Resummation of Superleading Logarithms

Toward understanding non-global observables at hadron colliders

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based on work with Thomas Becher and Ding Yu Shao
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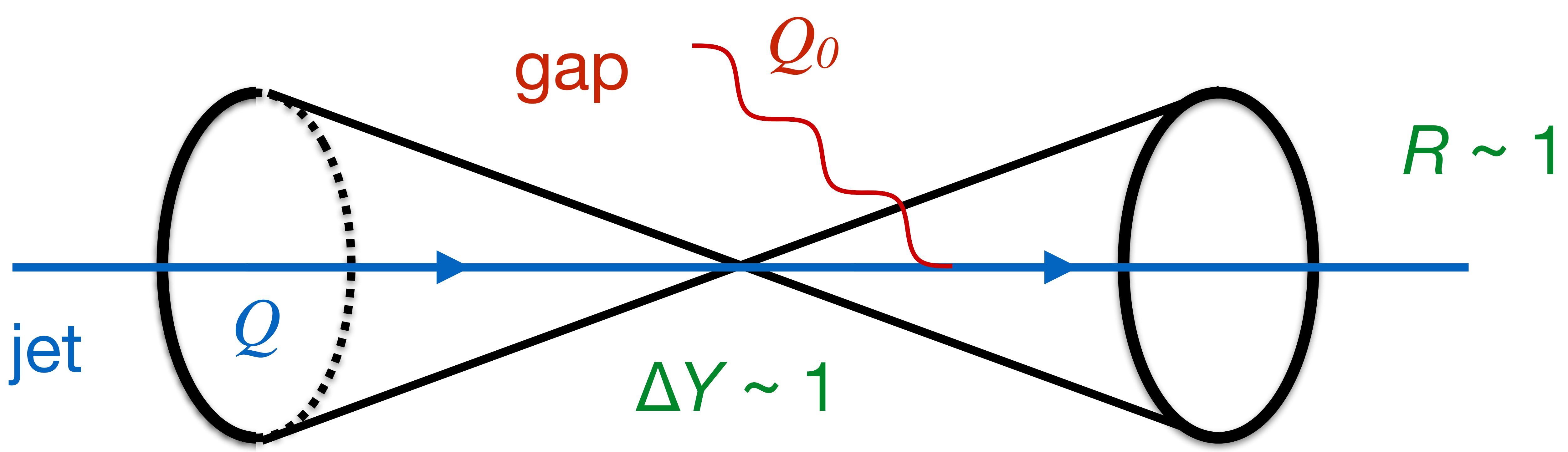
Introduction

- LHC cross sections are usually computed in terms of convolutions of partonic cross sections with parton distribution functions
- Partonic cross sections are calculated at some fixed order in perturbation theory (NNLO or N^{3}LO)
- In many cases this is insufficient, because in the presence of experimental cuts the cross sections are sensitive to very different energy scales
- Fixed-order results are affected by large logarithmic corrections, which need to be resummed to all orders; often this is done using parton showers (large-N_c approximation)
- This talk is about "strange logarithms" and a clever way to resum them …

Introduction

Consider the "gap between jets" observable (inter-jet energy flow):



(artwork by Thomas Becher)

The cross section contains large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

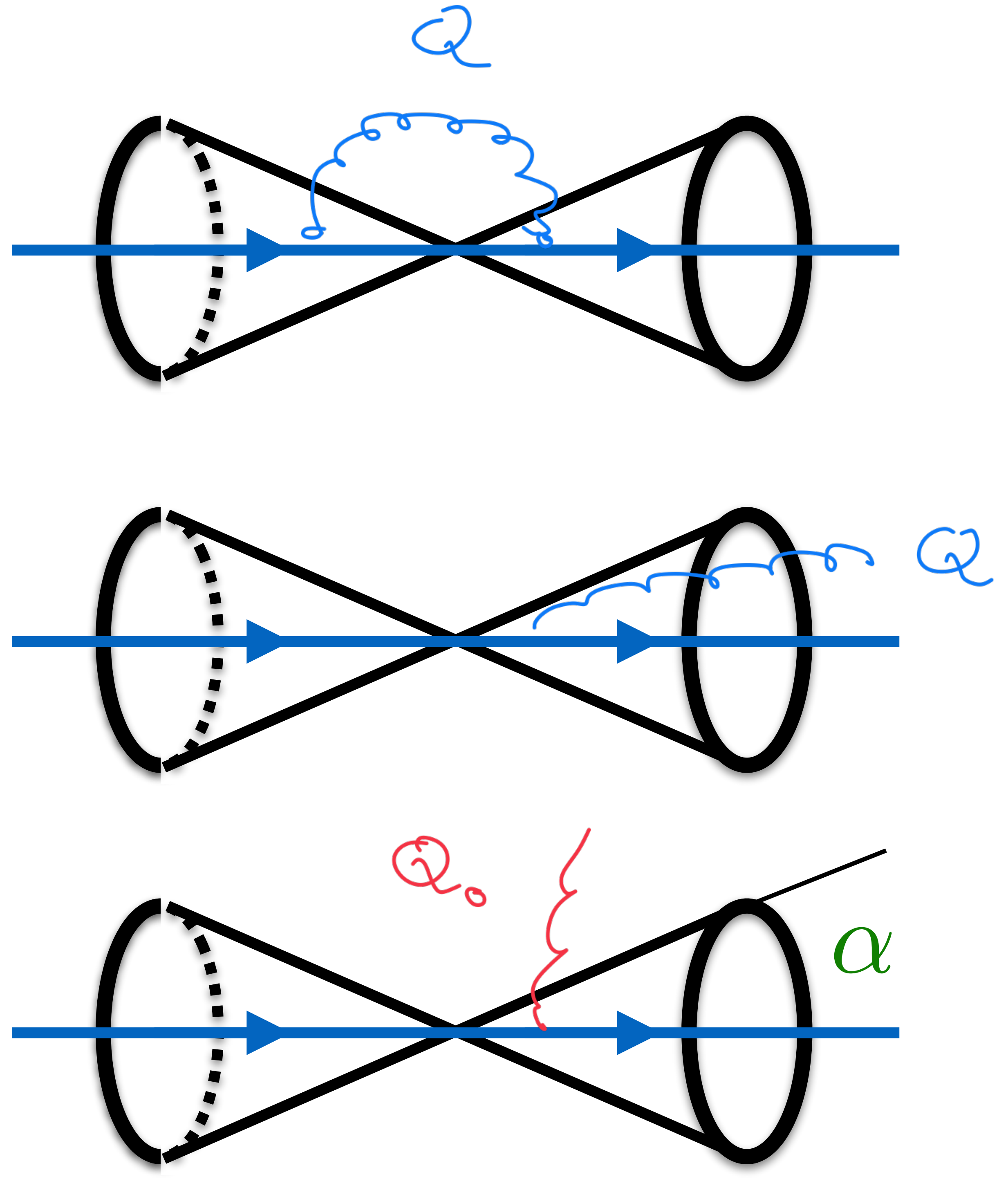
- for e^+e^- collisions: $m \leq n$, leading logs have $m = n$
- for hadron colliders: $\alpha_s L,\ \alpha_s^2 L^2,\ \alpha_s^3 L^3\ ,\boxed{\alpha_s^4 L^5 \dots ,\alpha_s^{3+n} L^{3+2n}}$

  superleading logs (SLLs)

  [Forshaw, Kyrieleis, Seymour (2006, 2008)]
- not contained in existing parton showers!

Introduction

In general, large logarithms arise from an incomplete cancellation of soft and collinear IR divergences:



$r = \tan^2(\alpha/2)$

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left[-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} + \dots \right]$$

soft+collinear divergence

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left[+\frac{4}{\epsilon^2} + \frac{6}{\epsilon} - \frac{4\ln(r)}{\epsilon} + \dots \right]$$

wide-angle soft

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q_0^2}\right)^\epsilon \left[\frac{4\ln(r)}{\epsilon} + \dots \right]$$

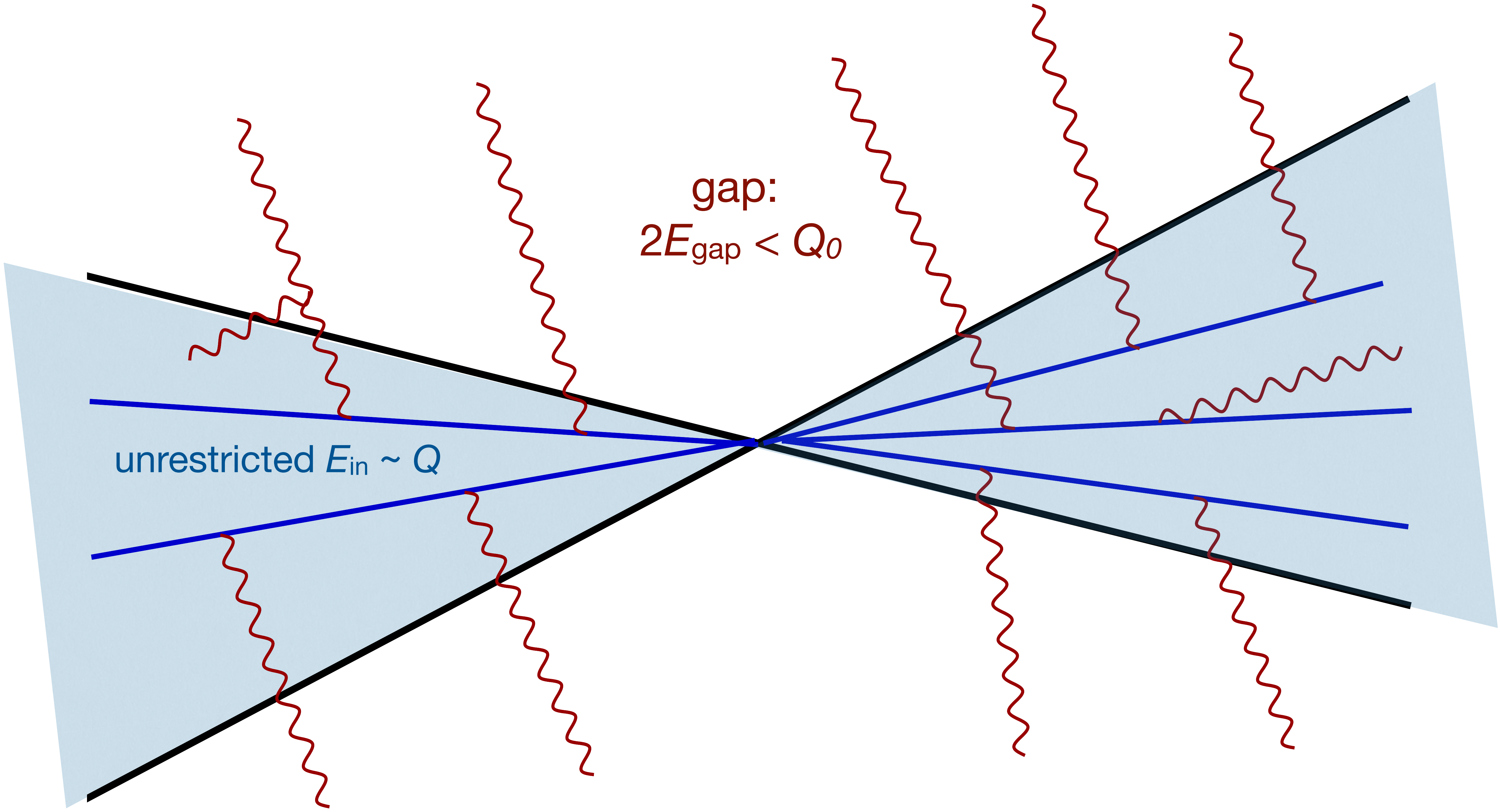
$$\sigma_0 \frac{\alpha_s C_F}{4\pi}\, 4\ln(r) \ln\frac{Q^2}{Q_0^2} + \dots$$

Introduction

Since the effect arises first at 4-loop order, little is known about SLLs:

- discovered in "gaps between jets” calculation for $qq \to qq$ [Forshaw, Kyrieleis, Seymour (2006)]
- calculation of first SLL $\sim \alpha_s^4 L^5$ for arbitrary $2 \to 2$ hard processes using the color-space formalism [Forshaw, Kyrieleis, Seymour (2008)]
- diagrammatic calculation of the first two SLLs for some selected 2-parton channels [Keates, Seymour (2009)]

All-order structure of SLLs, their contributions to other scattering processes, and their asymptotic behavior for $Q/Q_0 \to \infty$ are completely unknown!

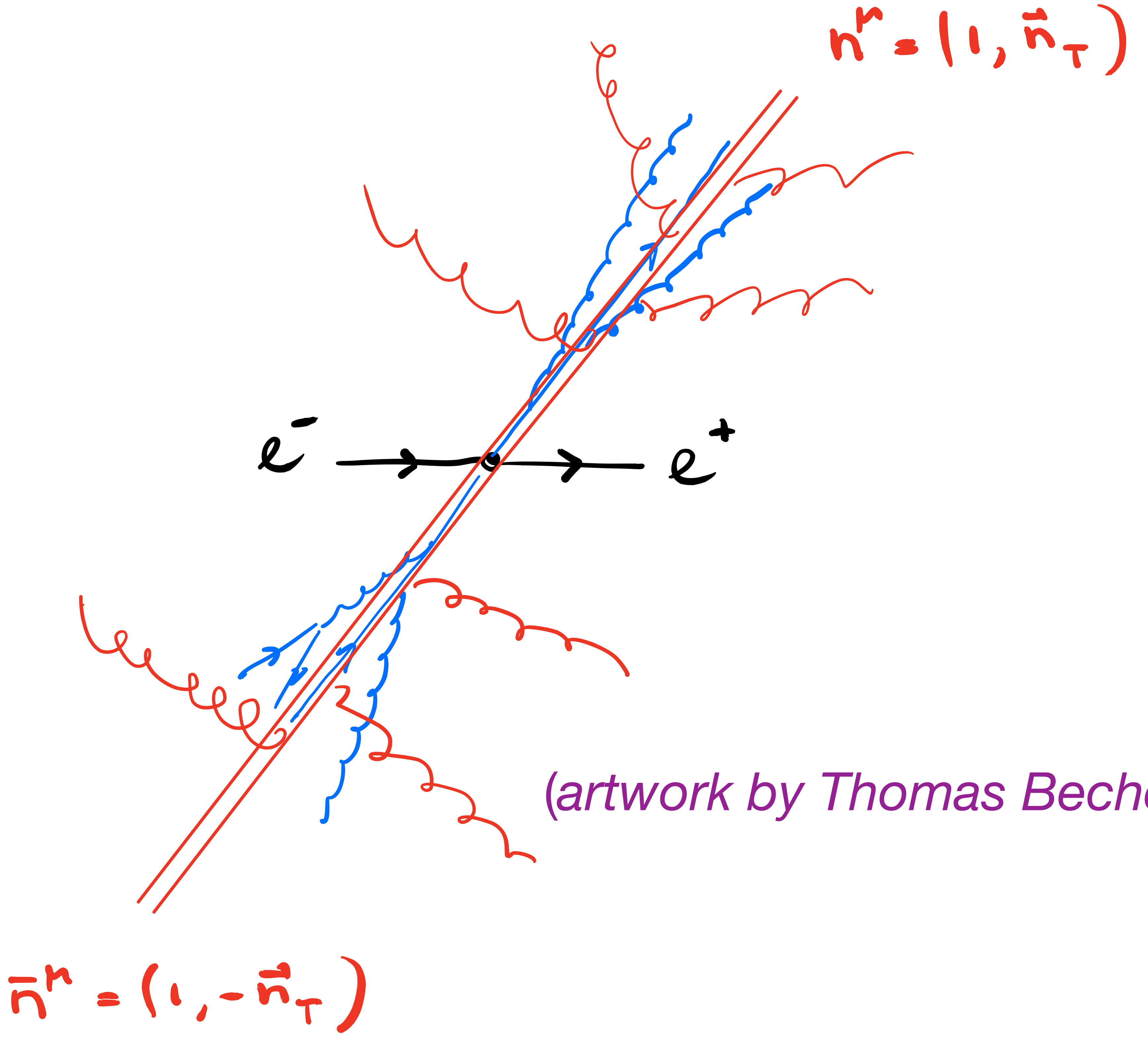


The SLLs are the parametrically leading contributions to exclusive LHC cross sections, yet these effects are currently not understood in higher orders and not included in existing parton showers!

We will argue that the contributions of SLLs can numerically be as large as a one-loop effect, and we will present a complete theory of their all-order contributions.

Soft radiation in global observables

Consider the thrust distribution in e^+e^- collisions near $T\sim1$:



(artwork by Thomas Becher)

$$\frac{d\sigma}{dT} = H\; J \otimes \bar{J} \otimes S$$

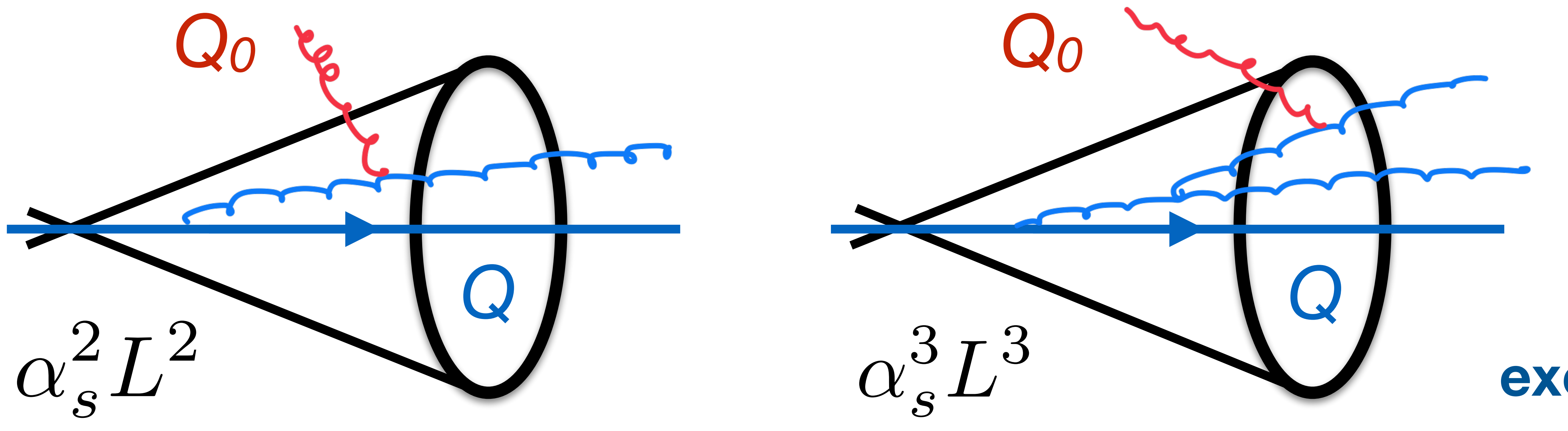
- soft radiation does not resolve individual energetic partons; sensitive only to the direction and total color charge of the jets
- soft function:

$$S \sim \sum_{X_s} \left| \langle X_s | \boldsymbol{S}(n) \boldsymbol{S}(\bar{n}) | 0 \rangle \right|^2$$

- simple structure → N³LL resummation

NGLs in e^+e^- collisions

Non-global observables are insensitive to radiation in certain regions of phase space:



exclusive jet cross sections

- soft radiation from secondary emissions inside the jets leads to a complicated pattern of large logs $\sim (\alpha_s L)^n$ with $L = \ln(Q/Q_0)$, which do not exponentiate [Dasgupta, Salam (2002)]

  non-global logs (NGLs)

- in large-N_c limit, the NGLs can be obtained by solving the non-linear BMS integral equation [Banfi, Marchesini, Smye (2002)]; $1/N_c$ corrections worked out in [Weigert (2003); Hatta, Ueda (2013); Hagiwara (2015)]
- **no generalization to hadron colliders exists!**

Factorization theorem for NGLs

For the "gap between jets" cross section at e^+e^- colliders, we have derived a factorization theorem using soft-collinear effective theory: [Becher, MN, Rothen, Shao (2015, 2016)]

Hard function
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with *m* Wilson lines

$$\sigma(Q,Q_0) = \sum_{m=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) \big\rangle$$

color trace

integration over directions

Factorization theorem for NGLs

Factorization theorem:

$$\sigma(Q,Q_0)=\sum_{m=2}^{\infty}\big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu)\otimes\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu)\big\rangle$$

- separates contributions from scales Q and Q_0
- operator definitions of all ingredients:

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu)=\frac{1}{2Q^2}\sum_{\text{spins}}\prod_{i=1}^{m}\int\frac{dE_i\,E_i^{d-3}}{(2\pi)^{d-2}}\,|\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|\;(2\pi)^d\,\delta\Big(Q-\sum_{i=1}^{m}E_i\Big)\,\delta^{(d-1)}(\vec{p}_{\text{tot}})\,\Theta_{\text{in}}\big(\{\underline{p}\}\big)$$

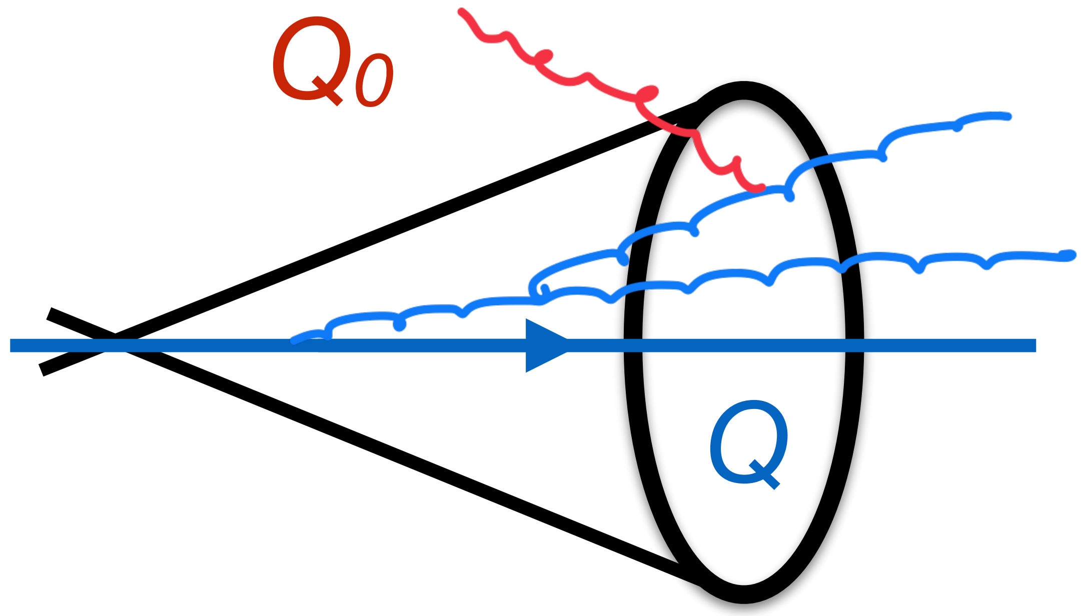
$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu)=\sum\!\!\!\!\!\!\!\int_{X_s}\langle 0|\,\boldsymbol{S}_1^\dagger(n_1)\,\dots\,\boldsymbol{S}_m^\dagger(n_m)\,|X_s\rangle\langle X_s|\,\boldsymbol{S}_1(n_1)\,\dots\,\boldsymbol{S}_m(n_m)\,|0\rangle\,\theta(Q_0-E_{\text{out}})$$

Factorization theorem for NGLs

Factorization theorem:

$$\sigma(Q,Q_0)=\sum_{m=2}^{\infty}\big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu)\otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu)\big\rangle$$

- separates contributions from scales Q and Q_0
- **crucial new ingredient is the sum over parton multiplicities!**
- provides a natural way to perform resummation using RG equations (including NGLs)
- not limited to leading logarithms or leading color
- sum over m accounts for possibility of branchings; hard and soft functions depend on all n_i vectors



Factorization theorem for NGLs

Similar formula holds for hadron colliders:

$$\sigma(Q,Q_0) = \sum_{a_1,a_2=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=4}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,x_1,x_2,\mu)_Q \rangle$$

Differences are:

- hard functions $\boldsymbol{\mathcal{H}}_m$ describe *m*-parton processes $a_1 + a_2 \to a_3 + \cdots + a_m$
- low-energy matrix elements $\boldsymbol{\mathcal{W}}_m$ now contain soft Wilson lines plus collinear fields for the incoming partons; they also contain single-logarithmic Q dependence from the collinear anomaly [Becher, MN (2010); Chiu, Jain, Neill, Rothstein (2012)]
- low-energy theory involves Glauber gluons, which mediate non-trivial interactions between soft and collinear particles [Rothstein, Stewart (2016)]

Factorization theorem for NGLs

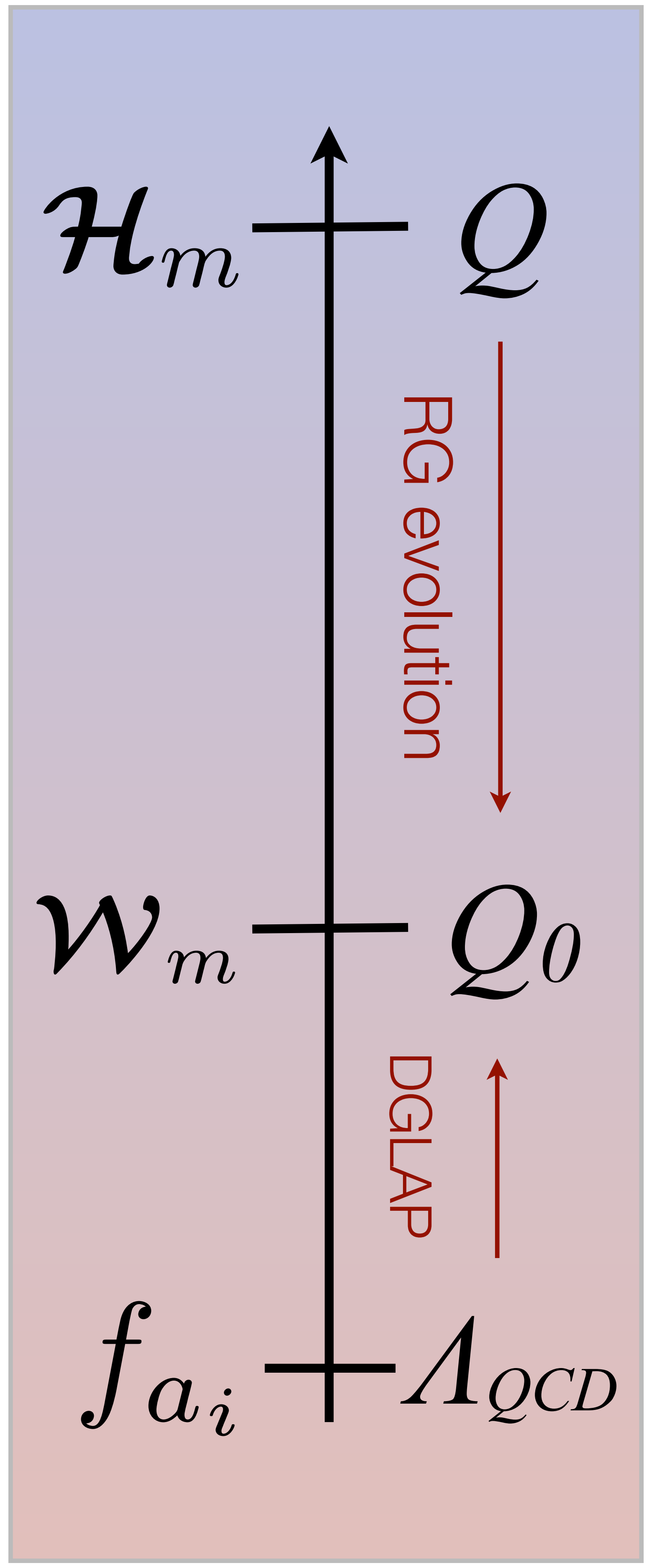
RG evolution of hard functions:

$$\frac{d}{d\ln\mu}\,\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=4}^{m} \boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

- key feature: hard functions with lower parton multiplicities mix into higher-multiplicity functions!

Strategy for resumming NGLs:

- compute hard functions at a scale $\mu_h \sim Q$
- evolve them to a low scale $\mu_s \sim Q_0$ by solving the RG equation
- evaluate low-energy matrix elements at the scale $\mu_s \sim Q_0$



Factorization theorem for NGLs

Formal solution:

$$\sigma(Q,Q_0)=\sum_{a_1,a_2=q,\bar{q},g}\int dx_1 dx_2 \sum_{m=4}^{\infty}\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu)\otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,x_1,x_2,\mu)_Q\rangle$$

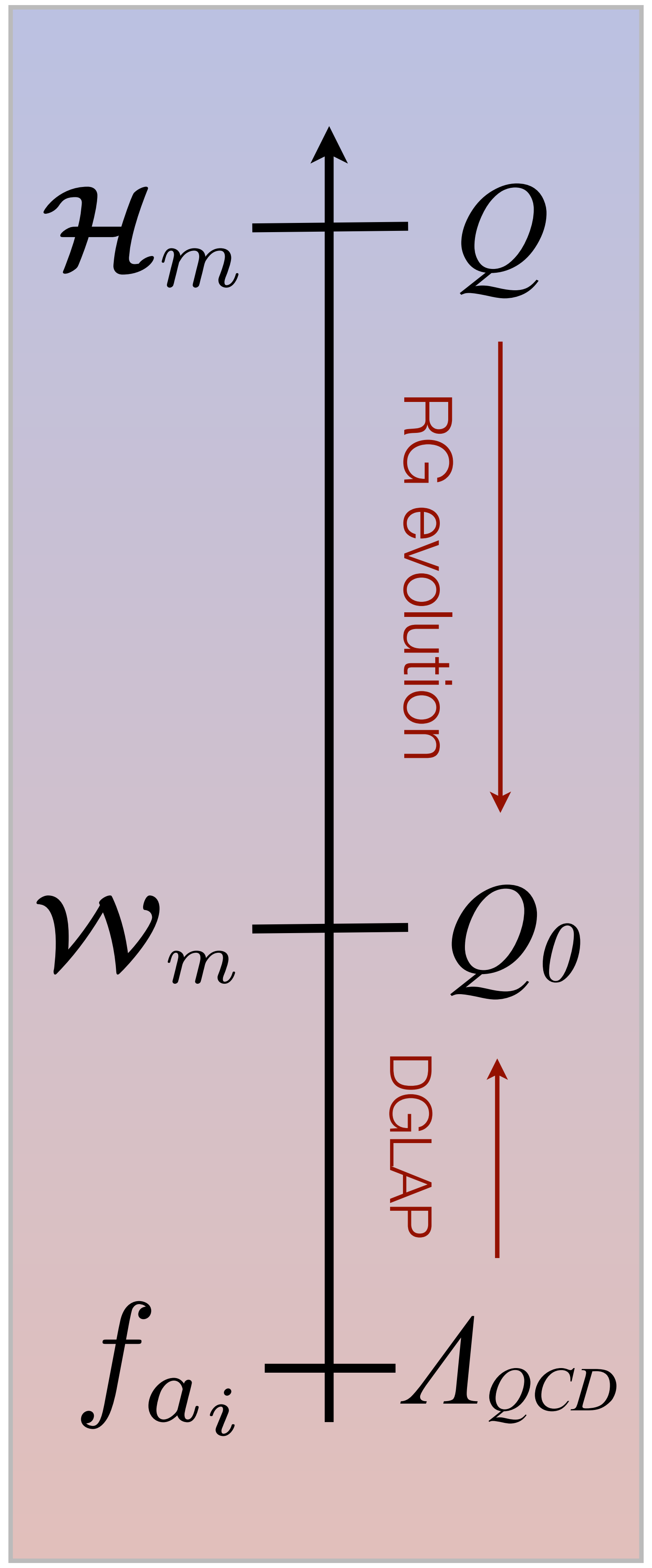
$$\sum_{m=4}^{\infty}\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu_h)\otimes \sum_{l=m}^{\infty}\boldsymbol{U}_{ml}(\{\underline{n}\},\mu_h,\mu_s)\,\hat{\otimes}\,\boldsymbol{\mathcal{W}}_l(\{\underline{n}\},Q_0,x_1,x_2,\mu_s)_Q\rangle$$

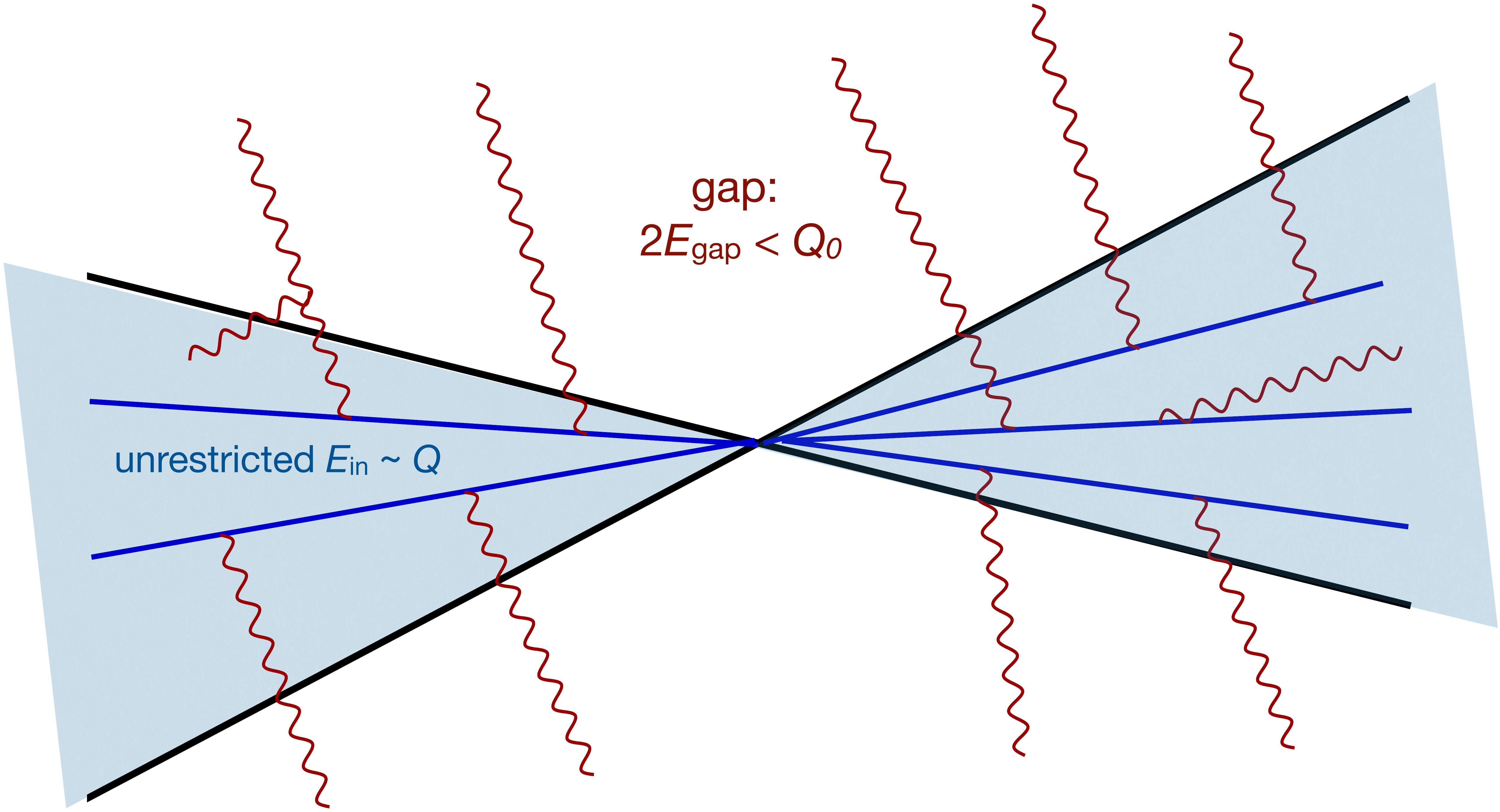
integration over (*l*-*m*) unresolved partons

with:

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h)=\mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\{\underline{n}\},\mu)\right]$$

This object is an operator in color space but also in the (infinite) space of parton multiplicities!





Our factorization formula provides a complete description of non-global observables at hadron colliders.

From the leading terms, we have obtained an all-order understanding of SLLs for arbitrary exclusive 2→*n* LHC cross sections.

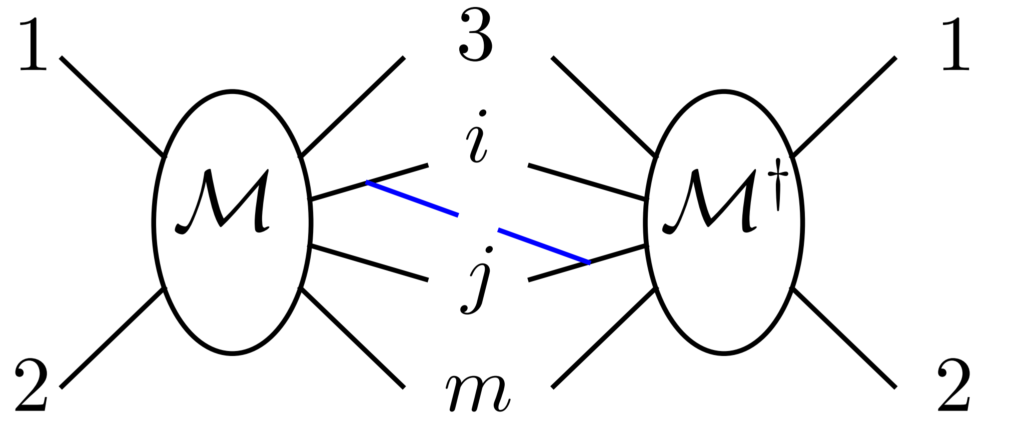
Superleading logarithms

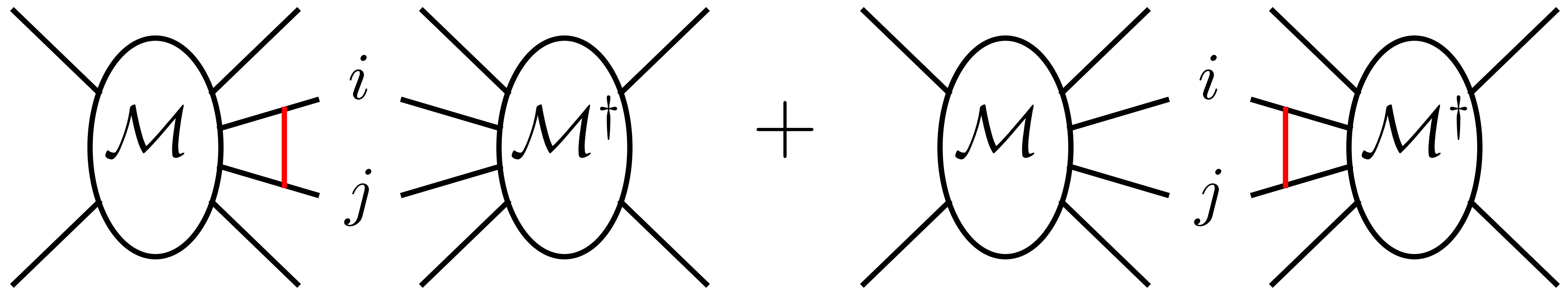
The leading logarithms are obtained using the lowest-multiplicity hard function $\boldsymbol{\mathcal{H}}_4$ (since $\boldsymbol{\mathcal{H}}_m \sim \alpha_s^{m-k}\,\boldsymbol{\mathcal{H}}_k$), the lowest-order result for the matrix elements

$$\boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a_1}(x_1)\, f_{a_2}(x_2)\, \mathbf{1}$$

and the soft anomalous dimension at 1-loop order (with $\boldsymbol{\Gamma}_H = \Gamma_C\, \mathbf{1} + \boldsymbol{\Gamma}_S$):

$$\boldsymbol{\Gamma}_S = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_4 & \boldsymbol{R}_4 & 0 & 0 & \cdots \\ 0 & \boldsymbol{V}_5 & \boldsymbol{R}_5 & 0 & \cdots \\ 0 & 0 & \boldsymbol{V}_6 & \boldsymbol{R}_6 & \cdots \\ 0 & 0 & 0 & \boldsymbol{V}_7 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \dots$$

$\boldsymbol{\mathcal{H}}_m \boldsymbol{R}_m = \sum_{(ij)}$ 

$\boldsymbol{\mathcal{H}}_m \boldsymbol{V}_m = \sum_{(ij)}$ 

Superleading logarithms

We obtain:

$$\sigma_{\rm SLL} = \sigma_{\rm Born}\,\Big\langle \boldsymbol{\mathcal{H}}_4(\{\underline{n}\},Q,\mu_h) \otimes \sum_{l=4}^{\infty} \boldsymbol{U}_{4l}(\{\underline{n}\},\mu_h,\mu_s)\,\hat{\otimes}\,\boldsymbol{1}\Big\rangle$$

where the evolved hard function can be expanded as:

$$\boldsymbol{\mathcal{H}}_4(\mu_h) + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\boldsymbol{\mathcal{H}}_4(\mu_h)\,\boldsymbol{\Gamma}_H(\mu)\; + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\int_{\mu}^{\mu_h}\frac{d\mu'}{\mu'}\,\boldsymbol{\mathcal{H}}_4(\mu_h)\,\boldsymbol{\Gamma}_H(\mu')\,\boldsymbol{\Gamma}_H(\mu) + \dots$$

For quark-initiated processes, we have succeeded to extract the infinite tower of SLLs from this expression!

Gluons are more complicated (work in progress …)

Superleading logarithms

SLLs are a subtle effect arising from Coulomb phases associated with soft gluon exchange between two energetic partons:

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\text{cusp}}(\alpha_s)\, \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

where $s_{ij} = 2p_i \cdot p_j + i0$ if particles *i* and *j* are both in the final state or in the initial state (→ imaginary part), and $s_{ij} = -2p_i \cdot p_j$ otherwise

This generates the imaginary part:

not relevant

$$i\,\text{Im}\,\boldsymbol{\Gamma}_{\text{S}}(\{\underline{p}\},\mu) = +2\pi\, i\, \gamma_{\text{cusp}}(\alpha_s)\, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + (\dots)\, \boldsymbol{1}$$

$$\frac{\alpha_s}{\pi} + \dots$$

non-trivial Coulomb phase only at hadron colliders

Superleading logarithms

The virtual and real contributions in the anomalous dimension contain collinear singularities, which must be regularized and subtracted. One finds:

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^G + \sum_i \boldsymbol{\Gamma}_i^c \ln\frac{\mu^2}{\hat{s}}$$

with:

$$\boldsymbol{V}^G = -8i\pi \left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

Coulomb phase

$$\left.\begin{aligned} \boldsymbol{V}_i^c &= 4C_i\,\boldsymbol{1} \\ \boldsymbol{R}_i^c &= -4\boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{i,R}\,\delta(n_k-n_i) \end{aligned}\right\}$$

Soft+collinear terms

Superleading logarithms

Comments on the notation:

- color generators $\boldsymbol{T}_{i,L}$ act on the amplitude and multiply the hard functions form the left
- color generators $\boldsymbol{T}_{i,R}$ act on the complex conjugate amplitude and multiply the hard functions form the right
- real-emission terms take an amplitude with *m* partons and turn it into an amplitude with (*m*+1) partons; then

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R} = \boldsymbol{T}_i^a \, \boldsymbol{\mathcal{H}}_m \, \boldsymbol{T}_j^{\tilde{a}}$$

where a and $\tilde{a}$ are the color indices of the emitted gluon; the symbol $\circ$ indicates the presence of the addition color space of the emitted gluon

Superleading logarithms

The SLLs arise from the terms with the highest number of insertions of $\mathbf{\Gamma}^c$

Three properties of the different components of the anomalous dimension greatly simplify the calculation of the SLLs

Superleading logarithms

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Three properties of the different components of the anomalous dimension greatly simplify the calculation of the SLLs

- **color coherence** — the sum of soft emissions off two collinear partons has the same effect as a single soft emission off the parent parton:

$$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{\Gamma}^c\, \overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m\, \overline{\boldsymbol{\Gamma}}\, \boldsymbol{\Gamma}^c$$

Superleading logarithms

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Three properties of the different components of the anomalous dimension greatly simplify the calculation of the SLLs

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- **collinear safety** — collinear singularities from real and virtual emissions cancel each other:

$$\langle \boldsymbol{\mathcal{H}}_m \, \mathbf{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$

Superleading logarithms

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$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$

- **cyclicity of the trace**:

$$\big\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \otimes \mathbf{1} \big\rangle = 0$$

Superleading logarithms

The SLLs arise from the terms with the highest number of insertions of $\boldsymbol{\Gamma}^c$

Under the color trace these insertions are non-zero only if they come in conjunction with (at least) two insertions of the Coulomb phase $\boldsymbol{V}^G$ and one insertion of $\overline{\boldsymbol{\Gamma}}$. The relevant color traces are:

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_4 \, (\boldsymbol{\Gamma}^c)^r \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{n-r} \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \boldsymbol{1} \right\rangle$$

For initial-state particles transforming in the fundamental representation of SU(N_c) — quarks or antiquarks — one can use

$$\{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\} = \frac{1}{N_c}\,\delta_{ab}\boldsymbol{1} + \sigma_i\, d_{abc}\,\boldsymbol{T}_i^c\,; \quad i = 1, 2$$

$\sigma = 1$ for q, $\sigma = -1$ for $\bar{q}$

to simplify the color algebra

Superleading logarithms

General result involving only three color traces:

$$C_{rn} = 2^{8-r}\pi^2\,(4N_c)^n \Big\{ \sum_{j=3,4} J_j\,\big\langle \boldsymbol{\mathcal{H}}_4 \big[(\boldsymbol{T}_2 - \boldsymbol{T}_1)\cdot\boldsymbol{T}_j + 2^{r-1} N_c\,(\sigma_1-\sigma_2)\,d_{abc}\,\boldsymbol{T}_1^a\,\boldsymbol{T}_2^b\,\boldsymbol{T}_j^c \big] \big\rangle + 2\,(1-\delta_{r0})\,J_2\,\big\langle \boldsymbol{\mathcal{H}}_4\,\big[C_F + (2^r-1)\,\boldsymbol{T}_1\cdot\boldsymbol{T}_2 \big] \big\rangle \Big\}$$

All angular information is contained in:

$$J_j = \int \frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\rm veto}(n_k)\,;\quad \text{with}\quad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\,n_j\cdot n_k}$$

From this formula we have rederived all that is known about SLLs!
But it contains infinitely much more information…

Superleading logarithms

The contribution of SLLs to a cross section is given by: $L = \ln(Q/Q_0)$

$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \, \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

SLLs start at 4-loop order, but *n*=0 term is of the same origin

Superleading logarithms

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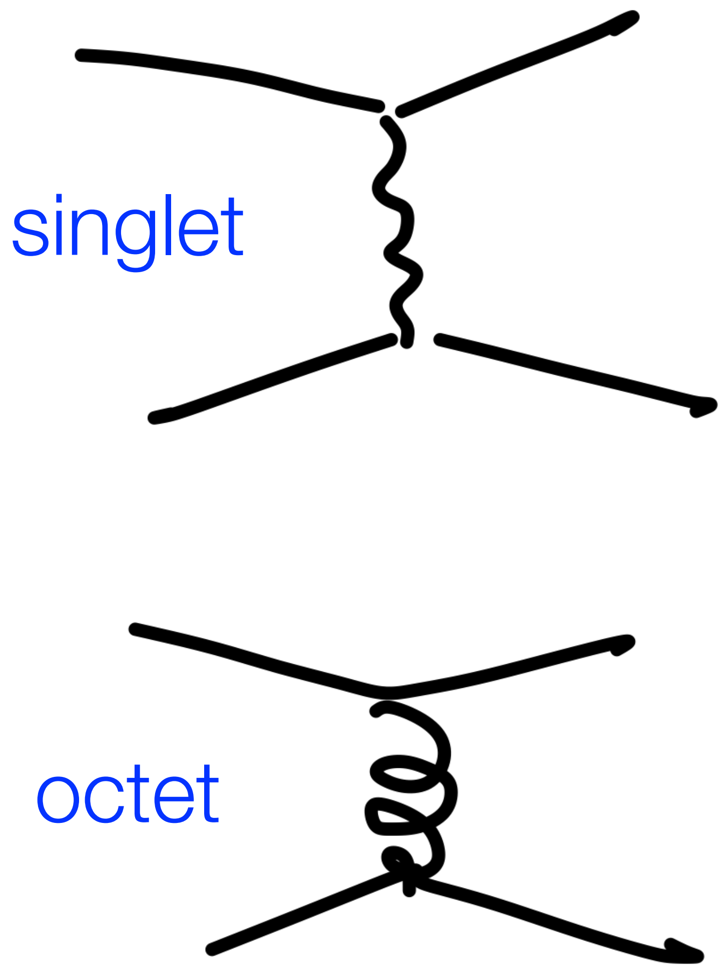
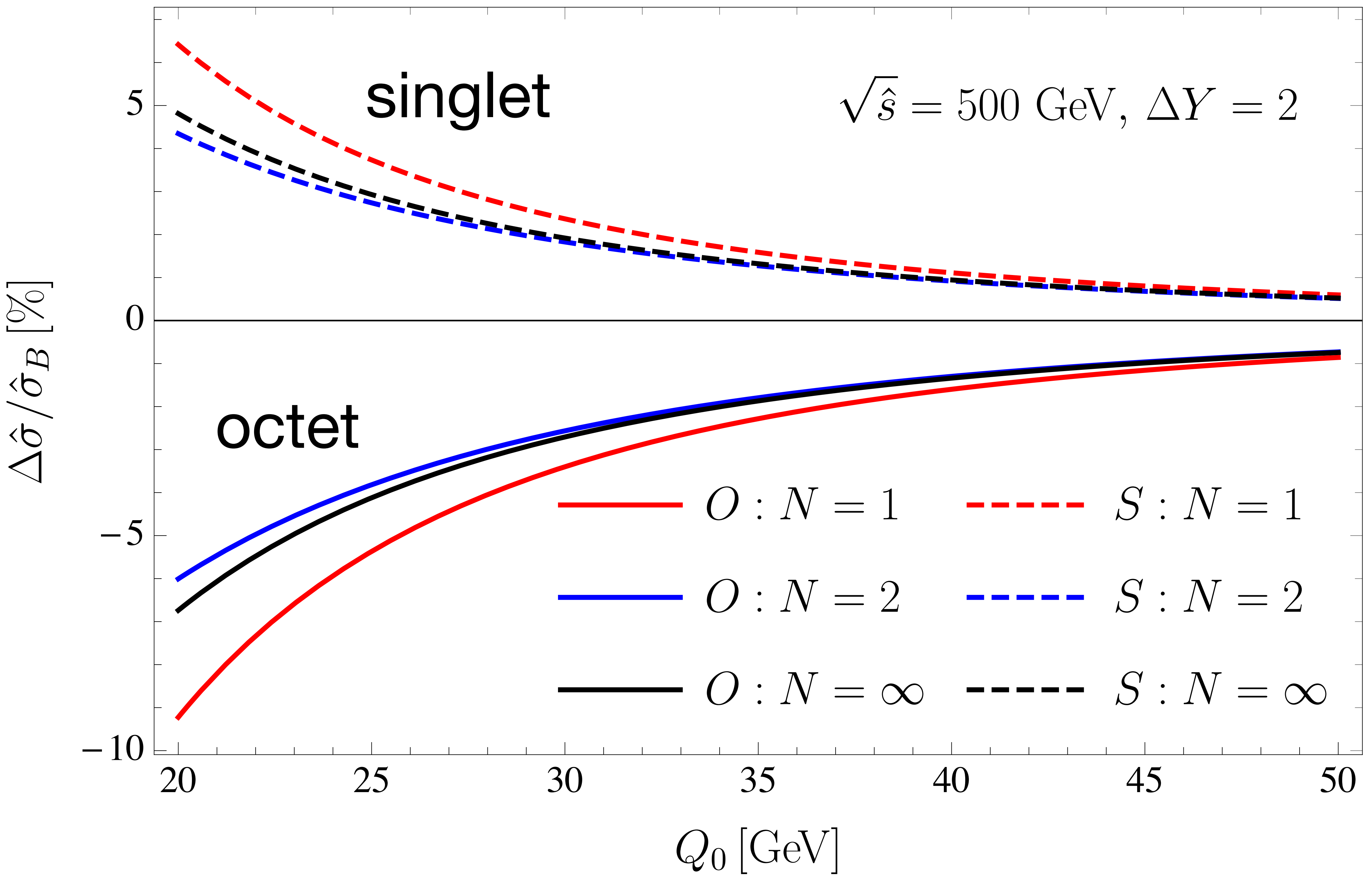
Example: $qq \to qq$ scattering





$$\sum_{n=1}^{N} \cdots$$

SLLs only

Superleading logarithms

The contribution of SLLs to a cross section is given by: $L = \ln(Q/Q_0)$

$$\sigma_{\rm SLL} = \sigma_{\rm Born} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^{n+3} L^{2n+3} \frac{(-4)^n\, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r\,(r!)^2}\, C_{rn}$$

Simple, closed-form expression for the color-singlet channel:

$$\sigma_{\rm SLL}^{(S)} = -\sigma_{\rm Born}\, \frac{2\alpha_s L}{3\pi N_c}\, \Delta Y \left(1 - \frac{1}{N_c^2}\right) w_\pi\, w\; {}_2F_2\big(1,1;2,\tfrac{5}{2};-w\big)$$

$$w_\pi = \frac{N_c\,\alpha_s}{\pi}\, \pi^2 \qquad w = \frac{N_c\,\alpha_s}{\pi}\, L^2$$

For $Q/Q_0 \to \infty$ we find:

$$w\; {}_2F_2\Big(1,1;2,\frac{5}{2};-w\Big) \to \frac{3}{2}\,\big[\ln(4w) + \gamma_E - 2\big]$$

not suppressed at all, unlike Sudakov double logs!

Superleading logarithms

The contribution of SLLs to a cross section is given by: $L = \ln(Q/Q_0)$

$$\sigma_{\text{SLL}} = \sigma_{\text{Born}} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

Simple, closed-form expression for the color-singlet channel:

$$\sigma_{\text{SLL}}^{(S)} = -\sigma_{\text{Born}} \underbrace{\frac{2\alpha_s L}{3\pi N_c}}_{\text{subleading color}} \Delta Y \left(1 - \frac{1}{N_c^2} \right) w_\pi \, w \; {}_2F_2\!\left(1, 1; 2, \tfrac{5}{2}; -w\right)$$

$$w_\pi = \frac{N_c \alpha_s}{\pi} \, \pi^2 \qquad w = \frac{N_c \alpha_s}{\pi} \, L^2$$

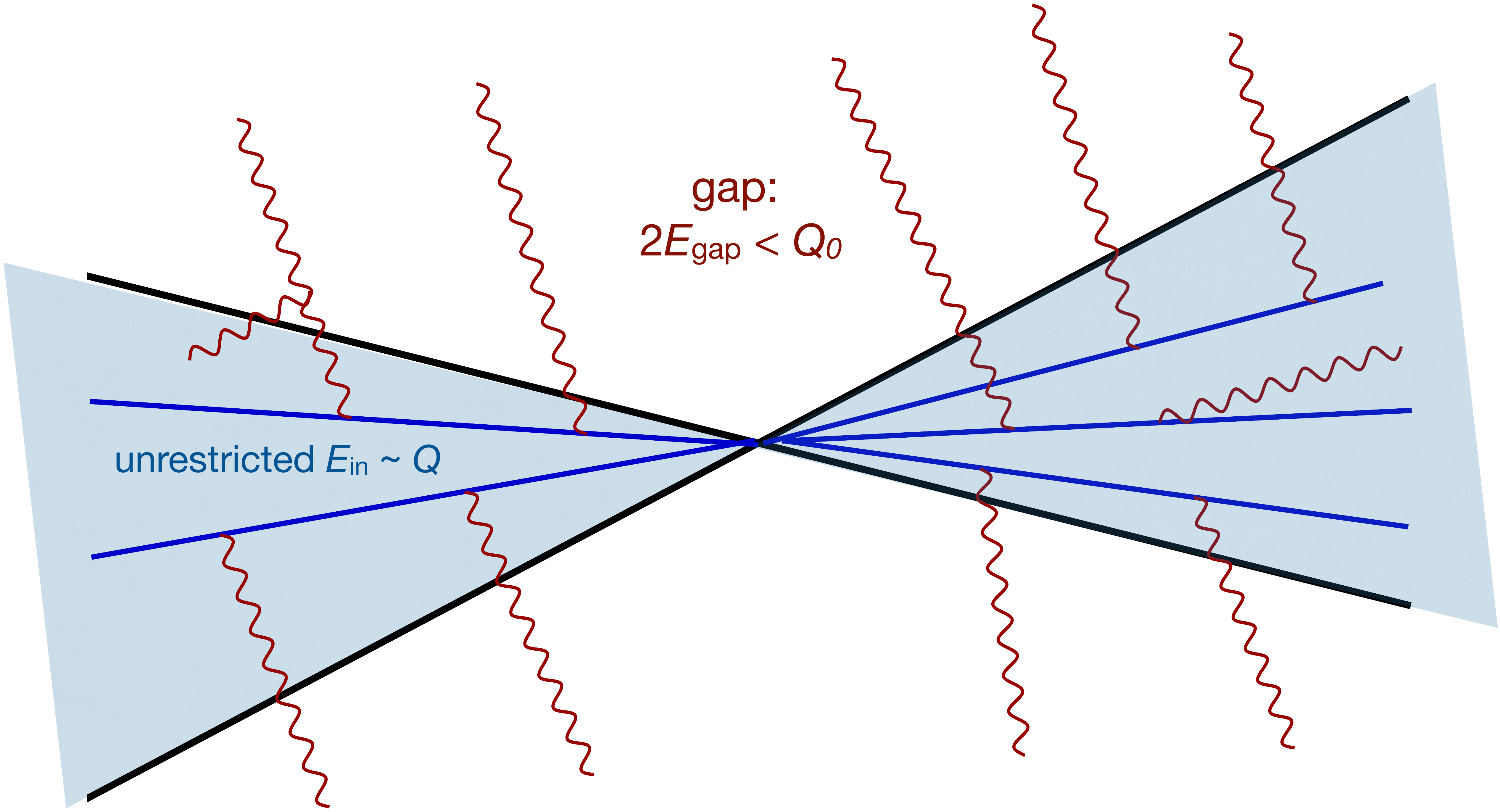
Even though it starts at 3-loop order, this corrections is numerically of the order of a typical one-loop correction → important for phenomenology!

Other $2 \to n$ processes

- While we have discussed $2 \to 2$ processes, our results holds for arbitrary $2 \to n$ hard-scattering processes, including the cases where $n = 0{,}1$ – a fact that was not appreciated before
- SLLs also arise in $qq \to Z + j$ as well as in $qq \to Z$ or $gg \to h$ with a central jet veto (starting at 5 loops, but with the same scaling as above)!
- They can thus play an important role for Higgs phenomenology at the LHC!

Conclusions and outlook

- First factorization theorem for non-global observables at hadron colliders
- First all-order resummation for SLLs, extending existing results by infinitely many orders and to arbitrary processes (quark-initiated, for now …)
- Derived asymptotic behavior of SLLs, finding no Sudakov-like behavior
- Important to go beyond the leading SLLs and analyze the structure of the low-energy matrix elements $\mathcal{W}_m$ in detail
- SCET-based approach offers a path toward a complete theory of non-global observables, including all logarithmically enhanced contributions
- Accurate calculation of these effects is of utmost importance!



Thank you!