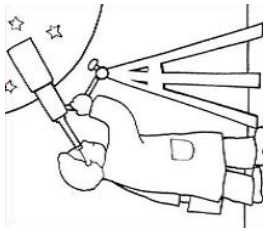
Probing Relaxion (Scalars) at the Precision Frontier



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Mainz, 2021

OUTLINE

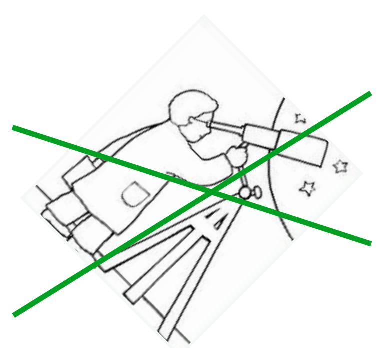
- Introduction
- Case study: Relaxion
- Sub-eV Scalar/relaxion dark-matter searches: using atomic spectroscopy
- Conclusions

MOTIVATION

We know that there is new physics (NP) but it is not associated with scale

For many years the common wisdom: Higgs hierarchy & DM => TeV NP

But LHC and direct searches (so far) fail to find this result



No NP!

Null results put pressure on the conventional symmetry based solution to the naturalness problem

MOTIVATION

New ideas have been recently proposed to think about the hierarchy problem

eg: "Cosmic attractors", "dynamical relaxation", "N-naturalness", "relating the weak-scale to the CC", "inflating the Weak scale", "sliding naturalness", "weak scale as a trigger" etc.

We used relaxion as a concrete example (many of others contains light scalar)

[Graham, kaplan, Rajendran 15]

I will focus on the phenomenology part and not discuss about relaxion in the context of multiverse/ anthropic (see e.g. Guidice, Kehagias, Riotto 19 for a discussion)

Bottomline below: relaxion is axion-like-particle (ALP), described as scalar mixes \w Higgs, but with weird (shallow) potential & unknown mass

[Espinosa, Grojean, Panico, Pomarol, Pujolàs, Servant 15] [Choi, Im 16] [Flacke, Frugiuele, Fuchs, Gupta, Perez 16]

[AB, Kim, Matsedonskyi, Perez, Safranova 20]

RELAXION MECHANISM

- A dynamical amelioration of the Higgs hierarchy problem

Add a scalar (relaxion) dependent mass term: $\underbrace{\left(\Lambda^2 - \frac{\Lambda^2}{f_{\text{eff}}}\phi\right)}_{\mu_H^2(\phi)} H^\dagger H$

Add some potential of relaxion: $V(\phi) \supset c\frac{\Lambda^4}{f_{\text{eff}}}\phi + \cdots$

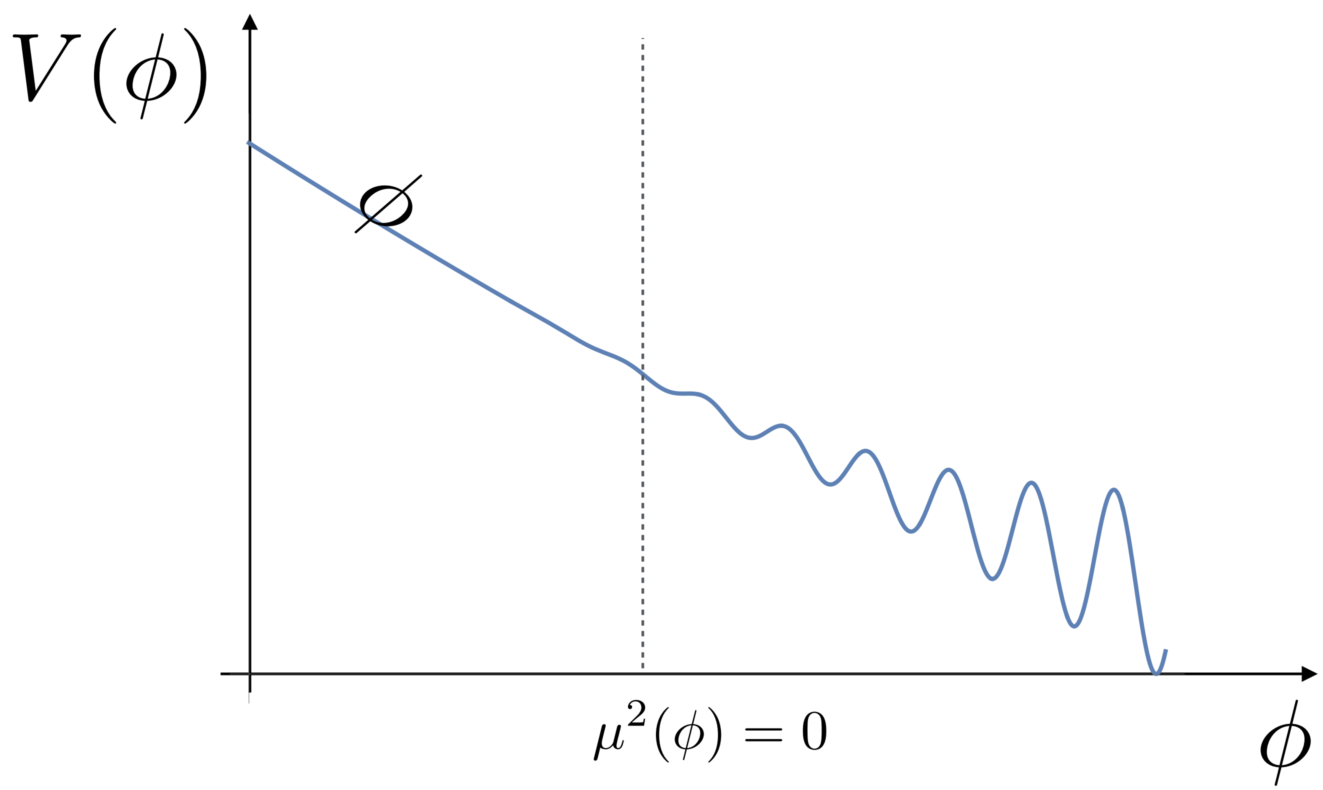
Add a Higgs dependent "back reaction", periodic potential for ϕ : $V_{\text{br}} = \mu_b^2 |H|^2 \cos(\phi/f)$

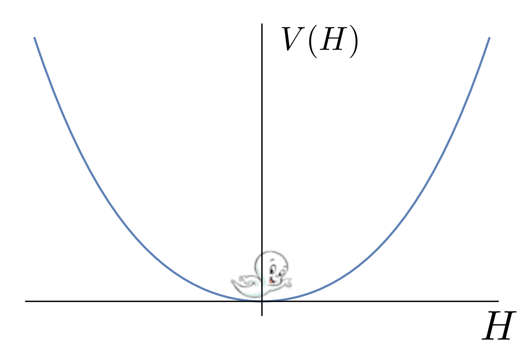
$$\boxed{\text{The hierarchy problem: we need to understand why today } \mu_H^2(\phi_{\text{today}}) \ll \Lambda^2}$$

RELAXION MECHANISM

Add a scalar (relaxion) dependent mass term: $\underbrace{\left(\Lambda^2 - \frac{\Lambda^2}{f_{\text{eff}}}\phi\right)}_{\mu_H^2(\phi)} H^\dagger H$

ϕ rolls till $\mu_H^2(\phi)$ changes sign $\Rightarrow$ $\langle H \rangle \neq 0 \Rightarrow$ stops rolling.





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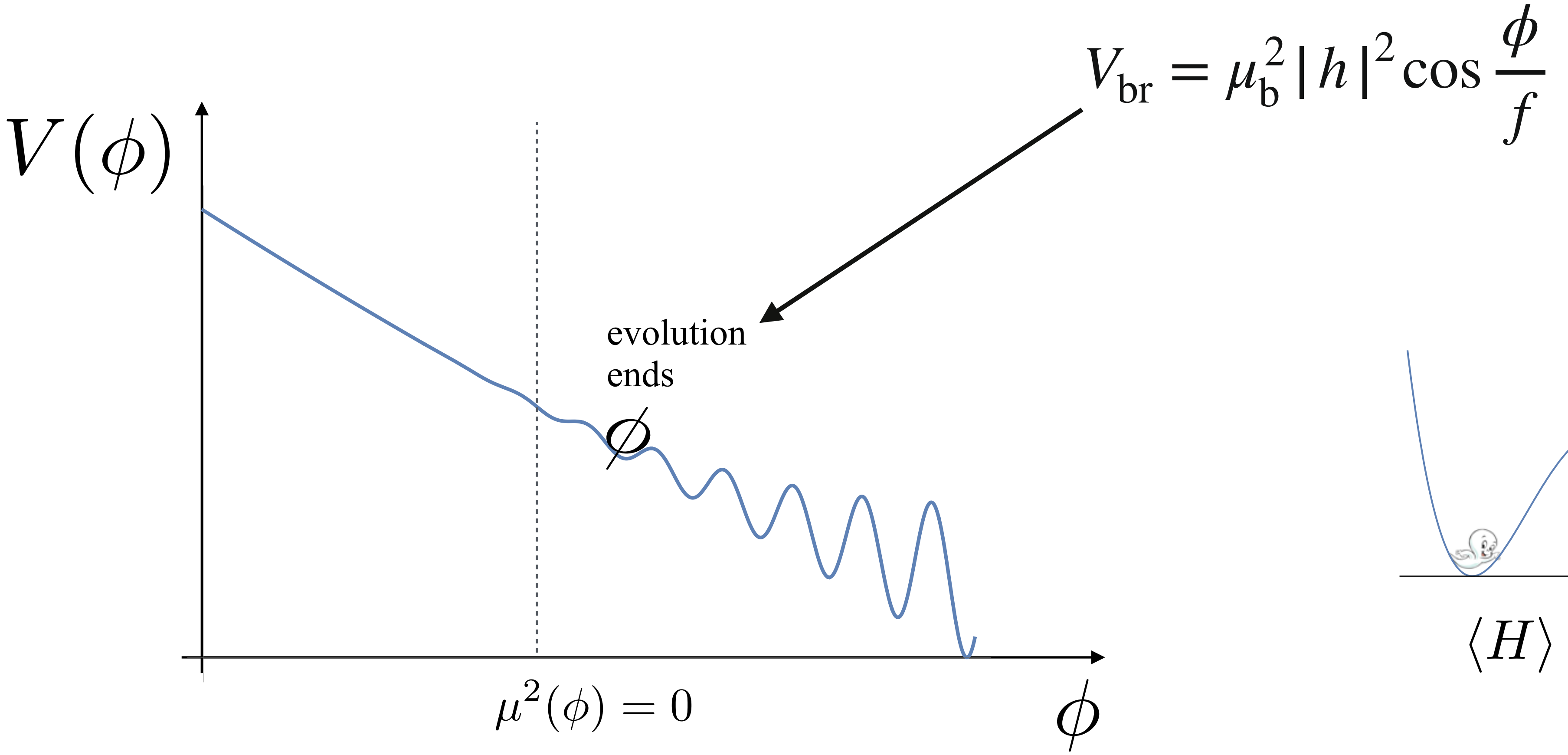
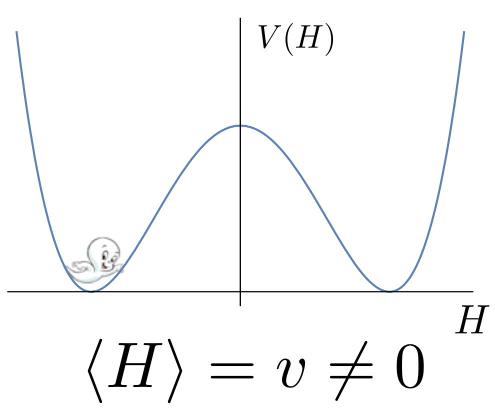


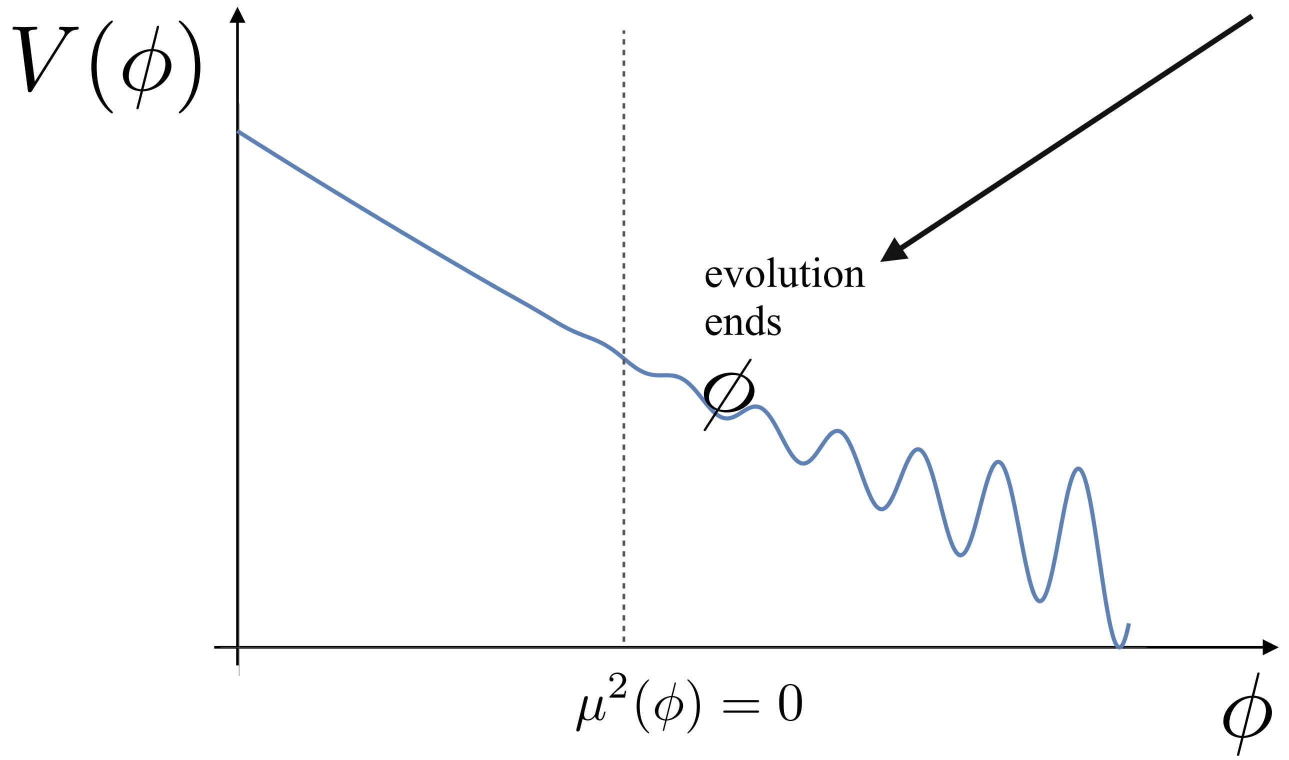


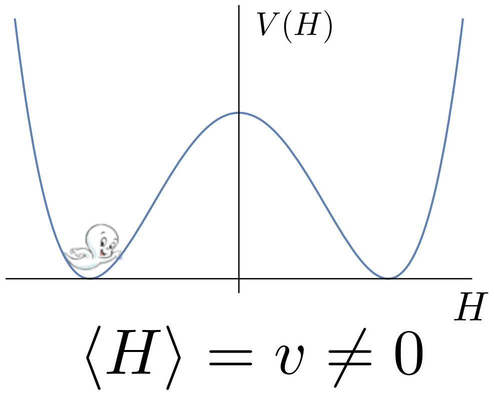
RELAXION MECHANISM

Relaxion stops rolling when: $V'(\phi) = 0 = c\dfrac{\Lambda^4}{f_{\rm eff}} = \dfrac{\mu_b^2 v_{\rm EW}^2}{f}\sin\dfrac{\phi_0}{f}$

$$\langle H \rangle \neq 0$$

$$\mu^2(\phi_0) = (\Lambda^2 - \Lambda^2\frac{\phi_0}{f_{\rm eff}}) \sim v_{\rm EW}^2$$





Relaxion Phenomenology

Smallness of the Higgs mass $v_{\rm EW}^2 \ll \Lambda^2$ is due to dynamical relaxation in the early universe

IR degrees of Freedom is the Relaxion! $m_{\rm IR} \lesssim v_{\rm EW}(v_{\rm EW}/\Lambda)^{3/2}$

[AB, Kim, Matsedonskyi, Perez, Safranova 20]

For low energy phenomenology, we need to know, its mass and interaction with SM

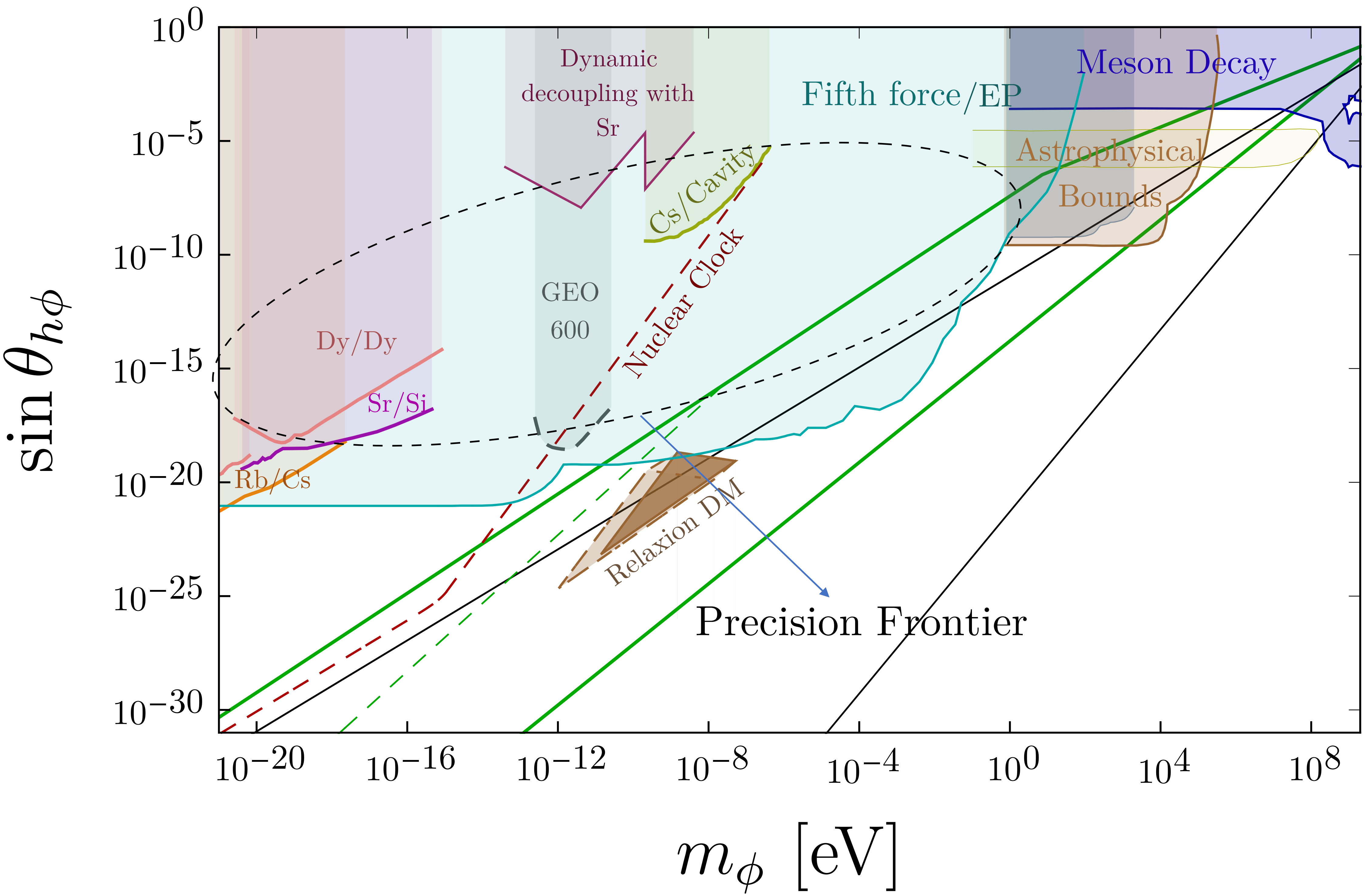
Most of the information from $V_{\rm br} = \mu_b^2 |H|^2 \cos(\phi/f)$

(It is much more interesting, relaxion mass is also relaxed and because of that usual relation between mass and mixing angle does not apply) [AB, Kim, Matsedonskyi, Perez, Safranova 20]

Due to the mixing with the Higgs the relaxion couples to the SM

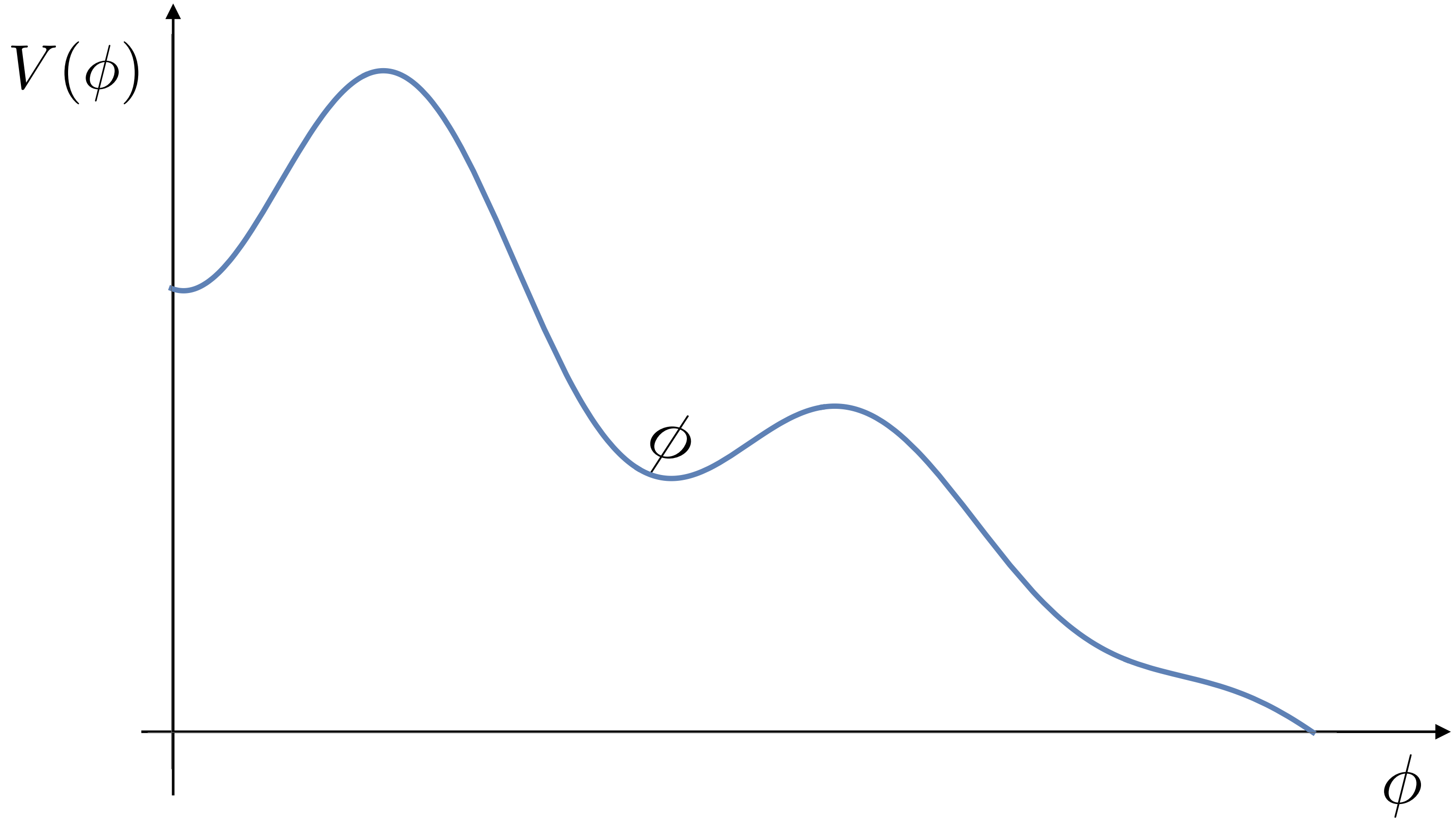
$$\mathcal{L}_{\rm eff} \supset -\sin\theta_{h\phi}\frac{\phi}{v}\left(\sum_f m_f \bar{f} f - c_{\gamma\phi}\frac{\alpha}{4\pi}F_{\mu\nu}F^{\mu\nu} - c_{g\phi}\frac{\alpha_s}{4\pi}G_{a\mu\nu}G^{a\mu\nu}\right)$$

OVERVIEW PLOT: 30 DECADE OPEN PARAMETER SPACE



Relaxion DM

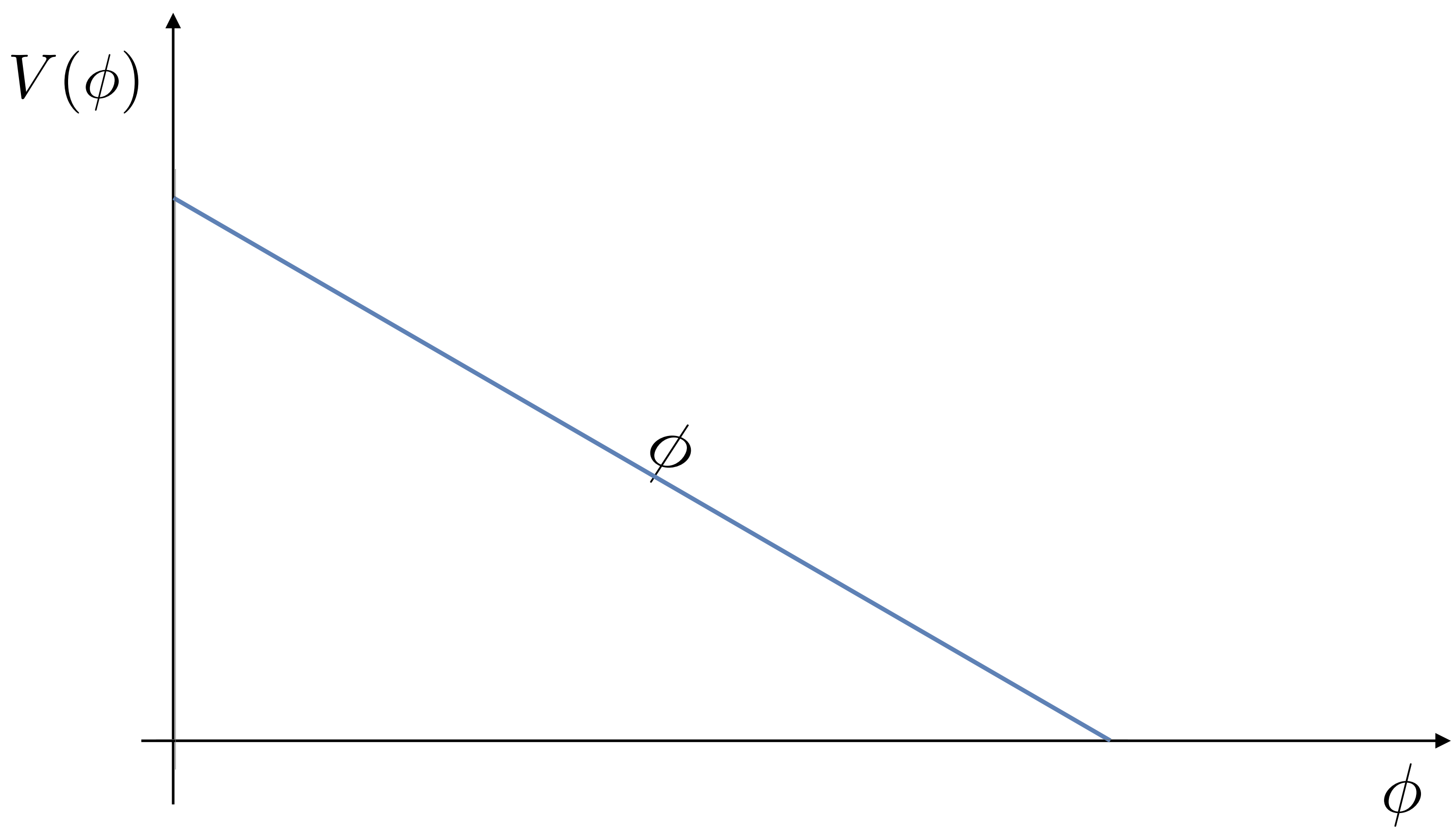
Basic idea is similar to axion DM (but avoiding misalignment problem):



Relaxion DM

Basic idea is similar to axion DM (but avoiding misalignment problem):

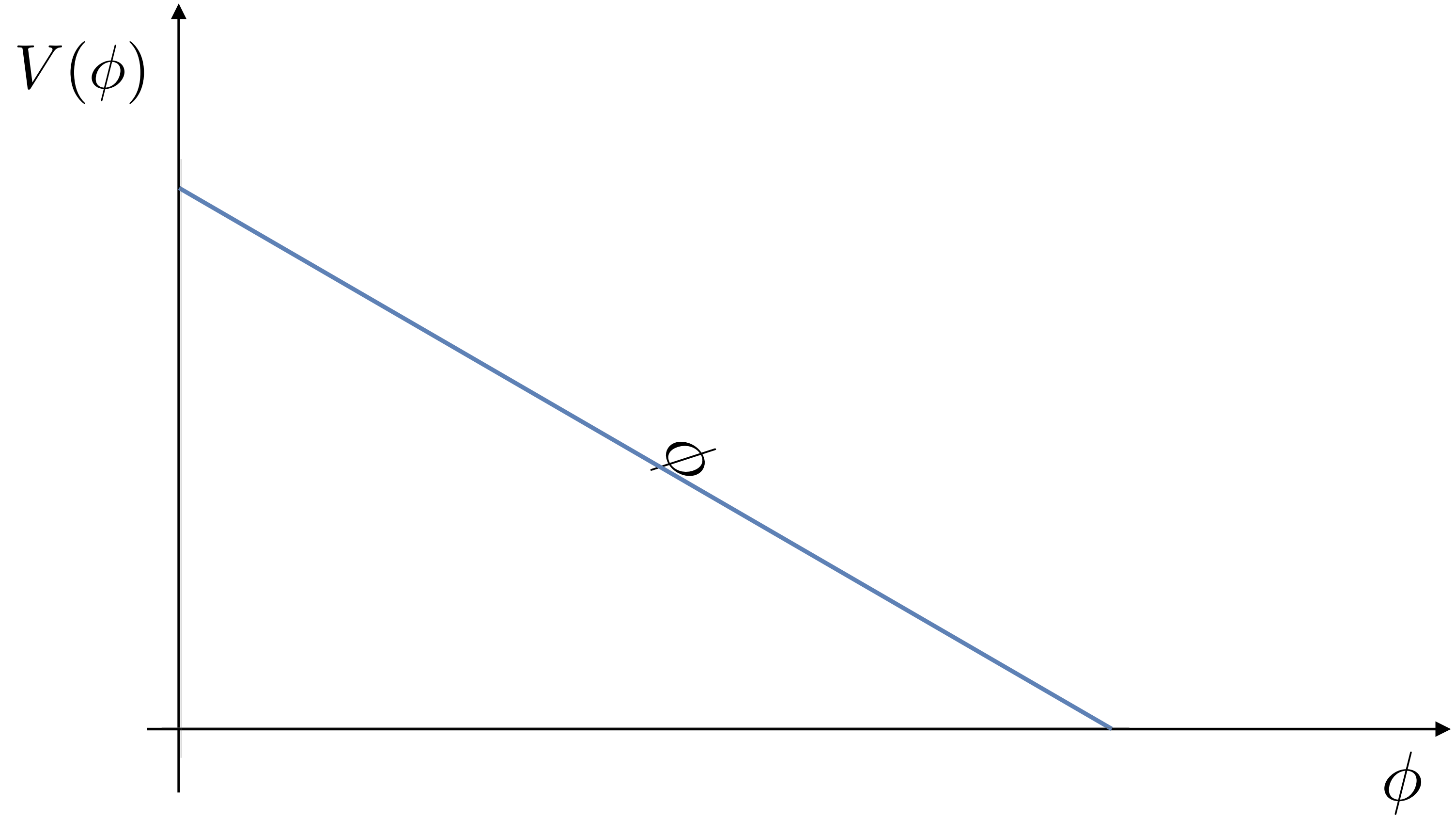
- After reheating the wiggles disappear: (symmetry restoration)



Relaxion DM

Basic idea is similar to axion DM (but avoiding misalignment problem):

- After reheating the wiggles disappear: (symmetry restoration) and the relaxion rolls a bit



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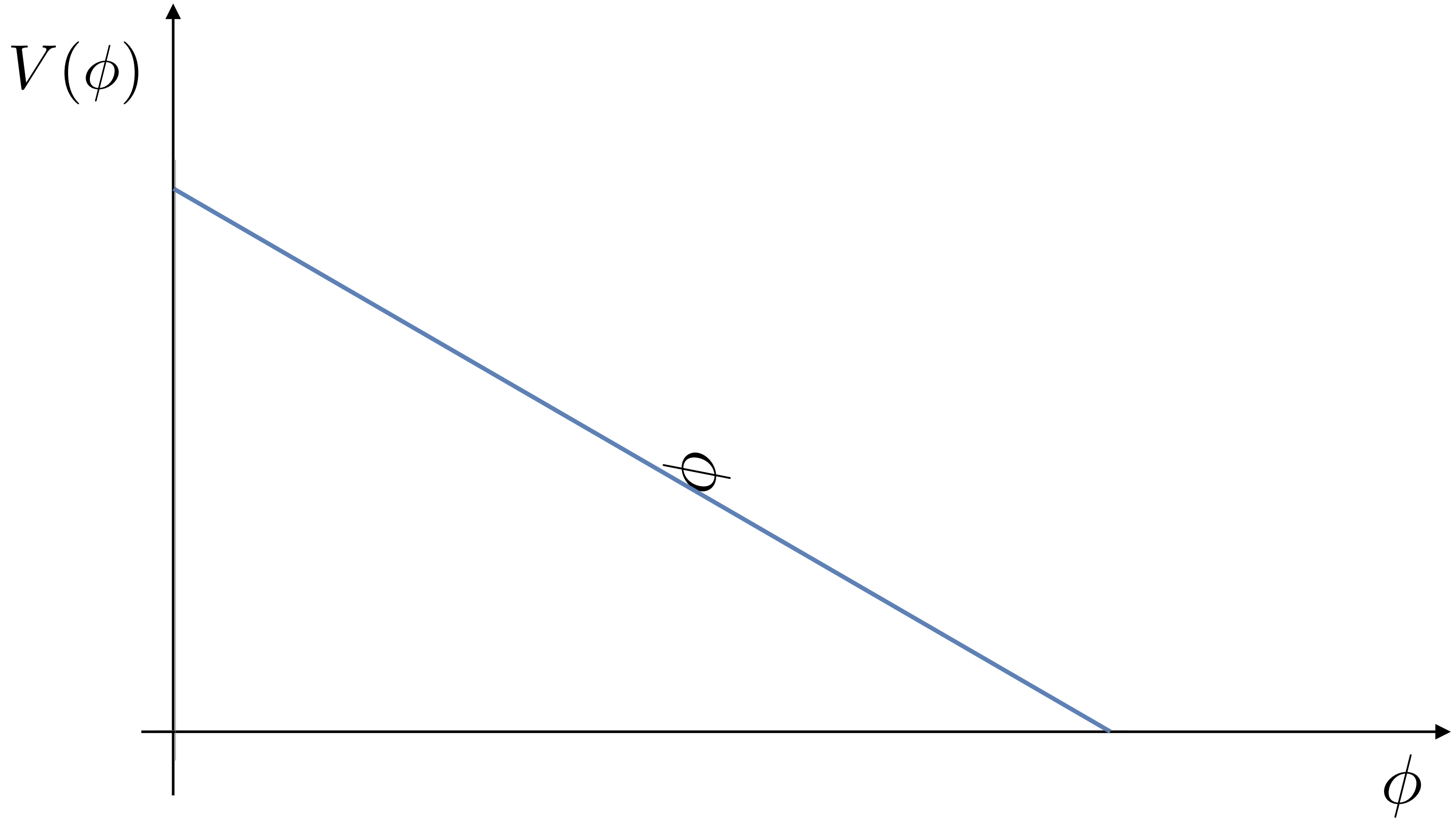
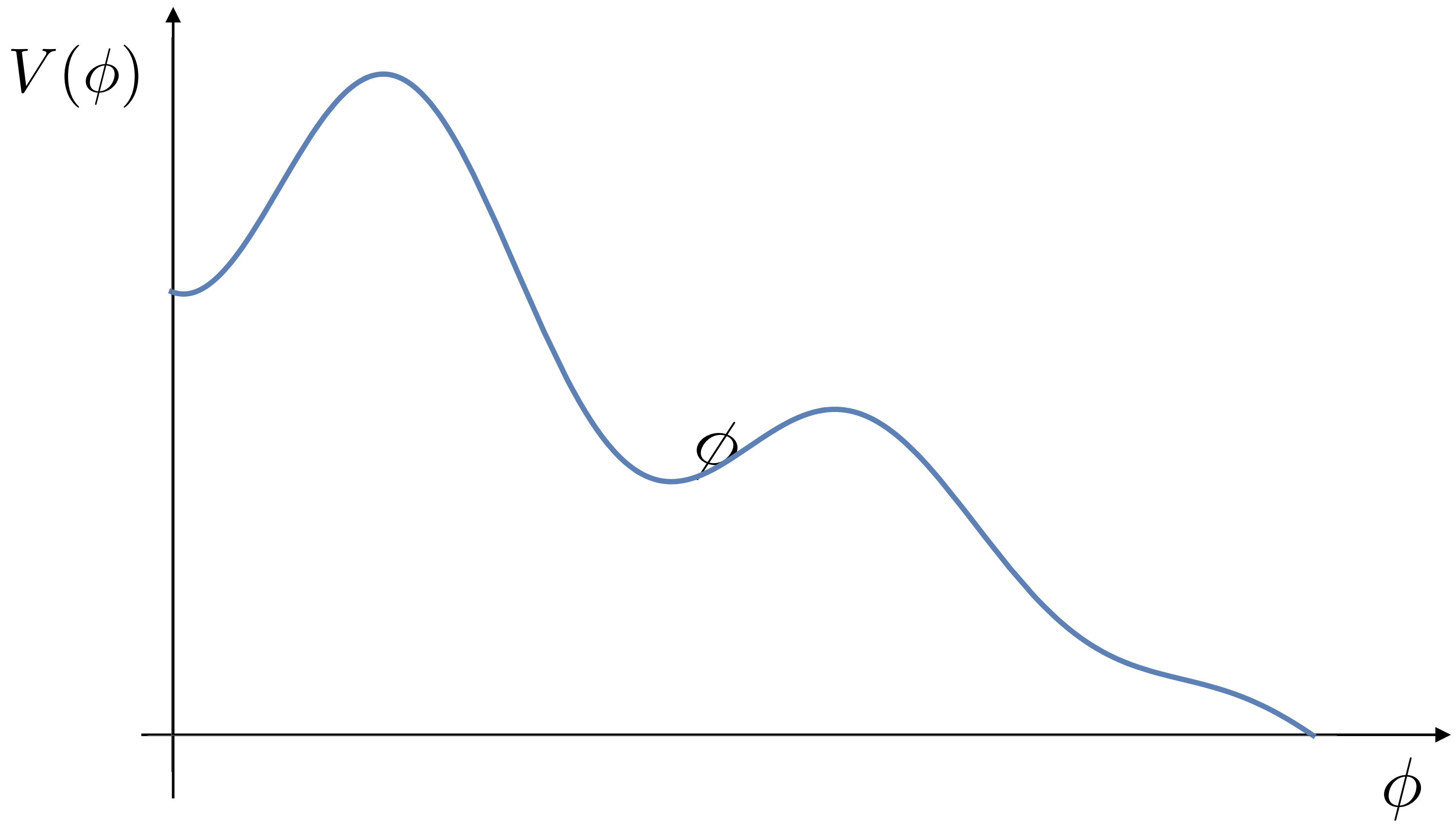


Relaxion DM

Basic idea is similar to axion DM (but avoiding misalignment problem):

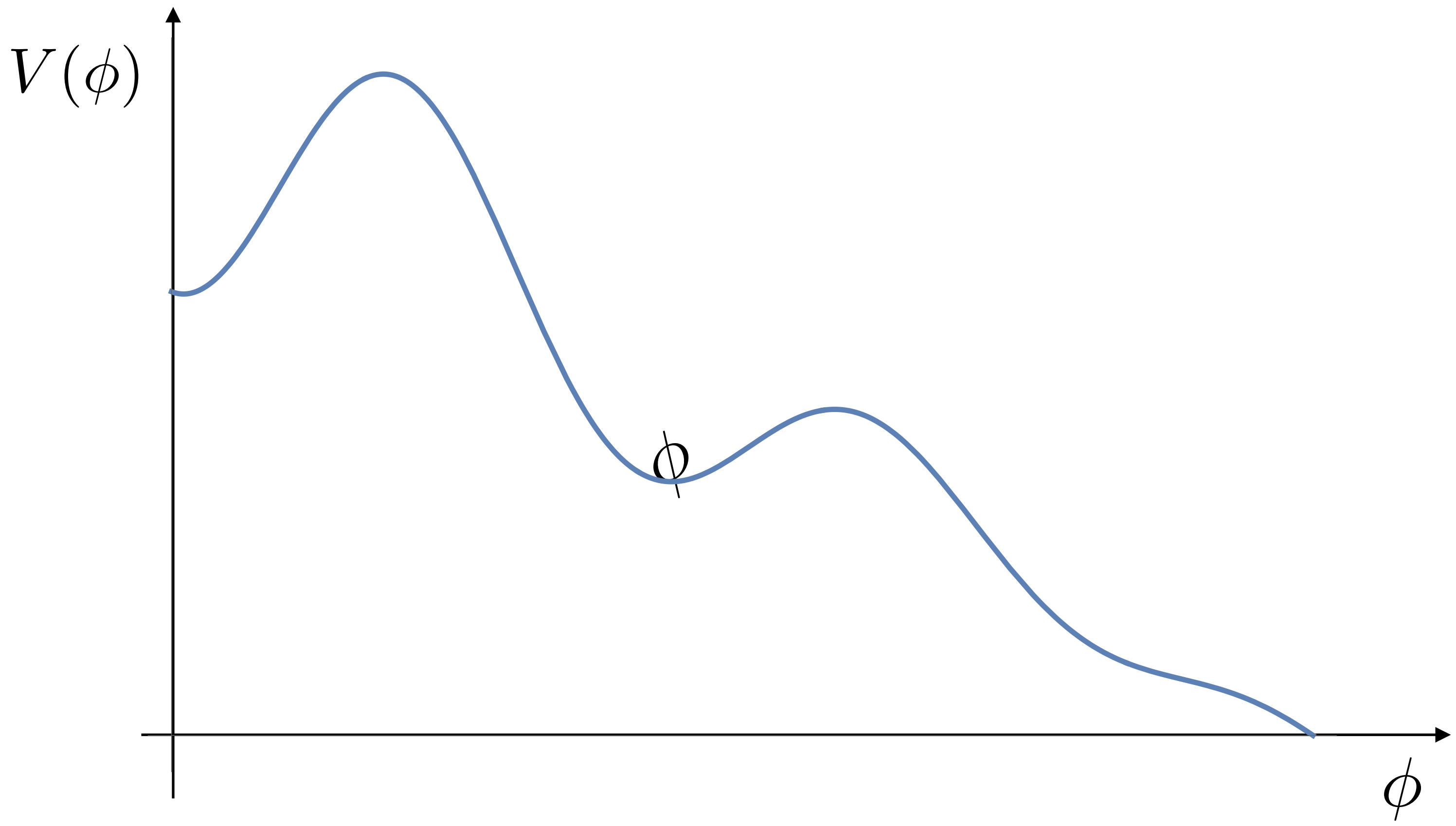
- Universe cools down and the wiggles re-appear.



Relaxion DM

Basic idea is similar to axion DM (but avoiding misalignment problem):

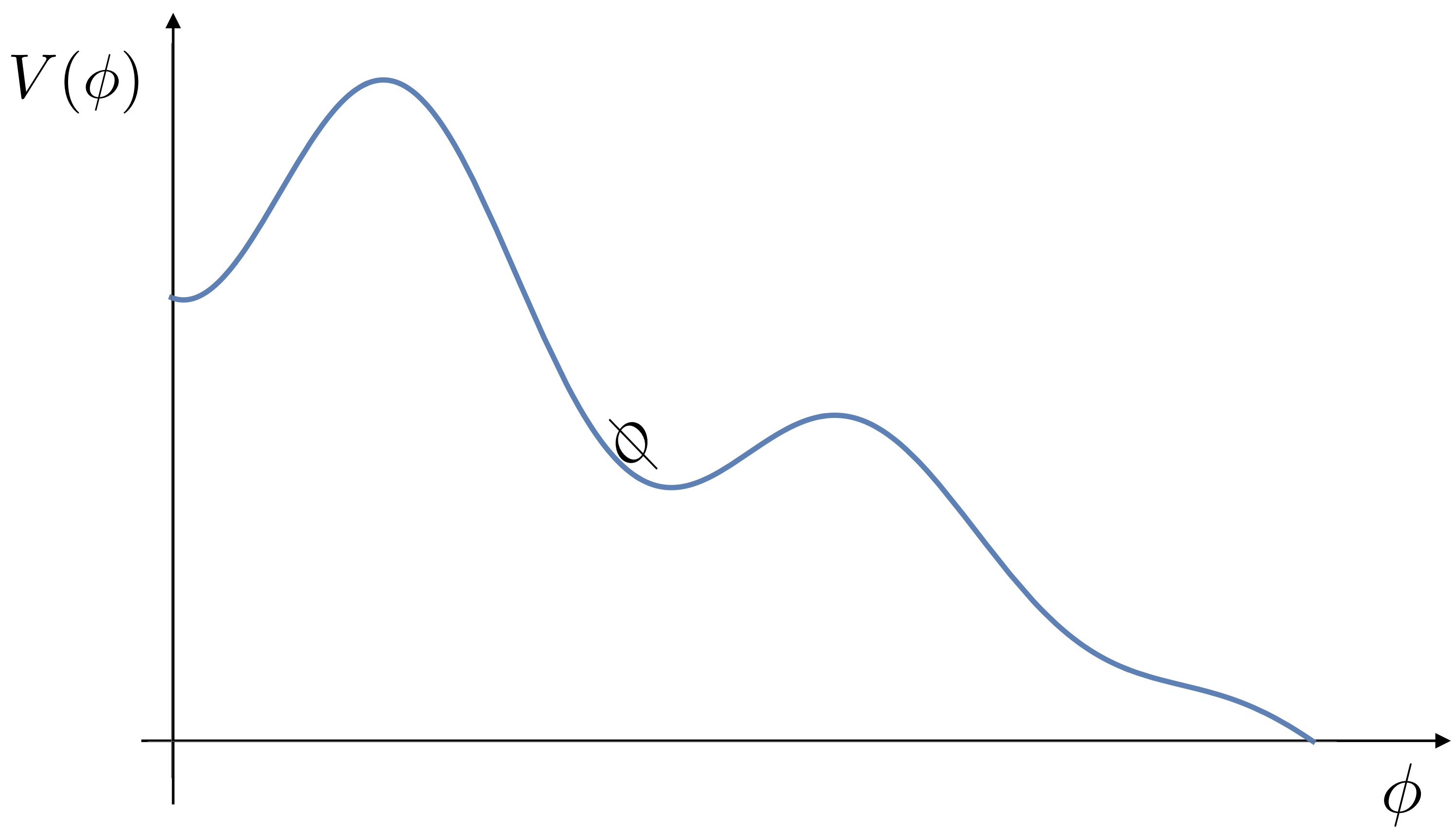
- If the relaxion is stopped, then it is not at the minimum and starts to oscillate = DM.



Relaxion DM

Basic idea is similar to axion DM (but avoiding misalignment problem):

- Now the relaxion not at the minimum and start to oscillate = DM.



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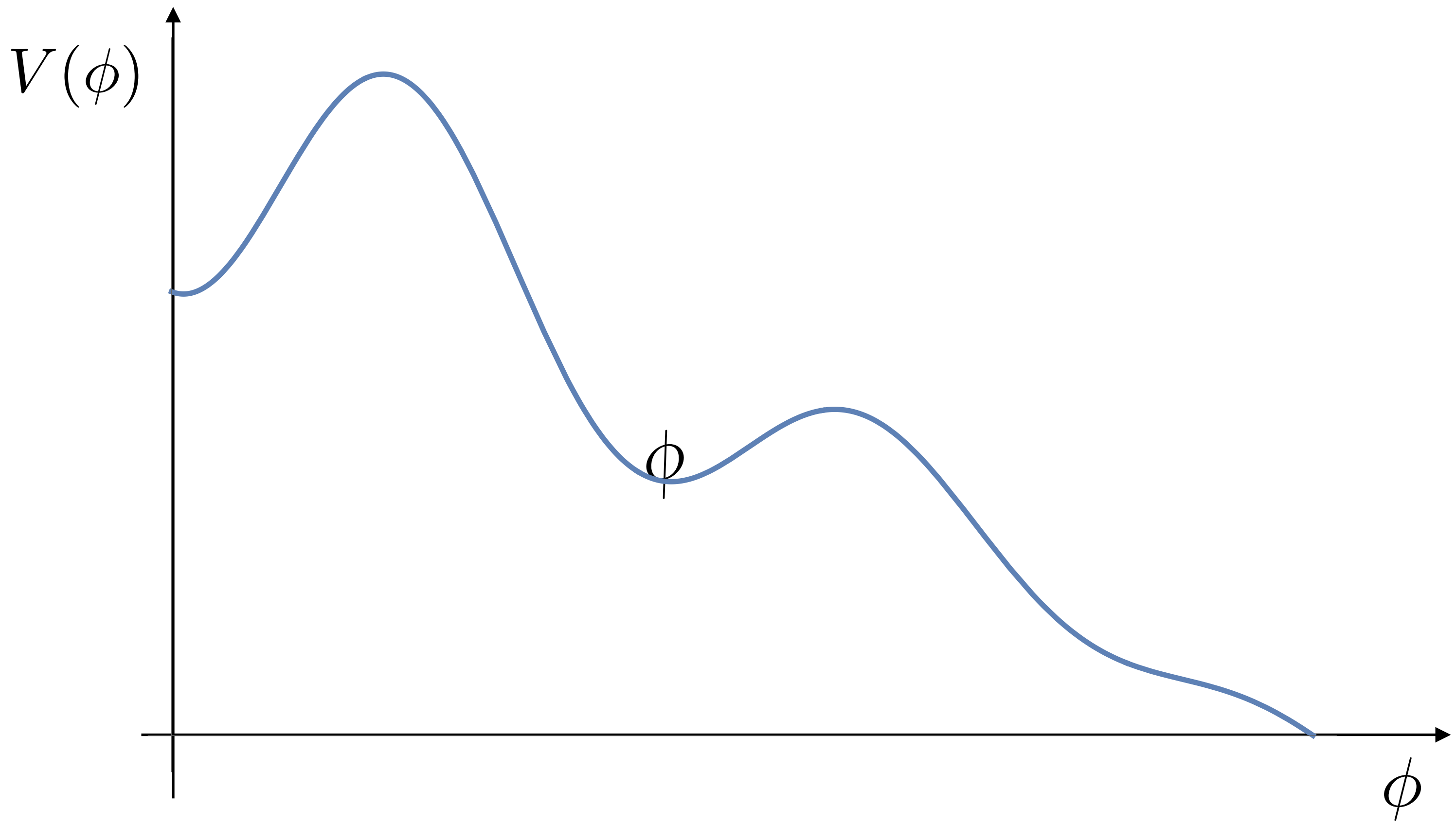

[AB, Kim, Perez 18]

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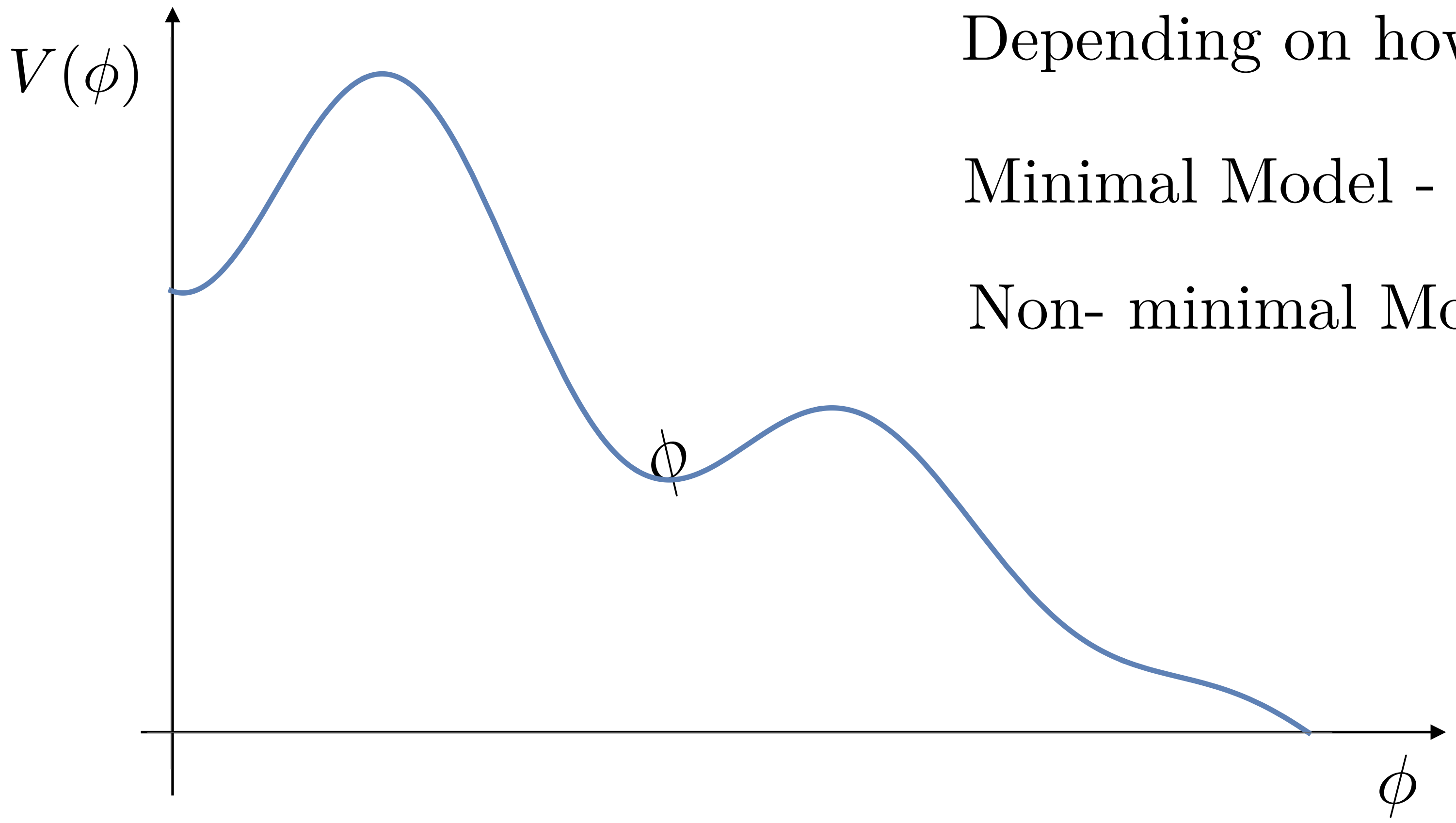


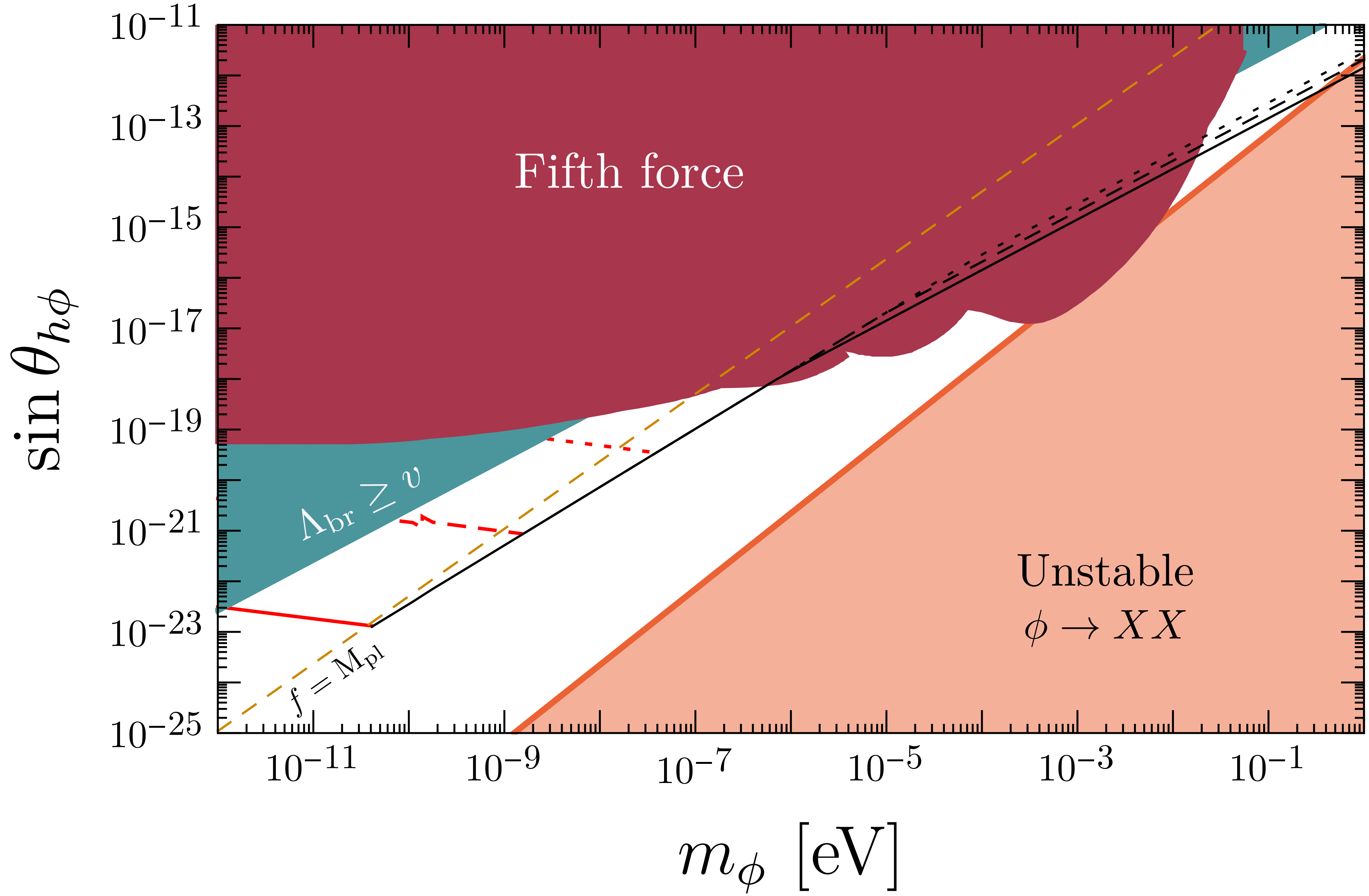
Depending on how the relaxion is stopped

Minimal Model - Hubble friction

Non- minimal Model - Requires Dark Photon

[AB, Kim, Perez 18]

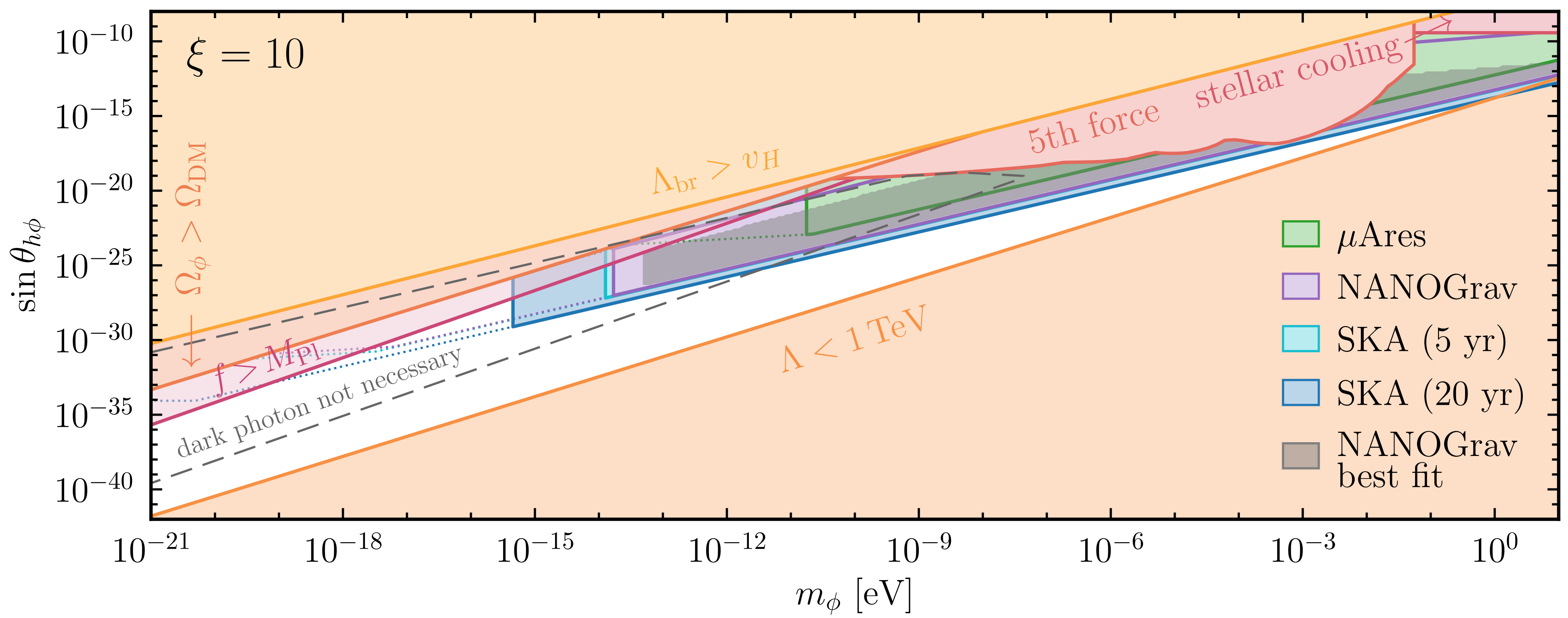
Relaxion DM Parameter Space



Gravitational Wave Echo of Relaxion Trapping

There are viable regions that requires coupling to dark photon to trap the relaxion

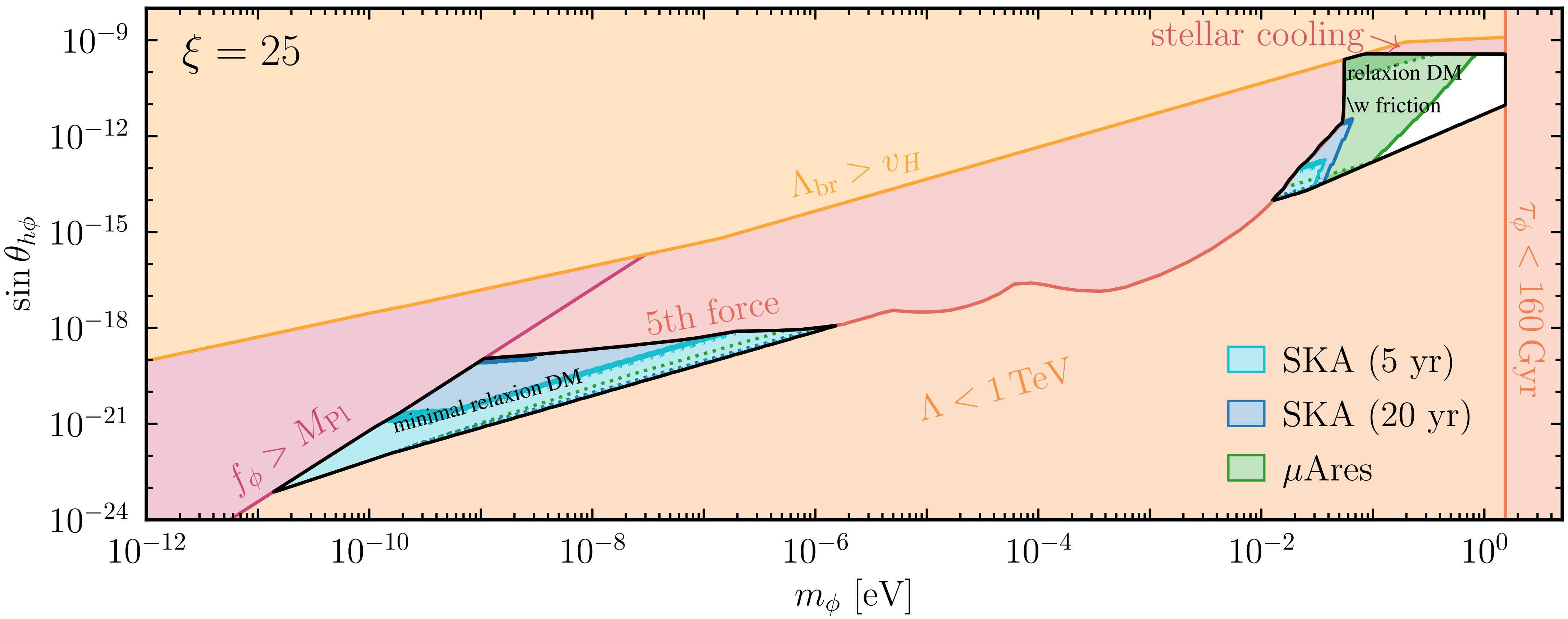
The friction is obtained by significant late production of coherent, anisotropic, dark photon field that efficiently sources gravitational waves



The red and orange shaded regions are excluded by the indicated constraints or combinations thereof. Above the red solid line, the relaxion decay constant becomes super-Planckian. The grey dashed line encloses the min. Relaxion case. The prospective GW sensitivity of µAres (green) as well as SKA after an observation period of 5 years (turquoise) and 20 years (blue) is indicated by the respective coloured regions. In the purple coloured region, a sub-range of the viable reappearance temperatures can be excluded using current NANOGrav data from the 11-year data set. The regions bounded by the coloured dotted lines need super-Planckian decay constants to be probed by the respective experiment. The grey shaded region can reproduce our best-fit spectrum (at $T_{\rm ra} \geq 20$ MeV) to the potential stochastic GW background seen in the recent NANOGrav data.

[AB, Madge, Banerjee, Perez, Ratzinger, Schwaller 21]

Relaxion DM Parameter Space



The black solid line encompass the DM relaxion parameter space. The coloured regions inside the viable DM space can be probed via GWs in µAres (green) or SKA (blue/turquoise). The light shading and solid lines indicate points that can be probed for a subrange of reappearance temperatures, whereas the darker shaded parts enclosed by dotted lines are accessible for all valid $T_{\rm ra}$.

[AB, Kim, Perez 18] [AB, Madge, Banerjee, Perez, Ratzinger, Schwaller 21]

Search for relaxion (scalar) DM

Coherently oscillating relaxion can be a viable DM candidate in the mass range

$$10^{-11}\,\mathrm{eV} \lesssim m_\phi \lesssim 10^{-6}\,\mathrm{eV} \qquad \text{(minimal model)}$$

$$10^{-2}\,\mathrm{eV} \lesssim m_\phi \lesssim 1\,\mathrm{eV} \qquad \text{(dark photon required)}$$

Relaxion has scalar interaction with the SM

How to search for relaxion (scalar) DM ?

Search for Sub-eV Scalar Dark Matter (DM)

$$\mathcal{L}_{\rm eff} \supset -\frac{\phi}{M_{\rm Pl}}\left(\sum_{f=e,u,d} d_{m_f} m_f \bar{f} f + \frac{d_\alpha}{4} FF + \frac{d_{g_s}\beta(g_s)}{2g_s} GG\right) \qquad \phi(t) \simeq \frac{\sqrt{2\rho_{\rm DM}}}{m_\phi}\cos(m_\phi t)$$

[CF early literature: Damour, Polyakov 94; Barrow 99] [For dilaton DM: Arvanitaki, Huang, Van Tilburg 15]

Scalar oscillation leads to profound implications:

Practically all constants of nature now oscillate at the frequency equal to its mass

$$m_f(t) \simeq m_f(1 + d_{m_f}\phi(t)/M_{\rm Pl})\,,\ \alpha(t) = \alpha(1 - d_\alpha \phi(t)/M_{\rm Pl})\,,\ \alpha_s(t) = \alpha_s(1 - 2d_g\beta(g_s)/g_s\,\phi(t)/M_{\rm Pl})$$

Can be searched in experiments which are looking for Direct time-varying constants

denoted as Direct Dark Matter (DDM) searches

atomic and nuclear clocks, gravitational wave detectors....

For relaxion we can express bounds $d_{m_f,\alpha,g_s} \Leftrightarrow \sin\theta_{h\phi}$ (relaxion-Higgs mixing angle)

DDM Searches: Strategy and Reach

General: find 2 systems with different dependence on the Fundamental Constants

$\Delta E_1 \uparrow f_1$

$$\Delta E_{1,2} \equiv f_{1,2} = f_{1,2}\left(\alpha^{\xi_\alpha^{1,2}}, m_e^{\xi_{m_e}^{1,2}}\right)$$

$\Delta E_2 \uparrow f_2$

[Safronova, Budker, DeMille, Kimball, Derevianko, Clark 18] [Antypas, Budker, Flambaum, Kozlov, Perez, Ye 20]

Fractional change of the frequency ratio: $$\frac{\Delta(f_1/f_2)}{(f_1/f_2)} = \frac{\Delta f_1}{f_1} - \frac{\Delta f_2}{f_2} = (\Delta\tilde{Q})_i^{12}\, d_i\, \frac{\phi}{M_{\rm Pl}}$$

$$\tilde{Q}_i^a = \partial \ln f_a / \partial \ln g_i = \xi_i^a$$

Classical ex: clock comparisons, clock-cavity comparisons

Electronic transition in an atom $\propto R_\infty \sim m_e \alpha^2$ $$\frac{\Delta f_1}{f_1} = 2\frac{\delta\alpha}{\alpha} + \frac{\delta m_e}{m_e} = (2d_\alpha + d_{m_e})\frac{\phi}{M_{\rm Pl}}$$

cavity $\propto$ bohr radius $\sim (m_e\alpha)^{-1}$ $$\frac{\Delta f_2}{f_2} = \frac{\delta\alpha}{\alpha} + \frac{\delta m_e}{m_e} = (d_\alpha + d_{m_e})\frac{\phi}{M_{\rm Pl}}$$

Clock cavity comparison test constraints $$\frac{\Delta(f_1/f_2)}{(f_1/f_2)} = \frac{\phi}{M_{\rm Pl}}\begin{cases} d_\alpha \text{ cavity active} \\ (2\,d_\alpha + d_{m_e}) \text{ cavity frozen} \end{cases}$$

The Equivalence Principle (EP) violation tests

EP violation tests measures differential acceleration between two test bodies in the presence of a source

$$\text{Eötvös parameter } \eta_{\rm EP}^{\rm AB} \equiv 2\frac{|\vec{a}_{\rm A} - \vec{a}_{\rm B}|}{|\vec{a}_{\rm A} + \vec{a}_{\rm B}|}$$

$$\eta_{\rm EP}^{\rm AB} \propto (\Delta Q)_i^{\rm AB} d_i \times Q_j^{\rm source} d_j$$

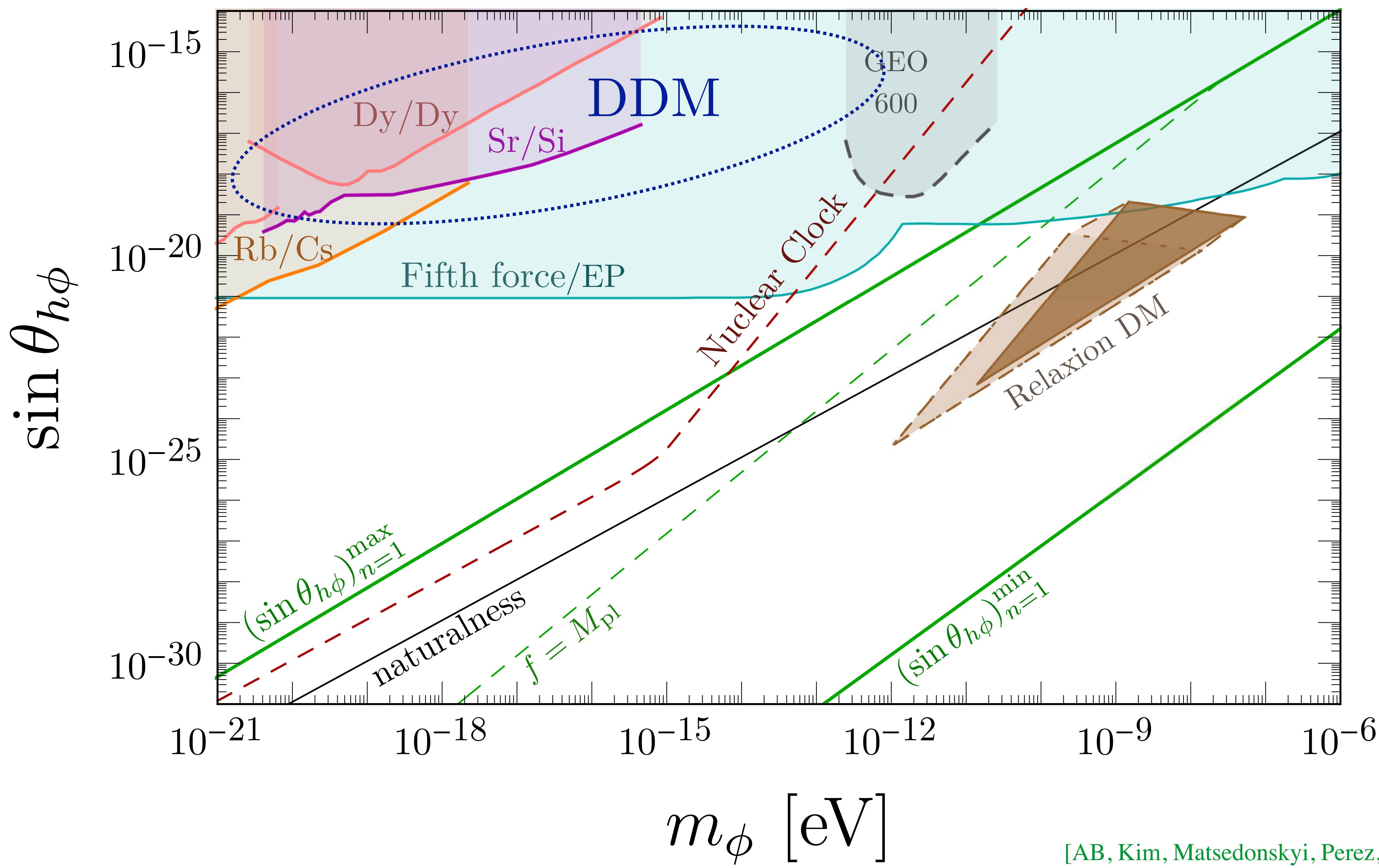
$$g_i \in (\alpha, \alpha_s, m_e, m_q)$$

$$d_i = M_{\rm Pl}\,\partial \ln g_i / \partial\phi$$

ith dilatonic charge of a body , $Q_i^{\rm X} = \partial \ln m^{\rm X} / \partial \ln g_i$

Examples: Microscope experiment (Pt alloy vs Ti alloy), comparing two test bodies Be-Ti, Cu-Pb, Be-Al,...

EP violation tests put strong constraint!



[AB, Kim, Matsedonskyi, Perez, Safranova 20]

Complementarity b/w EP tests and DDM Searches

DDM searches $\frac{\Delta(f_1/f_2)}{(f_1/f_2)} = \frac{\Delta f_1}{f_1} - \frac{\Delta f_2}{f_2} = (\Delta \tilde{Q})_i^{12}\, d_i\, \frac{\phi}{M_{\rm Pl}}$ $\tilde{Q}_i^a = \partial \ln f_a / \partial \ln g_i$

EP violation tests $\eta_{\rm EP}^{\rm AB} \propto (\Delta Q)_i^{\rm AB} d_i \times Q_j^{\rm source} d_j$

$\mathcal{L}_{\rm eff} \supset -\frac{\phi}{M_{\rm Pl}} \left(\sum_{f=e,u,d} d_{m_f} m_f \bar{f} f + \frac{d_\alpha}{4} FF + \frac{d_{g_s}\beta(g_s)}{2g_s} GG \right)$ 5 dimensional vector space of couplings

$\vec{X} = (X_\alpha, X_{m_e}, X_{g_s}, X_{\hat{m}}, X_{\delta m})$ $\hat{m}, \delta m/2 \equiv (m_u \pm m_d)/2$

Example: Microscope expt constraints $(\Delta \vec{Q})^{\rm Mic} \sim 10^{-3}(-1.94\,, 0.03\,, 0.8\,, -2.61\,, -0.19)$

Clock cavity comparison test constraints

$$(\Delta \vec{\tilde{Q}})^{\rm cc} = (1\,, 0\,, 0\,, 0\,, 0)$$

$$(\Delta \vec{\tilde{Q}})^{\rm cc} = (2\,, 1\,, 0\,, 0\,, 0)$$

EP tests and DDM searches constraint different linear combinations of couplings

Complementarity b/w EP tests and DDM Searches

EP tests and DDM searches constraint different linear combinations of couplings

We can find a direction which would be orthogonal to 4 EP tests

[Oswald, Nevsky, Vogt, Schiller, Figuerora, Zhang, Tretiak, Antypas, Budker, AB, Perez arXiv:2111.06883]

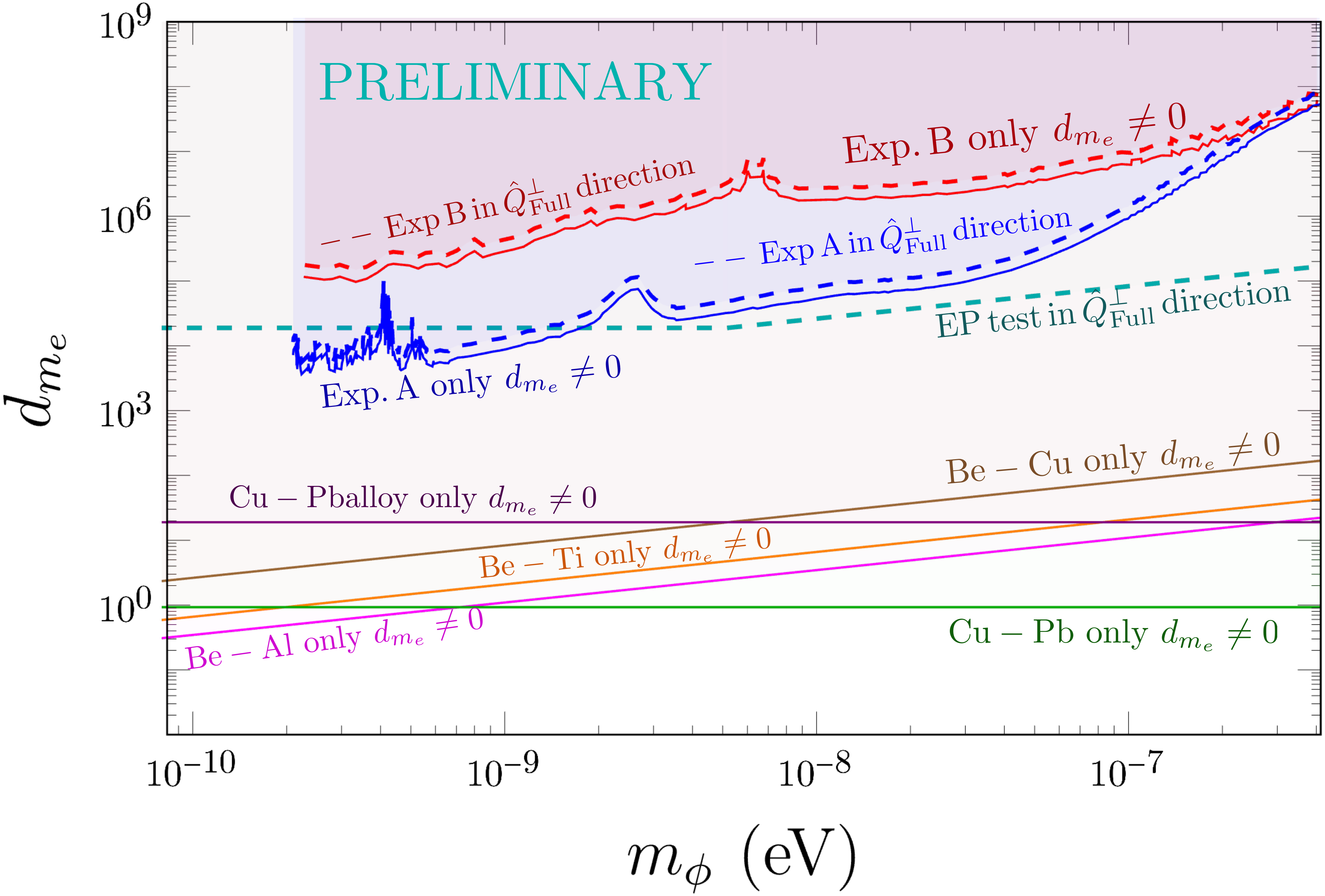
[Tretiak, Zhang, Figueroa, Antypas, Brogna, AB, Perez, Budker work in progress]

For four leading EP tests, orthogonal direction

$$\hat{Q}^{\perp}_{\mathrm{Full}} \sim \big(0.003\,,\; -0.987\,,\, 0.002\,,\, -0.001\,,\, -0.162\,\big)$$

Models of light scalar DM with coupling direction defined according to $\hat{Q}^{\perp}_{\mathrm{Full}} \cdot \vec{d}$ would not be constrained by these four leading EP bounds.

Complementarity b/w EP tests and DDM Searches



[Tretiak, Zhang, Figueroa, Antypas, Brogna, AB, Perez, Budker work in progress]

CONCLUSIONS

- Null results call for new theories (e.g. relaxion) and diverse experimental approaches
- Relaxion DM (scalar DM) can be probed by spectroscopy
- It is possible to bypass the EP tests constraints on scalar DM parameter space

UNCUT IT !


damn anti-gravity cat
always disproving ma theorem
ICANHASCHEEZBURGER.COM

RELAXION MASS AND MIXING ANGLE

$$V_{\rm br} = \mu_{\rm b}^2 |h|^2 \cos(\phi/f)$$

Relaxion mass $\quad m_\phi^2 \sim \partial_\phi^2 V_{br}(\phi, h) \sim \frac{\mu_b^2 v_{\rm EW}^2}{f^2} \cos \frac{\phi_0}{f}$

Mixing angle $\quad \sin\theta_{h\phi} \sim \partial_\phi \partial_h V_{br}(\phi, h)/v_{\rm EW}^2 \sim \frac{\mu_b^2}{f v_{\rm EW}} \sin \frac{\phi_0}{f}$

RELAXION MASS AND MIXING ANGLE

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RELAXION MASS AND MIXING ANGLE

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~ 1

Naive estimation

RELAXION MASS AND MIXING ANGLE

$$m_\phi^2 \sim \partial_\phi^2 V_{br}(\phi, h) \sim \frac{\mu_b^2 v_{\rm EW}^2}{f^2} \cos\frac{\phi_0}{f} \qquad \sim 1$$

Naive estimation

$$\sin\theta_{h\phi} \sim \partial_\phi \partial_h V_{br}(\phi, h)/v_{\rm EW}^2 \sim \frac{\mu_b^2}{f v_{\rm EW}} \sin\frac{\phi_0}{f}$$

How well we understand the stopping point?

RELAXION MASS AND MIXING ANGLE

$$m_\phi^2 \sim \partial_\phi^2 V_{br}(\phi, h) \sim \frac{\mu_b^2 v_{\mathrm{EW}}^2}{f^2} \times \frac{\mu_b}{\Lambda} \ll 1$$

$$\sin\theta_{h\phi} \sim \partial_\phi \partial_h V_{br}(\phi, h)/v_{\mathrm{EW}}^2 \sim \frac{\mu_b^2}{f v_{\mathrm{EW}}} \sin\frac{\phi_0}{f}$$

The relaxion is also relaxed

HOW IS RELAXION RELAXED ?

$$V(\phi, h) = \left(\Lambda^2 - \Lambda^2 \frac{\phi}{f_{\rm eff}}\right)|h|^2 - \underbrace{\frac{\Lambda^4}{f_{\rm eff}}\phi}_{V_{\rm roll}} - \underbrace{\mu_b^2 |h|^2 \cos(\phi/f)}_{V_{\rm br}} \qquad v^2(\phi) = \begin{cases} 0 \text{ when } \phi < f_{\rm eff} \\ > 0 \text{ when } \phi > f_{\rm eff} \end{cases}$$

Relaxion stops when $\quad 0 = V'_{\rm roll} + V'_{\rm br} = -\dfrac{\Lambda^4}{f_{\rm eff}} + \dfrac{\mu_b^2 |h|^2}{f} \sin(\phi/f)$

Stopping point determines the EW scale: $\quad \dfrac{\Lambda^4}{f_{\rm eff}} = \dfrac{\mu_b^2 v_{\rm EW}^2}{f}$

HOW IS RELAXION RELAXED ?

$$V(\phi,h) = \left(\Lambda^2 - \Lambda^2\frac{\phi}{f_{\rm eff}}\right)|h|^2 - \underbrace{\frac{\Lambda^4}{f_{\rm eff}}\phi}_{V_{\rm roll}} - \underbrace{\mu_b^2|h|^2\cos(\phi/f)}_{V_{\rm br}}$$

$$v^2(\phi) = \begin{cases} 0 \text{ when } \phi < f_{\rm eff} \\ > 0 \text{ when } \phi > f_{\rm eff} \end{cases}$$

Relaxion stopping point determines the EW scale $\displaystyle \frac{\Lambda^4}{f_{\rm eff}} = \frac{\mu_b^2 v_{\rm EW}^2}{f}$

Higgs mass is finely scanned

$$\text{for } \Delta\phi = 2\pi f\,, \quad \frac{\Delta v^2}{v_{\rm EW}^2} \sim \frac{\Lambda^2}{f_{\rm eff}}\frac{f}{v_{\rm EW}^2} \sim \frac{\mu_b^2}{\Lambda^2} \ll 1$$

resolution of the scanner

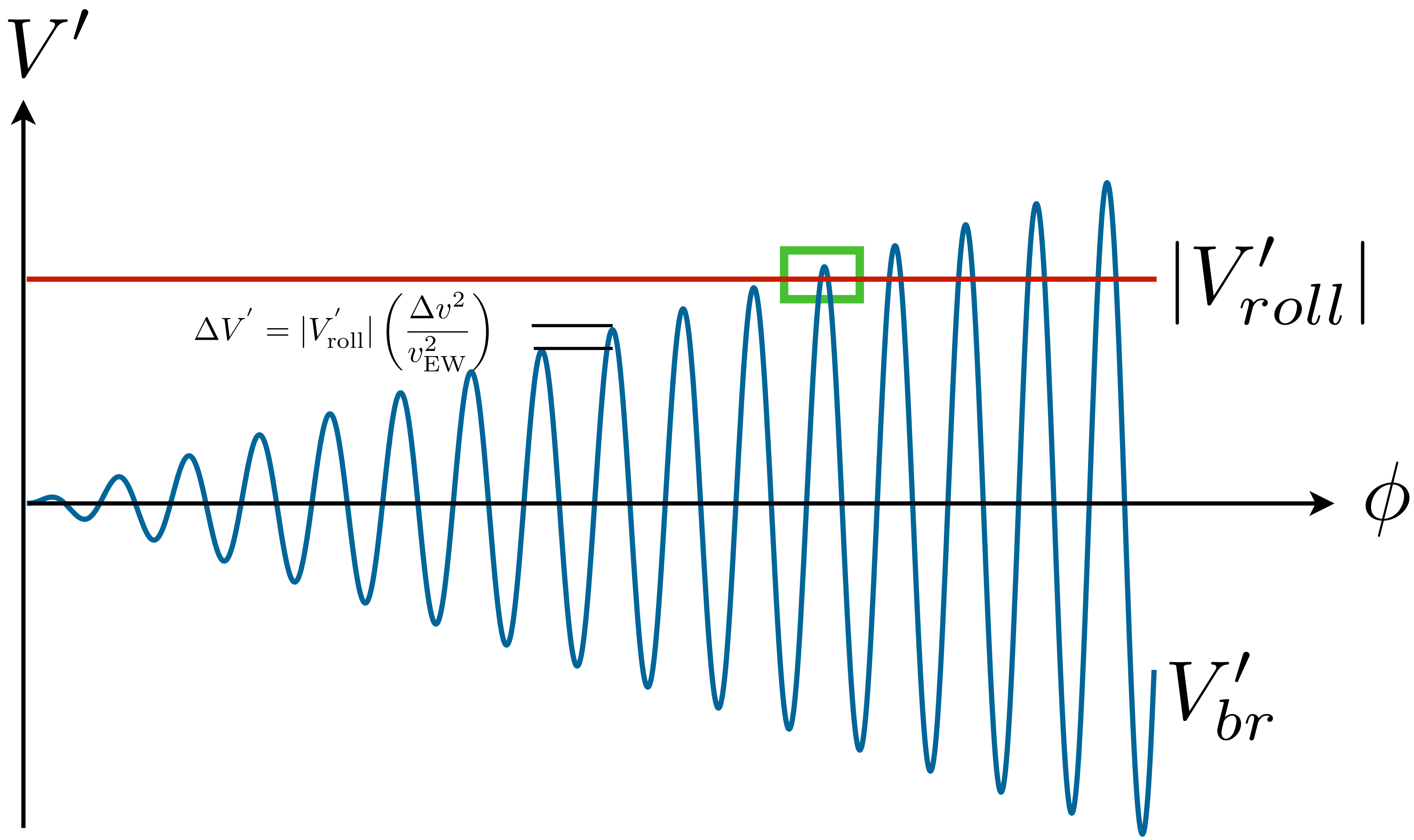
Potential height increases incrementally and relaxion stops at the shallow part of the potential

Slide by Hyungjin Kim

THE STOPPING POINT

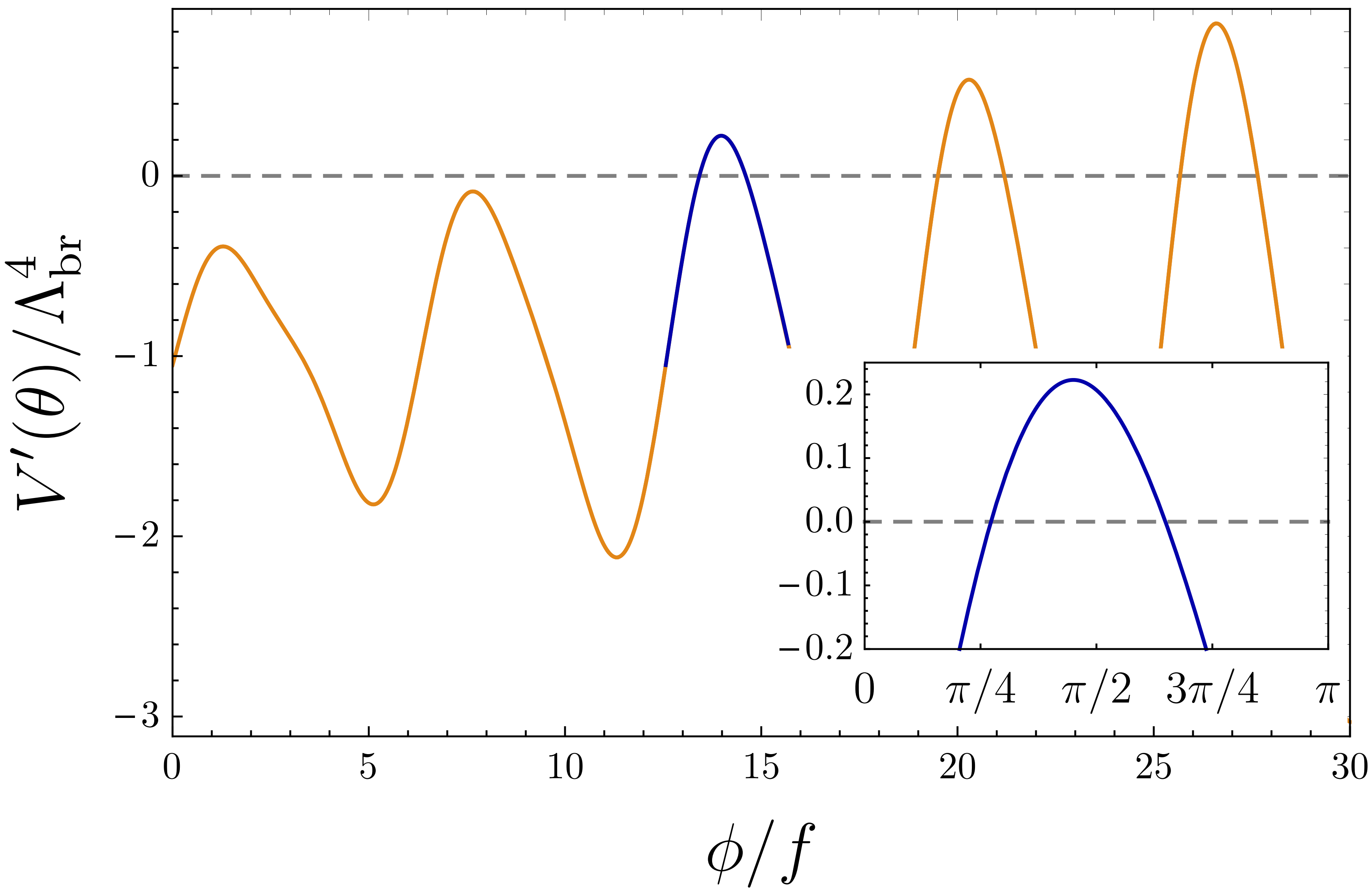
$$V'_\phi = 0 \Rightarrow \sin\theta = \frac{v_{\rm EW}^2}{v^2(\phi)} + \frac{v_{\rm EW}^2}{\Lambda^2}$$

$$\frac{\phi_0}{f} \sim \frac{\pi}{2} \text{ upto resolution factors}$$

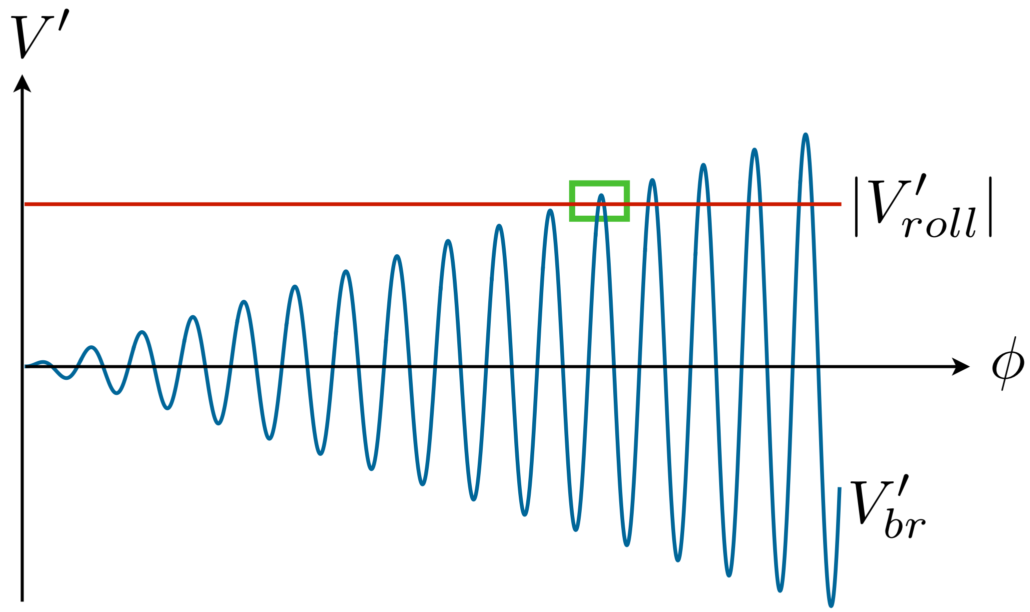


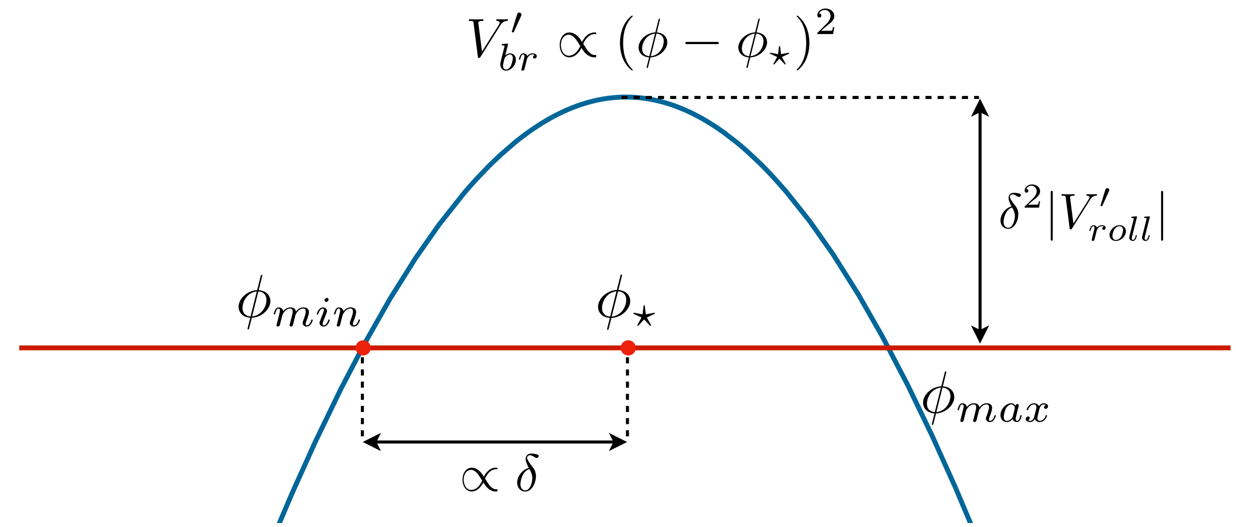
THE STOPPING POINT

Potential height increases incrementally and relaxion stops at the shallow part of the potential



THE STOPPING POINT



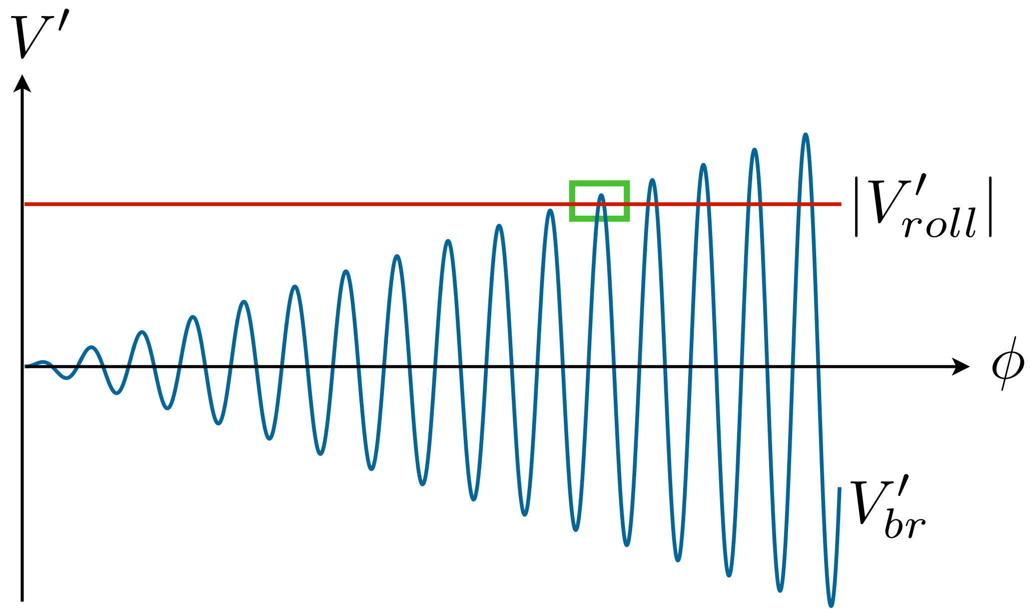


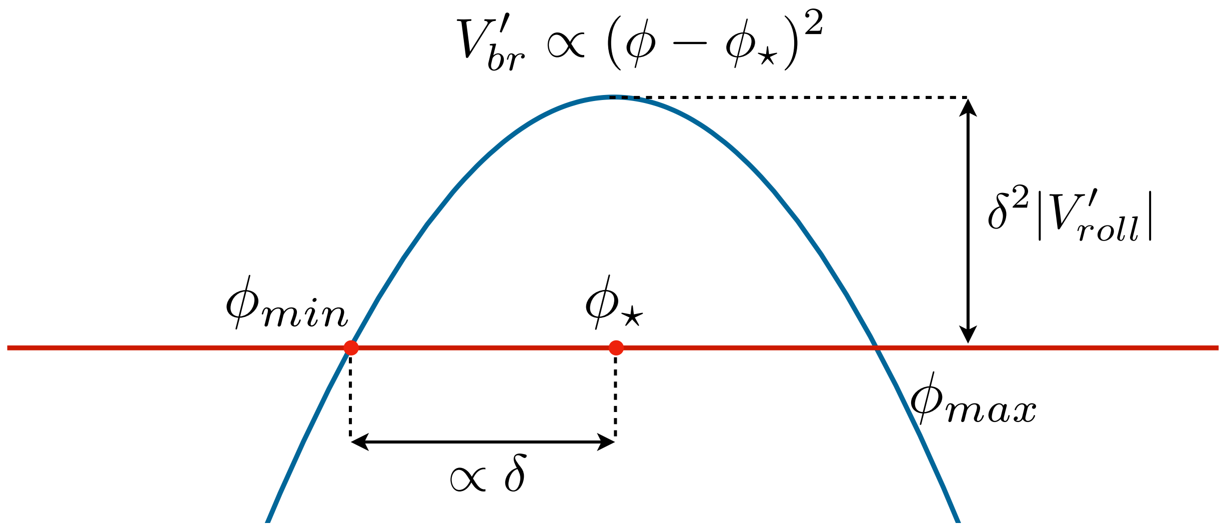
$$\frac{V'(\phi)}{|V'_{\rm roll}|} = \left(\frac{\Delta v^2}{v_{\rm EW}^2}\right) + \frac{1}{2}\left(\frac{\phi}{f} - \frac{\phi_*}{f}\right)^2 + \cdots$$

$$V'_{\rm roll} = \frac{\Lambda^4}{f_{\rm eff}} \sim \frac{\mu_{\rm b}^2 v_{\rm EW}^2}{f}$$

$$V'(\phi_{\rm min}) = 0 \Rightarrow \left(\frac{\phi_{\rm min}}{f} - \frac{\phi_*}{f}\right) \sim \sqrt{\frac{\Delta v^2}{v_{\rm EW}^2}}$$

THE STOPPING POINT





$$V'_{\rm roll} = \frac{\Lambda^4}{f_{\rm eff}} \sim \frac{\mu_{\rm b}^2 v_{\rm EW}^2}{f}$$

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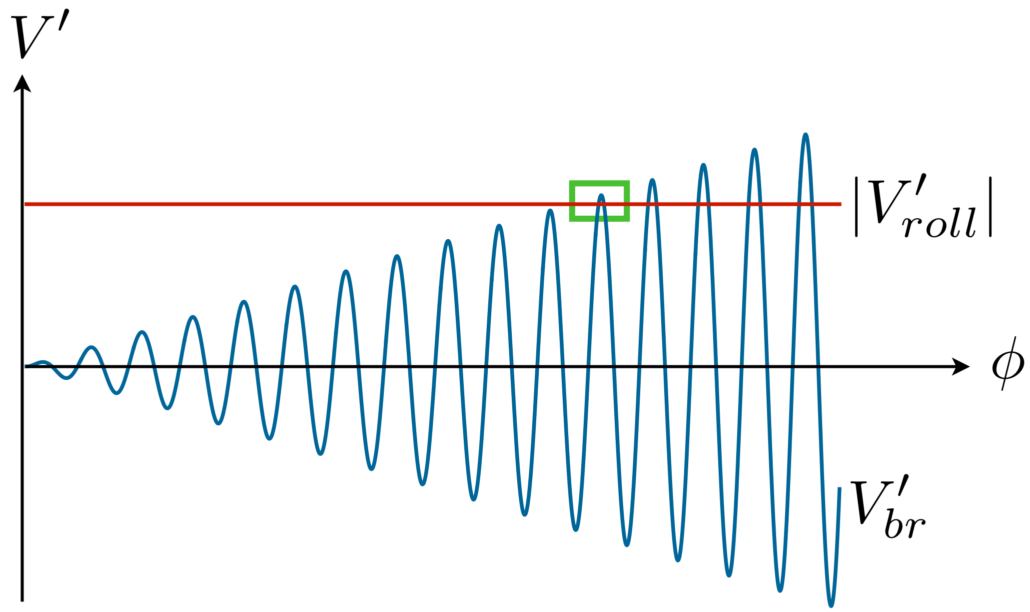
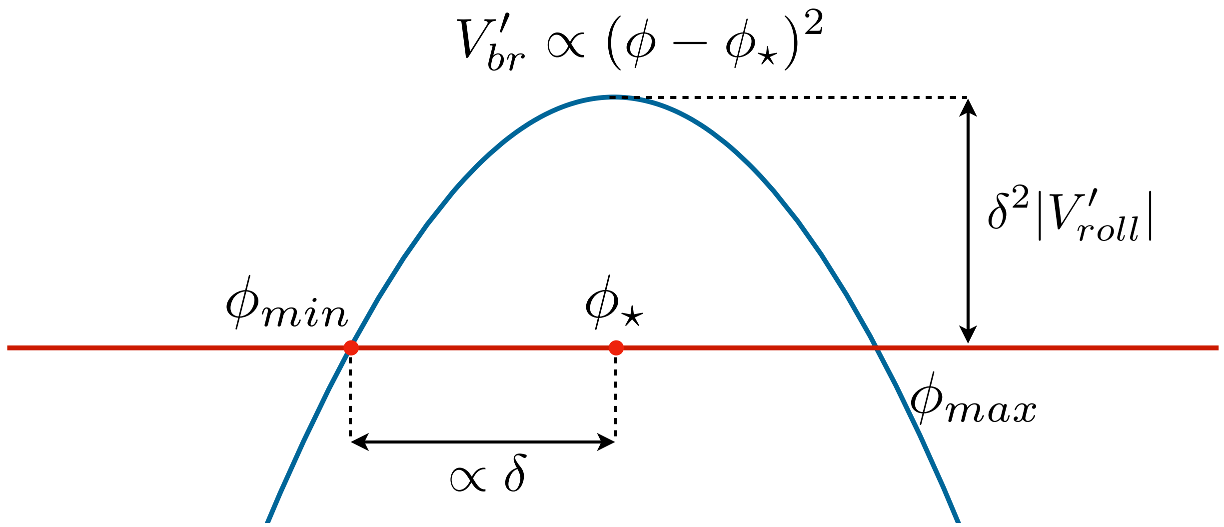
$$V'(\phi_{\rm min}) = 0 \Rightarrow \left(\frac{\phi_{\rm min}}{f} - \frac{\phi_*}{f}\right) \sim \sqrt{\frac{\Delta v^2}{v_{\rm EW}^2}}$$

$$\boxed{m_\phi^2 = V''(\phi_{\rm min}) = |V'_{\rm roll}|\frac{1}{f}\left(\frac{\phi_{\rm min}}{f} - \frac{\phi_*}{f}\right) = \frac{\mu_b^2 v_{\rm EW}^2}{f^2}\sqrt{\frac{\Delta v^2}{v_{\rm EW}^2}} = \frac{\mu_b^2 v_{\rm EW}^2}{f^2}\frac{\mu_b}{\Lambda}}$$

THE STOPPING POINT





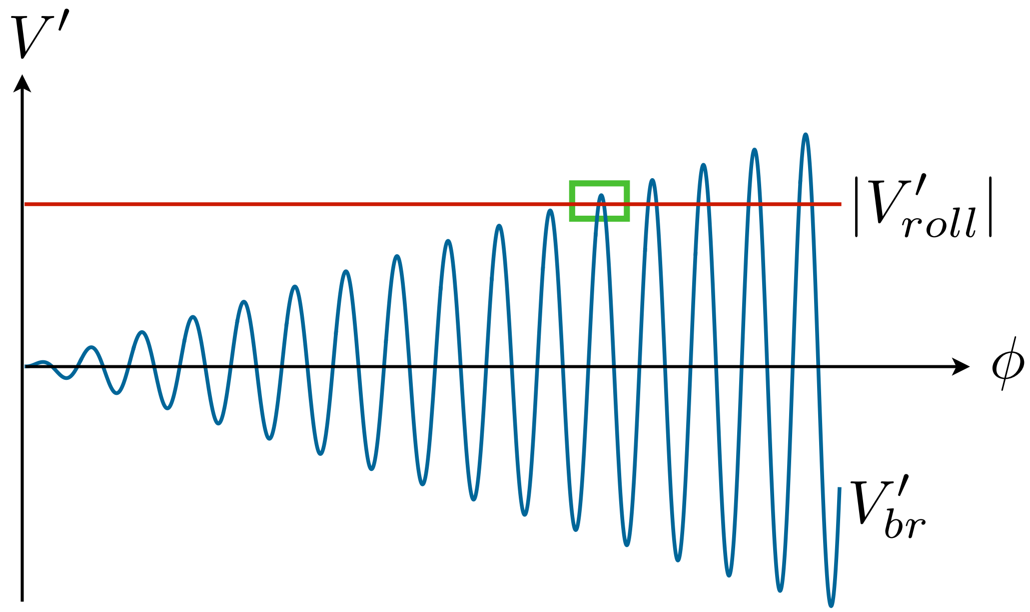
$$V'_{\text{roll}} = \frac{\Lambda^4}{f_{\text{eff}}} \sim \frac{\mu_{\text{b}}^2 v_{\text{EW}}^2}{f}$$

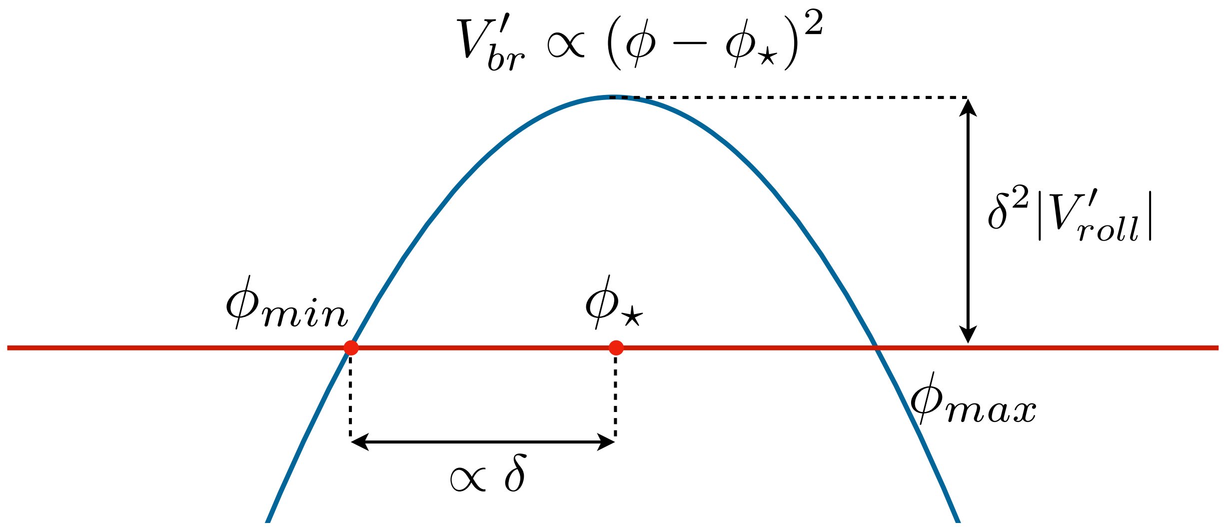
$$\frac{V'(\phi)}{|V'_{\text{roll}}|} = \left(\frac{\Delta v^2}{v_{\text{EW}}^2}\right) + \frac{1}{2}\left(\frac{\phi}{f} - \frac{\phi_*}{f}\right)^2 + \cdots$$

$$V'(\phi_{\text{min}}) = 0 \Rightarrow \left(\frac{\phi_{\text{min}}}{f} - \frac{\phi_*}{f}\right) \sim \sqrt{\frac{\Delta v^2}{v_{\text{EW}}^2}}$$

$$\boxed{m_\phi^2 = V''(\phi_{\text{min}}) = |V'_{\text{roll}}| \frac{1}{f}\left(\frac{\phi_{\text{min}}}{f} - \frac{\phi_*}{f}\right) = \frac{\mu_b^2 v_{\text{EW}}^2}{f^2}\sqrt{\frac{\Delta v^2}{v_{\text{EW}}^2}} = \frac{\mu_b^2 v_{\text{EW}}^2}{f^2}\frac{\mu_b}{\Lambda}}$$

THE STOPPING POINT





$$V'_{\text{roll}} = \frac{\Lambda^4}{f_{\text{eff}}} \sim \frac{\mu_b^2 v_{\text{EW}}^2}{f}$$

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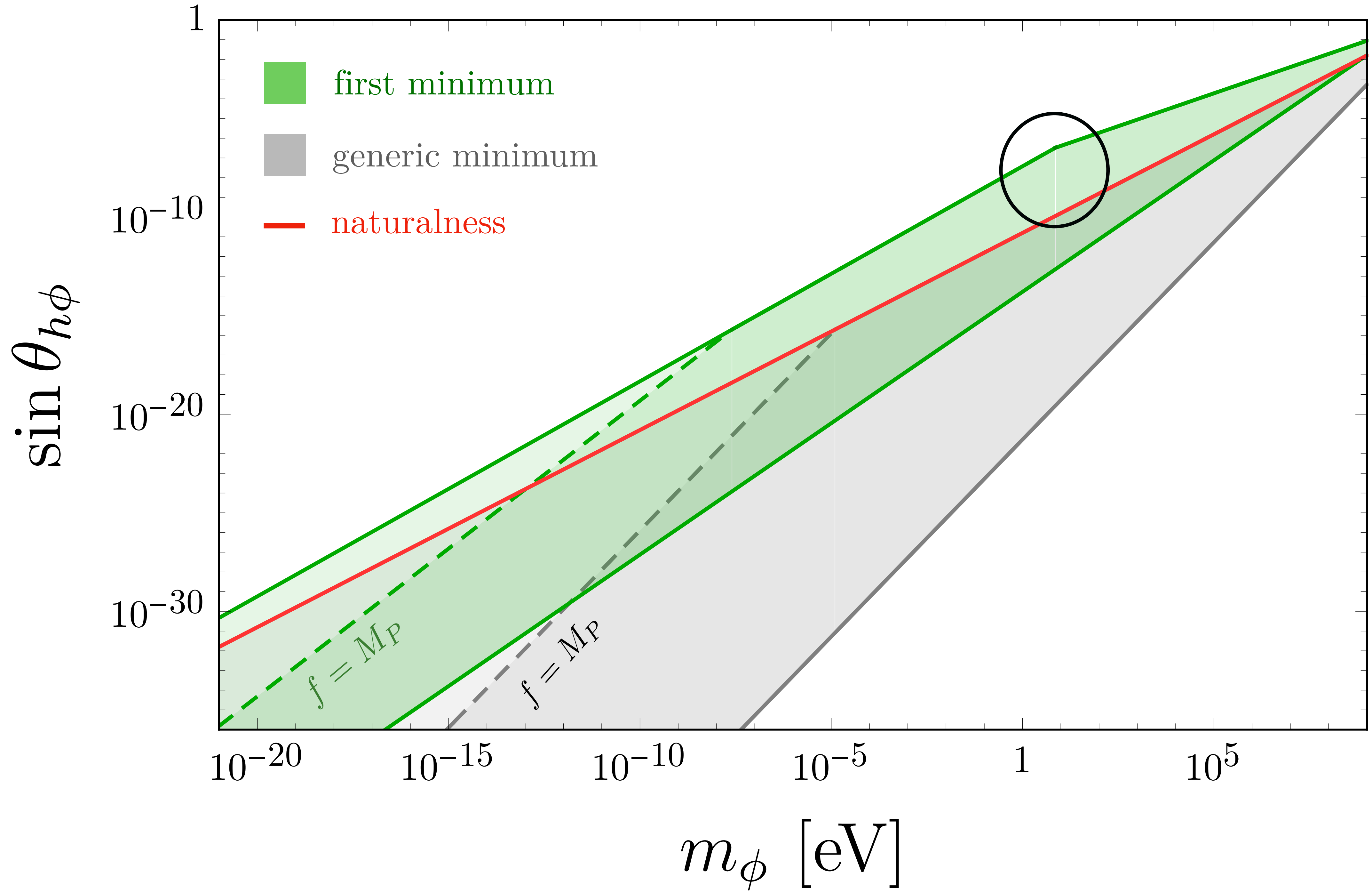
$$\boxed{\text{Mixing Angle} \quad \sin\theta_{h\phi} = \frac{\mu_b^2}{f v_{\text{EW}}}}$$

RELAXION PARAMETER SPACE



ENHANCEMENT FACTOR