1)}$; the latter can be directly compared to experiment!