
The story of V_{cb} - continued -

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in collaboration with M. Fael, Th. Mannel, K. Olschewsky and M. Rahimi
in collaboration with F. Bernlochner, M. Welsch, R. van Tonder, E. Persson

JHEP 1902 (2019) 177 and work in progress
[arXiv:1812.07472](#) and [arXiv:2105.02163](#)

Why V_{cb} ?

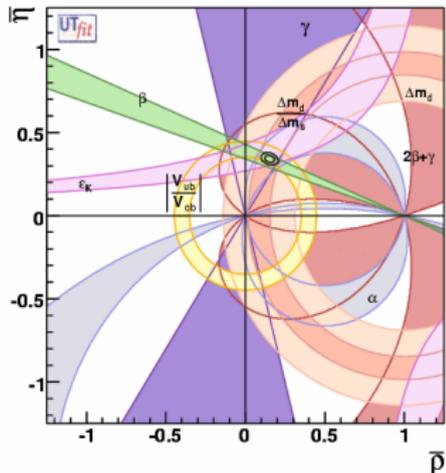
V_{cb} plays important role in CKM unitarity triangle

- Kaon CP violation via ϵ_K
- In Flavour-Changing-Neutral-Currents (FCNC)
- Aiming at the highest precision possible
 - SM test or NP probe?

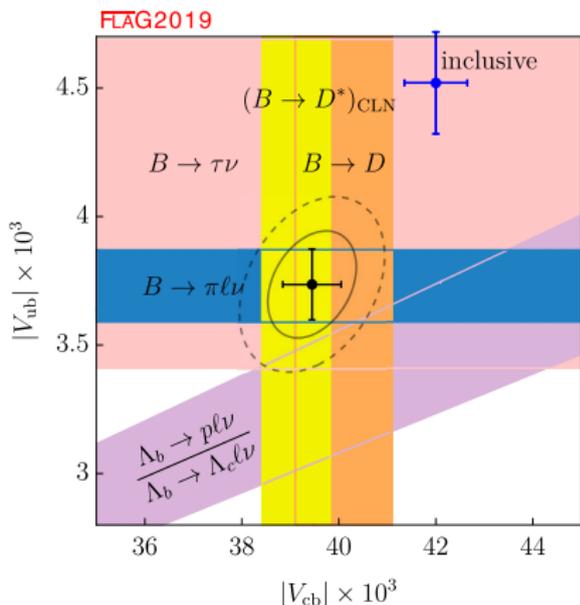
Inclusive versus Exclusive

- $B \rightarrow X_c \ell \nu$ versus $B \rightarrow D^{(*)} \ell \nu$
- use Heavy Quark Expansion
- V_{cb} puzzle
 - Discrepancy between both determinations

Focus on new ideas for more precise inclusive $|V_{cb}|$



Inclusive versus Exclusive decays



$|V_{cb}|$

- Exclusive $B \rightarrow D^{(*)} l \bar{\nu}$
- Inclusive $B \rightarrow X_c l \nu$

$|V_{ub}|$

- Exclusive $B \rightarrow \pi l \nu$ ($B \rightarrow \tau \nu$)
- Inclusive $B \rightarrow X_u l \nu$

$|V_{ub}|/|V_{cb}|$

- First determination in baryons

$$\frac{\mathcal{B}(\Lambda_b \rightarrow p l \nu)}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c l \nu)} = (1.5 \pm 0.1) \left| \frac{V_{ub}}{V_{cb}} \right|^2$$

LHCb'18; Detmold, Lehner, Meinel'15

Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner,

Bordone, van Dyk, Gubernari

Exclusive $B \rightarrow D^{(*)} l \bar{\nu}$

- Form factor required (only for $B \rightarrow D$ available at different kinematic points)
- Different parametrizations for form factors: CLN Caprini, Lellouch, Neubert [1997] and BGL Boyd, Grinstein, Lebed [1995]
 - BGL: model independent based on unitarity and analyticity
 - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]

Inclusive $B \rightarrow X_c l \nu$

- Determined fully data driven including $1/m_b$ power corrections

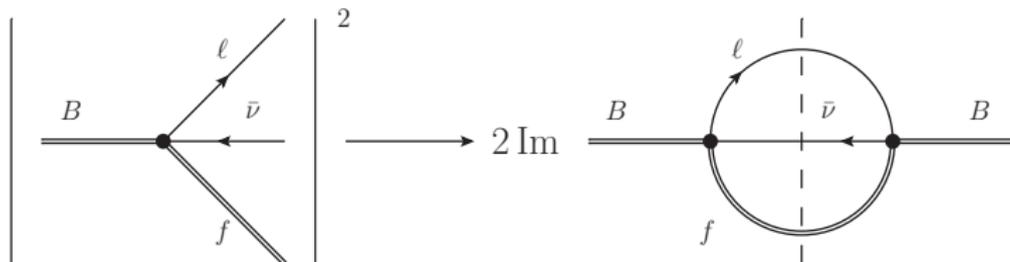
Puzzle needs to be scrutinized but seems to be disappearing....

Stay tuned!

Inclusive decays and the Heavy Quark Expansion

- Optical Theorem
- Heavy Quark Expansion (HQE)
- Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^3 + d\Gamma_4 \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^4 \\ + d\Gamma_5 \left[a_0 \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^5 + a_1 \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^3 \left(\frac{\Lambda_{\text{QCD}}}{m_c} \right)^2 \right] + \dots$$



Optical Theorem

$$\begin{aligned}
 \Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{\text{eff}} | B(v) \rangle|^2 \\
 &= \int d^4x \langle B(v) | \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) | B(v) \rangle \\
 &= 2 \text{Im} \int d^4x e^{-iq \cdot x} \langle B(v) | T \left\{ \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) \right\} | B(v) \rangle
 \end{aligned}$$

where $\mathcal{H}_{\text{eff}} = J_c^\mu L_\mu$, $J_c^\mu = \bar{b} \gamma^\mu P_L c$

Heavy Quark Expansion

- B meson: $p_B = m_B v$
- Split the momentum b quark: $p_b = m_b v + k$, expand in $k \sim iD Q_v$
- Field-redefinition of the heavy field $Q(x) = \exp(-im(v \cdot x)) Q_v(x)$

$$\begin{aligned}\Gamma &= 2 \operatorname{Im} \int d^4x e^{-iq \cdot x} \langle B(v) | T \{ \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}^\dagger(0) \} | B(v) \rangle \\ &= 2 \operatorname{Im} \int d^4x e^{i(m_b v - q) \cdot x} \langle B(v) | T \{ \tilde{\mathcal{H}}_{\text{eff}}(x) \tilde{\mathcal{H}}_{\text{eff}}^\dagger(0) \} | B(v) \rangle\end{aligned}$$

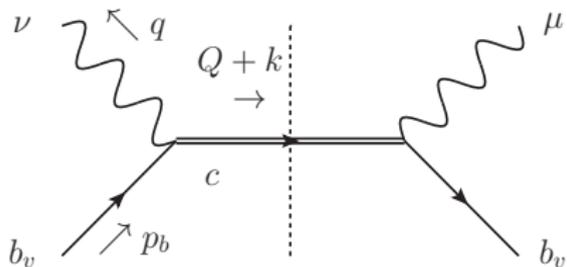
where $\tilde{\mathcal{H}}_{\text{eff}} = \tilde{J}_C^\mu L_\mu$, $\tilde{J}_C^\mu = \bar{b}_v \gamma^\mu P_L c$, $\Gamma \propto 2 \operatorname{Im} T^{\mu\nu} L_{\mu\nu}$

Inclusive Decays: the OPE

$$\Gamma(B \rightarrow X_c \ell \nu_\ell) \propto 2\text{Im} T^{\mu\nu} L_{\mu\nu}$$

$$T^{\mu\nu} = i \int d^4x e^{i(m_b v - q) \cdot x} T \{ \bar{b}_\nu(x) \gamma^\mu P_L c(x), \bar{c}(0) \gamma^\nu P_L b_\nu(0) \}$$

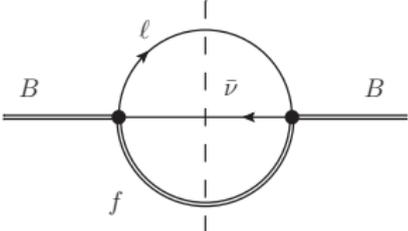
$$Q = m_b v - q$$



$$= \bar{b}_\nu \gamma_\mu P_L \left[\frac{i}{\not{Q} + i\not{D} - m_c} \right] \gamma_\nu P_L b_\nu$$

$$\frac{i}{\not{Q} + i\not{D} - m_c} = \frac{i}{\not{Q} - m_c} + \frac{i}{\not{Q} - m_c} (-i\not{D}) \frac{i}{\not{Q} - m_c} + \frac{i}{\not{Q} - m_c} (-i\not{D}) \frac{i}{\not{Q} - m_c} (-i\not{D}) \frac{i}{\not{Q} - m_c} + \dots$$

Operator Product Expansion (OPE)


$$2 \text{Im} \left[\text{Bubble Diagram} \right] = \sum_{n,i} \frac{C_i^{(n)}(\mu, \alpha_s)}{m_b^i} \langle B | \mathcal{O}_i^{(n)} | B \rangle_\mu$$

- $C_i(\mu)$: short distance, perturbative coefficients
- $\langle B | \mathcal{O}_i | B \rangle_\mu$: non-perturbative forward matrix elements of local operators
- operators contain chains of covariant derivatives

$$\langle B | \mathcal{O}_i^{(n)} | B \rangle = \langle B | \bar{b}_v (iD_\mu) \dots (iD_{\mu_n}) b_v | B \rangle$$

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b} \Gamma_1 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^3} \Gamma_3 \dots$$

- Γ_0 : decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2 : μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B \mu_\pi^2 = - \langle B | \bar{b}_\nu iD_\mu iD^\mu b_\nu | B \rangle$$

$$2M_B \mu_G^2 = \langle B | \bar{b}_\nu (-i\sigma^{\mu\nu}) iD_\mu iD_\nu b_\nu | B \rangle$$

- Γ_3 : ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B \rho_D^3 = \frac{1}{2} \langle B | \bar{b}_\nu [iD_\mu, [ivD, iD^\mu]] b_\nu | B \rangle$$

$$2M_B \rho_{LS}^3 = \frac{1}{2} \langle B | \bar{b}_\nu \{iD_\mu, [ivD, iD_\nu]\} (-i\sigma^{\mu\nu}) b_\nu | B \rangle$$

- Γ_4 : 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5 : 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Inclusive V_{cb} determination

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\text{cut}} = \frac{\int_{E_\ell > E_{\text{cut}}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\text{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\langle (M_X^2)^n \rangle_{\text{cut}} = \frac{\int_{E_\ell > E_{\text{cut}}} dM_X^2 (M_X^2)^n \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\text{cut}}} dM_X^2 \frac{d\Gamma}{dM_X^2}}$$

$$R^*(E_{\text{cut}}) = \frac{\int_{E_\ell > E_{\text{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \frac{d\Gamma}{dE_\ell}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use lepton energy cut

$$\begin{array}{c}
 R^*(E_{\text{cut}}) \quad \langle E^n \rangle_{\text{cut}} \quad \langle (M_X^2)^n \rangle_{\text{cut}} \\
 \downarrow \\
 \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, m_b, (m_c) \\
 \downarrow \\
 \text{Br}(\bar{B} \rightarrow X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_0 + \Gamma_{\mu\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho D} \frac{\rho_D^3}{m_b^3} \right] \\
 \downarrow \\
 V_{cb} = (42.21 \pm 0.78) \times 10^{-3}
 \end{array}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
 Alberti, Gambino et al, PRL 114 (2015) 061802

State-of-the-art

Ježabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma(\pi,0) + \frac{\alpha_s}{\pi} \Gamma(\pi,1) \right) + \frac{\mu_G^2}{m_b^2} \left(\Gamma(G,0) + \frac{\alpha_s}{\pi} \Gamma(G,1) \right) + \frac{\rho_D^3}{m_b^3} \Gamma(D,0) + \mathcal{O} \left(\frac{1}{m_b^4} \right) \dots \right]$$

- Includes all known α_s and α_s^2 corrections
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063
- Only uses mild external constraints
- Include terms up to $1/m_b^3$
- Assigned 1.4% theo. error due to missing higher orders

Towards the ultimate precision in inclusive V_{cb}

Ježabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma(\pi,0) + \frac{\alpha_s}{\pi} \Gamma(\pi,1) \right) \right. \\ \left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma(G,0) + \frac{\alpha_s}{\pi} \Gamma(G,1) \right) + \frac{\rho_D^3}{m_b^3} \Gamma(D,0) + \mathcal{O} \left(\frac{1}{m_b^4} \right) \dots \right]$$

- Proliferation of non-perturbative matrix elements

- 4 up to $1/m_b^3$
- 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
- 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

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$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma(\pi,0) + \frac{\alpha_s}{\pi} \Gamma(\pi,1) \right) \right. \\ \left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma(G,0) + \frac{\alpha_s}{\pi} \Gamma(G,1) \right) + \frac{\rho_D^3}{m_b^3} \Gamma(D,0) + \mathcal{O} \left(\frac{1}{m_b^4} \right) \dots \right]$$

- Proliferation of non-perturbative matrix elements

- 4 up to $1/m_b^3$
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The story of $|V_{cb}|$ continued:

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [2020]
- Full determination up to $1/m_b^4$ from data Fael, KKV, Bernlochner et al. [in progress]
- Reconsider how to deal with backgrounds Mannel, Rahimi, KKV [2021]

Reparametrization Invariance

Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel
Mannel, KKV, JHEP 1806 (2018) 115

- Choice of v not unique
- Reparametrization Invariant (RPI) under an infinitesimal change

$$v_\mu \rightarrow v_\mu + \delta v_\mu$$

$$\delta_{RP} v_\mu = \delta v_\mu \quad \text{and} \quad \delta_{RP} iD_\mu = -m_b \delta v_\mu$$

- Reparametrization invariance links different orders in $1/m_b$
 - Gives exact relations between different orders
 - Resums towers of operators
 - Reduces the number of independent parameters
- Up to $1/m_b^4$: 8 parameters versus previous 13

$$R = \sum_{n=0}^{\infty} C_{\mu_1 \dots \mu_n}^{(n)}(v) \otimes \bar{b}_v(iD_{\mu_1} \dots iD_{\mu_n}) b_v$$

$$\begin{aligned} \delta_{\text{RP}} R = 0 &= \sum_{n=0}^{\infty} \left[\delta_{\text{RP}} C_{\mu_1 \dots \mu_n}^{(n)} \right] \bar{b}_v(iD^{\mu_1} \dots iD^{\mu_n}) b_v \\ &\quad + \sum_{n=0}^{\infty} C_{\mu_1 \dots \mu_n}^{(n)} \left[\delta_{\text{RP}} \bar{b}_v(iD^{\mu_1} \dots iD^{\mu_n}) b_v \right] \end{aligned}$$

The RPI relation:

$$\delta_{\text{RP}} C_{\mu_1 \dots \mu_n}^{(n)} = m_b \delta v^\alpha \left[C_{\alpha \mu_1 \dots \mu_n}^{(n+1)} + C_{\mu_1 \alpha \mu_2 \dots \mu_n}^{(n+1)} + \dots + C_{\mu_1 \dots \mu_n \alpha}^{(n+1)} \right]$$

- $1/m_b^2$: $\mu_G^2 \rightarrow \underbrace{\eta(-i\sigma_{\mu\nu})}_{C_{\mu\nu}^{(2)}} \otimes \bar{b}_\nu (iD^\mu iD^\nu) b_\nu$
- $1/m_b^3$: $\rho_{LS}^3 \rightarrow \underbrace{\xi v_\alpha(-i\sigma_{\mu\nu})}_{C_{\mu\alpha\nu}^{(3)}} \otimes \bar{b}_\nu (iD^\mu iD^\alpha iD^\nu) b_\nu$

The RPI relation:

$$\begin{aligned}\delta_{\text{RP}} C_{\mu\nu}^{(2)} &= 0 \\ &= m_b \delta v^\alpha \left(C_{\mu\nu\alpha}^{(3)} + C_{\mu\alpha\nu}^{(3)} + C_{\alpha\mu\nu}^{(3)} \right) \\ &= -im_b \xi \delta v^\alpha (\sigma_{\mu\alpha} v_\nu + \sigma_{\alpha\nu} v_\mu) \\ &\Leftrightarrow \xi = 0\end{aligned}$$

Non-perturbative matrix elements

Mannel, KKV, JHEP 1806 (2018) 115

- 1:

$$- 2M_B \mu_3 = \langle B | \bar{b}_\nu b_\nu | B \rangle = 2M_B \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b} \right)$$

- $1/m_b^2$:

$$- 2M_B \mu_G^2 = \langle B | \bar{b}_\nu (-i\sigma^{\mu\nu}) iD_\mu iD_\nu b_\nu | B \rangle$$

- $1/m_b^3$:

$$- 2M_B \tilde{\rho}_D^3 = \frac{1}{2} \langle B | \bar{b}_\nu \left[iD_\mu, \left[(ivD + \frac{(iD)^2}{m_b}), iD^\mu \right] \right] b_\nu | B \rangle$$

- $1/m_b^4$:

$$- 2M_B r_G^4 \equiv \frac{1}{2} \langle B | \bar{b}_\nu [iD_\mu, iD_\nu] [iD^\mu, iD^\nu] b_\nu | B \rangle \propto \langle \vec{E}^2 - \vec{B}^2 \rangle$$

$$- 2M_B r_E^4 \equiv \frac{1}{2} \langle B | \bar{b}_\nu [ivD, iD_\mu] [ivD, iD^\mu] b_\nu | B \rangle \propto \langle \vec{E}^2 \rangle$$

$$- 2M_B s_B^4 \equiv \frac{1}{2} \langle B | \bar{b}_\nu [iD_\mu, iD_\alpha] [iD^\mu, iD_\beta] (-i\sigma^{\alpha\beta}) b_\nu | B \rangle \propto \langle \vec{\sigma} \cdot \vec{B} \times \vec{B} \rangle$$

$$- 2M_B s_E^4 \equiv \frac{1}{2} \langle B | \bar{b}_\nu [ivD, iD_\alpha] [ivD, iD_\beta] (-i\sigma^{\alpha\beta}) b_\nu | B \rangle \propto \langle \vec{\sigma} \cdot \vec{E} \times \vec{E} \rangle$$

$$- 2M_B s_{qB}^4 \equiv \frac{1}{2} \langle B | \bar{b}_\nu [iD_\mu, [iD^\mu, [iD_\alpha, iD_\beta]]] (-i\sigma^{\alpha\beta}) b_\nu | B \rangle \propto \langle \square \vec{\sigma} \cdot \vec{B} \rangle .$$

Up to $1/m_b^4$: 8 parameters versus previous 13

- Ratio between the rate with and without a cut

$$R^*(q_{\text{cut}}^2) = \int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2} \bigg/ \int_0 dq^2 \frac{d\Gamma}{dq^2}$$

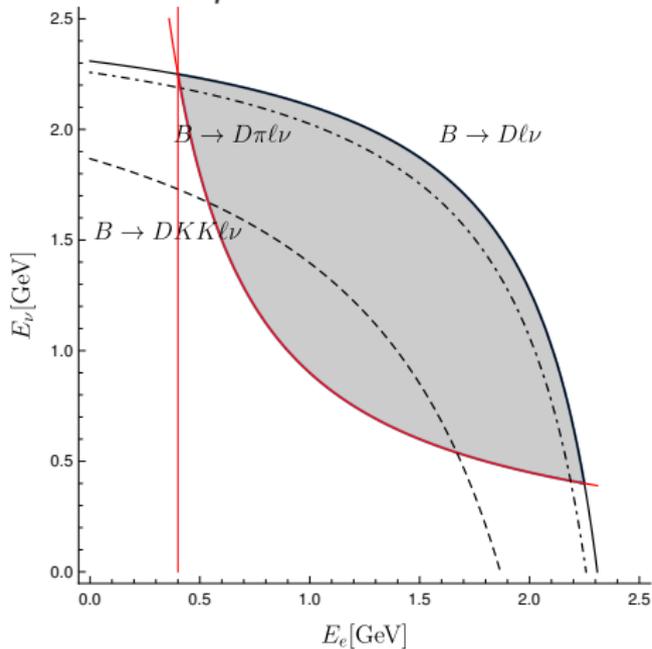
- q^2 moments

$$\langle (q^2)^n \rangle_{\text{cut}} = \int_{q^2 > q_{\text{cut}}^2} dq^2 (q^2)^n \frac{d\Gamma}{dq^2} \bigg/ \int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}$$

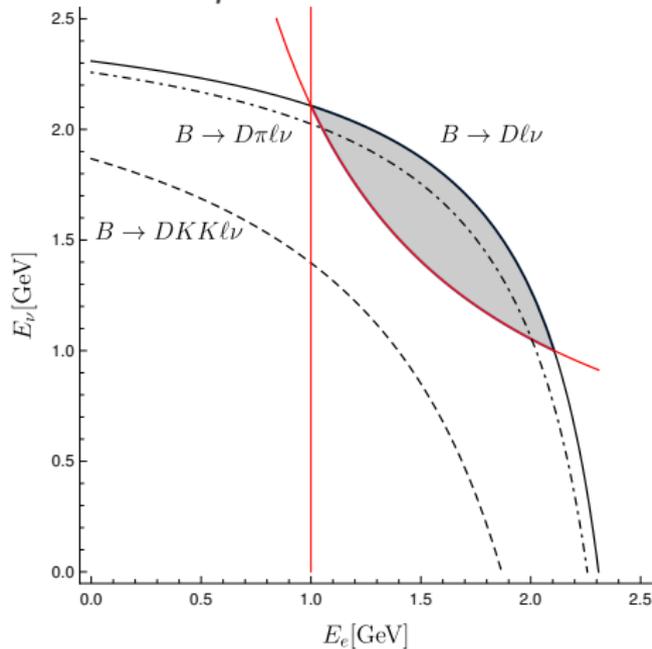
- Hadronic mass and lepton energy moments are NOT RPI
- Energy cut is not RPI, but q_{cut}^2 is RPI and can be superimposed

q^2 versus energy cut

$q^2 > 3.6 \text{ GeV}^2$



$q^2 > 8.4 \text{ GeV}^2$



Alternative V_{cb} determination

$$\begin{array}{c}
 R^*(q_{\text{cut}}^2) \quad \langle (q^2)^n \rangle_{\text{cut}} \\
 \downarrow \\
 \mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c \\
 \downarrow \\
 \text{Br}(\bar{B} \rightarrow X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} \right. \\
 \left. + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \right] \\
 \downarrow \\
 V_{cb} = ?
 \end{array}$$

Fael, Mannel, KKV, JHEP 02 (2019) 177

V_{cb} from q^2 moments

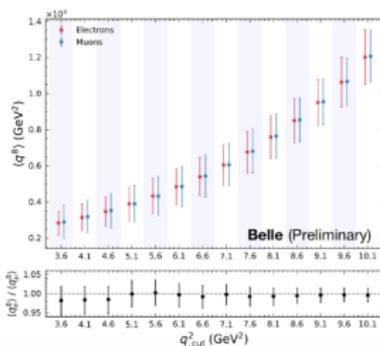
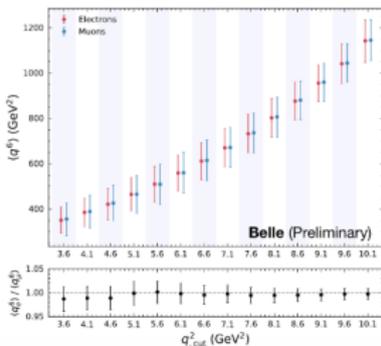
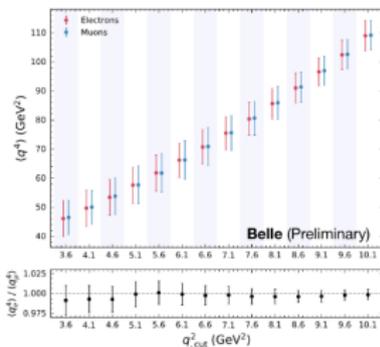
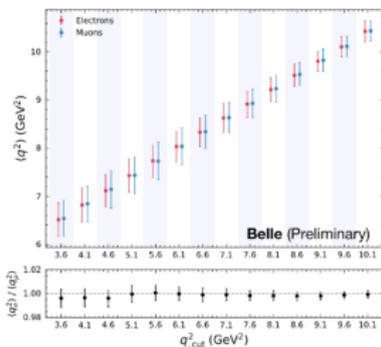
in collaboration with

F. Bernlochner, M. Welsch, M. Fael, K. Olschewsky, R. van Tonder

in progress

Preliminary Belle q^2 measurements!

See proceedings R. van Tonder [ArXiv:2105.08001]



Total statistical and systematic errors scaled by a factor of 10

Extracting both V_{cb} and HQE parameters up to $1/m_b^4$ from data

In progress: Software package

- Moments and centralized moments
 - Theoretical precision versus experimental precision
- α_s corrections $1/m_b^2$ not yet known
- α_s^3 to partonic rate included Fael, Schoenwald, Steinhauser [2020, 2021]
- kinetic scheme or on-shell scheme
- Flexible theoretical covariance matrix

Preliminary:

- V_{cb} rather insensitive to theory covariance
- Higher order corrections have small influence
- First determinations of V_{cb} very promising

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Stay Tuned!

Backgrounds in $B \rightarrow X_{cl}\nu$

in collaboration with

M. Rahimi and T. Mannel

arXiv: 2105.02163

Avoid background subtraction by calculating the full inclusive width:

$$d\Gamma(B \rightarrow X\ell) = d\Gamma(B \rightarrow X_c \ell \bar{\nu}) + d\Gamma(B \rightarrow X_u \ell \bar{\nu}) + d\Gamma(B \rightarrow X_c(\tau \rightarrow \ell \bar{\nu} \nu) \bar{\nu})$$

- $b \rightarrow u \ell \nu$ contribution: suppressed by V_{ub}/V_{cb}
- $b \rightarrow c(\tau \rightarrow \mu \nu \bar{\nu}) \bar{\nu}$ contribution: phase space suppressed
- QED effects
- Quark-hadron duality violation?

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:

estimate how much this description would improve V_{cb} determination

- Can be analyzed in local OPE as $B \rightarrow X_c l \nu$ by taking $m_c \rightarrow 0$ limit
- For V_{ub} determination
 - large charm background requires experimental cuts
 - reduces the inclusivity and local OPE no longer converges
 - spectrum described by non-local OPE
 - convolution of pert. coefficients with shape function

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2 + 1/m_b^3$
- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)
- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders
- Inputs HQE parameters from $B \rightarrow X_c l \nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

Monte Carlo:

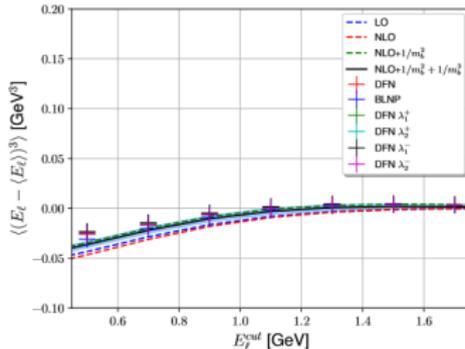
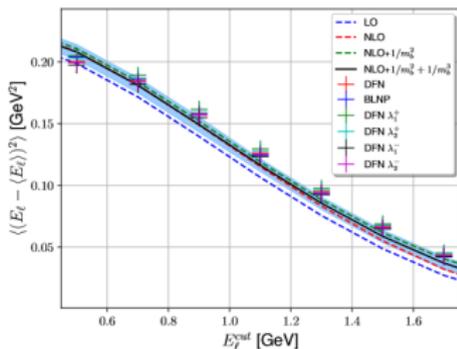
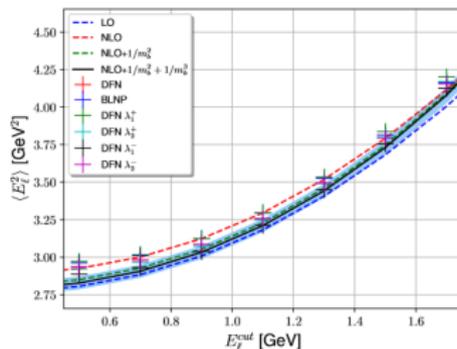
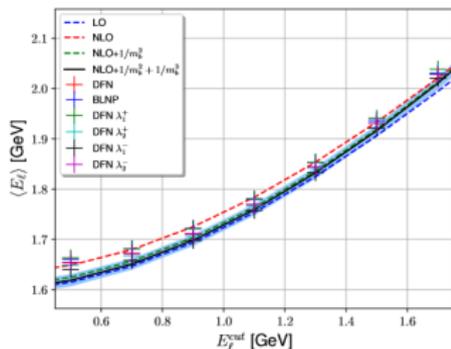
- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$
- DFN: α_s corrections convoluted with the exponential shape function model
 - Inputs from $B \rightarrow X_c \ell \nu$ and $B \rightarrow X_s \gamma$ data using KN-scheme Kagan, Neubert 1998
 - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: “hybrid Monte Carlo” Belle Collaboration [arXiv:2102.00020.]

- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \rightarrow \pi \ell \bar{\nu}$ and $\bar{B} \rightarrow \rho \ell \bar{\nu}$

Monte Carlo versus HQE

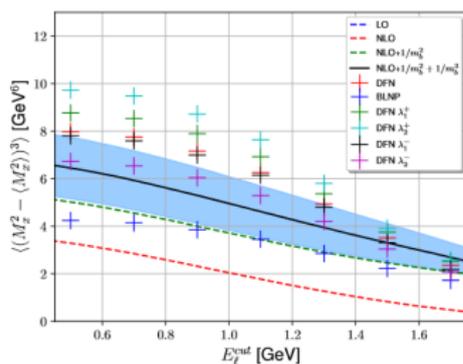
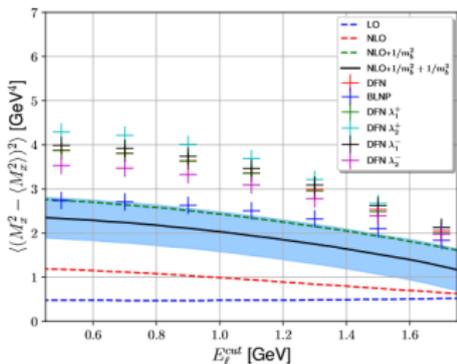
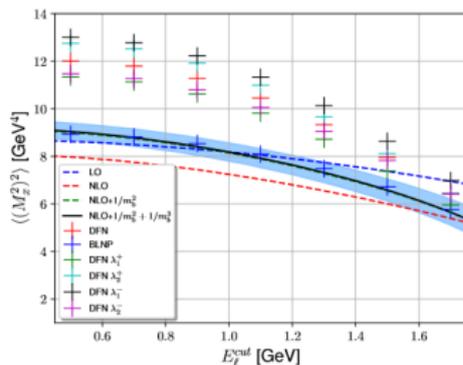
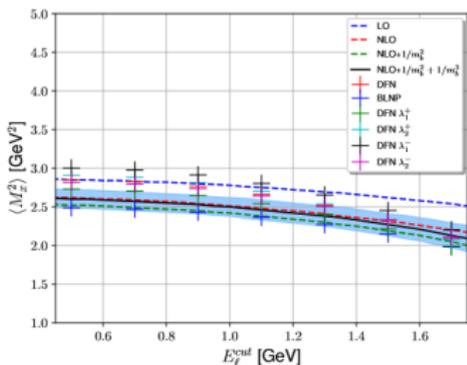
Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

Monte Carlo versus HQE

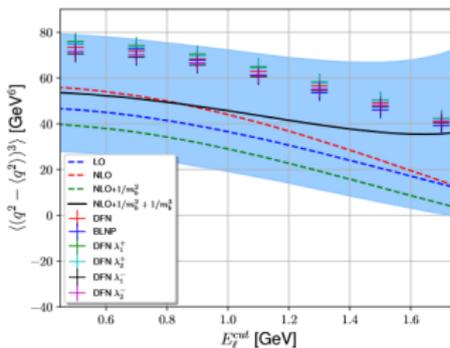
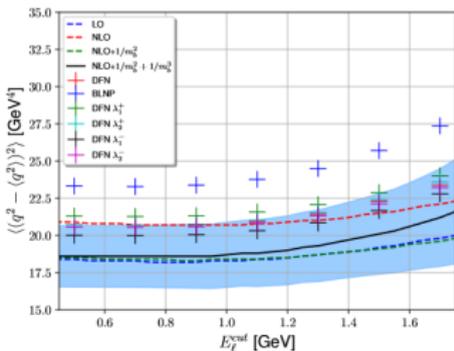
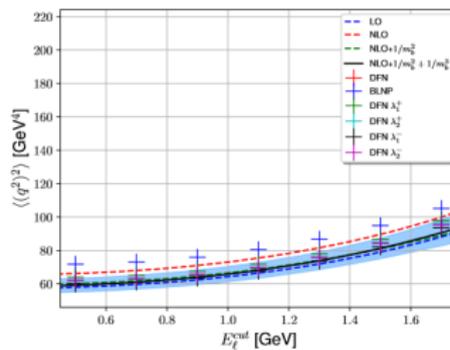
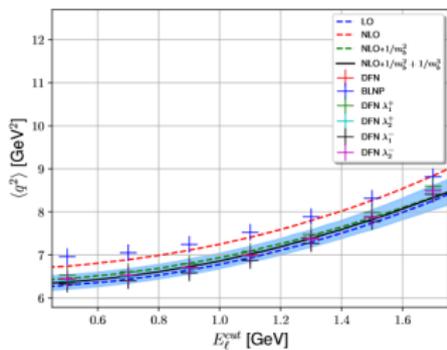
Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Wide spread between MC for higher moments

Monte Carlo versus HQE

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all
- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
 - should include higher moments of the shape-function model?
 - include subleading shape functions?
- our HQE: interesting to include α_s to HQE parameters, α_s^2 ?

Contribution from five-body charm decay to $b \rightarrow c \ell \nu$ via

$$B(p_B) \rightarrow X_c(p_{X_c})(\tau(q_{[\tau]} \rightarrow \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]}))$$

- Phase space suppressed:

$$\frac{\Gamma_{\text{tot}}(b \rightarrow c\tau(\rightarrow \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\text{tot}}(b \rightarrow c\ell\bar{\nu})} \sim 4.0\%$$

- Experimentally effects diminished by cutting on the invariant mass of the B
- Can be calculated exactly in the HQE

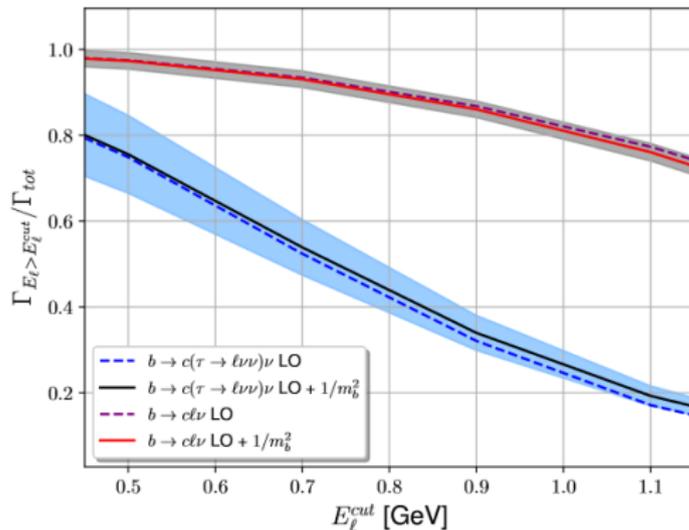
$$\frac{d^8\Gamma}{dq^2 dq_{\nu\bar{\nu}}^2 dp_{X_c}^2 d^2\Omega d\Omega^* d^2\Omega^{**}} = - \frac{3G_F^2 |V_{cb}|^2 \sqrt{\lambda}(q^2 - m_\tau^2)(m_\tau^2 - q_{\nu\bar{\nu}}^2) \mathcal{B}(\tau \rightarrow \mu\nu\nu)}{2^{17} \pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu} L^{\mu\nu}$$

- $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
- $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Five-body τ contribution

Rahimi, Mannel, KKV[arXiv: 2105.02163];



No MC data available to test with

Outlook

The story of V_{cb} continues:

- RPI reduces number of non-perturbative matrix elements
- Total rate and q^2 moments are RPI: 8 instead of 13 up to $1/m_b^4$
- Extract $|V_{cb}|$ up to $1/m_b^4$, completely data driven
- q^2 moments available soon!

In progress:

- Full machinery to obtain V_{cb} from q^2 moments
- Calculation of α_s terms
- Implementation of the full $B \rightarrow X\ell\nu$ (including $B \rightarrow X_u\ell\nu$ and $B \rightarrow X_c(\tau \rightarrow \mu\nu\bar{\nu})\bar{\nu}$)

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Close collaboration between theory and experiment necessary!