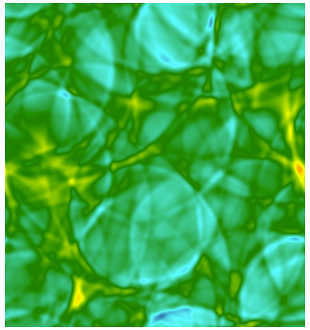
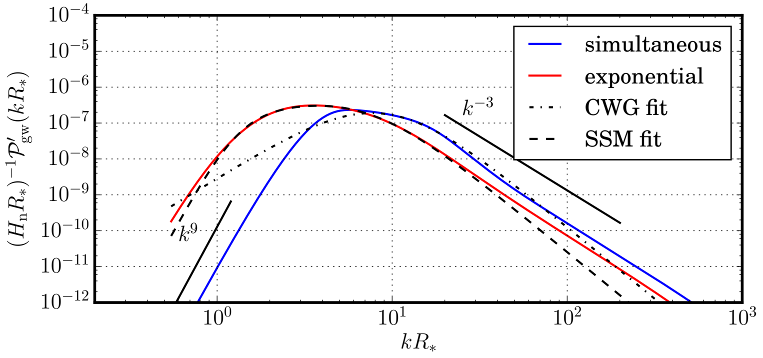
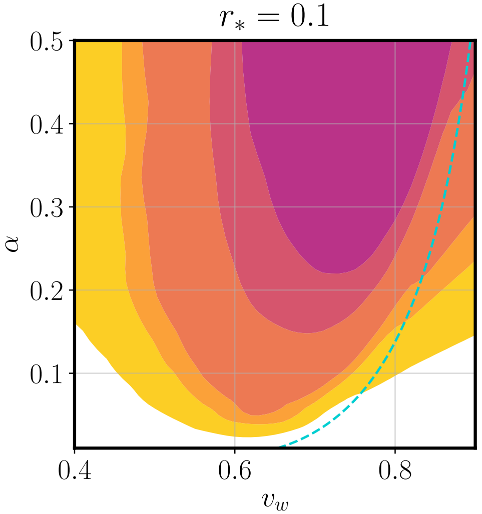
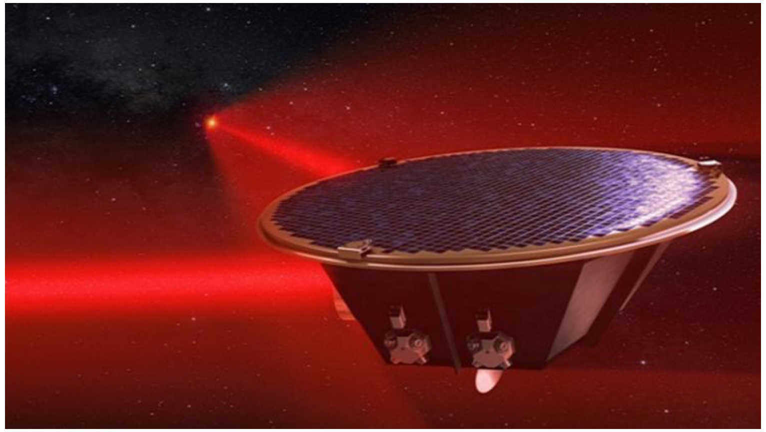








Probing phase transitions in the early universe with gravitational waves

Mark Hindmarsh

Helsinki Institute of Physics & Dept of Physics,
University of Helsinki

and

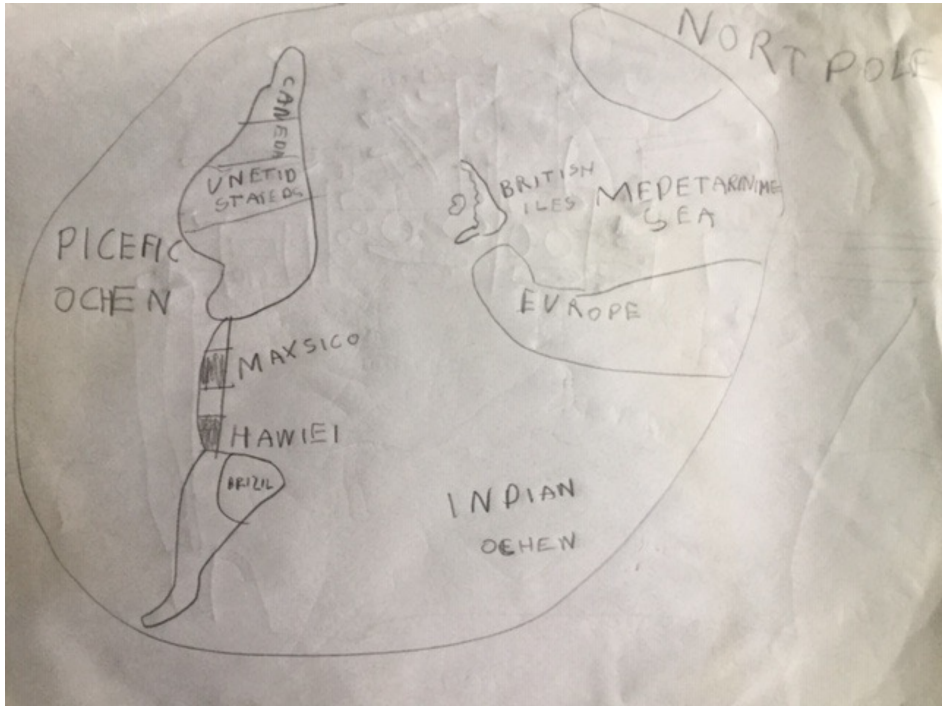
Department of Physics & Astronomy,
University of Sussex

PRISMA+ Colloquium
25. toukokuuta 2022

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

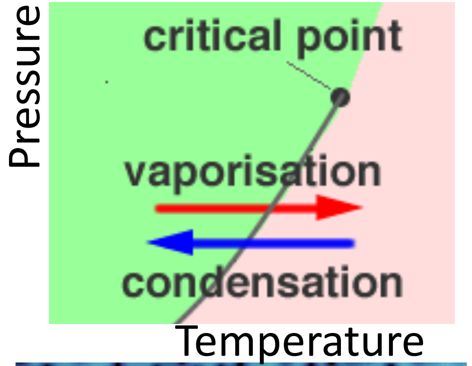
A brief career

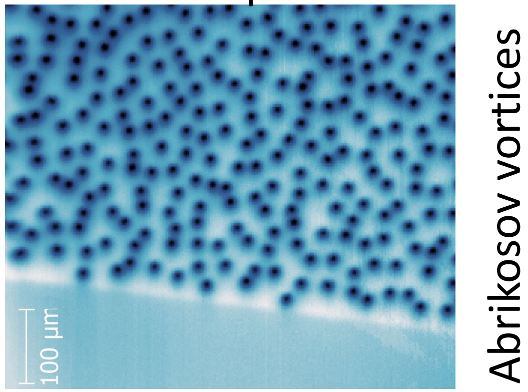


- Home town: Cambridge, UK
  - Early interest in the universe…
- Physics at Oxford
- PhD at Imperial College (with Tom Kibble)
  - Turned into interest in the early universe
- Postdocs at Los Alamos, Newcastle, Cambridge
- 5-year research fellowship at Sussex -> lectureship 1998
- 2012 sabbatical at University of Helsinki -> visiting professor
- Since 2018: 80% at Helsinki, 20% Sussex
- Lots of good fortune on the way …

Phase transitions in the early Universe

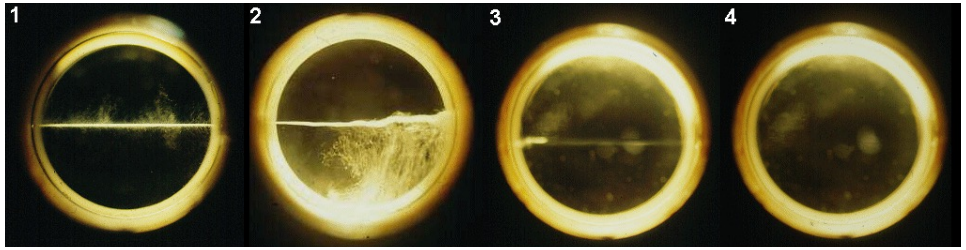
- At very high temperatures and pressures, the state of matter in the Universe changes
  - T_c ~ 100 MeV (1 ms) QCD
  - T_c ~ 100 GeV (10 ps) Electroweak
  - T_c >> 100 GeV new symmetries, interactions?
- Departures from equilibrium and homogeneity (-> shear stress)
  - First order phase transition: relativistic condensation or `fizz'
    Steinhardt (1982)
  - First or second order: formation of topological defects
    Kibble (1976)
  - First order: baryon asymmetry
    Sakharov 68; Kuzmin, Rubakov, Shaposhnikov (1985)
- First order phase transitions can produce GWs
  Witten (1984), Hogan (1986)

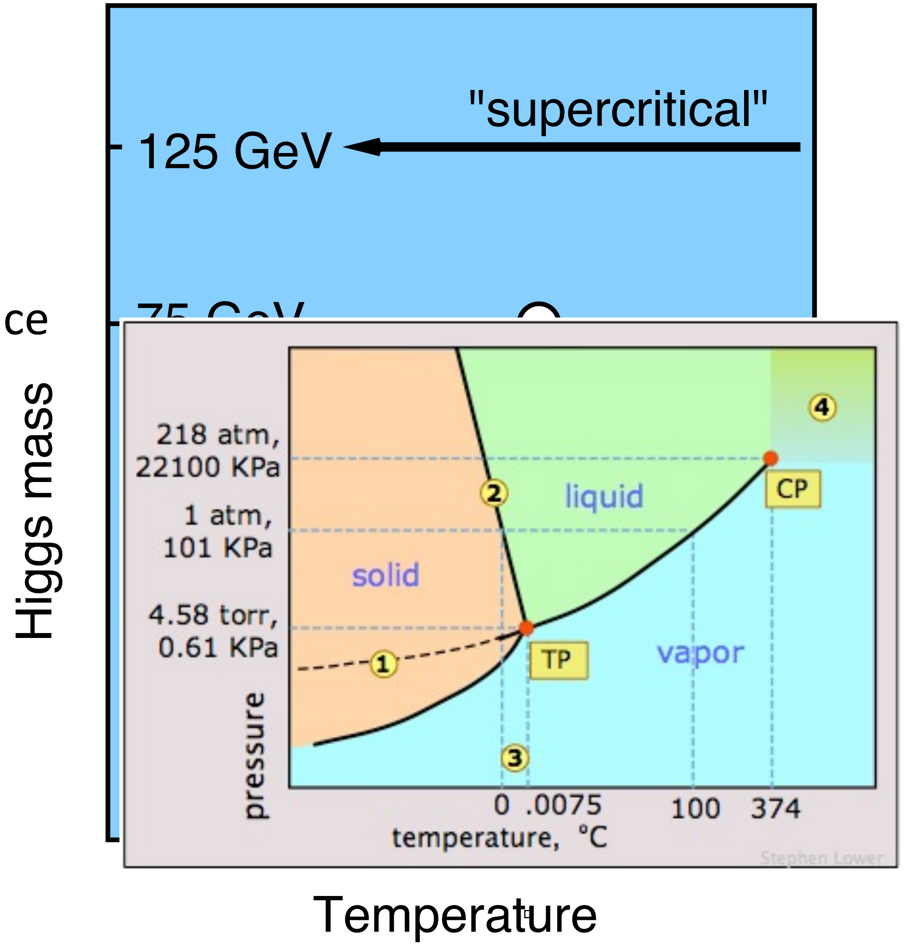




Electroweak transition: 100 GeV, 10 ps

- Perturbative: weakly first order transition
  Kirzhnitz, Linde (1972,4)
- But: SM is not weakly coupled at high T
  Linde (1980)
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over
  - a supercritical fluid





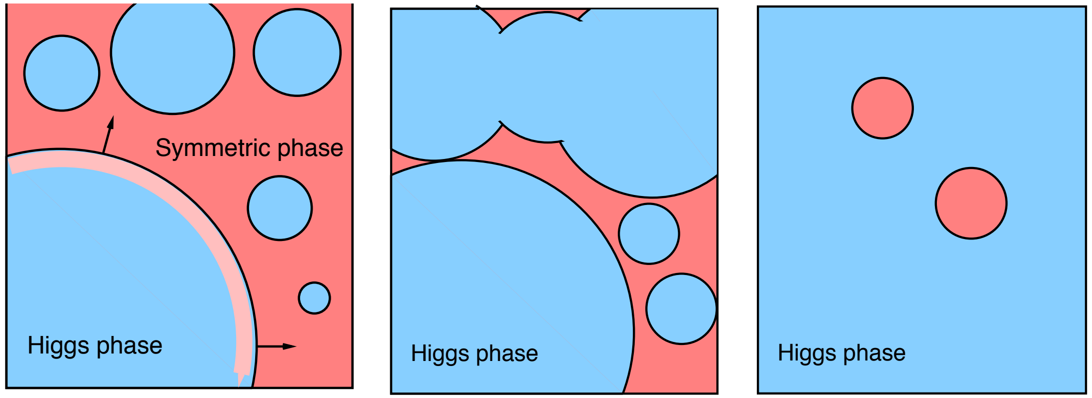
- Search for 1st order transition is a search for physics beyond SM

Little bangs in the Big Bang

- 1st order transition by nucleation of bubbles of low-*T* phase

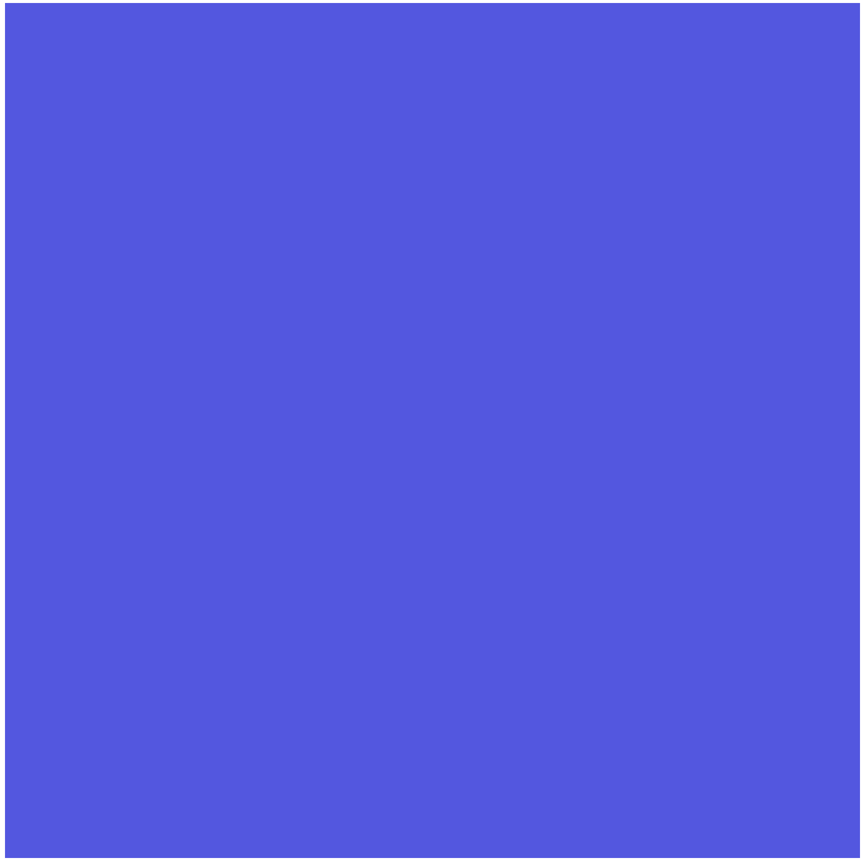
  Langer 1969, Coleman 1974, Linde 1983

- Nucleation rate/volume *p*(*t*) rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Universal "fizz"
- Gravitational wave production
- Spectrum has information about phase transition



Steinhardt (1982); Hogan (1983,86);
Gyulassy et al (1984); Witten (1984)

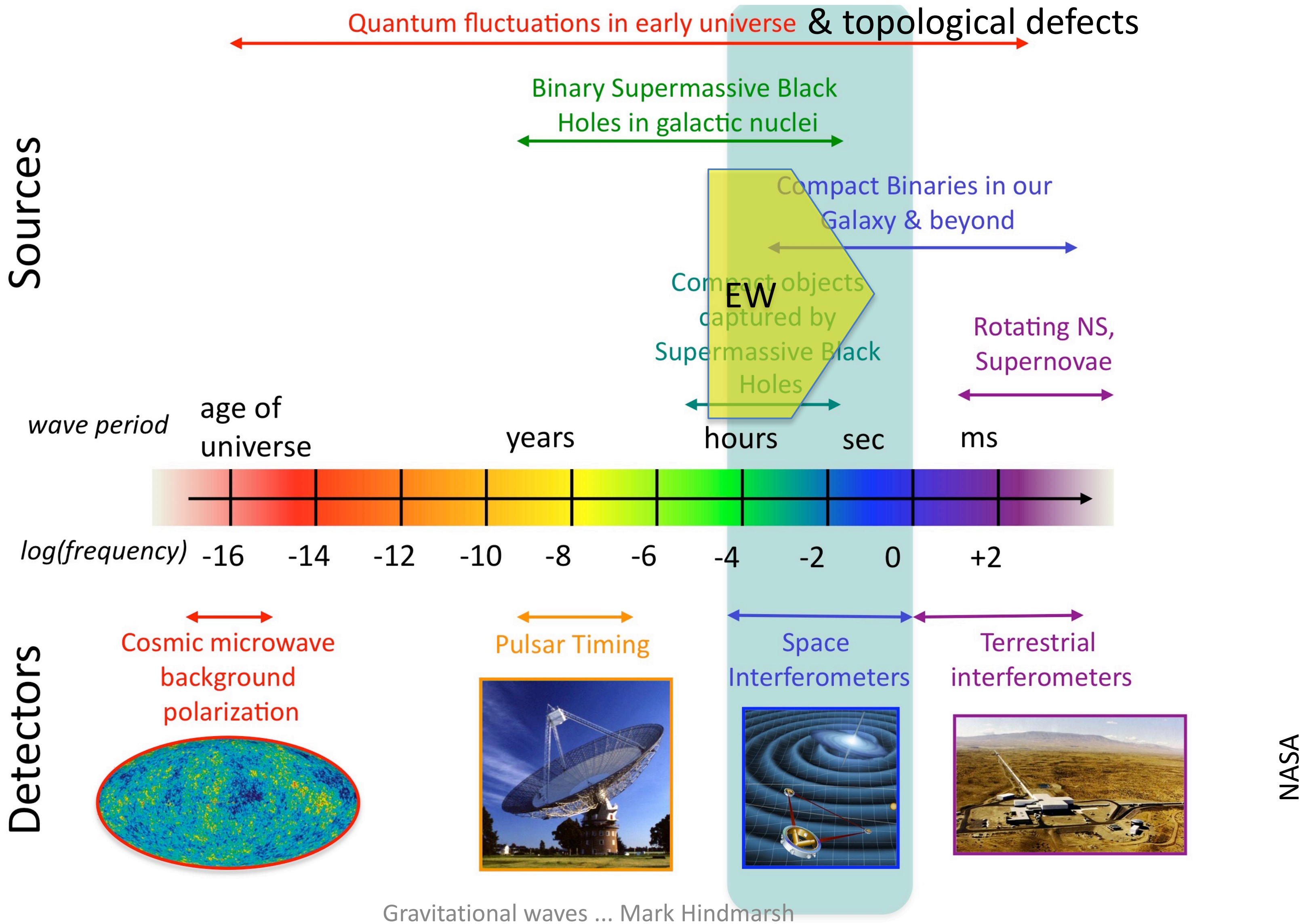
Fluid kinetic energy



David Weir

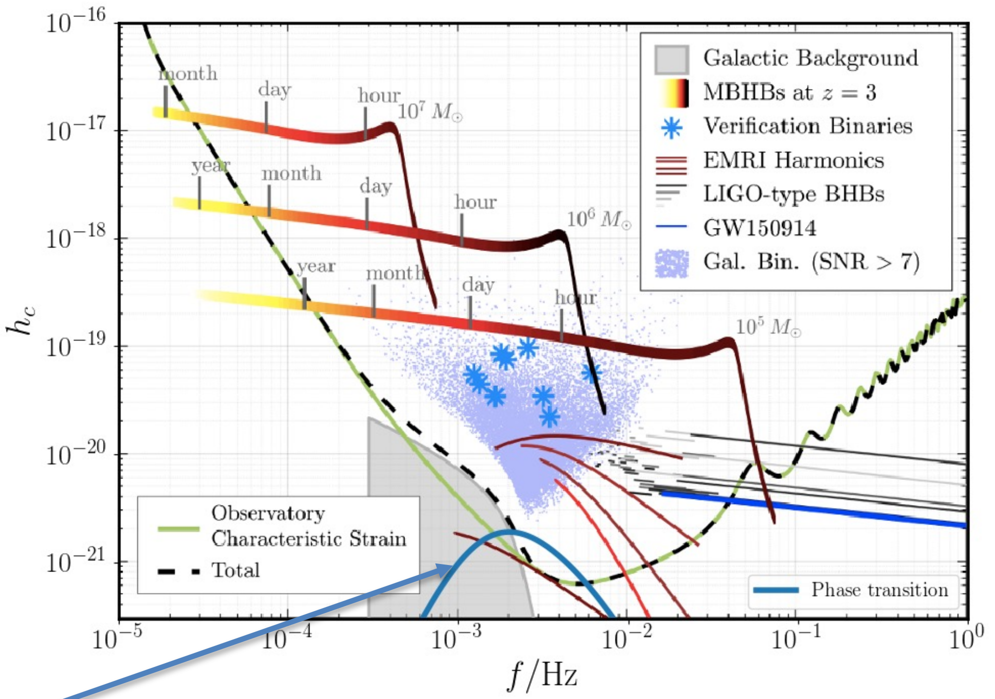
MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

Gravitational wave spectrum

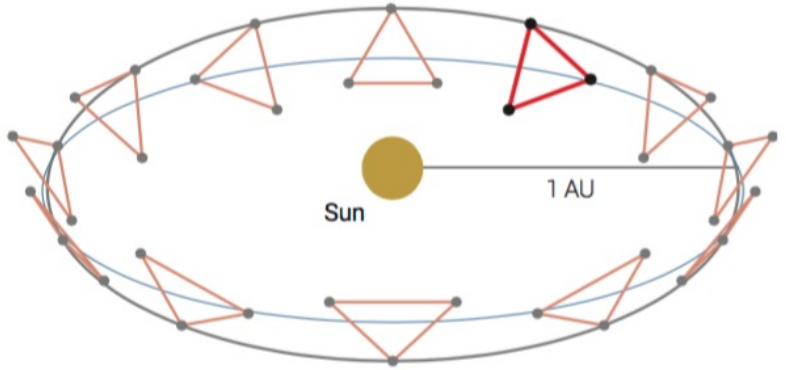


Laser Interferometer Space Antenna

- Launch mid 2030s
- 4-year mission (up to 10 years)
- 2.5M km arms
- Science objectives:
  - White dwarves
  - Black holes
  - Galaxy mergers
  - Extreme gravity
  - TeV-scale early Universe
- Other missions: DECIGO, Taiji, Tianqin

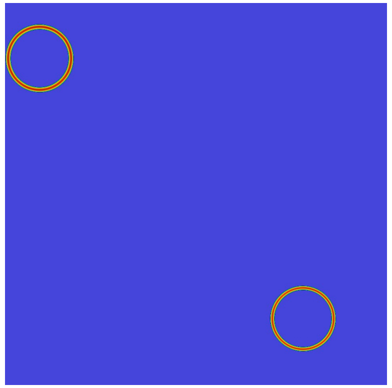


LISA sensitivity

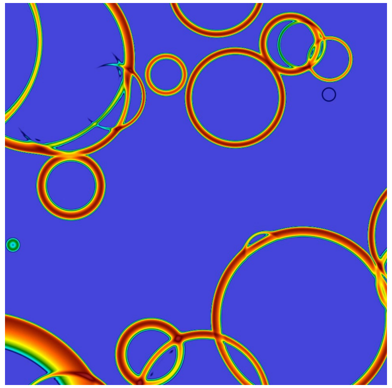


LISA

Phases of a phase transition



1



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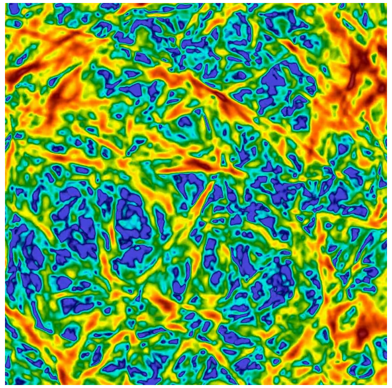


3

4

1. Bubble nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\rm nl} \sim L_f / \bar{U}_f$$

$L_{\rm f}$ – fluid flow length scale
$U_{\rm f}$ – RMS fluid velocity

'exponential' nucleation rate/volume p(t)

$$p(t) = p_n e^{\beta(t - t_n)}$$

$$\tau_{\rm co} = \beta^{-1}$$

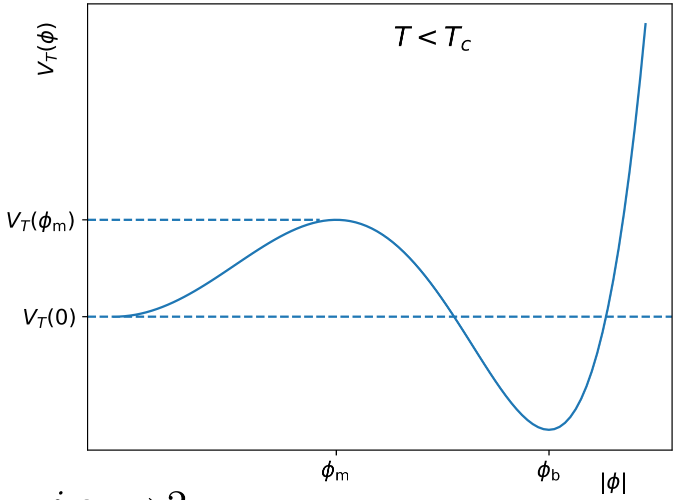
β – transition rate parameter
$\beta > H$ for successful transition

Guth, Weinberg 1983; Enqvist et al 1992;
Turner, Weinberg, Widrow 1992;

Review: MH, Lüben, Lumma, Pauly 2021

GWs from an early universe phase transition

Assume rapid transtion, β >> H, neglect expansion of universe



- Ingredients for theory: Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - **Higgs field** $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$$
    - *V(T,ϕ)* equation of state
    - *η(T,ϕ,W)* field-fluid coupling (models friction)
  - **Relativistic fluid**

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_iP + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - *E* energy density, Z_i momentum density, V_i velocity, *W* γ-factor
  - Discretisation — Wilson & Matthews (2003); Different approach: Brandenburg, Enqvist, Olesen (1996); Giblin, Mertens (2013)
  - **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \quad \Rightarrow \quad \tilde{h}_{ij}(\mathbf{k}) = \Lambda^{TT}_{ij,kl} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

Connection to fundamental theory

- Scalar hydrodynamics $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i \partial_i \phi)$$

- Scalar effective potential $V(\phi,T)$ → equilibrium, quasi-eqm. (T_n, α, β, c_s, $g_{\rm eff}$)
- Scalar-fluid coupling $\eta(\phi,T,\gamma)$ → non-equilibrium (v_w)

Phase transition parameters :

T_n = nucleation temperature
$g_{\rm eff}$ = effective d.o.f. in plasma
α ~ (latent heat)/(thermal energy)
c_s = sound speed(s)
β = transition rate
v_w = bubble wall speed

→ *Simulations, Modelling* →

$H_n(T_n, g_{\rm eff})$ (Hubble rate)
$K(v_w, \alpha, c_s)$ (kinetic energy fraction)
$R_*(\beta, v_w)$ (mean bubble separation)

GW parameters :

Ω_p = peak amplitude
f_p = peak frequency
σ_i = shape parameters

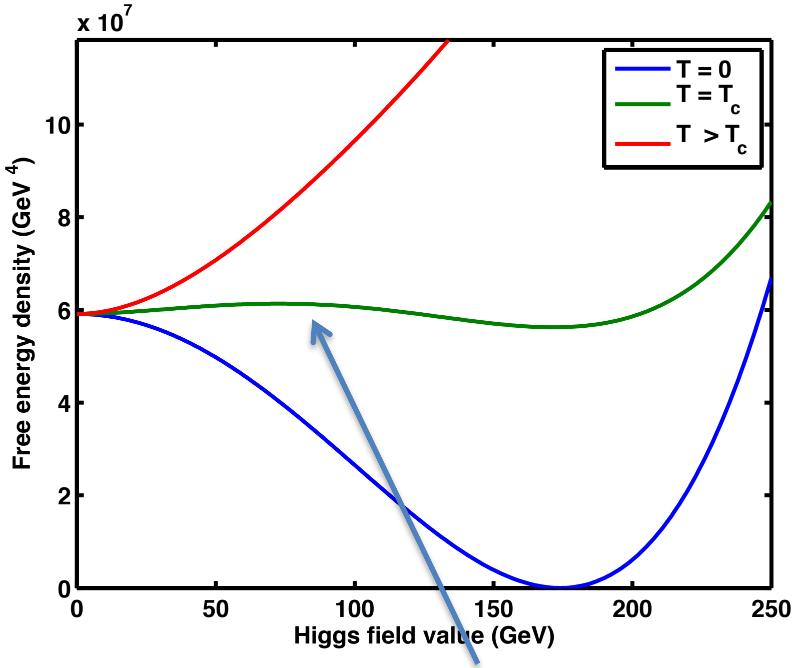
Phase transitions at weak coupling

- Phase transition in weakly coupled gauge theories:
  (Kirzhnits 1972, Kirzhnits & Linde 1972)
- Free energy density of plasma depends on
  - Temperature T
  - Particle masses $m_i(\ )$
- High T: reduce free energy by forcing Higgs ϕ to zero
- Electroweak transition: $T_c \approx v_{EW} \approx 100$ GeV (10^{15} K)
- High T (>> $m_i(\phi)$):

$$V_T(\phi) = \frac{1}{2}A(T^2 - T_0^2)\phi^2 - \frac{1}{3}ET\phi^3 + \frac{1}{4}\phi^4$$

- Non-abelian gauge theories ($T > 0$): simple methods fail Linde 1980
- Much recent work on $V(\phi,T)$ in Standard Model + extra Higgs

Anderson et al 2018; Gorda et al 2019; Niemi et al 2019, 2021; Kainulainen et al 2019; Niemi, Schicho, Tenkanen 2021



Potential barrier from cubic term in perturbative high-T expansion. First order transition?

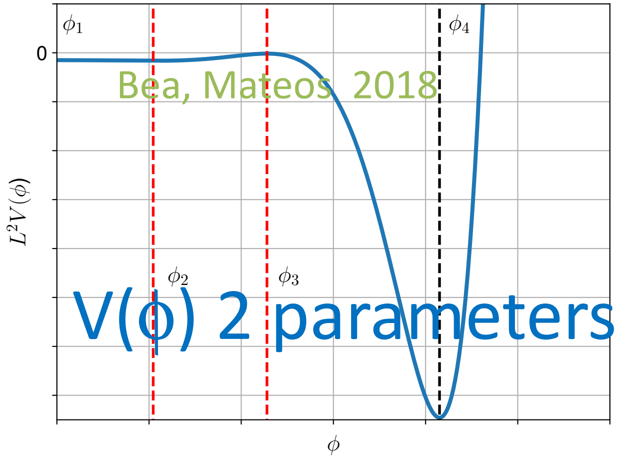
Phase transitions at strong coupling and holography

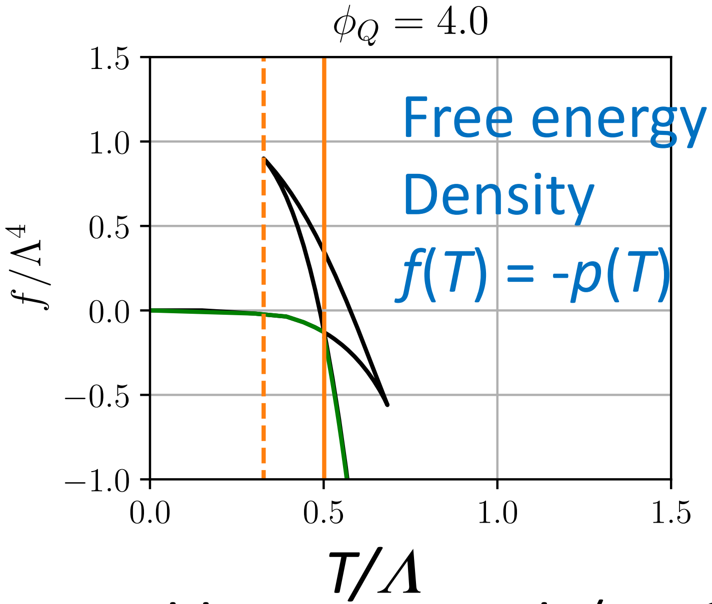
- Holography: a (4 + n)D gravity theory defines a quantum field theory in 4D

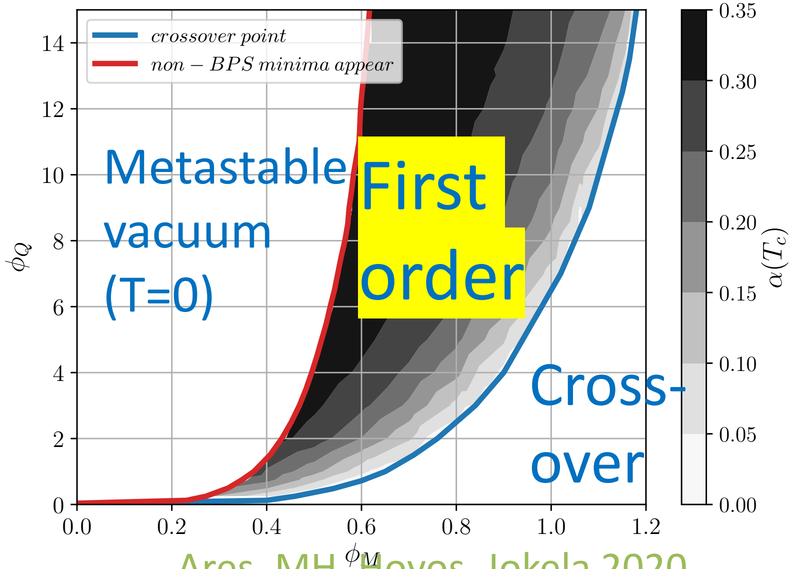
Witten 1998, Maldacena 1998

- ”Bottom-up” model

$$S_{\text{non-reg}} = \frac{2}{\kappa_5^2}\int \mathrm{d}^5x\sqrt{-g}\left(\frac{\mathcal{R}}{4} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) + \frac{1}{\kappa_5^2}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-\gamma}K\ ,$$







Ares, MH, Hoyos, Jokela 2020

- “Intermediate” strength phase transitions are generic (α ~ 0.1 – 0.3)
- Sound speed $c_s < 1/\sqrt{3}$

Creminelli, Nicolis, Rattazzi 2002; Nardini, Quiros, Wultzer 2007; Schwaller 2015; Dillon et al 2018; Bruggisser et al 2019; Bigazzi et al 2020, 2021;

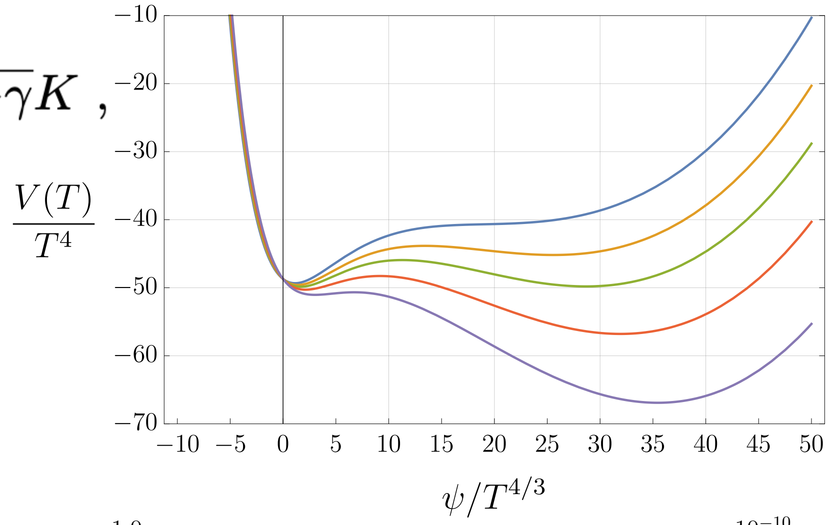
Holographic effective action for transition rates

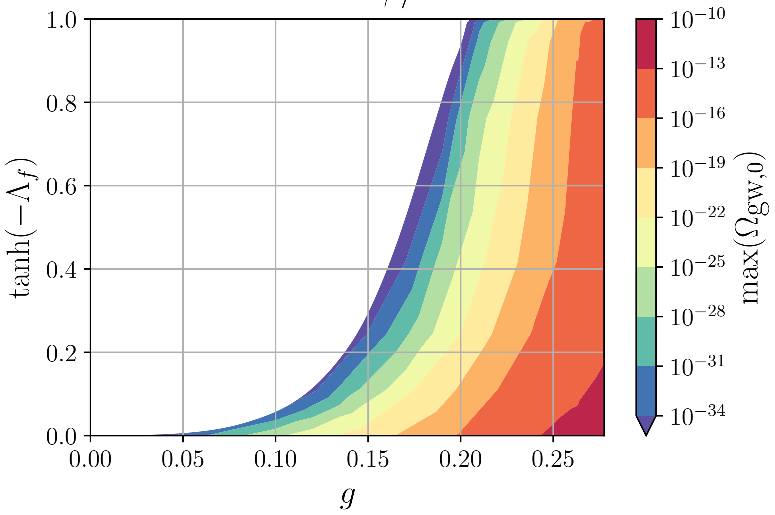
Ares, Henriksson, MH, Hoyos, Jokela 2022

- Bottom-up model

$$S_{\text{non-reg}} = \frac{2}{\kappa_5^2}\int \mathrm{d}^5x\sqrt{-g}\left(\frac{\mathcal{R}}{4} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) + \frac{1}{\kappa_5^2}\int_{\partial\mathcal{M}} \mathrm{d}^4x\sqrt{-\gamma}K\ ,$$

- From the holographic generating functional $W[J]$
  - Effective potential: $V(\psi) = \int J\, d\psi$ Hertog, Horowitz 2002
- From low-momentum expansion of 2-point function $<\psi\psi> = \ \delta\psi/\delta J$ Son, Starinets 2002
  - Kinetic term: $Z(\psi)\ (\nabla\psi)^2$
- Allows standard computation of transition rate β
- Avoids need to find solutions in 5D theory
- Future: wall speed
- Application: strongly-coupled Standard Model extensions





Hydrodynamic simulations of phase transitions

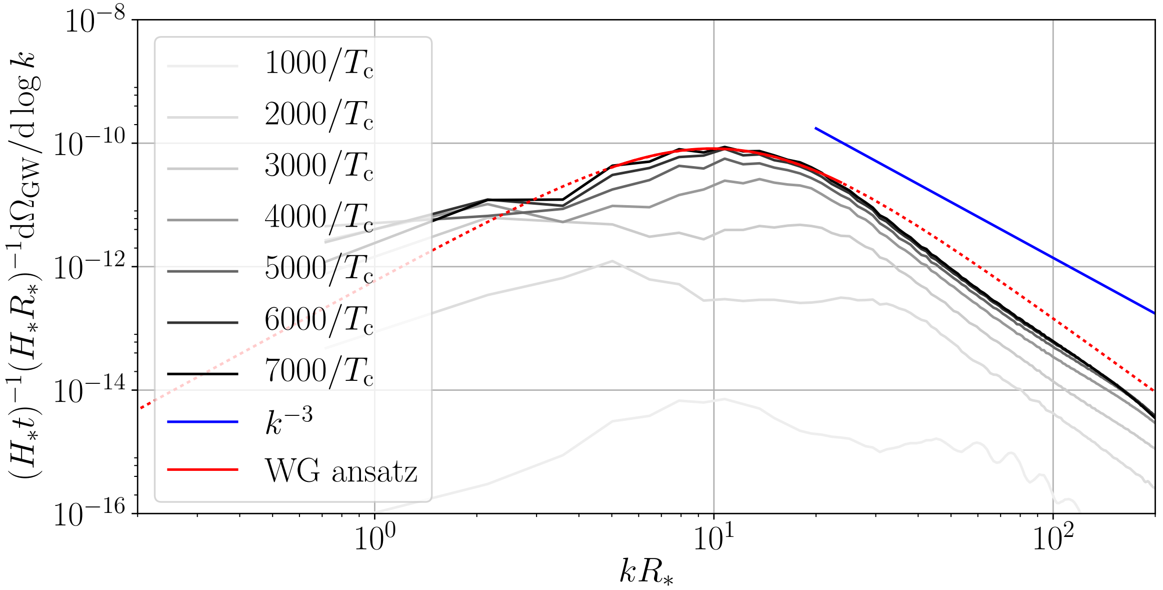
- 2015: 1M hrs CSC, Finland
- 2015/6: 17M CPU-hours Tier-0 PRACE
- 4200^3 lattice on 24k cores
- Output: GW power spectrum
(fraction GW energy density per log wavenumber)

$$\frac{d\Omega_{gw}}{d\ln k} = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\ln k} = \frac{1}{12H^2}\frac{k^3}{2\pi^2}P_{\dot{h}}(k)$$

- *P* - Plane wave spectral density

$$\langle \dot{\tilde{h}}_{ij}(\mathbf{k})\dot{\tilde{h}}_{ij}(\mathbf{k}')\rangle = P_{\dot{h}}(k)(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')$$

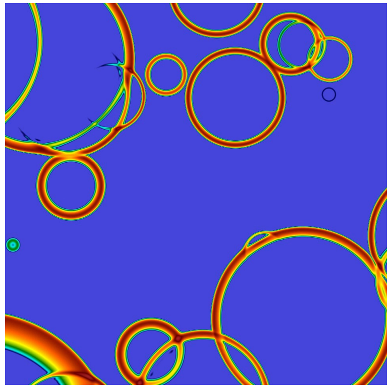
Transition strength: α = 0.0046
Wall speed: v_w = 0.44
Mean bubble spacing R_* = 2000/T_c

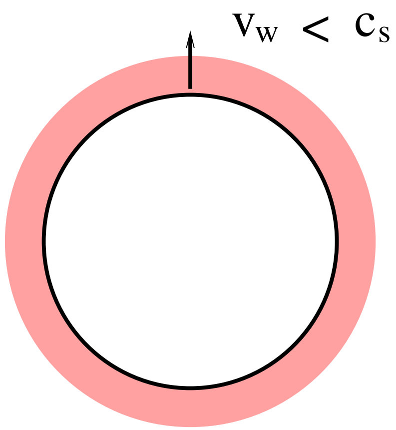


Generic features:
- "Domed" peak at kR_* ~ 10
- Approx k^{-3} spectrum at high k

Hindmarsh, Huber, Rummukainen, Weir 2017

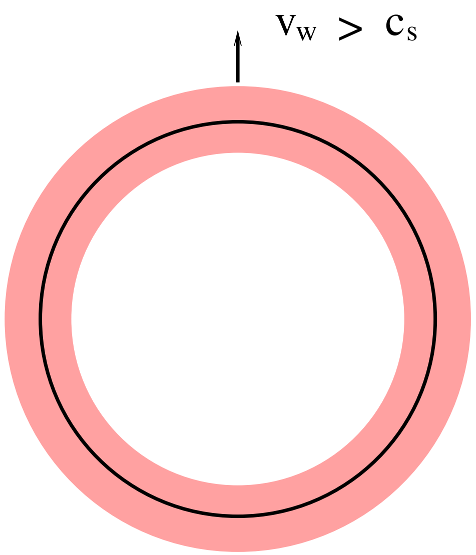
Towards a model: relativistic combustion



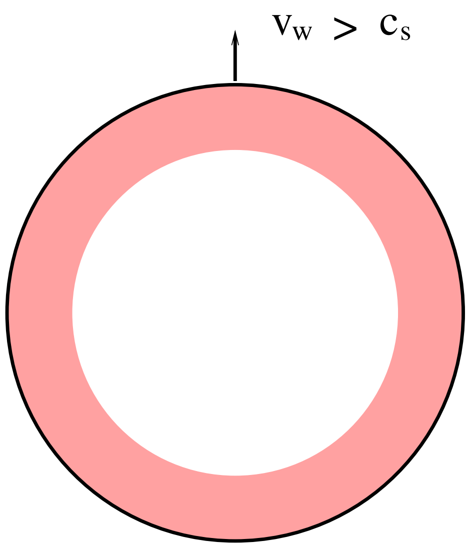


Deflagration

Landau & Lifshitz; Steinhardt (1984)
Kurki-Suonio, Laine (1991), Espinosa et al (2010)



Supersonic deflagration
("hybrid")

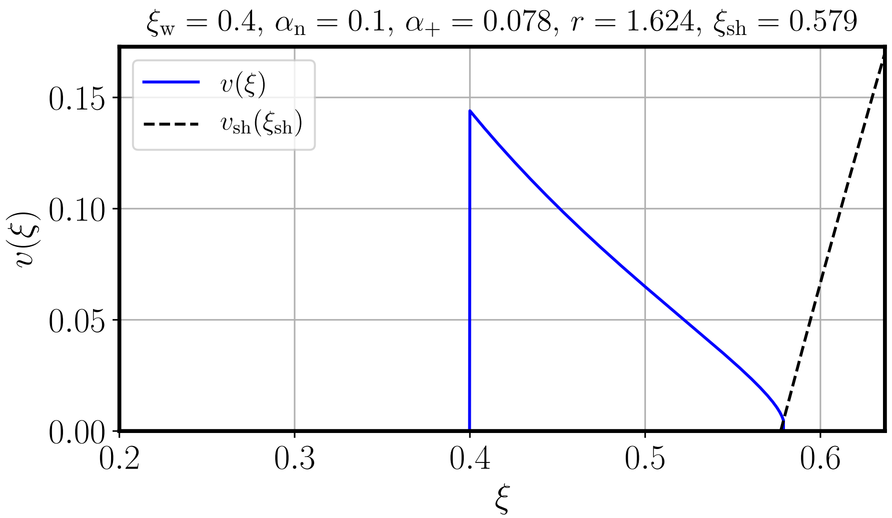


Detonation

- Large scales: ideal relativistic hydrodynamics
- Microphysics: $e(T)$, $p(T)$, v_w
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
  - **Similarity solution $v(\xi)$, $w(\xi)$: $\xi = r/t$**

- NB Weakly coupled near-vacuum transition … runaway ($v_w \to 1$)

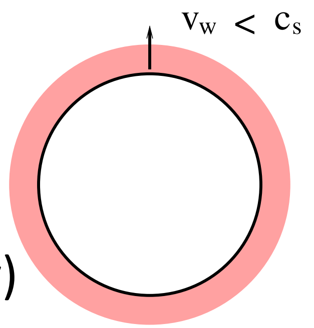
Bodeker Moore 2010, 2017

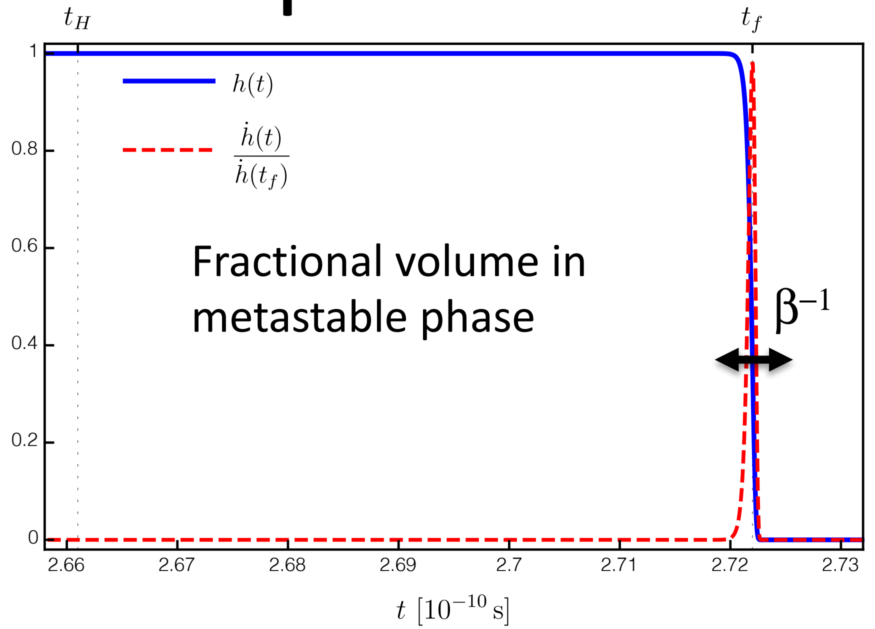


GWs from first order phase transitions: parameters

- Parameters of transition:
  - T_n = Temperature at nucleation
  - β = transition rate (= - d log p / dt)
  - v_w = Bubble wall speed
  - α = (Potential energy release)/(Heat energy)
  - c_s = sound speed

Giese et al 2020

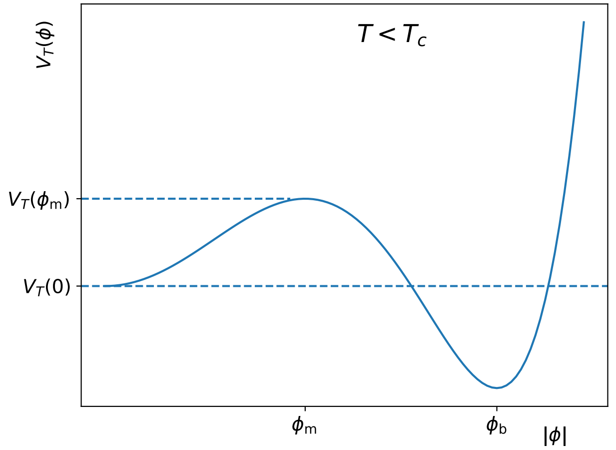


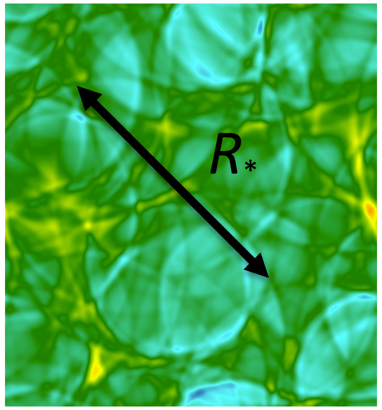


- Derived parameters:
  - r_* = (bubble centre spacing R_*)/Hubble length
  - K = fluid kinetic energy fraction

Steinhardt '84
Espinosa et al 2010

- Fluid kinetic energy makes GWs
  - Energy release via self-similar solutions

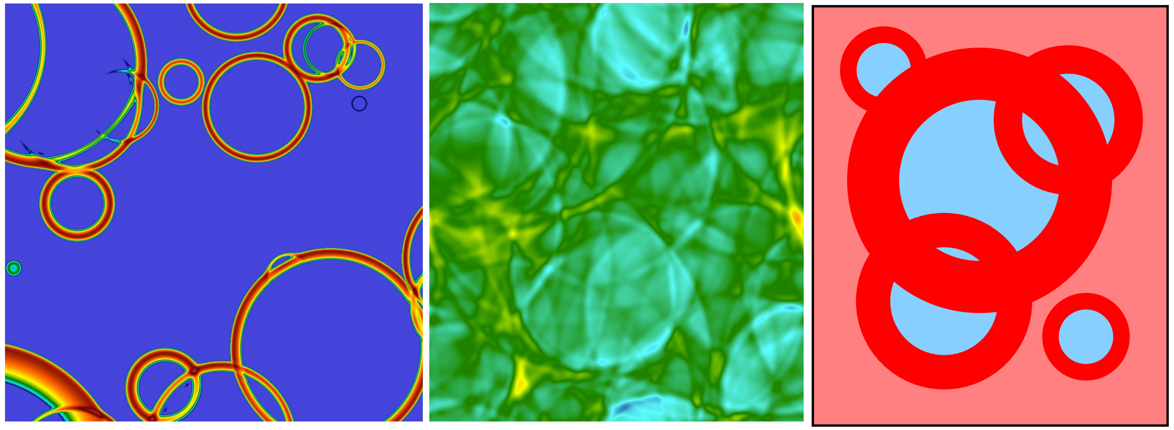


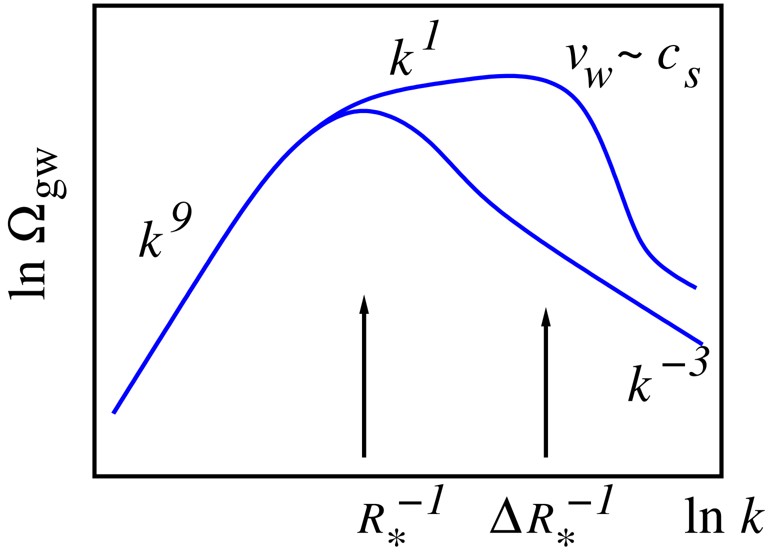


GWs from phase transitions: Sound shell model

- Gaussian velocity field from weighted addition of self-similar sound "shells" $\mathbf{v}_\mathbf{q}(t_\mathrm{i})$ MH 2017, MH, Hijazi 2019
- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_*|v_w - c_s|/c_s$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude proportional to:
  - Bubble spacing
  - Shear stress lifetime
  - (Kinetic energy)^2
- Similar: bulk flow model (real space)

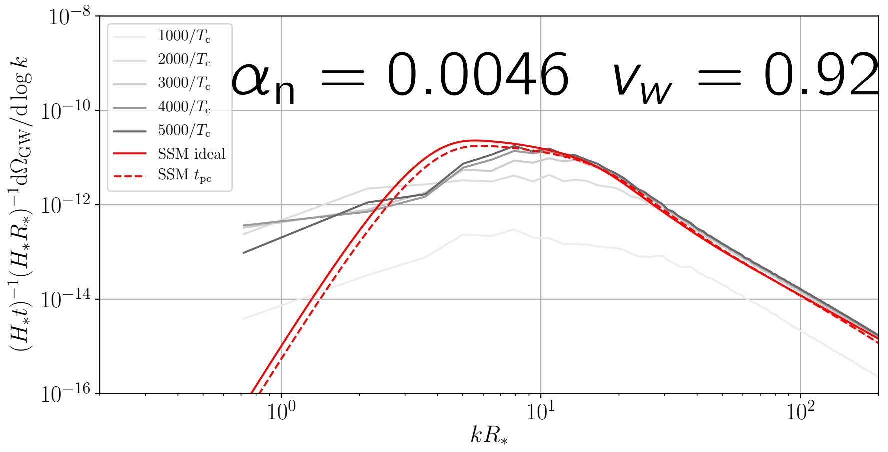
  Jinno, Konstandin, Rubira 2020



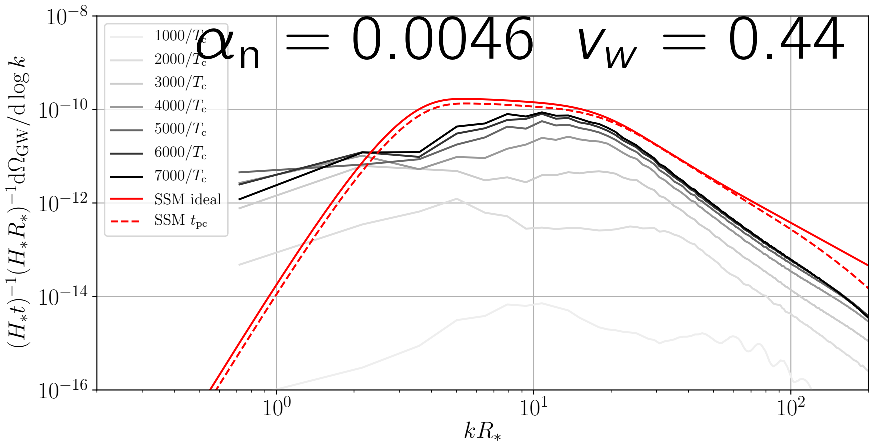


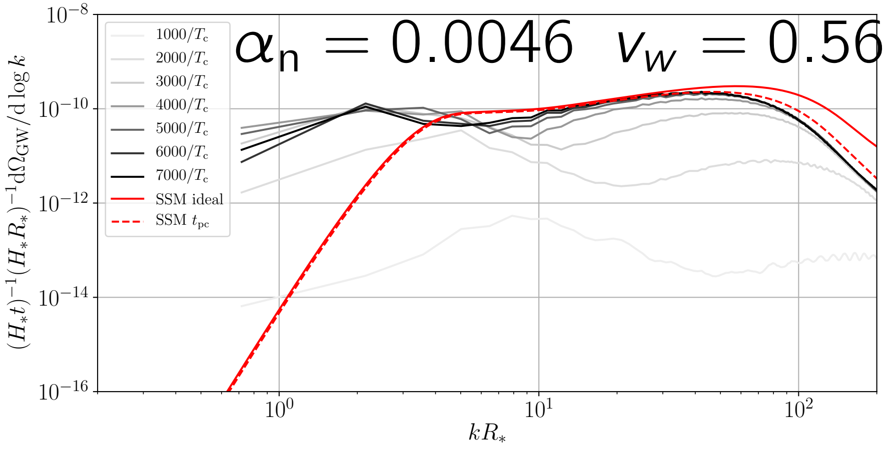
Sound shell model vs. simulations P_{gw}

- Solid: ideal self-similar sound shell
- Dash: evolving sound shell at peak collision time in 1+1D scalar hydro
- Simulations: simultaneous nucleation of bubbles

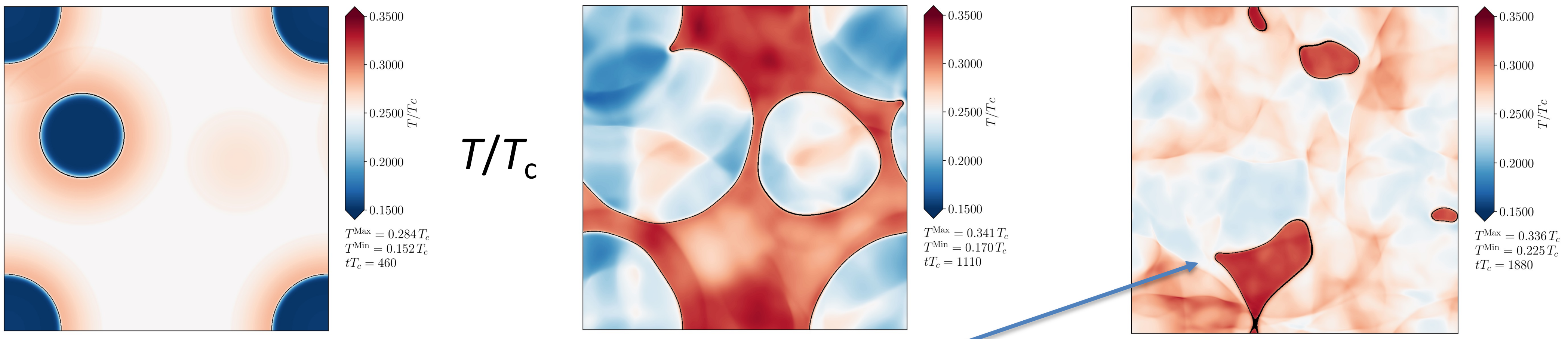


MH et al in prep





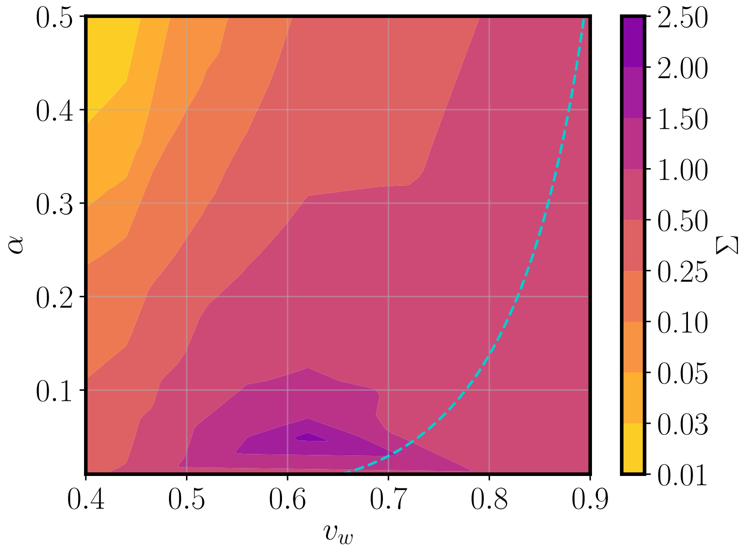
Nonlinearities 1: Kinetic energy & GW suppression



Cutting, MH, Weir, 2020

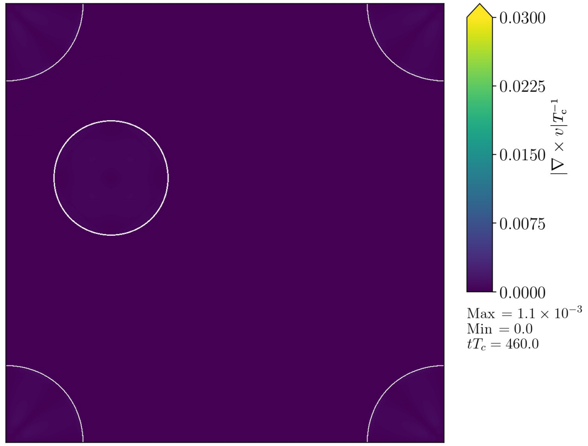
- **Deflagrations**: heat up fluid in front
- Pressure in front of wall increases, walls slow down
- Formation of hot droplets
- Less transfer into kinetic energy, more into heat.
- Include **GW suppression factor** as a numerical parameter (**right**)
- Also: nucleation suppression, boosts signal

Al-Ajmi, MH (in prep)

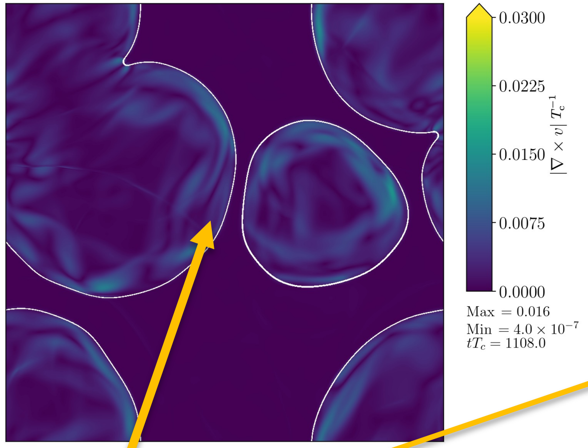


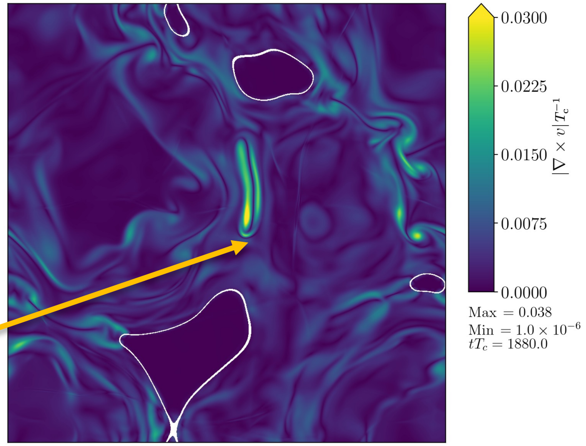
Gowling, MH (2021)

Nonlinearities 2: Vorticity and turbulence



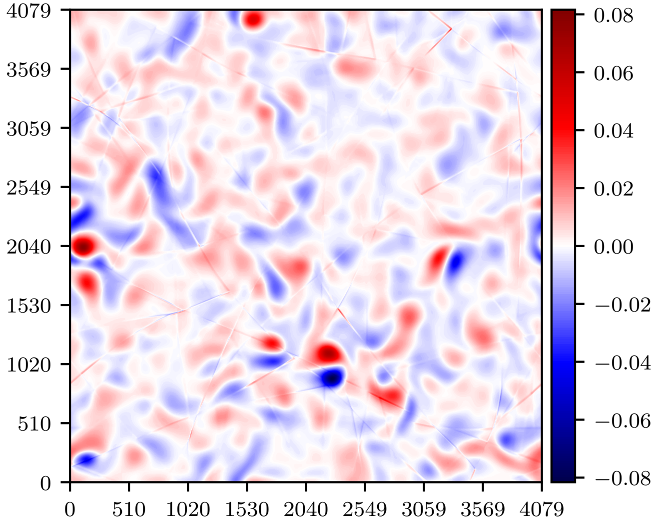
ω/T_c





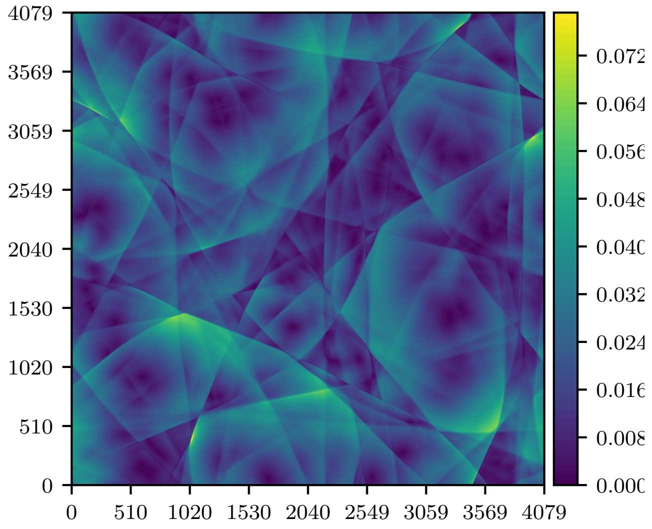
Cutting, MH, Weir, 2020

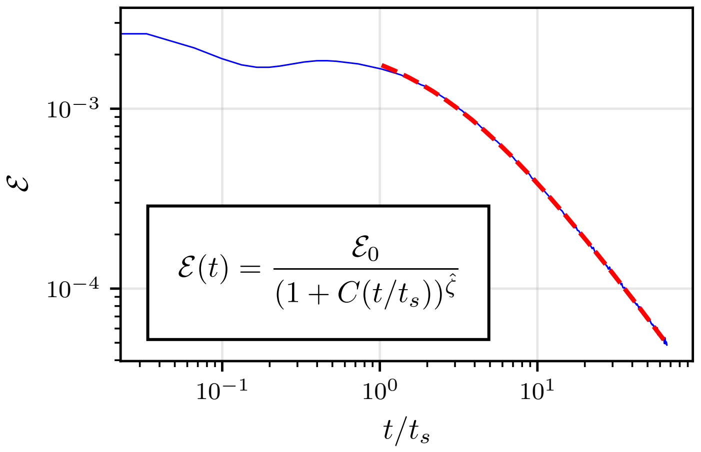
- **Deflagrations**
- Interaction between bubbles/shells generates vorticity
- Vorticity significant for slow walls in strong transitions
- Generation by later shock collision? Pen, Turok 2016
  - Small in 2D sims
- Larger, longer simulations needed Auclair et al 2022

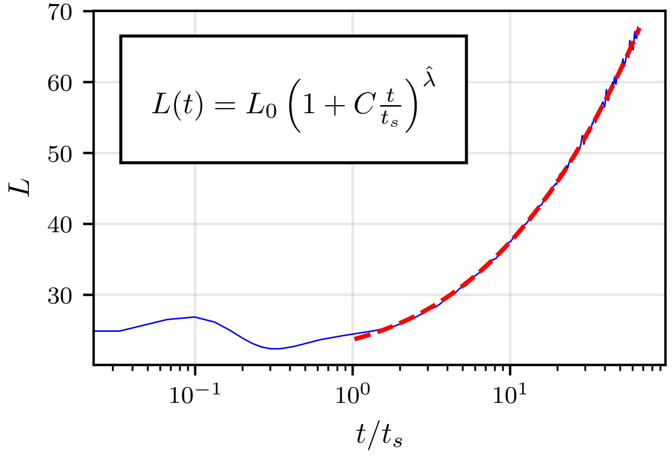


Dahl et al (2021)

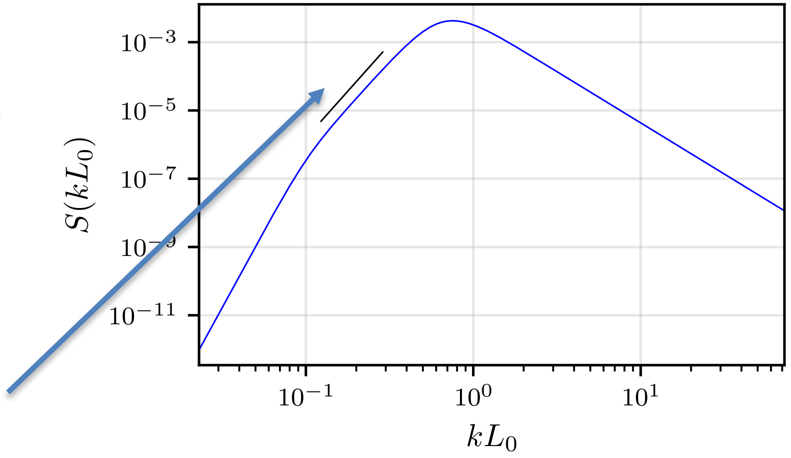
Nonlinearities 3: Shocks and kinetic energy decay







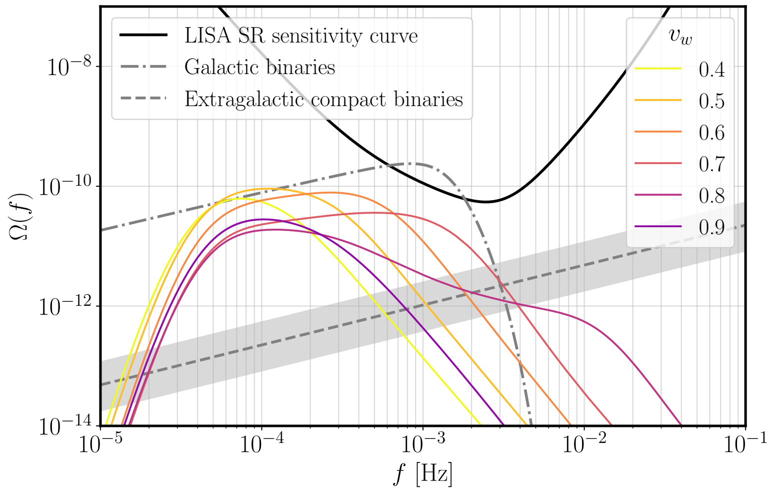
- Shocks develop from any sound wave
- Energy spectrum: k^{-2} at high k (any dimension)
- D=2 modelling can be applied to D=3
- KE decay, length scale growth: power laws
- GW spectrum: Intermediate slope change: k^9 to $k^{5.5}$



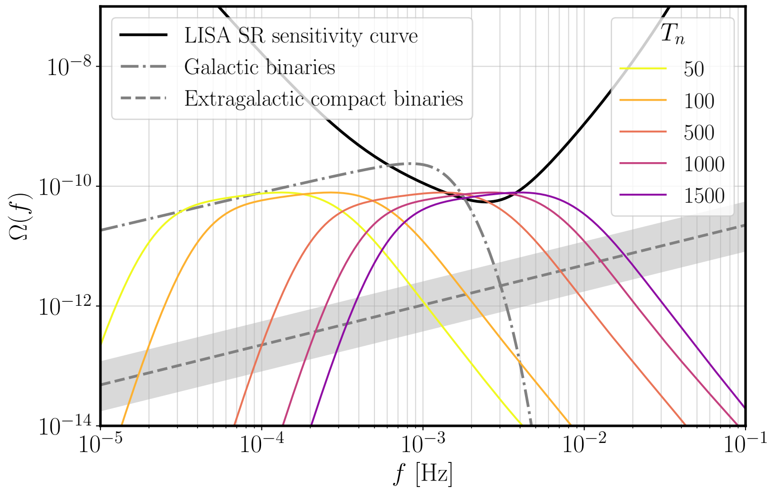
Dahl, MH, Rukkukainnen, Weir (2021)

GW power spectra in the SSM

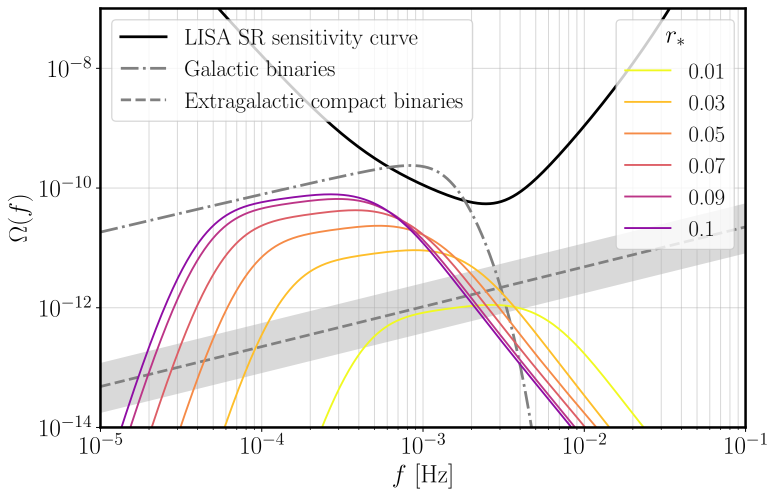
- **Sound shell model predictions, acceptable accuracy for** Gowling, MH (2021)
  - near-linear flows ($\alpha \leq 0.3$); fast walls: $v_w > 0.4$; sub-Hubble bubble separations ($r_* << 1$)



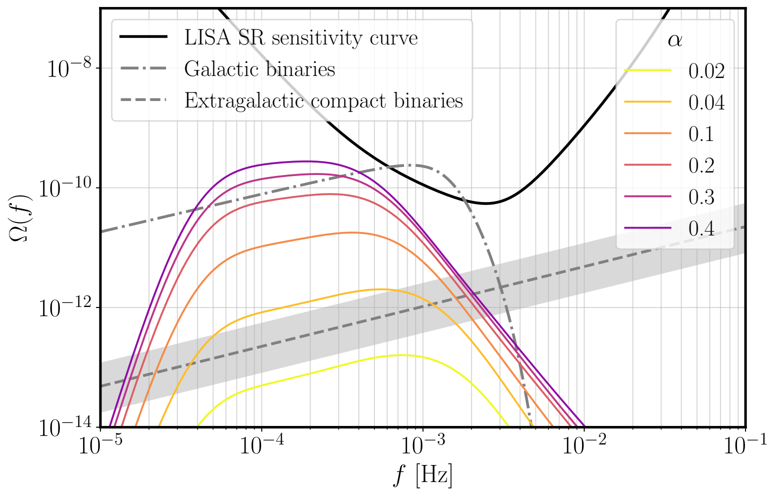
Vary wall speed



Vary transition temp

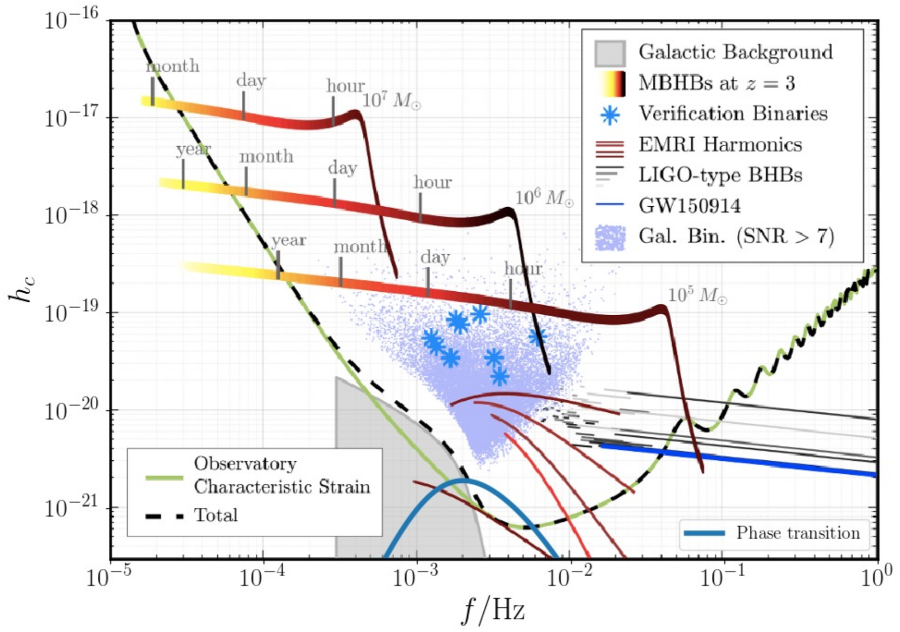


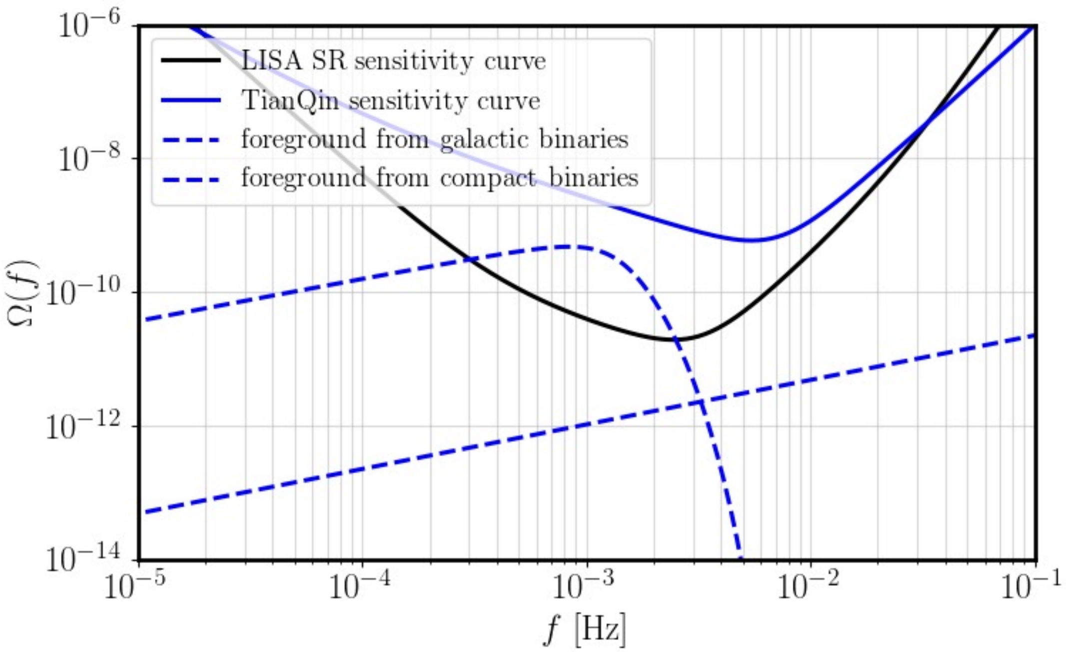
Vary bubble separation



Vary transition strength

Foregrounds in stochastic signal



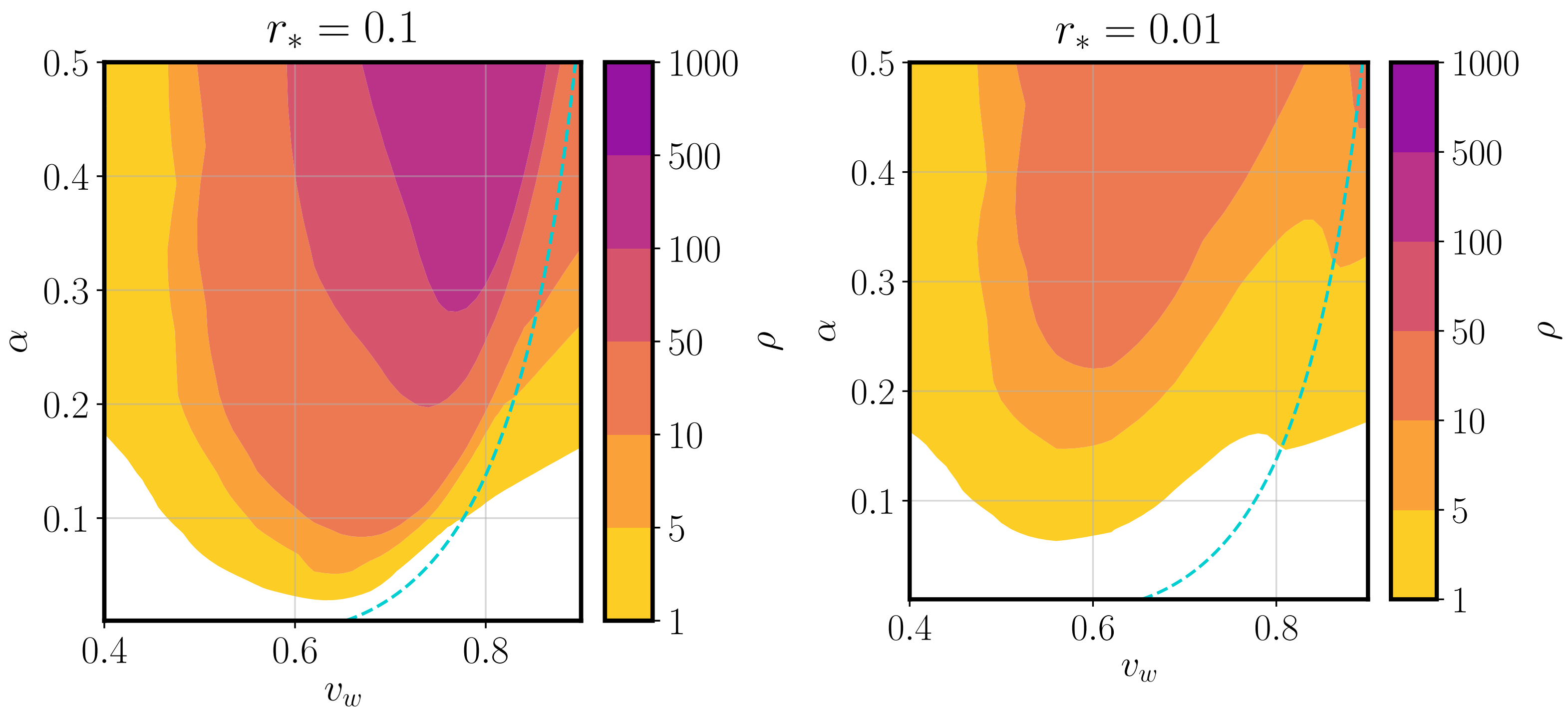


Gowling, MH

After detected objects (e.g. massive black hole binaries) are removed from signal:

- Unresolved white dwarf binaries in our galaxy (~ 20 million)
- Unresolved extra-galactic compact binaries
  - Mostly stellar origin black hole binaries ("LIGO-type")
- What can we hope to see?

Signal-to-noise ratios (LISA)



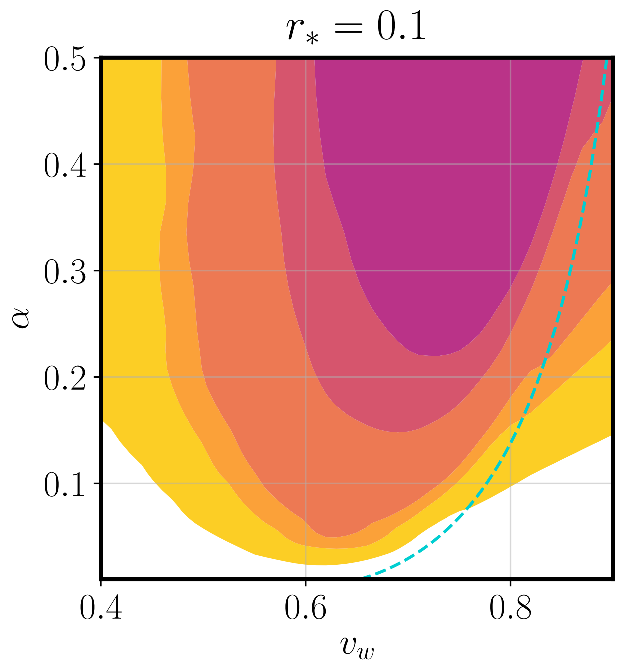
T_c = 100 GeV

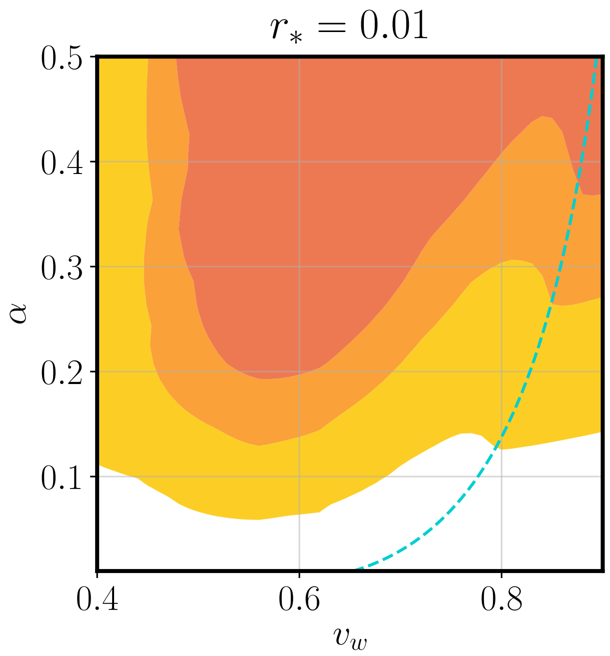
Gowling, MH (2021)

- Signal-to-noise ratio ρ (t_{obs} = 4 years)
- "Worst case" galactic binary foreground
  - (NB annual variation aids removal)
- "LISA science requirements" instrument noise

$$\rho^2(\vec{\theta}) = t_{\rm obs} \int df \left(\frac{\Omega_{\rm gw}(f;\vec{\theta})}{\Omega_{\rm noise}(f)} \right)^2$$

Signal-to-noise ratios (LISA)





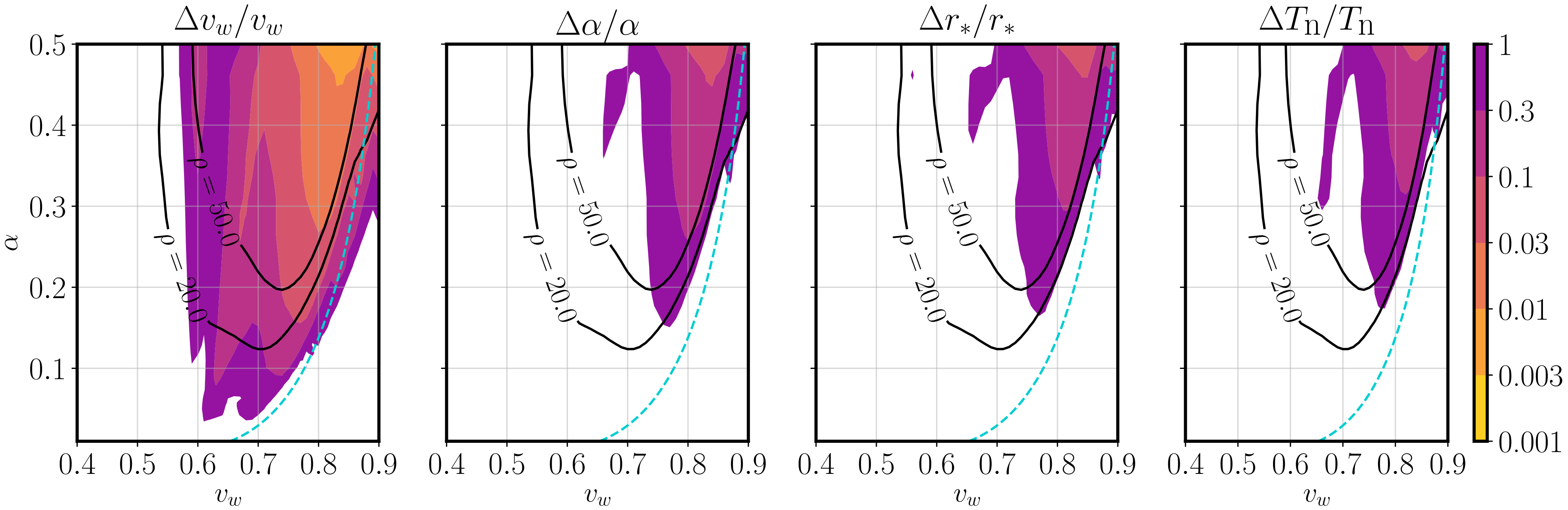
- T_n = 100 GeV
- $c_s = 1/\sqrt{3}$

Gowling, MH (2021)

- Signal-to-noise ratio ρ (t_{obs} = 4 years)
- Perfect removal of GB foreground
- "LISA science requirements" instrument noise

$$\rho^2(\vec{\theta}) = t_{obs} \int df \left(\frac{\Omega_{gw}(f;\vec{\theta})}{\Omega_{noise}(f)} \right)^2$$

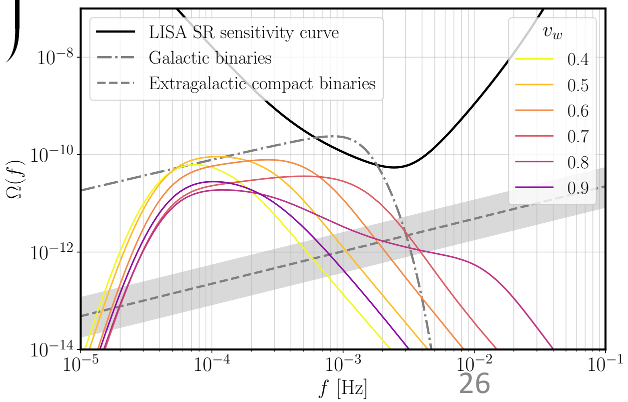
Observability of PT parameters: Fisher analysis



- T_c = 100 GeV
- $c_s = 1/\sqrt{3}$
- r_* = 0.1

Gowling, MH (2021)

- Covariance matrix ***C*** (t_{obs} = 4 years) $C_{ij}^{-1}(\vec{\theta}) = 2t_{\mathrm{obs}} \int df \frac{1}{\Omega^2_{\mathrm{noise}}(f)} \left(\frac{\partial\Omega_{\mathrm{gw}}(f;\vec{\theta})}{\partial\theta_i} \frac{\partial\Omega_{\mathrm{gw}}(f;\vec{\theta})}{\partial\theta_j} \right)$
- Uncertainty from eigenvalues of ***C***
  - "noise" = GB foreground + "LISA science requirements" instrument noise
- **Wall speed** is best determined parameter
  - shape dependence of power spectrum



A theory skeleton in the cupboard

- Is nucleation theory correct?
  - ^{3}He A/B transition rate puzzle

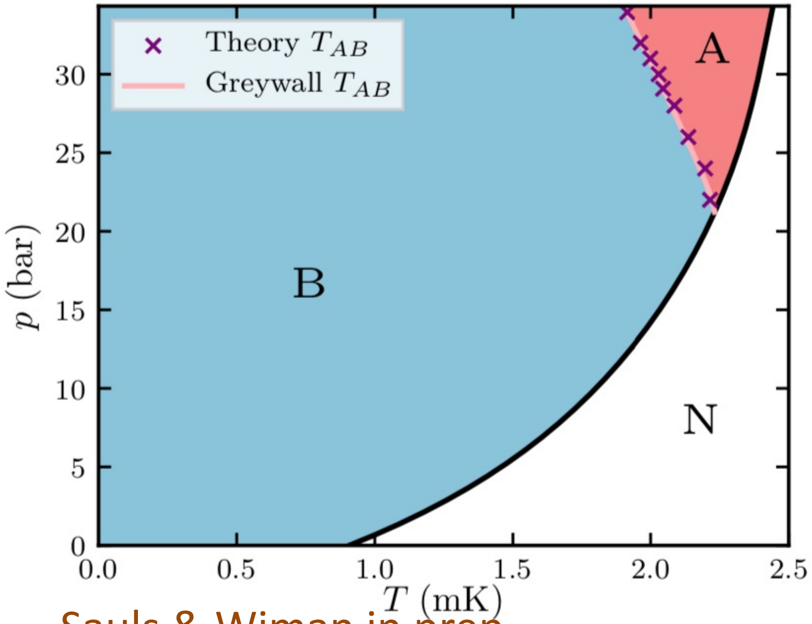
Kaul Kleinert 1980, Bailin, Love 1980, Leggett 1984, Tye Wohns 2011

- Cahn/Hilliard, Langer theory of nucleation rate:

Coleman 1974, Linde 1983

$$\frac{\Gamma}{\mathcal{V}} = \frac{\sqrt{V_T''(0)}}{\pi}\left[\frac{\det(-\vec{\nabla}^2+V_T''(0))}{|\det'(-\vec{\nabla}^2+V_T''(\bar{\phi}))|}\right]^{1/2}\left(\frac{\beta E_c}{2\pi}\right)^{3/2} e^{-\beta E_c}$$

E_c – energy of critical droplet/bubble



Sauls & Wiman in prep

- ^{3}He A/B theory prediction: $\beta E_c \sim 10^6$
- Lab: metastable ^{3}He A lasts hours/days.
- QUEST-DMC (Sussex, Royal Holloway UL, Lancaster) aims to resolve the puzzle

QUEST DMC

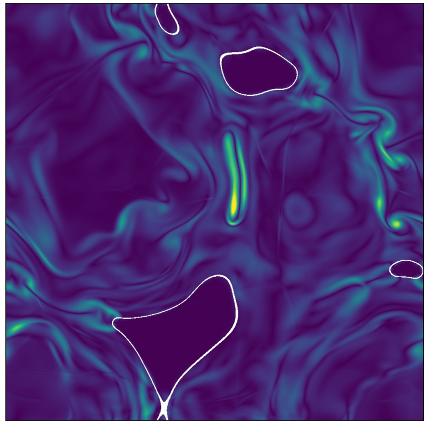
Future challenges: hydrodynamics

- Realistic equations of state
  - Sound speed is important

Giese et al 2020

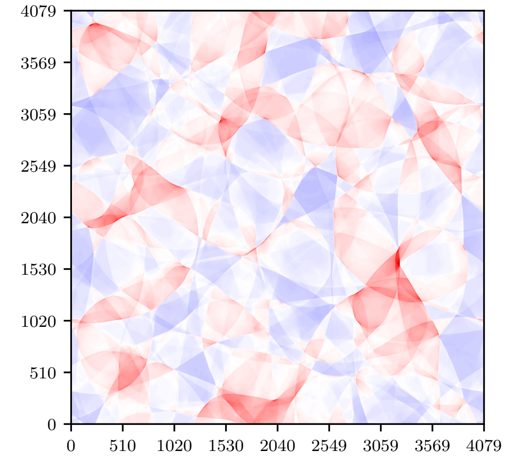
- Non-linear evolution of fluid
  - Longitudinal/compression modes
    - Kinetic energy suppression
    - Shocks, wave turbulence
  - Transverse/rotational modes
    - Vorticity generation
    - Turbulence
    - Turbulence less efficient at producing GWs? Roper Pol et al 2019
    - New! GWs from freely decaying turbulence in relativistic fluid

Auclair et al 2019



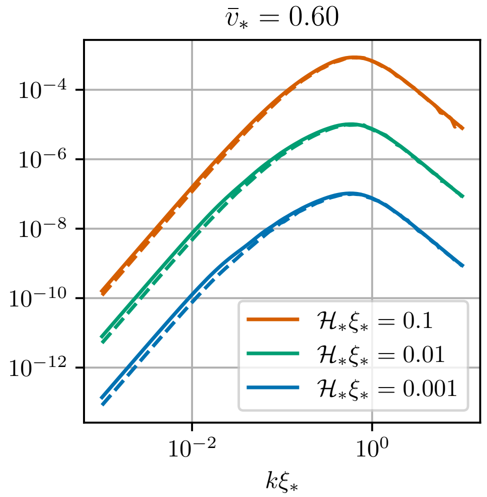
Vorticity, strong transition

Cutting, MH, Weir 2019



Shocks in 2D

Dahl, MH, Rummukainen, Weir (2022)

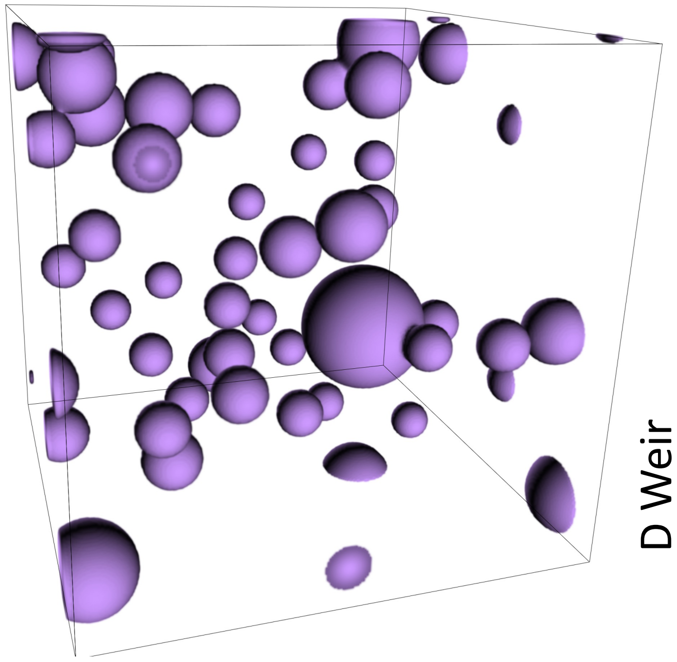


Future challenges: theory

- Scalar effective potential $V(\phi,T)$
  - Non-perturbative methods:
    - Dimensional reduction — Gould et al 2019, Croon et al 2020, Gould, Tenkanen, Lee 2021, Niemi et al 2020
    - Functional renormalization group — Einhorn et al 2020,
  - Strongly interacting fields
    - Lattice + Polyakov — Huang et al 2020, Reichert et al 2021
    - Holography — Ares et al 2020, 2021
- Scalar-fluid coupling $\eta(\phi,T,v)$ & wall speed — Moore, Prokopec 1994; John, Schmidt 2000; Laurent & Cline 2020
  - Perturbative estimates for SM and MSSM
  - Holography — Attems et al 2017, Bigazzi et al 2021, Janik et al 2021, Bea et al 2021
- Connection to phenomenology (e.g. λ_{hhh}) — Caprini et al 2019; …., Kozaczuk et al 2015; Ellis, Lewicki, No 2018; Fairbain et al 2019 … …
- Probing hidden sectors — Schwaller 2015; Jaeckel, Khoze, Spannowsky 2016; Addazi et al 2017, 2018; Baldes 2017; Croon, Sanz, White 2018, …

Conclusions

- LISA and other missions will probe physics of Higgs transition from mid-2030s
  - Measure/constrain phase transition parameters
    - Wall speed likely to be best determined
  - Parameters from underlying particle physics models
    - Wall speed the hardest (non-equilibrium)
- Towards accurate calculations of GW power spectrum from parameters
  - Non-linear evolution (turbulence, shocks) not well understood yet
- Ambition: make GWs as good a probe of the electroweak era as CMB is for the decoupling era



D Weir