Non-local contribution to exclusive $b \to s\ell\ell$ decays

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based on work in collaboration with

N. Gubernari & J. Virto [arXiv:2011.09813 (accepted for publication in JHEP)]

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Technische Universität München

Prelude

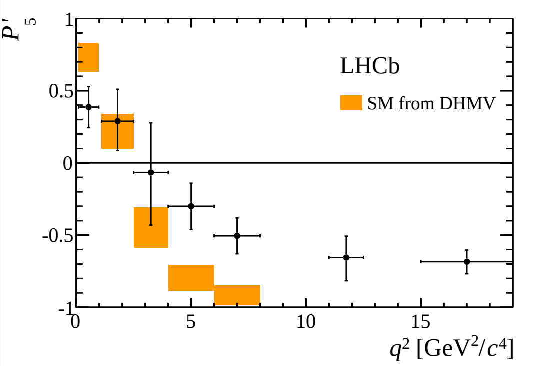
A beginning is the time for taking the most delicate care that the balances are correct.

[Herbert 1965]

- flavour physics has seen a lot of attention in the last five years with the emergence of what usually are called the flavour anomalies
- the anomalies are a set of tensions between Standard Model (SM) predictions and experimental measurements of various observables in B-meson decays
- to understand the impact of the anomalies, particularly the deviations in exclusive $b \to s\mu^+\mu^-$ decays, one requires are good understanding of the SM predictions from which the measurements are apparently deviating

The balances of this talk are tipped toward the SM, its assumptions and predictions.

My intention is to enable those members of the audience that are unfamiliar with flavour physics to have an understanding of how these measurements …



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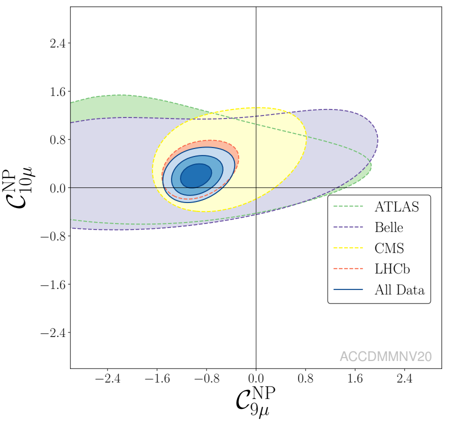
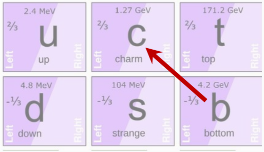


...can be accurately interpreted, validating assumption on present pheno analyses.



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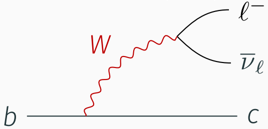
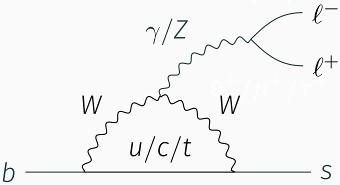
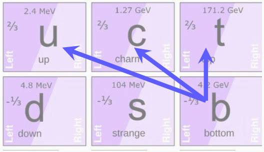


tree level contributions



only arises at loop level

Semileptonic b Decays



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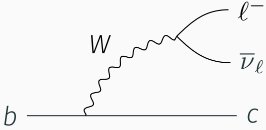

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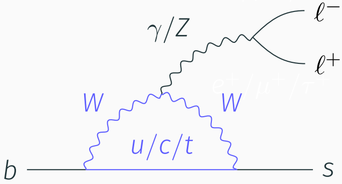

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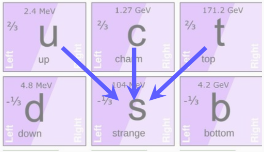

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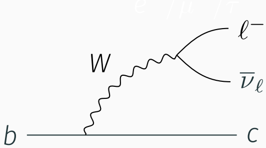

tree level contributions

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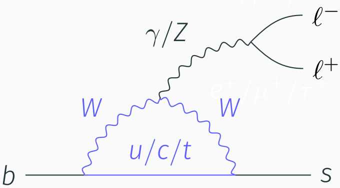

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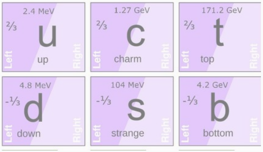

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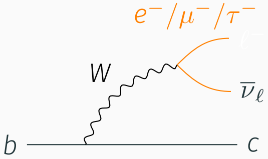
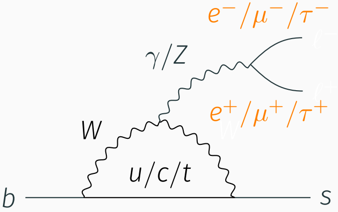


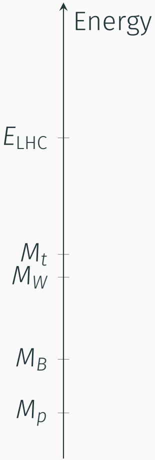
tree level contributions

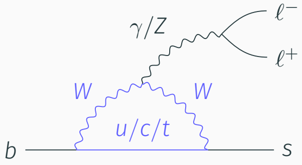


only arises at loop level

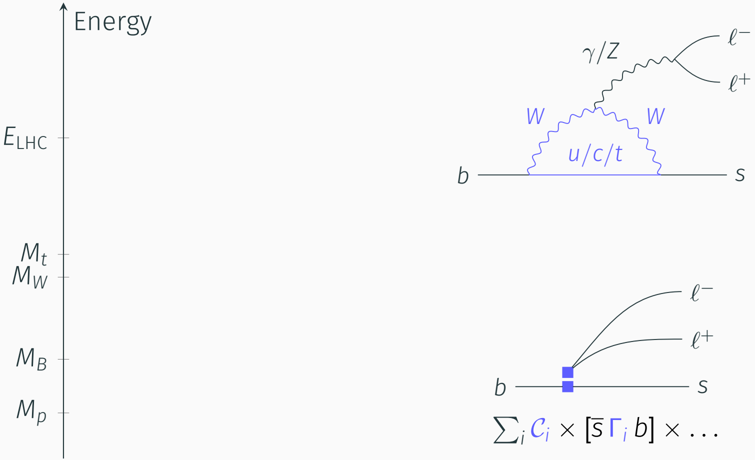
in both cases: **lepton-flavour-universal gauge couplings**!

- widely used tool of theoretical physics





- widely used tool of theoretical physics
- replaces dynamical degrees of freedom (here: t, W, Z) by coefficients $\mathcal{C}_i$ and static structures in local operators (here: Γ_i)



$\sum_i \mathcal{C}_i \times [\bar{s}\, \Gamma_i\, b] \times \ldots$

Weak Effective Theory: $b \to s\ell\ell$ SM Operators

in the SM the we find the following set of $D = 6$ effective operators

$$\mathcal{L}_{\text{SM}}^{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_i \mathcal{C}_i\, \mathcal{O}_i + \lambda_c \sum_i \mathcal{C}_i{}^c\, \mathcal{O}_i{}^c + \lambda_u \sum_i \mathcal{C}_i{}^u\, \mathcal{O}_i{}^u\right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b(\bar{s}\sigma^{\mu\nu} P_R b)F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b(\bar{s}\sigma^{\mu\nu} P_R T^A b)G^A_{\mu\nu}$$

$$\mathcal{O}_9 = \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10} = \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_1{}^q = (\bar{q}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L q) \qquad \mathcal{O}_2{}^q = (\bar{q}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a q)$$

$$\mathcal{O}_i = (\bar{s}\gamma_\mu P_X b)\sum_q (\bar{q}\gamma^\mu q)$$

with $\lambda_q \equiv V_{qb}V^*_{qs}$

- very complicated structure compared to the tree-level decays

SM contributions to $\mathcal{C}_i(\mu_b)$ known to NNLL [Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06] in the presence of NP effects, complete basis of semileptonic operators by adding

$$\mathcal{L}_{\rm BSM}^{\rm eff} = \mathcal{L}_{\rm SM}^{\rm eff} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_i \mathcal{C}_i\, \mathcal{O}_i\right]$$

with i running over $9'$, $10'$, S, S', P, P', T, $T5$:

$$\begin{aligned}
\mathcal{O}_{9'} &= \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell) & \mathcal{O}_{10'} &= \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)\\
\mathcal{O}_{S} &= \frac{\alpha}{4\pi}(\bar{s} P_R b)(\bar{\ell}\ell) & \mathcal{O}_{S'} &= \frac{\alpha}{4\pi}(\bar{s} P_L b)(\bar{\ell}\ell)\\
\mathcal{O}_{P} &= \frac{\alpha}{4\pi}(\bar{s} P_R b)(\bar{\ell}\gamma_5\ell) & \mathcal{O}_{P'} &= \frac{\alpha}{4\pi}(\bar{s} P_L b)(\bar{\ell}\gamma_5\ell)\\
\mathcal{O}_{T} &= \frac{\alpha}{4\pi}(\bar{s}\sigma^{\mu\nu} b)(\bar{\ell}\sigma_{\mu\nu}\ell) & \mathcal{O}_{T5} &= \frac{\alpha}{4\pi}(\bar{s}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu}\gamma_5\ell) \qquad (1)
\end{aligned}$$

- $\mathcal{C}_i = 0$ in the SM for all of these operator!

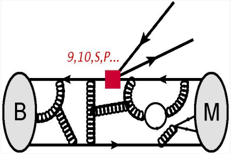
- WET makes calculation in the SM possible in the first place
  - separates long-distance from short-distance physics
  - resums potentially large logarithms

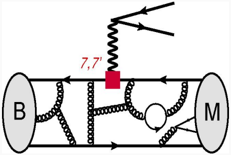
- “divide and conquer”

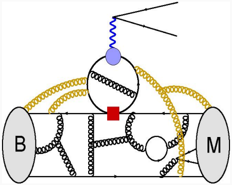
- transparently allows to account model-independently for the effects of physics beyond the SM
  - interface to model builders …
  - …although transitioning to SM Effective Field Theory, which can help to related constraints amongst the various Weak Effective Theories (*i.e.*, relate constraints in $b \to c\tau\nu$ with constraints in $b \to s\ell^+\ell^-$)

(Re)capitulation

Anatomy of Exclusive $b \to s\ell^+\ell^-$ Decay Amplitudes







$$\mathcal{A}_\lambda^X = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

nomenclature

- $\mathcal{F}_\lambda$ local form factors of dimension-three $\bar{s}\gamma^\mu b$ & $\bar{s}\gamma^\mu\gamma_5 b$ currents
- $\mathcal{F}_\lambda^T$ local dipole form factors of dimension-three $\bar{s}\sigma^{\mu\nu} b$ currents
- $\mathcal{H}_\lambda$ nonlocal form factors of dimension-five nonlocal operators

$$\int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\text{em}}(x), O_i(0) \}$$

all three needed for consistent description to leading-order in α_e

	$B \to K$	$B \to K^*$	$B_s \to \phi$	$\Lambda_b \to \Lambda$
# of FFs	3	7	7	10
$q^2 \lesssim 10\,\text{GeV}^2$ $q^2 \gtrsim 15\,\text{GeV}^2$	LCSR ($\times 1$) LQCD ($\times 2$)	LCSR ($\times 2$, *) LQCD ($\times 1$, *)	LCSR ($\times 2$) LQCD ($\times 1$)	LQCD (†) LQCD ($\times 1$)

LQCD Lattice QCD simulations, systematically improvable

LCSR Light-Cone Sum Rules calculations, with hard-to-quantify systematic uncertainties, with either

(*) assuming that the $K^*(892)$, which is a $K\pi$ resonance, can be replaced with a stable bound state

(†) large uncertainties due to extrapolation

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$q^2 \gtrsim 15\,\mathrm{GeV}^2$	LQCD ($\times 2$)	LQCD ($\times 1$, *)	LQCD ($\times 1$)	LQCD ($\times 1$)

- different excl. decay modes provide complementary systematic effects
- experimental data also provides information on the local form factors

$\Rightarrow$ global analyses: nontrivial crosschecks of the computation methods

! small q^2, which drives anomalies, dominated by LCSRs, which are least reliable method

✓ no conceptual problem for LQCD to reach small q^2

$\Rightarrow$ good prospects for improvement

$$\mathcal{H} \sim \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\, J^\mu_{\rm em}(x), O_i(0)\, \}\, |H_b\rangle$$

- focus on nonlocal form factors from $\bar{s}b\bar{c}c$ operators O_1^c and O_2^c
- numerically dominant effect: “charm loop effect”

	$B \to K$	$B \to K^*$	$B_s \to \phi$	$\Lambda_b \to \Lambda$
# of FFs	1	3	3	4
$q^2 \lesssim 1\,\mathrm{GeV}^2$ $q^2 \gtrsim 15\,\mathrm{GeV}^2$	LCOPE OPE	LCOPE OPE	LCOPE OPE	LCOPE (*) OPE

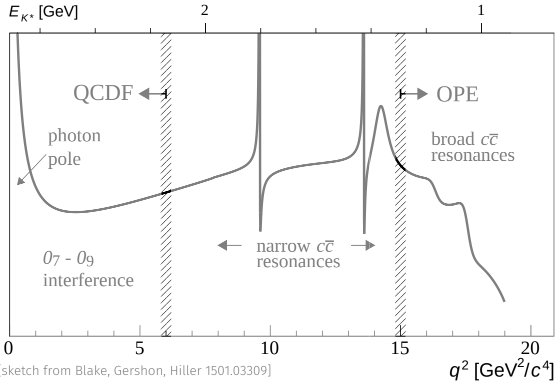
OPE reduction to local operators $x^\mu = 0$

LCOPE reduction to operators on the light-cone $x^2 \simeq 0$

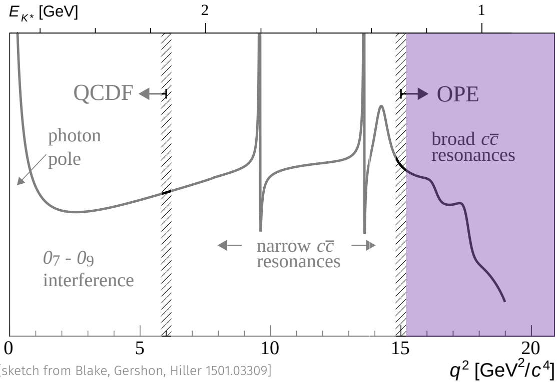
(*) next-to-leading power matrix elements cannot presently be computed

both cases: matrix elements of the leading operators are the local form factors

Spectrum



[sketch from Blake, Gershon, Hiller 1501.03309]

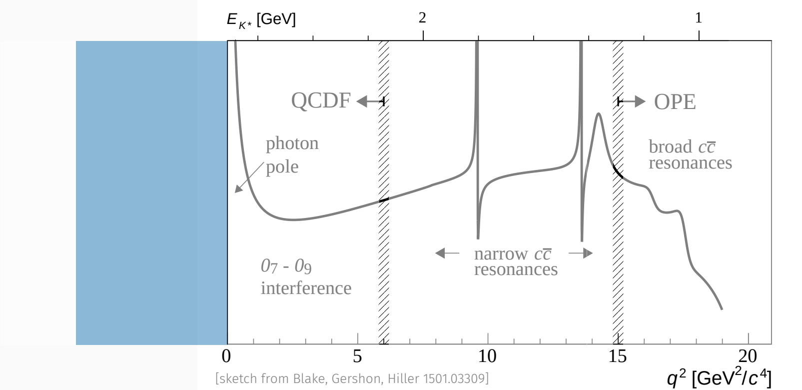


[sketch from Blake, Gershon, Hiller 1501.03309]

- if $|q^2| = \mathcal{O}\left(m_b^2\right)$, expand T-product in local operators
- leading operators have mass dimenion three, with universal matching coefficient $c_3(q^2)$

$\Rightarrow \mathcal{H}_\lambda = c_3(q^2) F_\lambda(q^2) + \ldots$

! usually applied in integrated form to $q^2 \leq 4M_D^2$



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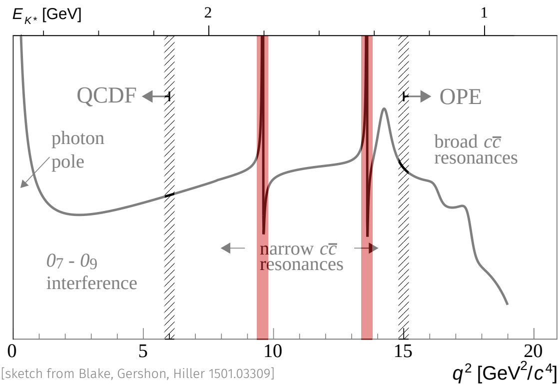
- if $q^2 - 4m_c^2 \ll \Lambda_{\text{had}} m_b$, expand T-product in light-cone operators
- leading operators have mass dimenion three, with universal matching coefficient $c_3(q^2)$

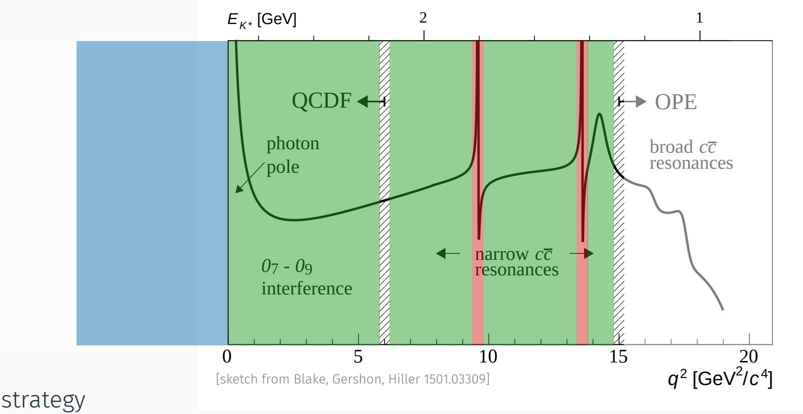
$\Rightarrow \mathcal{H}_\lambda = c_3(q^2) F_\lambda(q^2) + \dots$

Spectrum



[sketch from Blake, Gershon, Hiller 1501.03309]

- for $q^2 = M^2_{J/\psi}$ and $q^2 = M^2_{\psi(2S)}$, spectrum dominated by hadronic decays
- residues of nonlocal form factors model-independently relate to hadronic decay amplitudes



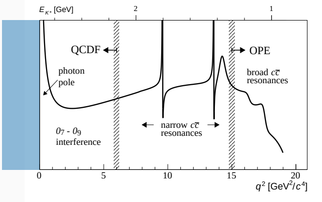
[sketch from Blake, Gershon, Hiller 1501.03309]

strategy

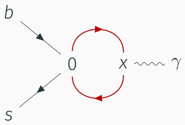
- compute $\mathcal{H}$ at spacelike q^2
- extrapolate to timelike $q^2 \leq 4M_D^2$
- include information from hadronic decays $\Lambda_b \to \Lambda\psi_n$

Development

$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$

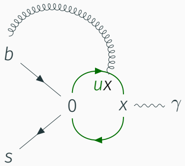


- ▶ expansion in operators at light-like distances $x^2 \simeq 0$ [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- ▶ employing light-cone expansion of charm propagator [Balitsky, Braun 1989]



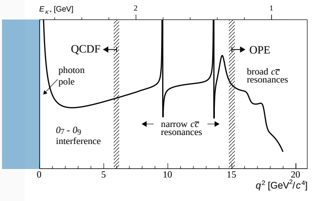
$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$+ (\text{coeff \#2}) \times [\,\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,]$$



$$0 \le u \le 1$$

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$



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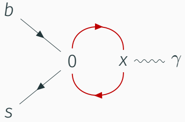
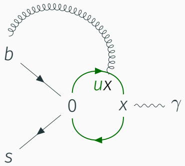


$$\Rightarrow \mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}}$$
$$+ \text{coeff \#2} \times \tilde{\mathcal{V}}$$

▶ leading part identical to QCD Fact. results [Beneke, Feldmann, Seidel 2001&2004]



▶ subleading matrix element $\tilde{\mathcal{V}}$ can be inferred from B-LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]

▶ recalculating this step we obtain full agreement! Also cast result in more convenient form

$$0 \le u \le 1$$

at subleading power in the OPE, need matrix elements of a non-local operator

$$\tilde{\mathcal{V}} \sim \langle M | \bar{s}(0) \gamma^\rho P_L G^{\alpha\beta}(-u n^\mu) b(0) | B \rangle$$

for $B \to K^{(*)}$ and $B_s \to \phi$ transitions

- matrix element has been calculated in light-cone sum rules

  [Khodjamirian et al, 1006.4945]
- physical picture provides that the soft gluon field originates from the B meson
  - analytical results independent of two-particle $b\bar{q}$ Fock state inside the B
  - expressions start with three-particle $b\bar{q}G$ Fock state, and their light-cone distribution amplitudes (LCDAs)

$$\langle 0 | \bar{q}(x) G^{\mu\nu}(ux) \Gamma h_v^b(0) | \bar{B}(v M_B) \rangle$$

  - original results missing out on four out of eight three-particle LCDAs

- we recalculate the soft-gluon contributions to the full set of $B \to V$ and $B \to P$ non-local form factors using light-cone sum rules
  - analytic results for restricted set of LCDAs in full agreement with KMPW2010 [Khodjamirian, Mannel, Pivovarov, Wang 2010]
  - result of restricted set fails to reproduce duality thresholds obtained from local form actor sum rules [Gubernari, Kokulu, DvD '18]
  - using the full set of LCDAs, our results reproduce the duality thresholds!
  - our numerical results differ significantly from KMPW2010, but are well understood!

Soft gluon matrix elements

Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\text{GeV}^2)$	GvDV2020	KMPW2010
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\text{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\text{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\text{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\text{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\text{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\text{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\text{GeV}$	—

reduction by a factor of ~ 200

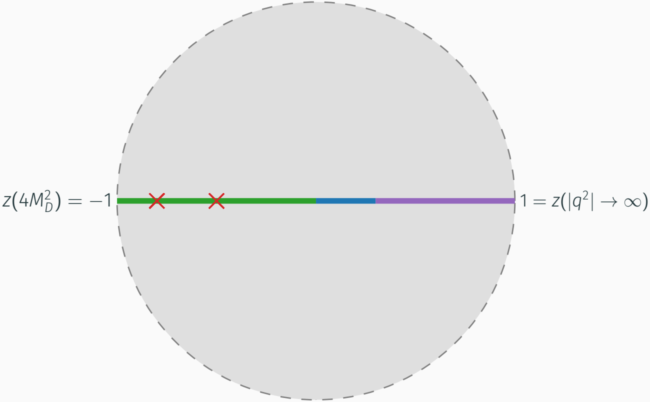
- new structures in three-particle LCDAs account for factor 10
- updated inputs that enter the sum rules (mostly) linearly account for further factor 10
- similar relative uncertainties, but absolute uncertainties reduced by $\mathcal{O}(100)$

- Taylor expand $\mathcal{H}_\lambda$ in q^2/M_B^2 around 0 [Ciuichini et al. '15]
  - \+ simple to use in a fit
  - \- incomaptible with analyticity properties, does not reproduce resonances
  - \- expansion coefficients unbounded!
- use information from hadronic intermediate states in a dispersion relation [Khodjamirian et al. '10]

$$\mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(q^2) = \int ds \frac{\text{Im}\,\mathcal{H}_\lambda(s)}{(s-s_0)(s-q^2)} + \dots$$

  - \+ reproduces resonances
  - \- hadronic information above the threshold must be modelled
  - \- complicated to use in a fit, relies on theory input in single point s_0
- expand the matrix elements in variable $z(q^2)$ that develops branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]
  - \+ resonances can be included through explicit poles (Blaschke factors)
  - \+ easy to use in a fit
  - \+ compatible with analyticitiy properties
  - \- expansion coefficients unbounded!

Tangent: z expansion



light-cone OPE

SL phase space

$J/\psi, \psi(2S)$

local OPE

matrix elements $\mathcal{H}$ arise from non-local operator

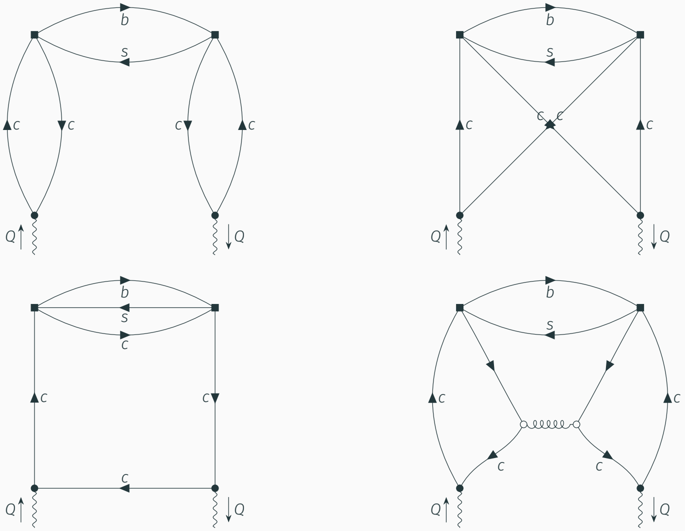
$$O^\mu(Q;x) \sim \int e^{iQ\cdot y}\, T\{J^\mu_{\text{em}}(x+y), [C_1 O_1 + C_2 O_2](x)\}$$

construct four-point operator to derive a dispersive bound

- define matrix element of “square“ operator

$$\left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right] \Pi(Q^2) \equiv \int e^{iQ\cdot x} \langle 0|\, T\{O^\mu(Q;x) O^{\dagger,\nu}(Q;0)\}\, |0\rangle$$

- as hermiatian operator, vacuum eigenvalues are positive definite!
- for $Q^2 < 0$ we find that $\Pi(Q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)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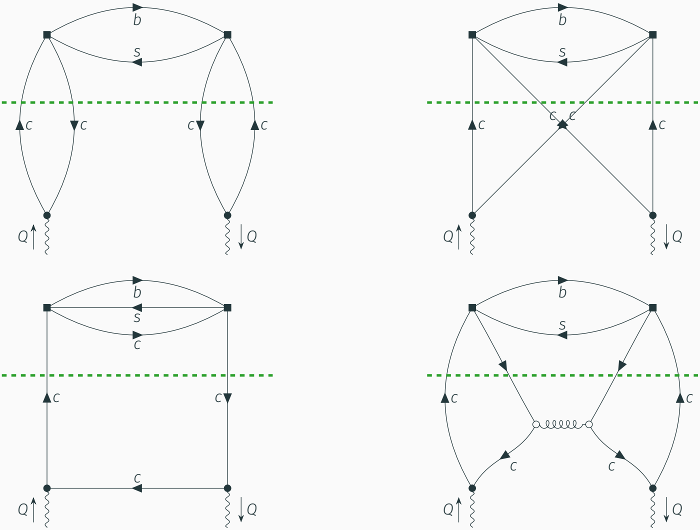
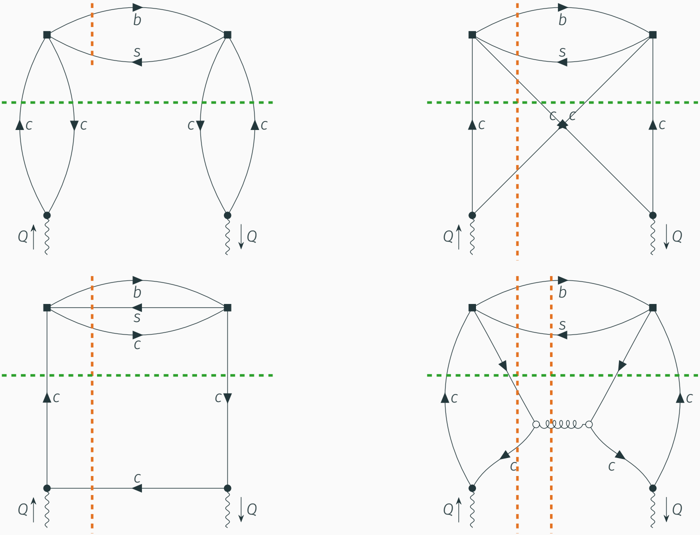


- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)



- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
- from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

dispersive representation of the $b\bar{s}$ contribution to derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi} \int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\,\Pi(s)}{s - Q^2}$$

positive definite for $Q^2 < 0$

- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be computed in the local OPE
  $\rightarrow$ yields $\chi^{\mathrm{OPE}}(Q^2)$
- OPE result indicates that two derivatives are needed for convergence of dispersive integral
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be expressed in terms of the matrix elements $\mathcal{H}_\lambda$
  $\rightarrow$ yields $\chi^{\mathrm{had}}(Q^2)$
- global quark hadron duality suggests that both quantities are equal
  $\rightarrow$ yields a dispersive bound

the hadronic represenation reads schematically:

$$\chi^{\mathrm{OPE}}(Q^2) \geq \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \int_{(m_b+m_s)^2}^{\infty} ds \sum_\lambda \frac{\omega_\lambda(s)\,|\mathcal{H}_\lambda(s)|^2}{s-Q^2}$$

- aim: diagonalize this expression

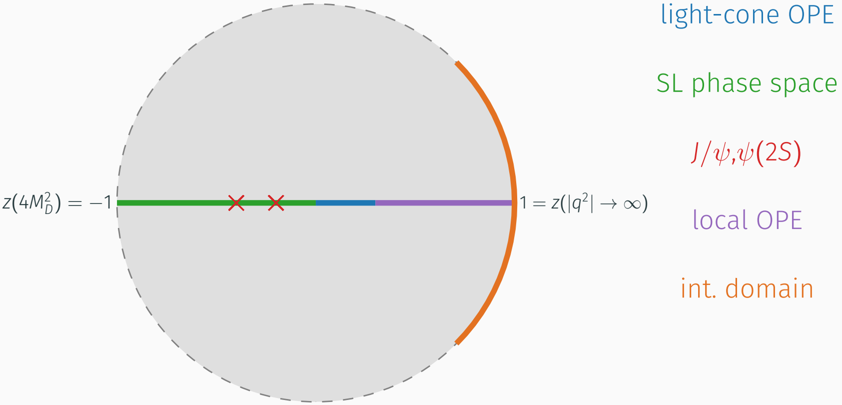
Ansatz:

$$\hat{\mathcal{H}}_\lambda(q^2) \equiv P(q^2) \times \phi_\lambda(q^2) \times \mathcal{H}_\lambda(q^2) \equiv \sum_n a_{\lambda,n} f_n(q^2)$$

- Blaschke factor $P(q^2)$ remove narrow charmonia poles
- outer functions ϕ_λ account for weight function ω_λ and Cauchy integration kernel
- orthonormal functions $f_n(q^2)$ diagonalizes remainder of the expression

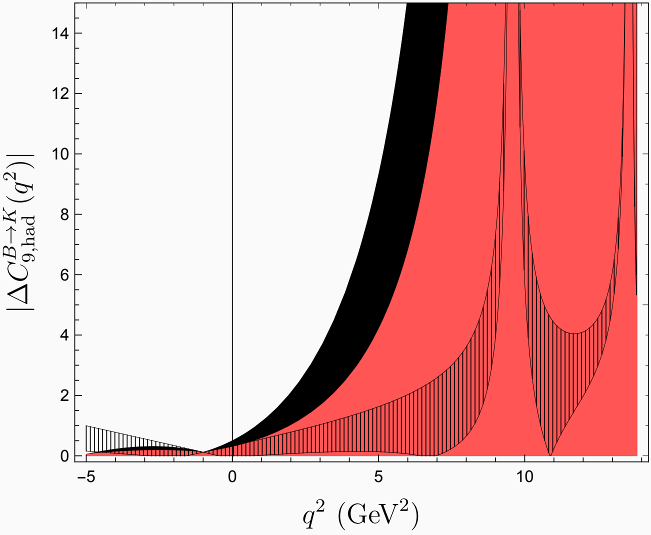
normalisation to χ^{OPE} leads to a diagonal bound

$$1 \geq \sum_\lambda \sum_n |a_{\lambda,n}|^2$$



$z(4M_D^2) = -1$
$1 = z(|q^2| \to \infty)$
light-cone OPE
SL phase space
$J/\psi, \psi(2S)$
local OPE
int. domain

simple exercise: bound on the shift to C_9 from nonlocal form factors, assuming only two data points at negative q^2



$$\frac{1}{2} > \sum_n |a_n|^2$$

$$\frac{1}{11} > \sum_n |a_n|^2$$

Truncation error

- drawback: basis of orthonormal polynomials $f_n(z)$ behaves pathologically for $\mathrm{Re}\, z < 0$
  - $|f_n(-1)| \sim C^n$ with $C \geq 1$
  - can be partially alleviated by chosing free parameter t_0 in definition of z
- we do not currently claim control of the truncation error, rather, only a handle
- actively looking into alternative formulations of the dispersive bound that evade the pathological behaviour

Summary and Outlook

- nonlocal form factors contribute the single-largest systematic uncertainty in exclusive $b \to s\ell\ell$ decays

- I think there is a clear road toward a reliable description of these objects, but much work still needs to be done

- key is a combined theory + data driven approach
  - new dispersive bound will provide handle on truncation errors

- phenomenological applications of the new parametrisation and the bound are forthcoming [Gubernari/Reboud/DvD/Virto]