Functional Prescription for STrEAMlined EFT Matching

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Based on 2011.02484, 2012.07851 (w/ Tim Cohen, Xiaochuan Lu)

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EFT matching

m_Φ heavy fields (e.g. BSM states $\sim$ TeV)

energy scales of experimental interest (e.g. LHC energies)

m_ϕ light fields (e.g. SM particles)

$\mathscr{L}_{\mathrm{UV}}[\Phi, \phi]$

match & run (resum $\log \frac{m_\Phi}{m_\phi}$)

$\mathscr{L}_{\mathrm{EFT}}[\phi]$

Many pheno applications (e.g. SMEFT).

Highly desirable to have efficient algorithms for EFT matching.

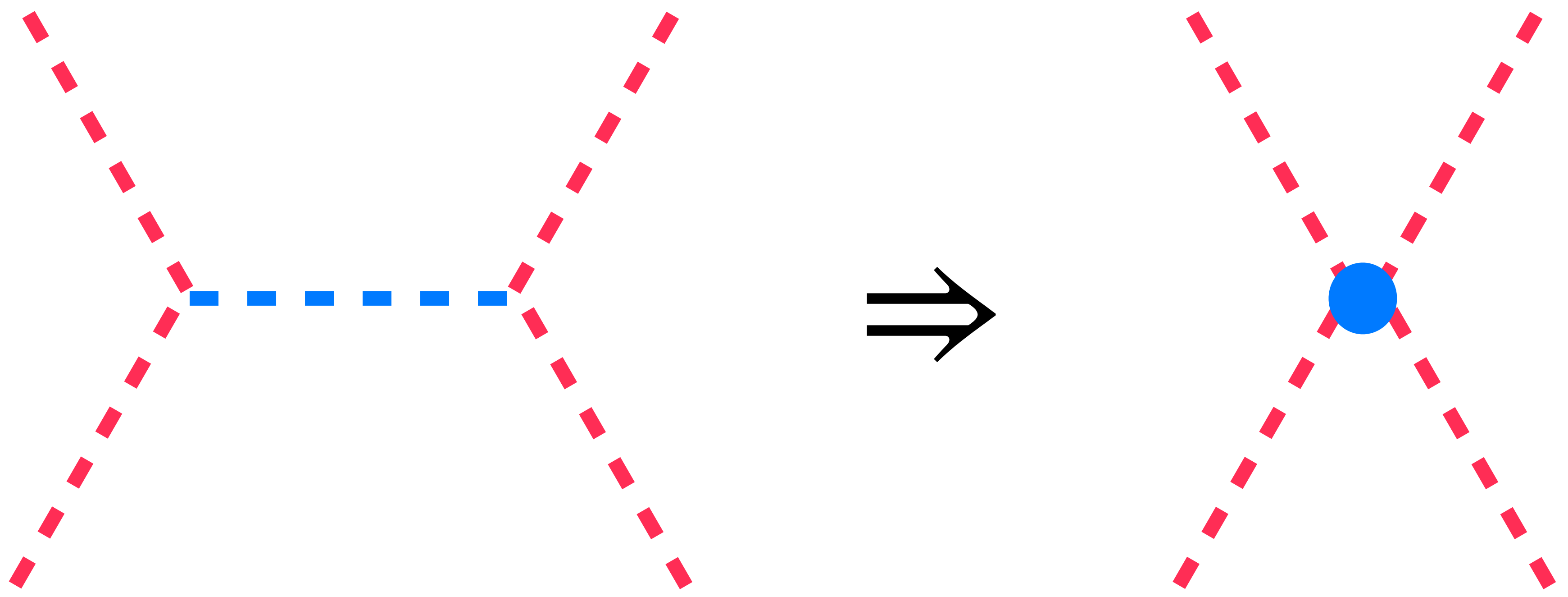
Outline

- What is functional matching, and what is new?
- The prescription.
- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.
- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

A toy model

- $\mathscr{L}_{\text{UV}} = \underbrace{\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2}_{\text{heavy scalar}} + \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{light scalar}} + \underbrace{\frac{1}{2}\kappa\,\Phi\,\phi^2}_{\text{interaction}}$

- $\mathscr{L}_{\text{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \boxed{\frac{1}{4!}c_1\,\phi^4 + \frac{1}{4}c_2\,\phi^2\partial^2\phi^2 + \ldots}$



A toy model

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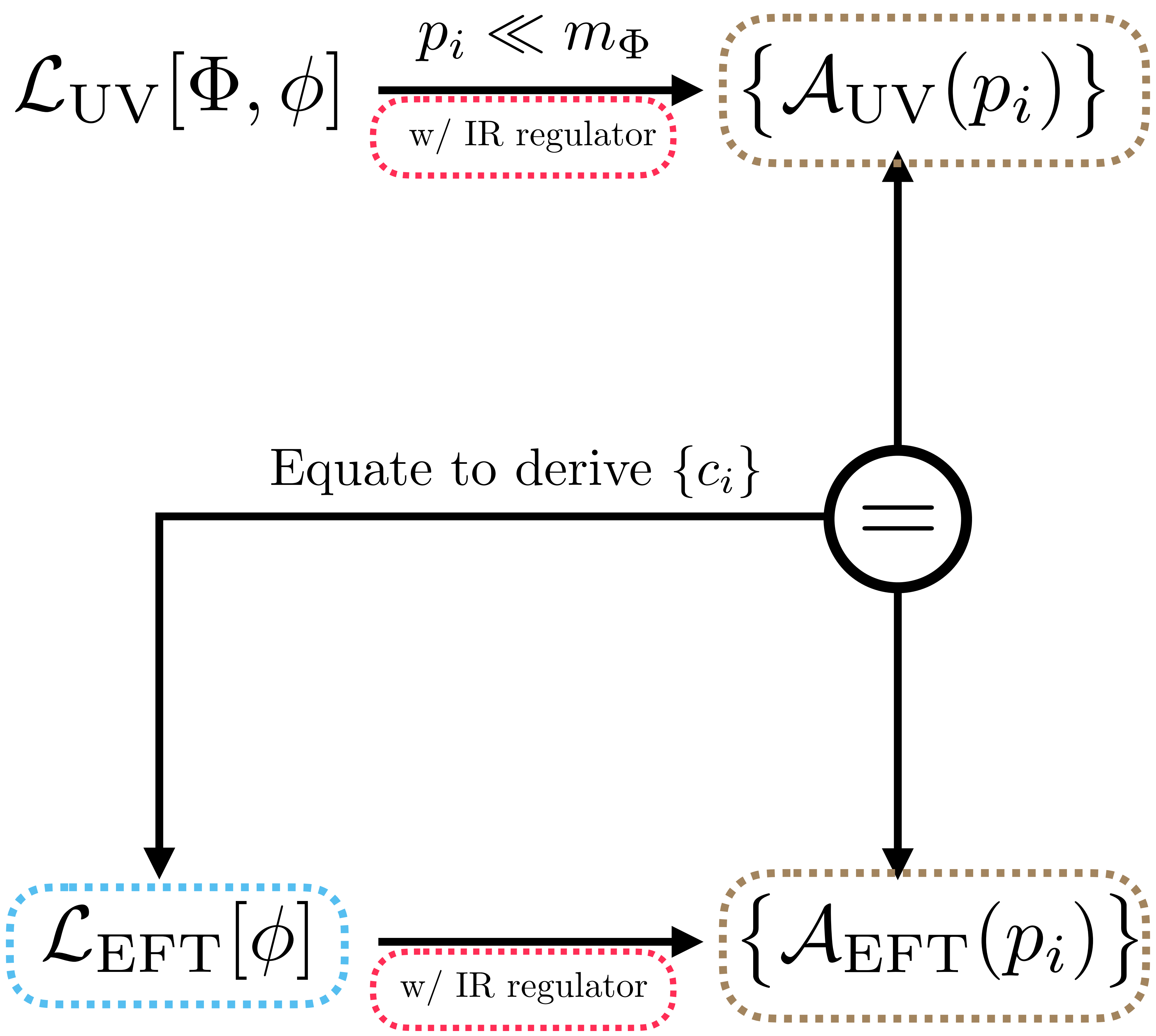
- $$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \boxed{\frac{1}{4!}c_1\,\phi^4 + \frac{1}{4}c_2\,\phi^2\partial^2\phi^2 + \ldots}$$

$$\mathscr{A}^{(\mathrm{tree})}_{\mathrm{UV}}(\phi\phi \to \phi\phi) = -\kappa^2\left(\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2}\right) = \frac{3\kappa^2}{M^2} + \frac{\kappa^2}{M^4}(s+t+u) + \ldots$$

$$\mathscr{A}^{(\mathrm{tree})}_{\mathrm{EFT}}(\phi\phi \to \phi\phi) = c_1 - c_2\,(s+t+u) + \ldots$$

$$\Rightarrow \quad c_1^{(\mathrm{tree})} = \frac{3\kappa^2}{M^2}, \quad c_2^{(\mathrm{tree})} = -\frac{\kappa^2}{2M^4}.$$

What we have just done is “amplitude matching”



Identify a set of amplitudes.

Work out a basis of EFT operators.

Keep track of IR details.

➤ Becomes cumbersome with more (higher-spin) fields, at higher operator dimensions, and higher loop orders.

A (familiar) more efficient approach

$$\mathcal{L}_{\text{UV}}[\Phi, \phi]$$

$$\Big\downarrow \quad \text{Equation of motion (EOM): } \frac{\delta \mathscr{L}_{\text{UV}}}{\delta \Phi} = 0\,.$$

$$\Phi_{\text{c}}[\phi] \qquad \text{(EOM solution)}$$

$$\Big\downarrow \quad \text{Set } \Phi = \Phi_{\text{c}}[\phi]$$

$$\mathcal{L}_{\text{EFT}}[\phi]$$

A (familiar) more efficient approach

$$\mathscr{L}_{\rm UV} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\kappa\,\Phi\,\phi^2$$

$\mathcal{L}_{\rm UV}[\Phi, \phi]$

Equation of motion (EOM): $\dfrac{\delta \mathscr{L}_{\rm UV}}{\delta \Phi} = 0\,.$ $\Rightarrow \left(\partial^2 + M^2\right)\Phi = \frac{1}{2}\kappa\,\phi^2$

$\Phi_{\rm c}[\phi]$ (EOM solution) $\Rightarrow \Phi_{\rm c}[\phi] = \frac{1}{2}\kappa\,\frac{1}{\partial^2 + M^2}\,\phi^2 = \frac{\kappa}{2M^2}\left(\phi^2 - \frac{1}{M^2}\partial^2\phi^2 + \dots\right)$

Set $\Phi = \Phi_{\rm c}[\phi]$

$\mathcal{L}_{\rm EFT}[\phi]$

$$\mathscr{L}_{\rm EFT}^{\rm (tree)} = \mathscr{L}_{\rm UV}\left[\Phi_{\rm c}[\phi]\,,\,\phi\right] = \frac{3\kappa^2}{M^2}\,\frac{1}{4!}\,\phi^4 - \frac{\kappa^2}{2M^4}\,\frac{1}{4}\,\phi^2\partial^2\phi^2 + \dots$$

➤ This is nothing but functional matching at tree level.

Functional matching

- Match generating functionals of amplitudes, rather than amplitudes themselves.
- Path integral:

$$Z[J] = e^{-iE[J]} = \int D\varphi \, e^{i\int d^dx\,(\mathscr{L}[\varphi]+J\varphi)}$$

Generating functional of connected correlation functions: $\langle\varphi^n\rangle_{\text{conn}} = (-i)^{n+1}\dfrac{\delta^n E}{\delta J^n}$.

- Simpler to work with its Legendre transform $\Rightarrow$ 1PI effective action:

$$\Gamma\big[\langle\varphi\rangle\big] = -E[J] - \int d^dx\, J\,\langle\varphi\rangle \qquad \text{where } \langle\varphi\rangle \text{ and } J \text{ are related by } \langle\varphi\rangle = -\frac{\delta E}{\delta J}\,, \quad J = -\frac{\delta\Gamma}{\delta\langle\varphi\rangle}$$

Generating functional of 1PI correlation functions: $\langle\varphi^n\rangle_{\text{1PI}} = i\,\dfrac{\delta^n\Gamma}{\delta\langle\varphi\rangle^n}$.

Wish to match between UV theory and EFT.

Matching 1PI effective actions

tree level matching

one-loop matching

$$\Gamma_{\mathrm{UV,L}}[\phi] = \boxed{\int d^dx\, \mathscr{L}_{\mathrm{UV}}\big[\Phi_c[\phi],\phi\big]} + \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) + \dots$$

1LPI

$$\Gamma_{\mathrm{EFT}}[\phi] = \boxed{\int d^dx\, \mathscr{L}^{(\mathrm{tree})}_{\mathrm{EFT}}[\phi]} + \int d^dx\, \mathscr{L}^{(1-\mathrm{loop})}_{\mathrm{EFT}}[\phi] + \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}^{(\mathrm{tree})}_{\mathrm{EFT}}}{\delta\phi^2}\right) + \dots$$

one-loop-generated operators used in tree graphs

tree-level-generated operators used in one-loop graphs

$$\Rightarrow \int d^dx\, \mathscr{L}^{(1-\mathrm{loop})}_{\mathrm{EFT}}[\phi] = \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) - \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}^{(\mathrm{tree})}_{\mathrm{EFT}}}{\delta\phi^2}\right)$$

$$\varphi = \{\,\Phi\,,\,\phi\,\}$$

Functional matching at one loop

$$\int d^dx\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] = \frac{i}{2}\log \text{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\text{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) - \boxed{\frac{i}{2}\log \text{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\text{EFT}}^{(\text{tree})}}{\delta\phi^2}\right)}$$

- If $-\dfrac{\delta^2\mathscr{L}_{\text{UV}}}{\delta\varphi^2} = \begin{array}{cc} \begin{array}{cc} \Phi & \phi \end{array} & \\ \begin{pmatrix} \blacksquare & 0 \\ 0 & \blacksquare \end{pmatrix} & \begin{array}{c} \Phi \\ \phi \end{array} \end{array}$

  $\Rightarrow$ heavy loops

  $\Rightarrow$ light loops (also present in the EFT)

- Just compute heavy fields' contributions to the functional superdeterminant.

B. Henning, X. Lu, H. Murayama, 1412.1837.

Functional matching at one loop

$$\int d^dx\, \mathscr{L}_{\mathrm{EFT}}^{(1-\mathrm{loop})}[\phi] = \frac{i}{2} \log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right) - \boxed{\frac{i}{2} \log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{EFT}}^{(\mathrm{tree})}}{\delta \phi^2} \right)}$$

- If $-\dfrac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2} = \begin{array}{cc} \begin{array}{cc} \Phi & \phi \end{array} & \\ \begin{pmatrix} \blacksquare & ? \\ ? & \blacksquare \end{pmatrix} & \begin{array}{l} \Phi \\ \phi \end{array} \end{array}$

  Φ ⇒ heavy loops

  ϕ ⇒ light loops (also present in the EFT)

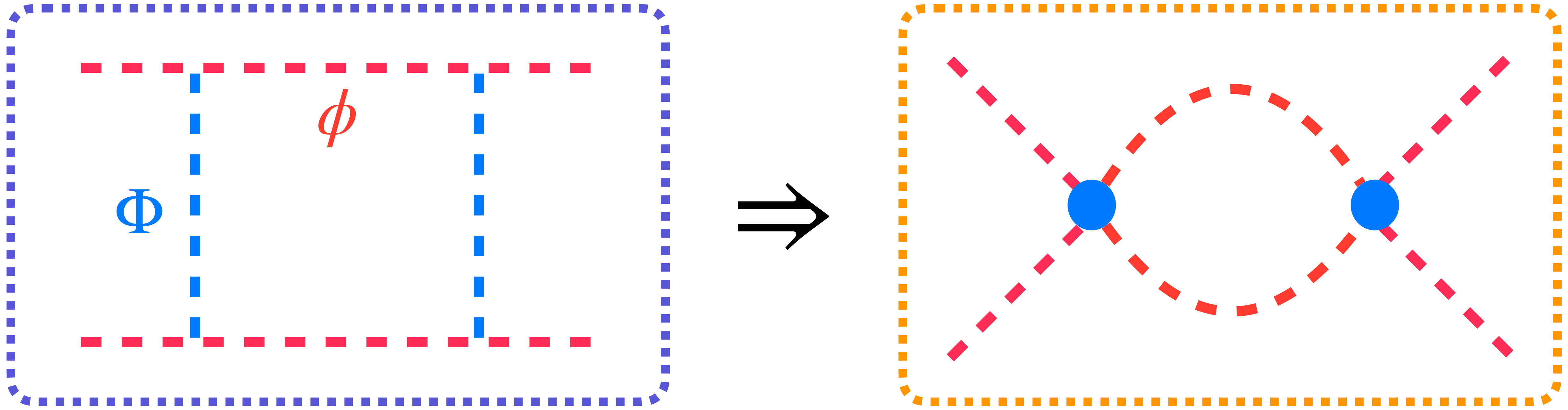
- Just compute heavy fields' contributions to the functional superdeterminant.
- More work is needed to include mixed heavy-light contributions.

B. Henning, X. Lu, H. Murayama, 1604.01019. S. A. R. Ellis, J. Quevillon, T. You, ZZ, 1604.02445.
J. Fuentes-Martin, J. Portoles, P. Ruiz-Femenia, 1607.02142. ZZ, 1610.00710.

Functional matching at one loop

$$\int d^d x\, \mathscr{L}_{\rm EFT}^{(1-\rm loop)}[\phi] = \boxed{\frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right)} - \boxed{\frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\rm EFT}^{(\rm tree)}}{\delta \phi^2}\right)}$$

➤ A closer look at the second term (tree-generated operators used in 1-loop graphs).



Functional matching at one loop

$$\int d^d x\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] = \boxed{\frac{i}{2}\log \text{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right)} - \boxed{\frac{i}{2}\log \text{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\text{EFT}}^{(\text{tree})}}{\delta \phi^2}\right)}$$

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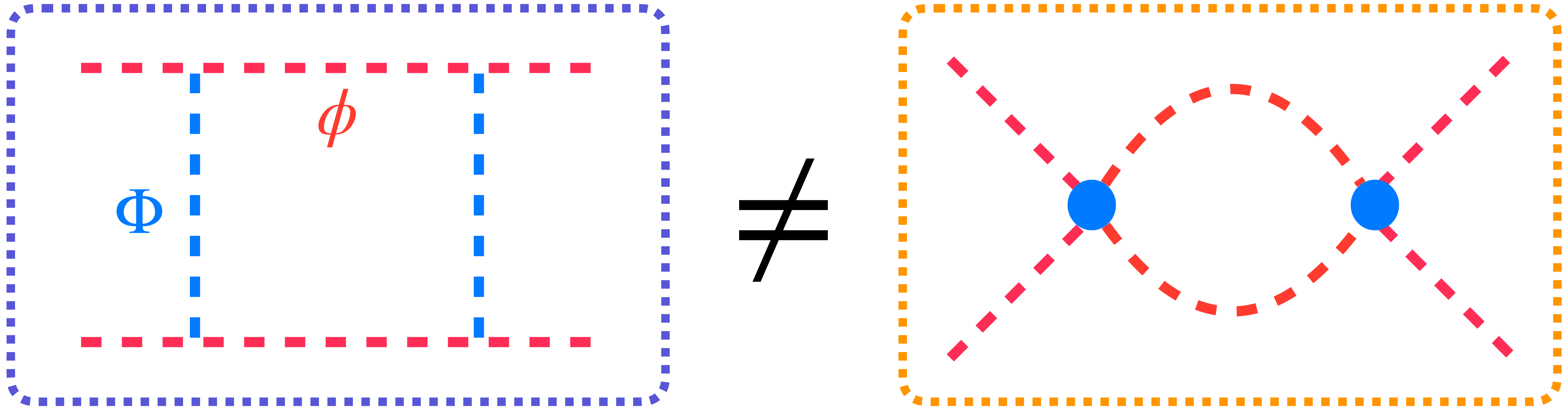

Difference goes into $\mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi]$.

Functional matching at one loop

$$\int d^d x\, \mathscr{L}_{\mathrm{EFT}}^{(1-\mathrm{loop})}[\phi] = \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) - \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{EFT}}^{(\mathrm{tree})}}{\delta \phi^2}\right)$$

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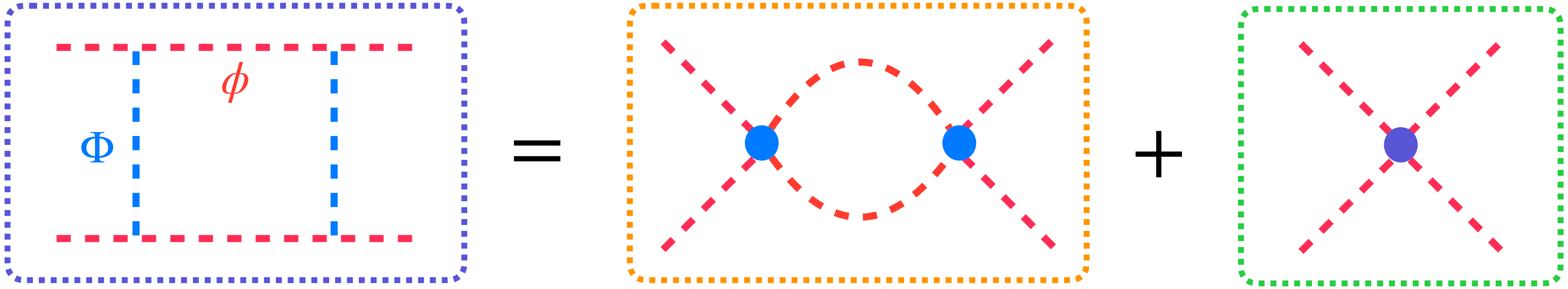

- This is nothing but the **method of regions**.
  - $\int d^d q = \int d^d q \Big|_{\mathrm{soft}} + \int d^d q \Big|_{\mathrm{hard}}$. Expand then integrate over full q space using DimReg.

$q \sim m_\phi$ $\Rightarrow$ heavy propagator shrinks to a point.

$q \sim m_\Phi$ $\Rightarrow$ entire loop shrinks to a point $\Rightarrow$ local operators in the EFT.

Functional matching at one loop

➤ Central formula:

$$\boxed{\int d^d x\, \mathscr{L}_{\mathrm{EFT}}^{(1-\mathrm{loop})}[\phi] \;=\; \frac{i}{2}\,\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_{\mathrm{c}}[\phi]}\right)\bigg|_{\mathrm{hard}}}$$

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Functional matching at one loop

$$\int d^dx\, \mathscr{L}^{(1-\mathrm{loop})}_{\mathrm{EFT}}[\phi] \;=\; \frac{i}{2}\,\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right)\bigg|_{\mathrm{hard}}$$

- What makes the functional approach powerful is an elegant procedure to compute this superdeterminant, which crucially relies upon:
  - **Method of regions.**
    - Just need hard region, no IR details. ☺
  - **Covariant derivative expansion (CDE).**
    - Work with D_μ , $\phi \Rightarrow$ gauge-invariant operators directly, no momentum-space Feynman rules. ☺
    - Extension of Coleman-Weinberg including derivatives.

Functional matching at one loop

$$\boxed{\int d^dx\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] \;=\; \frac{i}{2}\, \log \text{Sdet}\left(- \frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta\varphi^2} \bigg|_{\Phi=\Phi_c[\phi]} \right) \bigg|_{\text{hard}}}$$

- What makes the functional approach powerful is an elegant procedure to compute this superdeterminant, which crucially relies upon:
  - **Method of regions.**
  - **Covariant derivative expansion (CDE).**
- Quite some freedom on how to put these ingredients together + CDE can be tedious. ☹
  - Motivated new development: streamlined prescription + automated CDE calculation.

T. Cohen, X. Lu, ZZ, 2011.02484. T. Cohen, X. Lu, ZZ, 2012.07851.

see also: J. Fuentes-Martin, M. König, J. Pagès, A. E. Thomsen, F. Wilsch, 2012.08506.

Outline

- What is functional matching, and what is new? ✅
- The prescription.
- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.
- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

Two types of supertraces

$$
\begin{aligned}
\int d^dx\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] &= \frac{i}{2}\log \text{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\text{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right)\bigg|_{\text{hard}} \\
&= \frac{i}{2}\,\text{STr}\log\left(-\frac{\delta^2\mathscr{L}_{\text{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right)\bigg|_{\text{hard}} \\
&= \frac{i}{2}\,\text{STr}\log\boxed{\left(\mathbf{K}-\mathbf{X}\right)}\Big|_{\text{hard}} \\
&= \frac{i}{2}\,\text{STr}\log\mathbf{K}\Big|_{\text{hard}} + \frac{i}{2}\,\text{STr}\log\left(\mathbf{1}-\mathbf{K}^{-1}\mathbf{X}\right)\Big|_{\text{hard}} \\
&= \frac{i}{2}\boxed{\text{STr}\log\mathbf{K}\Big|_{\text{hard}}} - \frac{i}{2}\sum_{n=1}^{\infty}\boxed{\frac{1}{n}\,\text{STr}\left[\left(\mathbf{K}^{-1}\mathbf{X}\right)^n\right]\Big|_{\text{hard}}}
\end{aligned}
$$

“log-type” “power-type”

generic form of the quadratic action:

inverse propagator (**K**) – interaction (**X**)

K, X matrices in relativistic theories

- $\mathbf{K}$ is block-diagonal with elements:

$$K_i = \begin{cases} \boxed{P^2} - m_i^2 & \text{(spin-0)} \\ \gamma^\mu \boxed{P_\mu} - m_i & \text{(spin-1/2)} \\ -\eta^{\mu\nu}\left(\boxed{P^2} - m_i^2\right) & \text{(spin-1, Feynman gauge } \xi = 1) \end{cases}$$

where $P_\mu = iD_\mu$ is the hermitian covariant derivative.

- $\mathbf{X}$ admits a derivative expansion:

$$\mathbf{X}\left[\phi\,,\,P_\mu\right] \;=\; U[\phi] \;+\; \left(\boxed{P_\mu}\mathbf{Z}^\mu[\phi] + \overline{\mathbf{Z}}^\mu\boxed{P_\mu}\right) \;+\; \text{higher-derivative interactions}$$

These P_μ's are "open" covariant derivatives (act openly to the right),

as opposed to "closed" covariant derivatives [enclosed by "()"], e.g. $(P_\mu\phi) = i\,(D_\mu\phi) \equiv [P_\mu, \phi]$ (act just on ϕ).

Log-type supertraces

$$\frac{i}{2}\,\mathrm{STr}\log\mathbf{K}\Big|_{\text{hard}} = \frac{i}{2}\sum_i \mathrm{STr}\log K_i\Big|_{\text{hard}}$$

- Heavy fields: full STr = hard + $\cancel{\text{soft}}^{\,0\ (\text{scaleless})}$.
- Light fields: full STr = $\cancel{\text{hard}}^{\,0}$ + soft.

$$\Rightarrow \quad \frac{i}{2}\,\mathrm{STr}\log\mathbf{K}\Big|_{\text{hard}} = \frac{i}{2}\sum_{i\in\{\Phi\}} \mathrm{STr}\log K_i$$

(Add up contributions from all heavy fields Φ)

Log-type supertraces

- Results are universal. Using CDE (later in the talk) to carry out the **functional part** of the supertraces, we obtain:

$$\frac{i}{2}\,\mathrm{STr}\log\left(P^2-m^2\right)=\int \mathrm{d}^dx\,\frac{1}{16\pi^2}\,\boxed{\mathrm{tr}}\Bigg\{-\left(\frac{2}{\bar\epsilon}-\log\frac{m^2}{\mu^2}\right)\frac{1}{24}\,F_{\mu\nu}F^{\mu\nu} + \frac{1}{m^2}\left[-\frac{1}{120}\left(D^\mu F_{\mu\nu}\right)\left(D_\rho F^{\rho\nu}\right)-\frac{1}{180}\,i\,F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right]+\ldots\Bigg\},$$

$$\frac{i}{2}\,\mathrm{STr}\log\left(\not{P}-m\right)=\int \mathrm{d}^dx\,\frac{1}{16\pi^2}\,\boxed{\mathrm{tr}}\Bigg\{-\left(\frac{2}{\bar\epsilon}-\log\frac{m^2}{\mu^2}\right)\frac{1}{24}\,F_{\mu\nu}F^{\mu\nu} + \frac{1}{m^2}\left[-\frac{1}{60}\left(D^\mu F_{\mu\nu}\right)\left(D_\rho F^{\rho\nu}\right)+\frac{1}{360}\,i\,F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right]+\ldots\Bigg\}.$$

Remaining trace is over field components.

Log-type supertraces

Integrate out a heavy ...	Operator coefficients $\times 16\pi^2$		
	$\mathrm{tr}_G\big(F_{\mu\nu}F^{\mu\nu}\big)$	$\mathrm{tr}_G\big[(D^\mu F_{\mu\nu})\,(D_\rho F^{\rho\nu})\big]$	$\mathrm{tr}_G\big(iF_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\big)$
real scalar	$\frac{1}{24}\log\frac{m^2}{\mu^2}$	$-\frac{1}{120}\frac{1}{m^2}$	$-\frac{1}{180}\frac{1}{m^2}$
complex scalar	$\frac{1}{12}\log\frac{m^2}{\mu^2}$	$-\frac{1}{60}\frac{1}{m^2}$	$-\frac{1}{90}\frac{1}{m^2}$
Majorana fermion	$\frac{1}{6}\log\frac{m^2}{\mu^2}$	$-\frac{1}{15}\frac{1}{m^2}$	$\frac{1}{90}\frac{1}{m^2}$
Dirac fermion	$\frac{1}{3}\log\frac{m^2}{\mu^2}$	$-\frac{2}{15}\frac{1}{m^2}$	$\frac{1}{45}\frac{1}{m^2}$
real vector	$\frac{1}{6}\left(\log\frac{m^2}{\mu^2}+\frac{1}{2}\right)$	$-\frac{1}{30}\frac{1}{m^2}$	$-\frac{1}{45}\frac{1}{m^2}$
ghost	$-\frac{1}{12}\log\frac{m^2}{\mu^2}$	$\frac{1}{60}\frac{1}{m^2}$	$\frac{1}{90}\frac{1}{m^2}$

Table 1. Universal results for log-type supertraces up to dimension six.

trace over gauge indices:

$$\mathrm{tr}_G\big(F_{\mu\nu}F^{\mu\nu}\big) = C_\Phi\, g^2\, G^a_{\mu\nu}G^{a\mu\nu}\,,$$

$$\mathrm{tr}_G\big[(D^\mu F_{\mu\nu})\,(D_\rho F^{\rho\nu})\big] = C_\Phi\, g^2\big(D^\mu G^a_{\mu\nu}\big)^2\,,$$

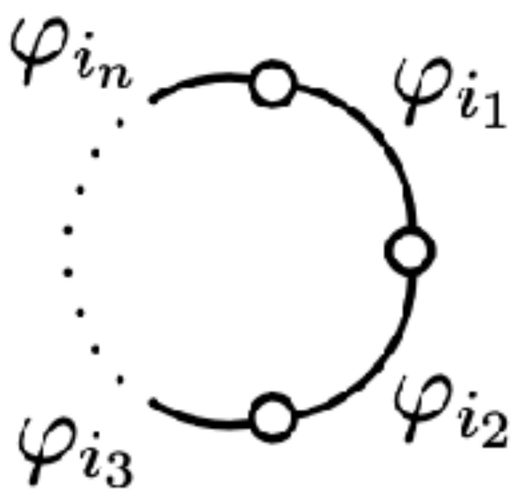
$$\mathrm{tr}_G\big(iF_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\big) = -C_\Phi\,\frac{1}{2}\,g^3 f^{abc} G^{a\nu}_\mu G^{b\rho}_\nu G^{c\mu}_\rho\,,$$

where C_Φ is defined by $\mathrm{tr}_G\big(T^a_\Phi\, T^b_\Phi\big) = C_\Phi\,\delta^{ab}$.

Power-type supertraces

$$-\frac{i}{2}\frac{1}{n}\,\mathrm{STr}\left[\left(\mathbf{K}^{-1}\mathbf{X}\right)^{n}\right]\Big|_{\mathrm{hard}} = -\frac{i}{2}\frac{1}{n}\sum_{i_1,\ldots,i_n}\mathrm{STr}\left(\boxed{\frac{1}{K_{i_1}}}\,\boxed{X_{i_1 i_2}}\,\boxed{\frac{1}{K_{i_2}}}\,\boxed{X_{i_2 i_3}}\cdots\boxed{\frac{1}{K_{i_n}}}\,\boxed{X_{i_n i_1}}\right)\Big|_{\mathrm{hard}}$$

- This has the structure of a 1-loop graph (propagators & vertices).



$$\equiv -\frac{i}{2}\boxed{\frac{1}{r}}\mathrm{STr}\left[\frac{1}{K_{i_1}}X_{i_1 i_2}\frac{1}{K_{i_2}}X_{i_2 i_3}\cdots\frac{1}{K_{i_n}}X_{i_n i_1}\right]\Big|_{\mathrm{hard}}$$

symmetry factor (graph has $\mathbb{Z}_r$ symmetry under rotation)

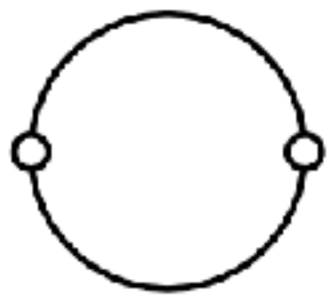
- Enumerate distinct graphs (just 1 topology).
- Finite # of graphs with $\dim X_{i_1 i_2} + \dim X_{i_2 i_3} + \cdots + \dim X_{i_n i_1} \leq$ desired operator dim in EFT.

Power-type supertraces

- Results depend on specific forms of X_{ij} .
  - Example:

Suppose: $K_i = P^2 - M^2$ (heavy), $K_j = P^2$ (massless),

$$X_{ij} = U_1 + \bar{Z}^\mu P_\mu \,, \;\; X_{ji} = U_2 + P_\mu Z^\mu .$$



$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{K_i}X_{ij}\,\frac{1}{K_j}X_{ji}\right)\bigg|_{\mathrm{hard}}$$

$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}U_2\right)\bigg|_{\mathrm{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}P_\mu Z^\mu\right)\bigg|_{\mathrm{hard}}$$

$$- \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}U_2\right)\bigg|_{\mathrm{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}P_\nu Z^\nu\right)\bigg|_{\mathrm{hard}} .$$

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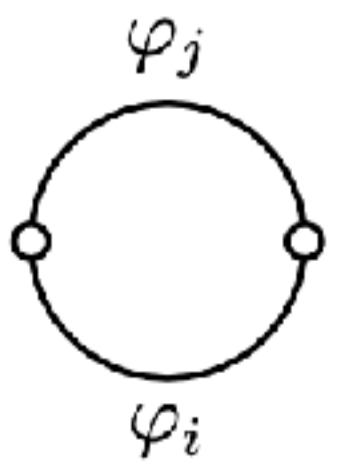

$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{K_i}X_{ij}\,\frac{1}{K_j}X_{ji}\right)\Big|_{\text{hard}}$$

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$$- \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}U_2\right)\Big|_{\text{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}P_\nu Z^\nu\right)\Big|_{\text{hard}}$$

1) carry out the functional part of the supertraces with CDE.

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\,U_1\,\frac{1}{P^2}\,U_2\right)\Big|_{\text{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\frac{1}{2}\left(1-\log\frac{M^2}{\mu^2}\right)U_1U_2 + \frac{1}{4M^2}\,(D^\mu U_1)\,(D_\mu U_2) + \ldots\right\},$$

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\,U_1\,\frac{1}{P^2}\,P_\mu Z^\mu\right)\Big|_{\text{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\frac{1}{4}\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)i\,U_1\,(D_\mu Z^\mu) + \ldots\right\},$$

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\,\bar{Z}^\mu P_\mu\,\frac{1}{P^2}\,U_2\right)\Big|_{\text{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{-\frac{1}{4}\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)i\,(D_\mu\bar{Z}^\mu)\,U_2 + \ldots\right\},$$

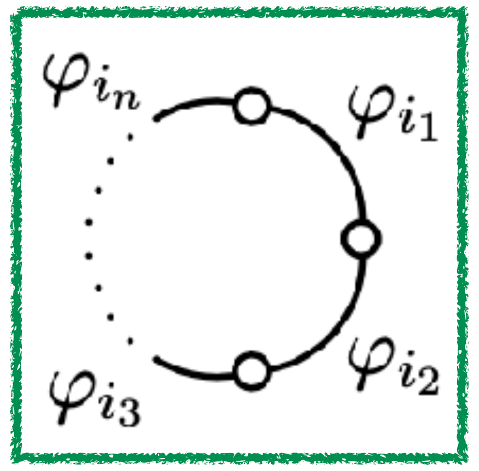
$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\,\bar{Z}^\mu P_\mu\,\frac{1}{P^2}\,P_\nu Z^\nu\right)\Big|_{\text{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\frac{1}{8}M^2\left(\frac{3}{2}-\log\frac{M^2}{\mu^2}\right)\bar{Z}^\mu Z_\mu\right.$$
$$-\frac{1}{8}\,i\,\bar{Z}^\mu F_{\mu\nu}Z^\nu - \frac{1}{24}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)(D^\mu\bar{Z}^\nu)\,(D_\mu Z_\nu)$$
$$\left.+\frac{1}{12}\left(\frac{1}{3}-\log\frac{M^2}{\mu^2}\right)\left[(D_\mu\bar{Z}^\mu)(D_\nu Z^\nu) + (D_\nu\bar{Z}^\mu)(D_\mu Z^\nu)\right] + \ldots\right\}.$$

2) substitute in explicit expressions of U_1, U_2, Z^μ, $\bar{Z}^\mu$ that are derived from $\mathscr{L}_{\text{UV}}$.

Summary of the prescription

$$
\begin{tikzcd}[column sep=small]
 & \mathcal{L}_{\mathrm{UV}}[\Phi, \phi] \arrow[dl] \arrow[dr] & \\
\boxed{\text{EOM}} \quad \Phi_{\mathrm{c}}[\phi] \arrow[rr] \arrow[dd] & & \boldsymbol{K}, \boldsymbol{X} \arrow[d, "\text{Enumerate}"] \\
 & & \text{Functional supertraces} \arrow[d, "\text{Evaluate}"] \\
\mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}[\phi] & & \mathcal{L}_{\mathrm{EFT}}^{(\text{1-loop})}[\phi]
\end{tikzcd}
$$

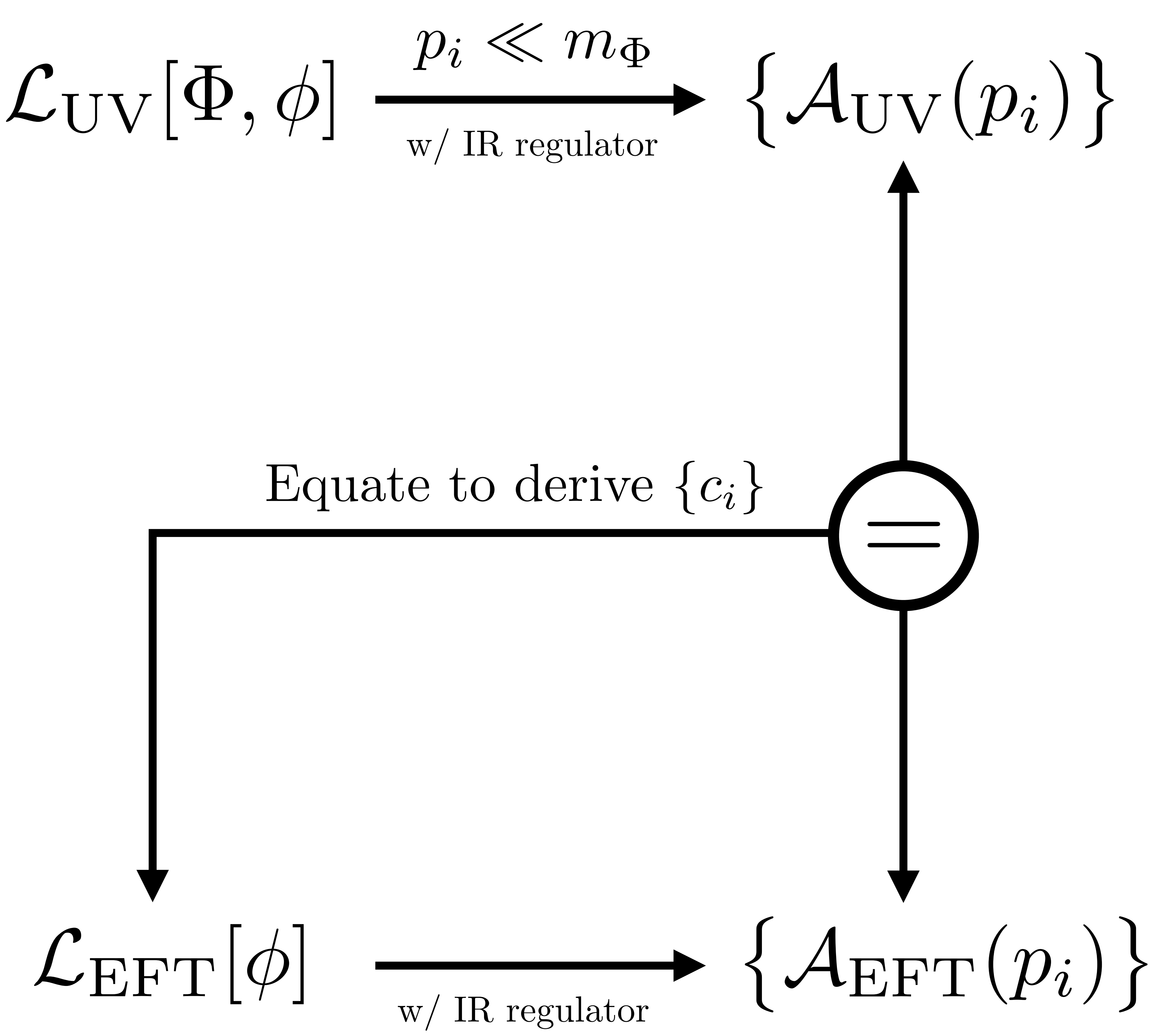
$$\boxed{-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_{\mathrm{c}}[\phi]} = \mathbf{K} - \mathbf{X}}$$



(automated) CDE + matrix algebra

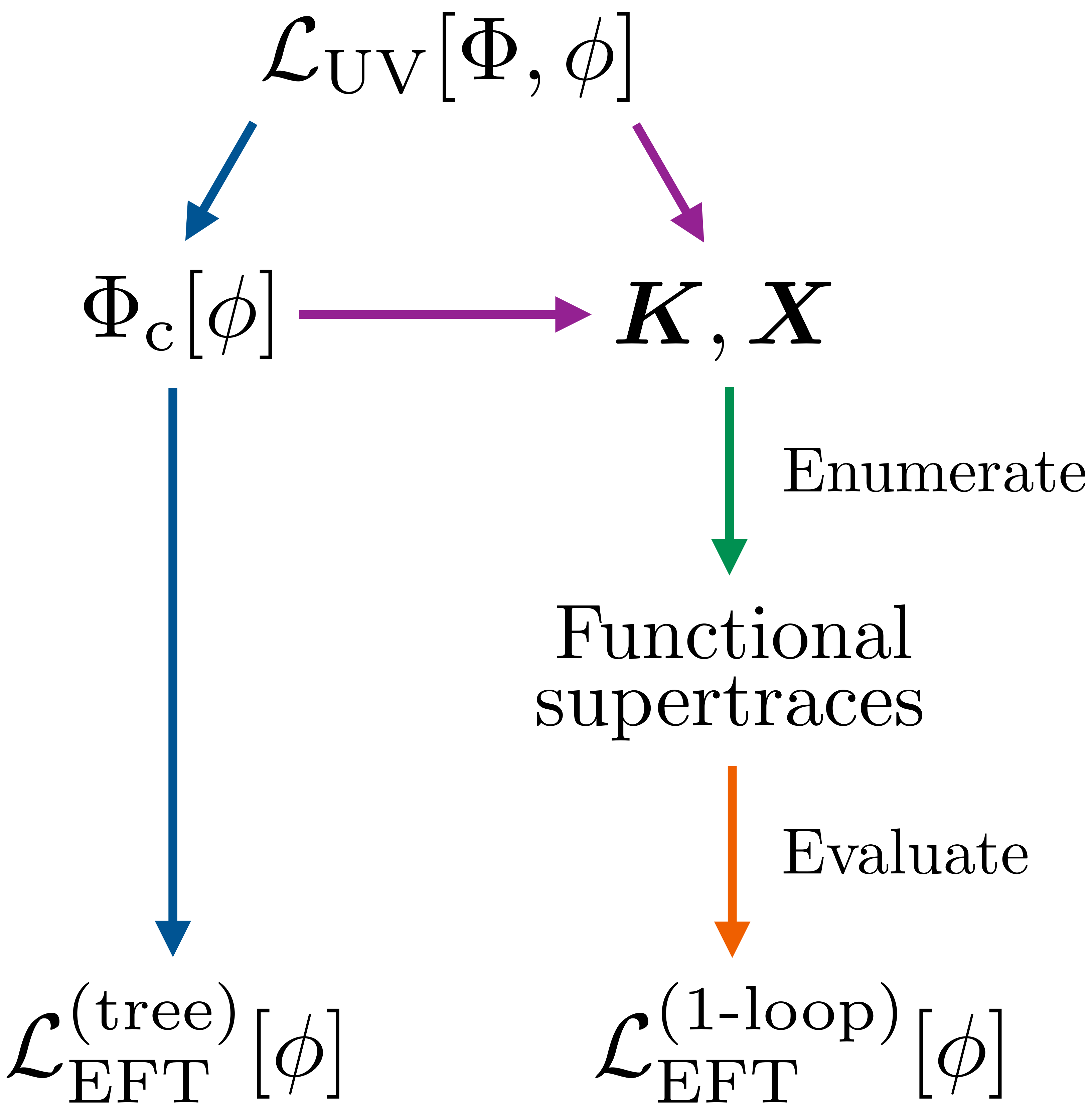
Amplitude matching

(with Feynman diagrams)



Functional matching

(our prescription)



No prior determination of operator basis.
No (amplitude) calculations within EFT.
No keeping track of IR details.

Outline

- What is functional matching, and what is new? ✓
- The prescription. ✓
- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.
- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

SM + heavy singlet scalar S

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

S. A. R. Ellis, J. Quevillon, T. You, ZZ, 1706.07765 (bosonic sector, functional matching)
M. Jiang, N. Craig, Y. Li, D. Sutherland, 1811.08878 (full dim-6, functional + amplitude matching)
U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini, A. Weiler, 2003.05936 (full dim-6, amplitude matching)

Step 1: tree-level matching

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

➤ Heavy field's EOM:

$$\frac{\delta S_{\rm UV}}{\delta S} = -A|H|^2 + \left(P^2 - M^2 - \kappa|H|^2\right) S - \frac{1}{2}\mu_S S^2 - \frac{1}{3!}\lambda_S S^3 = 0\,.$$

➤ Solve order by order:

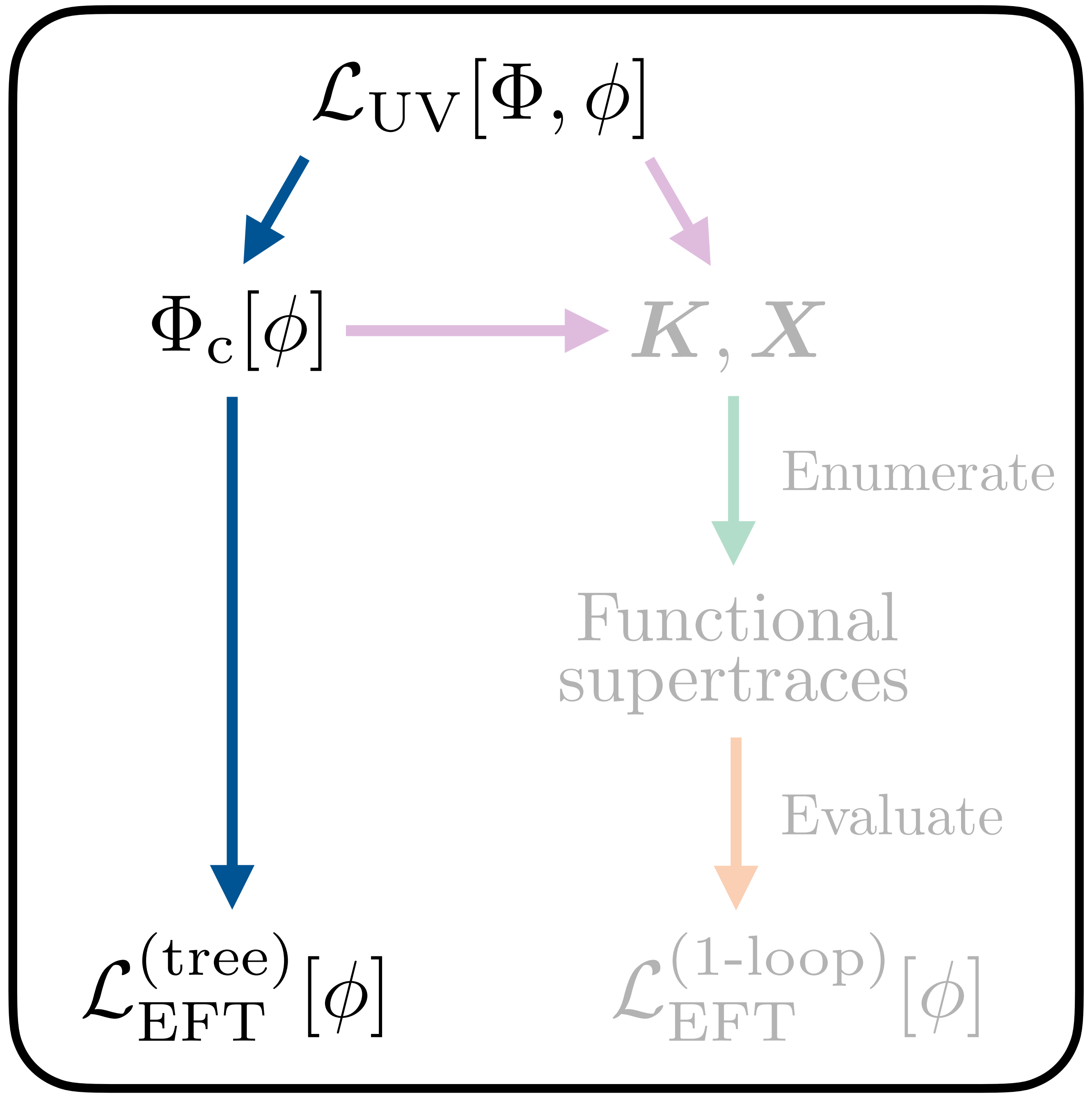
$$S_{\rm c} = S_{\rm c}^{(2)} + S_{\rm c}^{(4)} + S_{\rm c}^{(6)} + \ldots$$

$$S_{\rm c}^{(2)} = -\tfrac{A}{M^2}|H|^2\,,$$

$$S_{\rm c}^{(4)} = \tfrac{A}{M^4}\left[\left(\partial^2|H|^2\right) + \left(\kappa - \tfrac{\mu_S A}{2M^2}\right)|H|^4\right]\,,$$

$$S_{\rm c}^{(6)} = -\tfrac{A}{M^6}\left\{\left(\kappa - \tfrac{\mu_S A}{M^2}\right)|H|^2\left(\partial^2|H|^2\right) + \left[\left(\kappa - \tfrac{\mu_S A}{M^2}\right)\left(\kappa - \tfrac{\mu_S A}{2M^2}\right) - \tfrac{\lambda_S A^2}{6M^2}\right]|H|^6 + \partial^2\left[\left(\partial^2|H|^2\right) + \left(\kappa - \tfrac{\mu_S A}{2M^2}\right)|H|^4\right]\right\}.$$

$$\Rightarrow \quad \mathcal{L}_{\rm EFT}^{(\rm tree)} = \mathcal{L}_{\rm SM} + \frac{A^2}{2M^2}|H|^4 - \frac{A^2}{2M^4}|H|^2\left(\partial^2|H|^2\right) - \frac{A^2}{2M^4}\left(\kappa - \frac{\mu_S A}{3M^2}\right)|H|^6\,.$$



Step 2: derive K and X matrices

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Recall:

$$-\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\varphi^2} = \mathbf{K} - \mathbf{X} \qquad K_i = \begin{cases} P^2 - m_i^2 & \text{(spin-0)} \\ \gamma^\mu P_\mu - m_i & \text{(spin-1/2)} \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) & \text{(spin-1, Feynman gauge } \xi = 1) \end{cases}$$

- Field content:

$$\varphi_i \in \{\varphi_S\,, \quad \varphi_H\,, \quad \varphi_q\,, \quad \varphi_u\,, \quad \varphi_d\,, \quad \varphi_l\,, \quad \varphi_e\,, \quad \varphi_G\,, \quad \varphi_W\,, \quad \varphi_B\}\,,$$

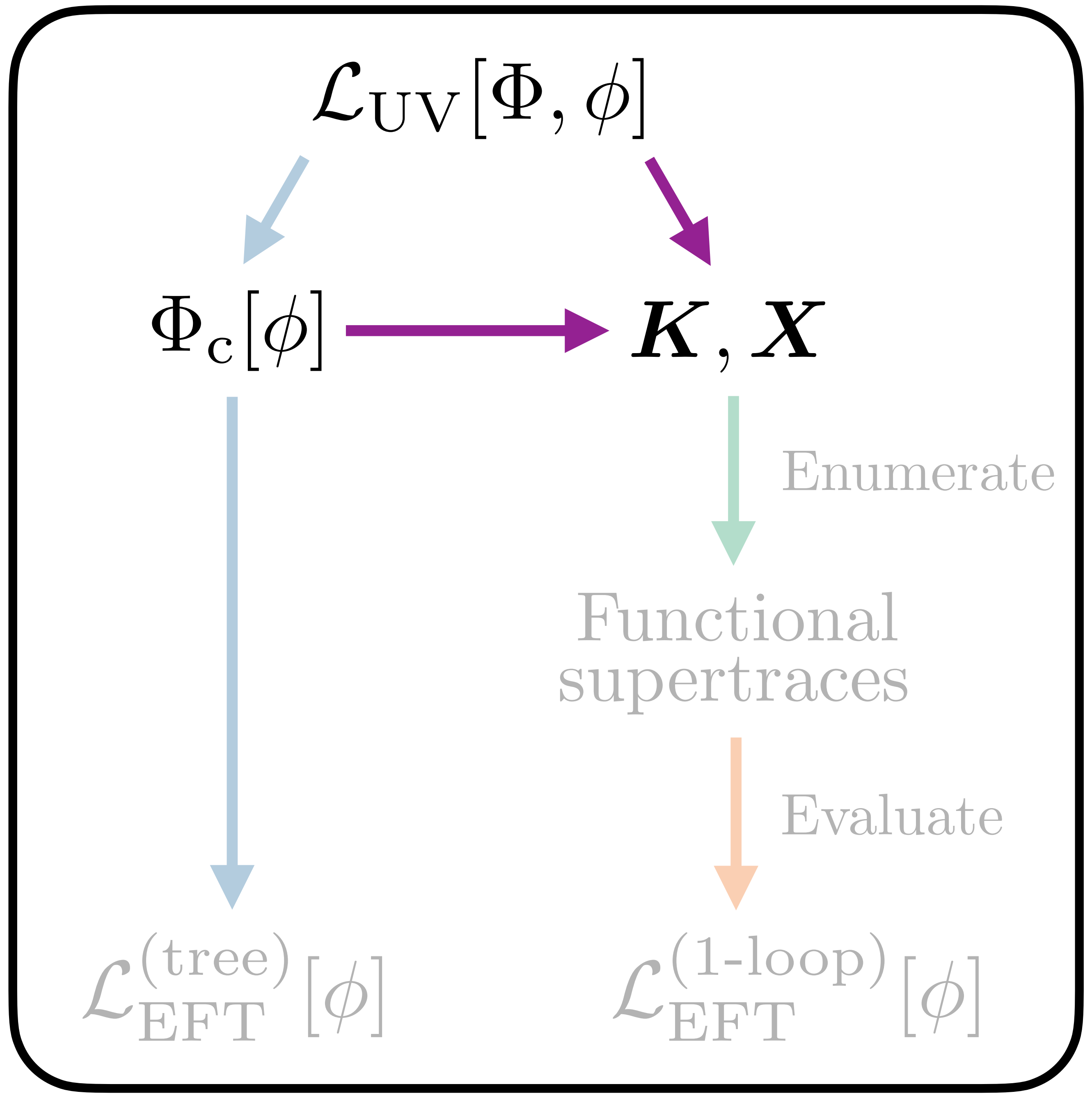
$$\boxed{\bar\varphi_i \in \{\bar\varphi_S\,, \quad \bar\varphi_H\,, \quad \bar\varphi_q\,, \quad \bar\varphi_u\,, \quad \bar\varphi_d\,, \quad \bar\varphi_l\,, \quad \bar\varphi_e\,, \quad \bar\varphi_G\,, \quad \bar\varphi_W\,, \quad \bar\varphi_B\}}\,,$$

where conjugate fields defined s.t. **K** has the assumed block-diagonal form

$$\varphi_S = S\,, \qquad \varphi_H = \begin{pmatrix} H \\ H^* \end{pmatrix}, \qquad \boxed{\varphi_f = \begin{pmatrix} f \\ f^c \end{pmatrix}}, \qquad \varphi_V = V\,,$$

$$\bar\varphi_S = S\,, \qquad \bar\varphi_H = \begin{pmatrix} H^\dagger & H^T \end{pmatrix}, \qquad \bar\varphi_f = \begin{pmatrix} \bar f & \bar{f^c} \end{pmatrix}, \qquad \bar\varphi_V = V\,,$$

with $f = q, u, d, l, e,$ and $V = G, W, B.$



Dirac spinors including auxiliary wrong-chirality fields (denoted with prime):

$$q = \begin{pmatrix} q_a \\ q'^{\dagger\dot a} \end{pmatrix}, \qquad q^c = \begin{pmatrix} q'_a \\ q^{\dagger\dot a} \end{pmatrix}$$

$$u = \begin{pmatrix} u'_a \\ u^{\dagger\dot a} \end{pmatrix}, \qquad u^c = \begin{pmatrix} u_a \\ u'^{\dagger\dot a} \end{pmatrix}$$

Step 2: derive K and X matrices

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Example: SS block.

$$\delta^2 \mathcal{L}_{\text{UV}} \supset \delta^2 \left[\frac{1}{2}\, S\,(P^2 - M^2)\, S - \frac{1}{2}\,\kappa\,|H|^2 S^2 - \frac{1}{3!}\,\mu_S\, S^3 - \frac{1}{4!}\,\lambda_S\, S^4\right]$$

$$\supset \delta S\, \underbrace{(P^2 - M^2)}_{K_S}\, \delta S - \delta S\, \underbrace{\left(\kappa\,|H|^2 + \mu_S\, S + \frac{1}{2}\,\lambda_S\, S^2\right)}_{X_{SS} = U_{SS} \text{ (non-derivative interactions)}}\, \delta S\,.$$

Step 2: derive K and X matrices

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Example: *SH* and *HS* blocks.

$$\delta^2 \mathcal{L}_{\text{UV}} \supset \delta^2 \left[-A\,|H|^2 S - \frac{1}{2}\,\kappa\,|H|^2 S^2 \right]$$

$$\supset -2\,(\delta H^\dagger H + H^\dagger \delta H)\,(A + \kappa\, S)\,\delta S$$

$$= -(\delta H^\dagger H + H^T \delta H^* + H^\dagger \delta H + \delta H^T H^*)\,(A + \kappa\, S)\,\delta S$$

$$= -\delta S \boxed{\begin{pmatrix} (A + \kappa\, S)\, H^\dagger & (A + \kappa\, S)\, H^T \end{pmatrix}} \begin{pmatrix} \delta H \\ \delta H^* \end{pmatrix}$$

$X_{SH} = U_{SH}$

$$- \begin{pmatrix} \delta H^\dagger & \delta H^T \end{pmatrix} \boxed{\begin{pmatrix} (A + \kappa\, S)\, H \\ (A + \kappa\, S)\, H^* \end{pmatrix}} \delta S\,.$$

$X_{HS} = U_{HS}$

Step 2: derive K and X matrices

$$\mathcal{L}_{\mathrm{UV}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Example: *HW* and *WH* blocks. (The only derivative interactions in the SM are these and *HB*, *BH*.)

$$\begin{aligned}
\delta^2\mathcal{L}_{\mathrm{UV}} &\supset \delta^2\left(|D_\mu H|^2\right) \\
&= \left[\delta^2(D_\mu H)^\dagger\right](D^\mu H) + (D_\mu H^\dagger)\left[\delta^2(D^\mu H)\right] + 2\left[\delta\,(D_\mu H)^\dagger\right]\left[\delta\,(D^\mu H)\right] \\
&\supset ig_2\,\delta W^I_\mu\,\delta H^\dagger\,\sigma^I\,(D^\mu H) - ig_2\,(D_\mu H)^\dagger\,\sigma^I\,\delta H\,\delta B^\mu \\
&\quad -ig_2\,(D_\mu \delta H)^\dagger\,\sigma^I\,H\,\delta W^I_\mu + ig_2\,\delta W^I_\mu\,H^\dagger\,\sigma^I\,(D^\mu\delta H) \\
&= -\frac{g_2}{2}\,\delta W^I_\mu\,\big[-i\,\delta H^\dagger\,\sigma^I\,(D^\mu H) - i\,(D^\mu H)^T\,\sigma^{I*}\,\delta H^* \\
&\qquad +i\,(D_\mu H)^\dagger\,\sigma^I\,\delta H + i\,\delta H^T\,\sigma^{I*}\,(D_\mu H)^* \\
&\qquad +i\,(D_\mu \delta H)^\dagger\,\sigma^I\,H + i\,H^T\,\sigma^{I*}\,(D_\mu\delta H)^* \\
&\qquad -i\,H^\dagger\,\sigma^I\,(D^\mu\delta H) - i\,(D^\mu\delta H)^T\,\sigma^{I*}\,H^*\big] \\
&\overset{\mathrm{IBP}}{=} -\begin{pmatrix}\delta H^\dagger & \delta H^T\end{pmatrix}\boxed{\left[\begin{pmatrix}-\frac{ig_2}{2}\,\sigma^J\,(D^\nu H) \\ \frac{ig_2}{2}\,\sigma^{J*}\,(D^\nu H)^*\end{pmatrix} + iD_\rho\begin{pmatrix}-\eta^{\rho\nu}\,\frac{g_2}{2}\,\sigma^J\,H \\ \eta^{\rho\nu}\,\frac{g_2}{2}\,\sigma^{J*}\,H^*\end{pmatrix}\right]}\,\delta W^J_\nu \\
&\quad -\delta W^I_\mu\boxed{\Big[\begin{pmatrix}\frac{ig_2}{2}\,(D^\mu H)^\dagger\,\sigma^I & -\frac{ig_2}{2}\,(D^\mu H)^T\sigma^{I*}\end{pmatrix} \\ \quad + \begin{pmatrix}-\eta^{\rho\mu}\,\frac{g_2}{2}\,H^\dagger\,\sigma^I & \eta^{\rho\mu}\,\frac{g_2}{2}\,H^T\,\sigma^{I*}\end{pmatrix} iD_\rho\Big]}\begin{pmatrix}\delta H \\ \delta H^*\end{pmatrix},
\end{aligned}$$

$$X_{HW} = U_{HW} + P_\rho Z^\rho_{HW}$$

$$X_{WH} = U_{WH} + \bar{Z}^\rho_{WH} P_\rho$$

Step 2: derive K and X matrices

- Full results:

Scalar sector entries

$$U_{SS} = \kappa\,|H|^2 + \mu_S\,S_c + \frac{1}{2}\,\lambda_S\,S_c^2\,. \tag{B.5}$$

$$U_{SH} = (A + \kappa\,S_c)\begin{pmatrix} H^\dagger & H^T \end{pmatrix}\,, \quad U_{HS} = (A + \kappa\,S_c)\begin{pmatrix} H \\ H^* \end{pmatrix}\,. \tag{B.6}$$

$$U_{HH} = \left(A\,S_c + \frac{1}{2}\,\kappa\,S_c^2\right)\begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} + \lambda_H\begin{pmatrix} |H|^2\mathbb{1} + H H^\dagger & H H^T \\ H^* H^\dagger & |H|^2\mathbb{1} + H^* H^T \end{pmatrix}. \tag{B.7}$$

Fermion-fermion entries

$$U_{qu} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_u\,\frac{1+\gamma^5}{2}\,\widetilde{H} & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2}\,\widetilde{H}^* \end{pmatrix}, \quad U_{uq} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2}\,\widetilde{H}^\dagger & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2}\,\widetilde{H}^T \end{pmatrix}. \tag{B.8}$$

$$U_{qd} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_d\,\frac{1+\gamma^5}{2}\,H & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_d^*\,\frac{1-\gamma^5}{2}\,H^* \end{pmatrix}, \quad U_{dq} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2}\,H^\dagger & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2}\,H^T \end{pmatrix}. \tag{B.9}$$

$$U_{le} = \begin{pmatrix} \boldsymbol{y}_e\,\frac{1+\gamma^5}{2}\,H & 0 \\ 0 & \boldsymbol{y}_e^*\,\frac{1-\gamma^5}{2}\,H^* \end{pmatrix}, \qquad U_{el} = \begin{pmatrix} \boldsymbol{y}_e^\dagger\,\frac{1-\gamma^5}{2}\,H^\dagger & 0 \\ 0 & \boldsymbol{y}_e^T\,\frac{1+\gamma^5}{2}\,H^T \end{pmatrix}. \tag{B.10}$$

Vector-vector entries

$$U_{GG}^{\mu A,\,\nu B} = 2\,g_3\,f^{ABC}G^{C\mu\nu}\,. \tag{B.11}$$

$$U_{WW}^{\mu I,\,\nu J} = 2\,g_2\,\epsilon^{IJK}W^{K\mu\nu} - \frac{g_2^2}{2}\,\eta^{\mu\nu}\,\delta^{IJ}\,|H|^2\,. \tag{B.12}$$

$$U_{BB}^{\mu,\,\nu} = -\frac{g_1^2}{2}\,\eta^{\mu\nu}\,|H|^2\,. \tag{B.13}$$

$$U_{WB}^{\mu I,\,\nu} = -\frac{g_1 g_2}{2}\,\eta^{\mu\nu}\,H^\dagger\sigma^I H\,, \qquad U_{BW}^{\mu,\,\nu J} = -\frac{g_1 g_2}{2}\,\eta^{\mu\nu}\,H^\dagger\sigma^J H\,. \tag{B.14}$$

Higgs-fermion entries

$$U_{Hq} = \begin{pmatrix} \mathbb{1}\,\bar{d}\,\boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2} & -\boldsymbol{\epsilon}\,\bar{u}^c\,\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2} \\ -\boldsymbol{\epsilon}\,\bar{u}\,\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2} & \mathbb{1}\,\bar{d}^c\,\boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2} \end{pmatrix}, \quad U_{qH} = \begin{pmatrix} \boldsymbol{y}_d\,\frac{1+\gamma^5}{2}\,d\,\mathbb{1} & \boldsymbol{y}_u\,\frac{1+\gamma^5}{2}\,u\,\boldsymbol{\epsilon} \\ \boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2}\,u^c\,\boldsymbol{\epsilon} & \boldsymbol{y}_d^*\,\frac{1-\gamma^5}{2}\,d^c\,\mathbb{1} \end{pmatrix}. \tag{B.15}$$

$$U_{Hu} = \begin{pmatrix} (\bar{q}\,\boldsymbol{\epsilon})_{\bar{\alpha}}\,\boldsymbol{y}_u\,\frac{1+\gamma^5}{2} & 0 \\ 0 & (\bar{q}^c\,\boldsymbol{\epsilon})_{\alpha}\,\boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2} \end{pmatrix}, \quad U_{uH} = \begin{pmatrix} -\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2}\,(\boldsymbol{\epsilon}\,q)_{\beta} & 0 \\ 0 & -\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2}\,(\boldsymbol{\epsilon}\,q^c)_{\bar{\beta}} \end{pmatrix}. \tag{B.16}$$

$$U_{Hd} = \begin{pmatrix} 0 & \bar{q}^c_{\bar{\alpha}}\,\boldsymbol{y}_d^*\,\frac{1-\gamma^5}{2} \\ \bar{q}_{\alpha}\,\boldsymbol{y}_d\,\frac{1+\gamma^5}{2} & 0 \end{pmatrix}, \qquad U_{dH} = \begin{pmatrix} 0 & \boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2}\,q_{\beta} \\ \boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2}\,q^c_{\bar{\beta}} & 0 \end{pmatrix}. \tag{B.17}$$

$$U_{Hl} = \begin{pmatrix} \mathbb{1}\,\bar{e}\,\boldsymbol{y}_e^\dagger\,\frac{1-\gamma^5}{2} & 0 \\ 0 & \mathbb{1}\,\bar{e}^c\,\boldsymbol{y}_e^T\,\frac{1+\gamma^5}{2} \end{pmatrix}, \qquad U_{lH} = \begin{pmatrix} \boldsymbol{y}_e\,\frac{1+\gamma^5}{2}\,e\,\mathbb{1} & 0 \\ 0 & \boldsymbol{y}_e^*\,\frac{1-\gamma^5}{2}\,e^c\,\mathbb{1} \end{pmatrix}. \tag{B.18}$$

$$U_{He} = \begin{pmatrix} 0 & \bar{l}^c_{\bar{\alpha}}\,\boldsymbol{y}_e^*\,\frac{1-\gamma^5}{2} \\ \bar{l}_{\alpha}\,\boldsymbol{y}_e\,\frac{1+\gamma^5}{2} & 0 \end{pmatrix}, \qquad U_{eH} = \begin{pmatrix} 0 & \boldsymbol{y}_e^\dagger\,\frac{1-\gamma^5}{2}\,l_{\beta} \\ \boldsymbol{y}_e^T\,\frac{1+\gamma^5}{2}\,l^c_{\bar{\beta}} & 0 \end{pmatrix}. \tag{B.19}$$

Higgs-vector entries

$$U_{HW}^{\nu J} = \frac{ig_2}{2}\begin{pmatrix} -\sigma^J(D^\nu H) \\ \sigma^{J*}\,(D^\nu H)^* \end{pmatrix}, \quad U_{WH}^{\mu I} = \frac{ig_2}{2}\begin{pmatrix} (D^\mu H)^\dagger\,\sigma^I & -(D^\mu H)^T\,\sigma^{I*} \end{pmatrix}. \tag{B.20}$$

$$Z_{HW}^{\rho\nu J} = \eta^{\rho\nu}\,\frac{g_2}{2}\begin{pmatrix} -\sigma^J H \\ \sigma^{J*}\,H^* \end{pmatrix}, \qquad \bar{Z}_{WH}^{\rho\mu I} = \eta^{\rho\mu}\,\frac{g_2}{2}\begin{pmatrix} -H^\dagger\sigma^I & H^T\,\sigma^{I*} \end{pmatrix}. \tag{B.21}$$

$$U_{HB}^{\nu} = \frac{ig_1}{2}\begin{pmatrix} -D^\nu H \\ (D^\nu H)^* \end{pmatrix}, \qquad U_{BH}^{\mu} = \frac{ig_1}{2}\begin{pmatrix} (D^\mu H)^\dagger & -(D^\mu H)^T \end{pmatrix}. \tag{B.22}$$

$$Z_{HB}^{\rho\nu} = \eta^{\rho\nu}\,\frac{g_1}{2}\begin{pmatrix} -H \\ H^* \end{pmatrix}, \qquad \bar{Z}_{BH}^{\rho\mu} = \eta^{\rho\mu}\,\frac{g_1}{2}\begin{pmatrix} -H^\dagger & H^T \end{pmatrix}. \tag{B.23}$$

Fermion-vector entries

$$U_{qG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B q \\ \gamma^\nu\lambda^{B*}\,q^c \end{pmatrix}, \qquad U_{Gq}^{\mu A} = \frac{g_3}{2}\begin{pmatrix} -\bar{q}\,\gamma^\mu\lambda^A & \bar{q}^c\,\gamma^\mu\lambda^{A*} \end{pmatrix}, \tag{B.24}$$

$$U_{uG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B u \\ \gamma^\nu\lambda^{B*}\,u^c \end{pmatrix}, \qquad U_{Gu}^{\mu A} = \frac{g_3}{2}\begin{pmatrix} -\bar{u}\,\gamma^\mu\lambda^A & \bar{u}^c\,\gamma^\mu\lambda^{A*} \end{pmatrix}, \tag{B.25}$$

$$U_{dG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B d \\ \gamma^\nu\lambda^{B*}\,d^c \end{pmatrix}, \qquad U_{Gd}^{\mu A} = \frac{g_3}{2}\begin{pmatrix} -\bar{d}\,\gamma^\mu\lambda^A & \bar{d}^c\,\gamma^\mu\lambda^{A*} \end{pmatrix}, \tag{B.26}$$

$$U_{qW}^{\nu J} = \frac{g_2}{2}\begin{pmatrix} -\gamma^\nu\sigma^J q \\ \gamma^\nu\sigma^{J*}\,q^c \end{pmatrix}, \qquad U_{Wq}^{\mu I} = \frac{g_2}{2}\begin{pmatrix} -\bar{q}\,\gamma^\mu\sigma^I & \bar{q}^c\,\gamma^\mu\sigma^{I*} \end{pmatrix}, \tag{B.27}$$

$$U_{lW}^{\nu J} = \frac{g_2}{2}\begin{pmatrix} -\gamma^\nu\sigma^J l \\ \gamma^\nu\sigma^{J*}\,l^c \end{pmatrix}, \qquad U_{Wl}^{\mu I} = \frac{g_2}{2}\begin{pmatrix} -\bar{l}\,\gamma^\mu\sigma^I & \bar{l}^c\,\gamma^\mu\sigma^{I*} \end{pmatrix}, \tag{B.28}$$

$$U_{qB}^{\nu} = \frac{g_1}{6}\begin{pmatrix} -\gamma^\nu q \\ \gamma^\nu q^c \end{pmatrix}, \qquad U_{Bq}^{\mu} = \frac{g_1}{6}\begin{pmatrix} -\bar{q}\,\gamma^\mu & \bar{q}^c\,\gamma^\mu \end{pmatrix}, \tag{B.29}$$

$$U_{uB}^{\nu} = \frac{2g_1}{3}\begin{pmatrix} -\gamma^\nu u \\ \gamma^\nu u^c \end{pmatrix}, \qquad U_{Bu}^{\mu} = \frac{2g_1}{3}\begin{pmatrix} -\bar{u}\,\gamma^\mu & \bar{u}^c\,\gamma^\mu \end{pmatrix}, \tag{B.30}$$

$$U_{dB}^{\nu} = -\frac{g_1}{3}\begin{pmatrix} -\gamma^\nu d \\ \gamma^\nu d^c \end{pmatrix}, \qquad U_{Bd}^{\mu} = -\frac{g_1}{3}\begin{pmatrix} -\bar{d}\,\gamma^\mu & \bar{d}^c\,\gamma^\mu \end{pmatrix}, \tag{B.31}$$

$$U_{lB}^{\nu} = -\frac{g_1}{2}\begin{pmatrix} -\gamma^\nu l \\ \gamma^\nu l^c \end{pmatrix}, \qquad U_{Bl}^{\mu} = -\frac{g_1}{2}\begin{pmatrix} -\bar{l}\,\gamma^\mu & \bar{l}^c\,\gamma^\mu \end{pmatrix}, \tag{B.32}$$

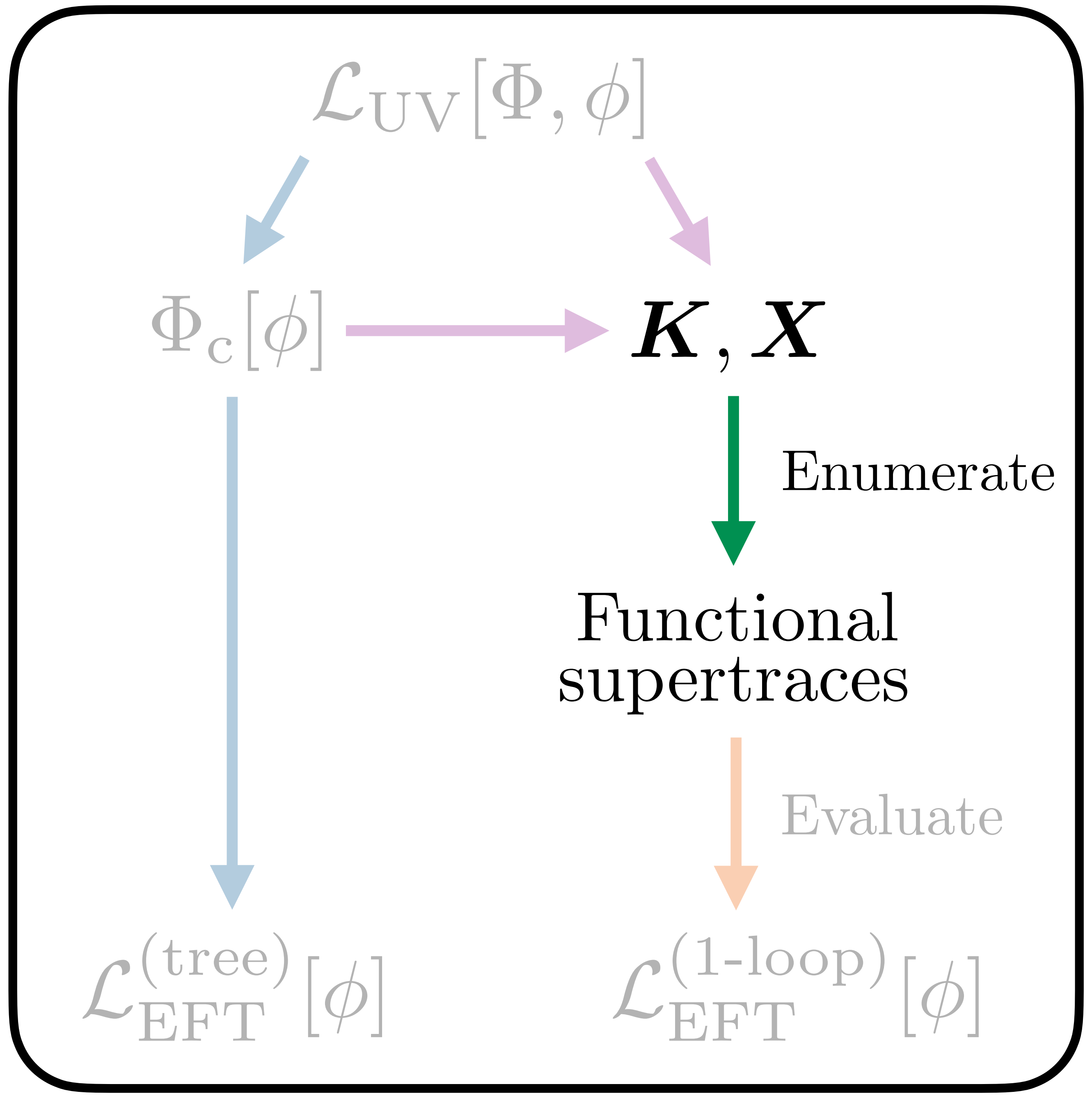
$$U_{eB}^{\nu} = -g_1\begin{pmatrix} -\gamma^\nu e \\ \gamma^\nu e^c \end{pmatrix}, \qquad U_{Be}^{\mu} = -g_1\begin{pmatrix} -\bar{e}\,\gamma^\mu & \bar{e}^c\,\gamma^\mu \end{pmatrix}. \tag{B.33}$$

- A bit tedious, but the SM part is done once and for all.

Step 3: enumerate supertraces

- No log-type supertraces (S is a gauge singlet).
- Enumerate power-type supertraces with $\sum \dim(X_{ij}) \leq 6$.

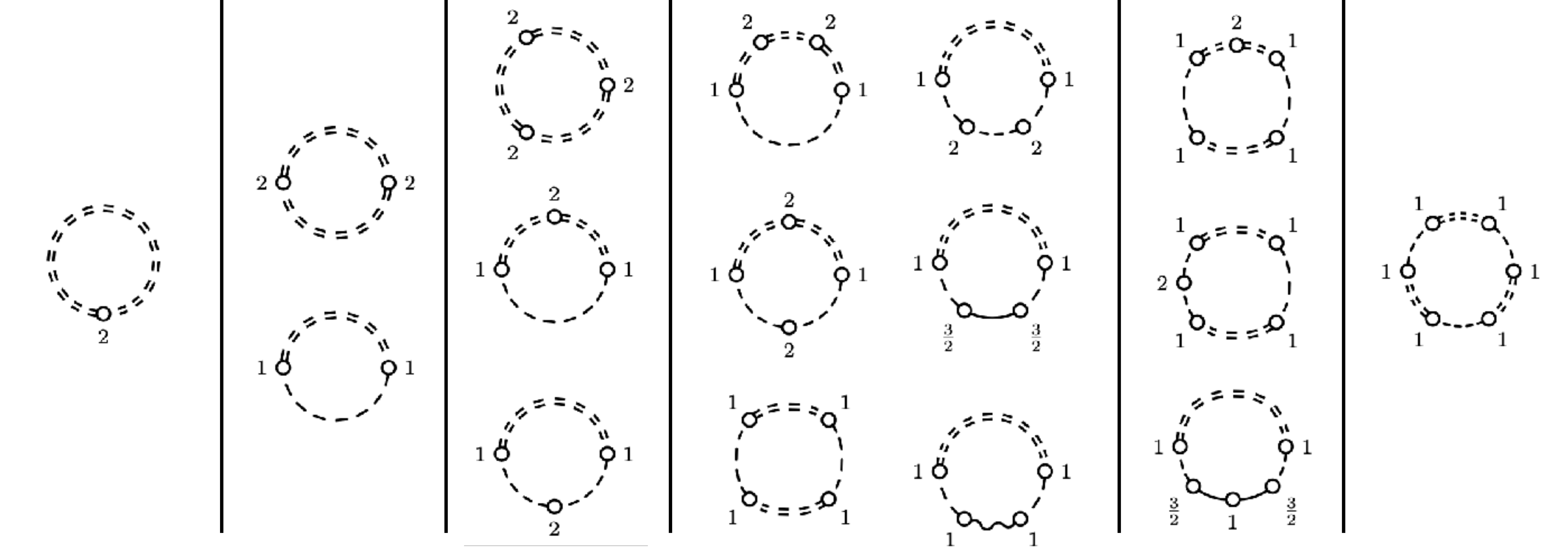
$$
\dim(\boldsymbol{X}) \geq
\begin{array}{c}
\begin{array}{cccccccccc} S & H & q & u & d & l & e & G & W & B \end{array} \\
\begin{array}{c} S \\ H \\ q \\ u \\ d \\ l \\ e \\ G \\ W \\ B \end{array}
\begin{pmatrix}
2 & 1 & & & & & & & & \\
1 & 2 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & & 1 & 1 \\
 & \frac{3}{2} & & 1 & 1 & & & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \\
 & \frac{3}{2} & 1 & & & & & \frac{3}{2} & & \frac{3}{2} \\
 & \frac{3}{2} & 1 & & & & & \frac{3}{2} & & \frac{3}{2} \\
 & \frac{3}{2} & & & & & 1 & & \frac{3}{2} & \frac{3}{2} \\
 & \frac{3}{2} & & & & 1 & & & & \frac{3}{2} \\
 & & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & & & 2 & & \\
 & 1 & \frac{3}{2} & & & \frac{3}{2} & & & 2 & 2 \\
 & 1 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & & 2 & 2
\end{pmatrix}
\end{array} .
$$



Step 3: enumerate supertraces

- No log-type supertraces (S is a gauge singlet).
- Enumerate power-type supertraces with $\sum \dim(X_{ij}) \leq 6$.

# of propagators	1	2	3	4	5	6



Step 3: enumerate supertraces

- ➤ No log-type supertraces (S is a gauge singlet).
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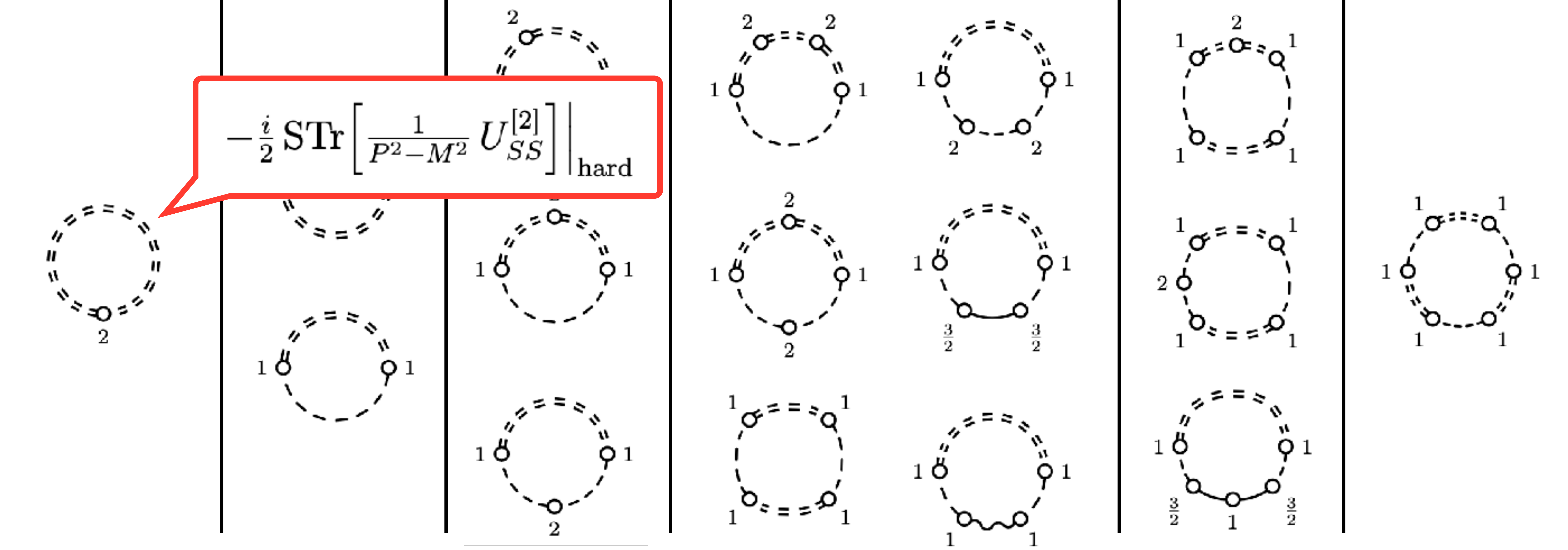

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U_{SS}^{[2]}\right]\Big|_{\text{hard}}$$

Step 3: enumerate supertraces

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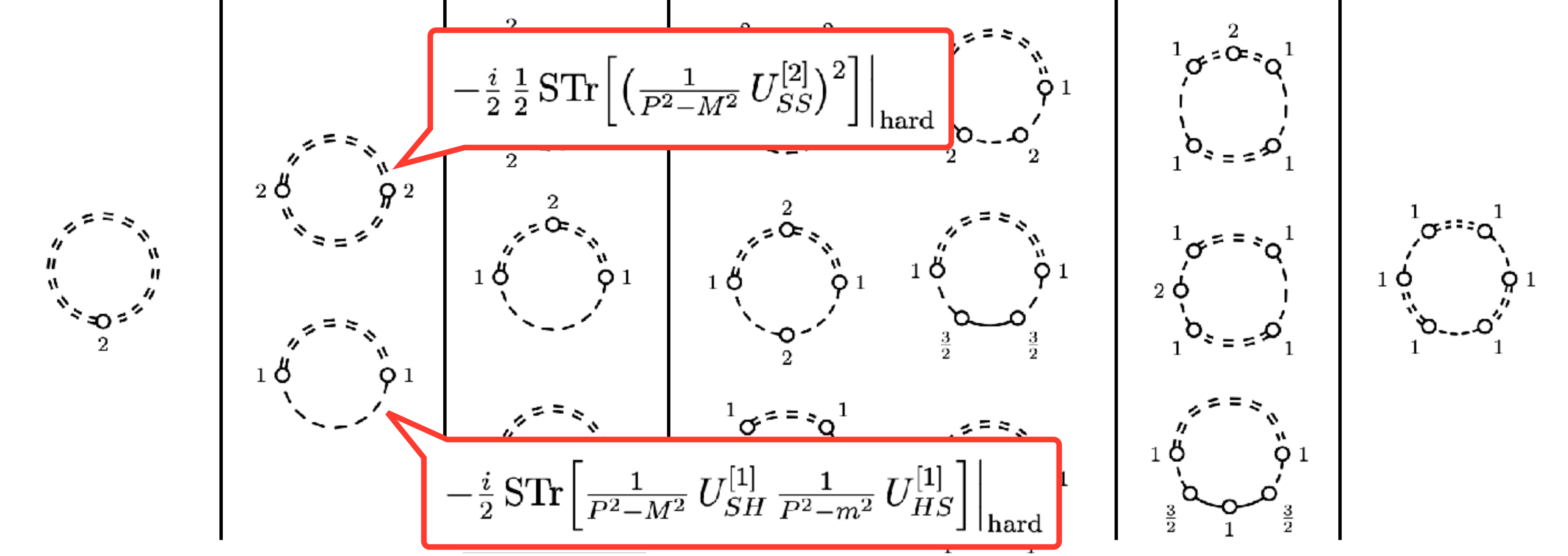
# of propagators	1	2	3	4	5	6



Step 3: enumerate supertraces

- No log-type supertraces (S is a gauge singlet).
- Enumerate power-type supertraces with $\sum \dim(X_{ij}) \leq 6$.

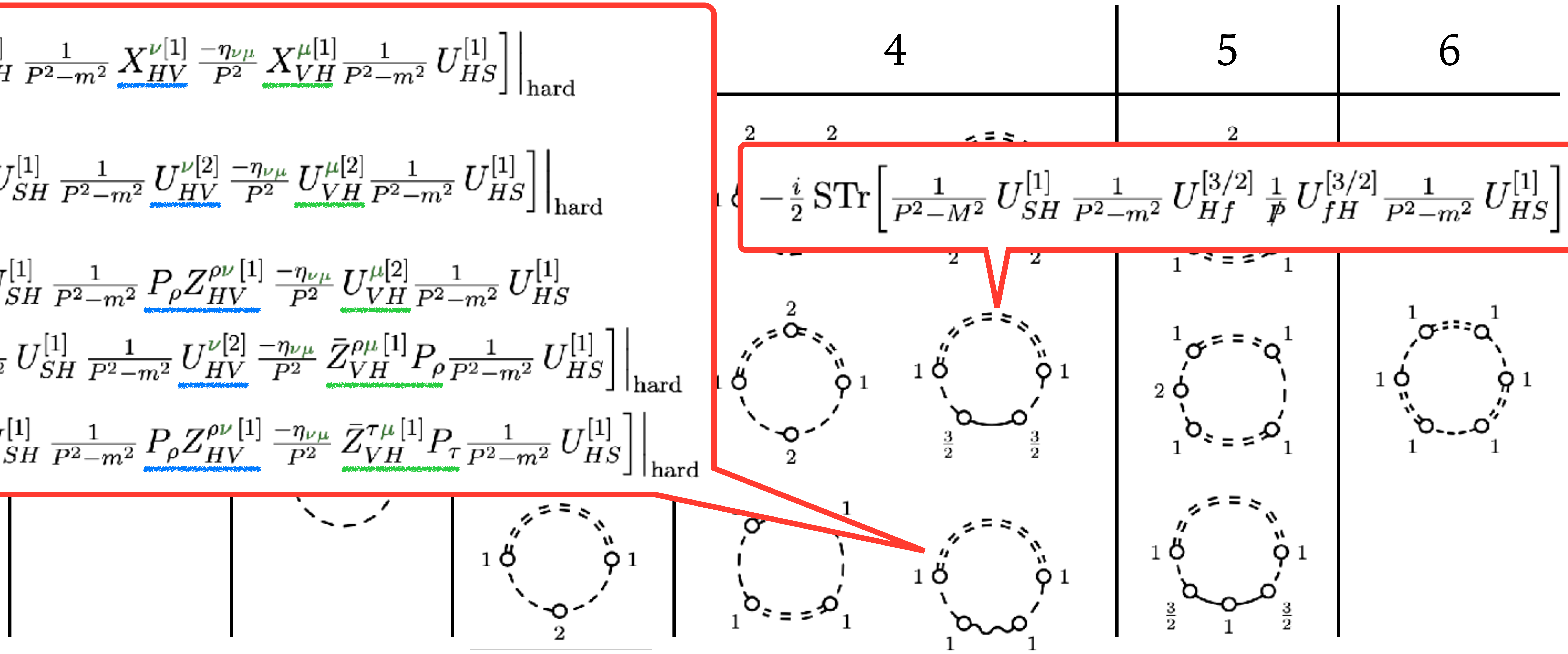
$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,\underline{X^{\nu[1]}_{HV}}\,\frac{-\eta_{\nu\mu}}{P^2}\,\underline{X^{\mu[1]}_{VH}}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}$$

$$=-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,\underline{U^{\nu[2]}_{HV}}\,\frac{-\eta_{\nu\mu}}{P^2}\,\underline{U^{\mu[2]}_{VH}}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}$$

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,\underline{P_\rho Z^{\rho\nu[1]}_{HV}}\,\frac{-\eta_{\nu\mu}}{P^2}\,\underline{U^{\mu[2]}_{VH}}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right.$$
$$\left.+\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,\underline{U^{\nu[2]}_{HV}}\,\frac{-\eta_{\nu\mu}}{P^2}\,\underline{\bar{Z}^{\rho\mu[1]}_{VH}P_\rho}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}$$

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,\underline{P_\rho Z^{\rho\nu[1]}_{HV}}\,\frac{-\eta_{\nu\mu}}{P^2}\,\underline{\bar{Z}^{\tau\mu[1]}_{VH}P_\tau}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}$$

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,U^{[3/2]}_{Hf}\,\frac{1}{\not{P}}\,U^{[3/2]}_{fH}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}$$



Step 4: evaluate supertraces

- First, use CDE to evaluate STr with generic X_{ij} .
- Then, substitute in $X_{ij}[\phi\,, P_\mu]$ and perform matrix algebra.

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\boxed{U_1^{[2]}}\right]\Bigg|_{\text{hard}}$$

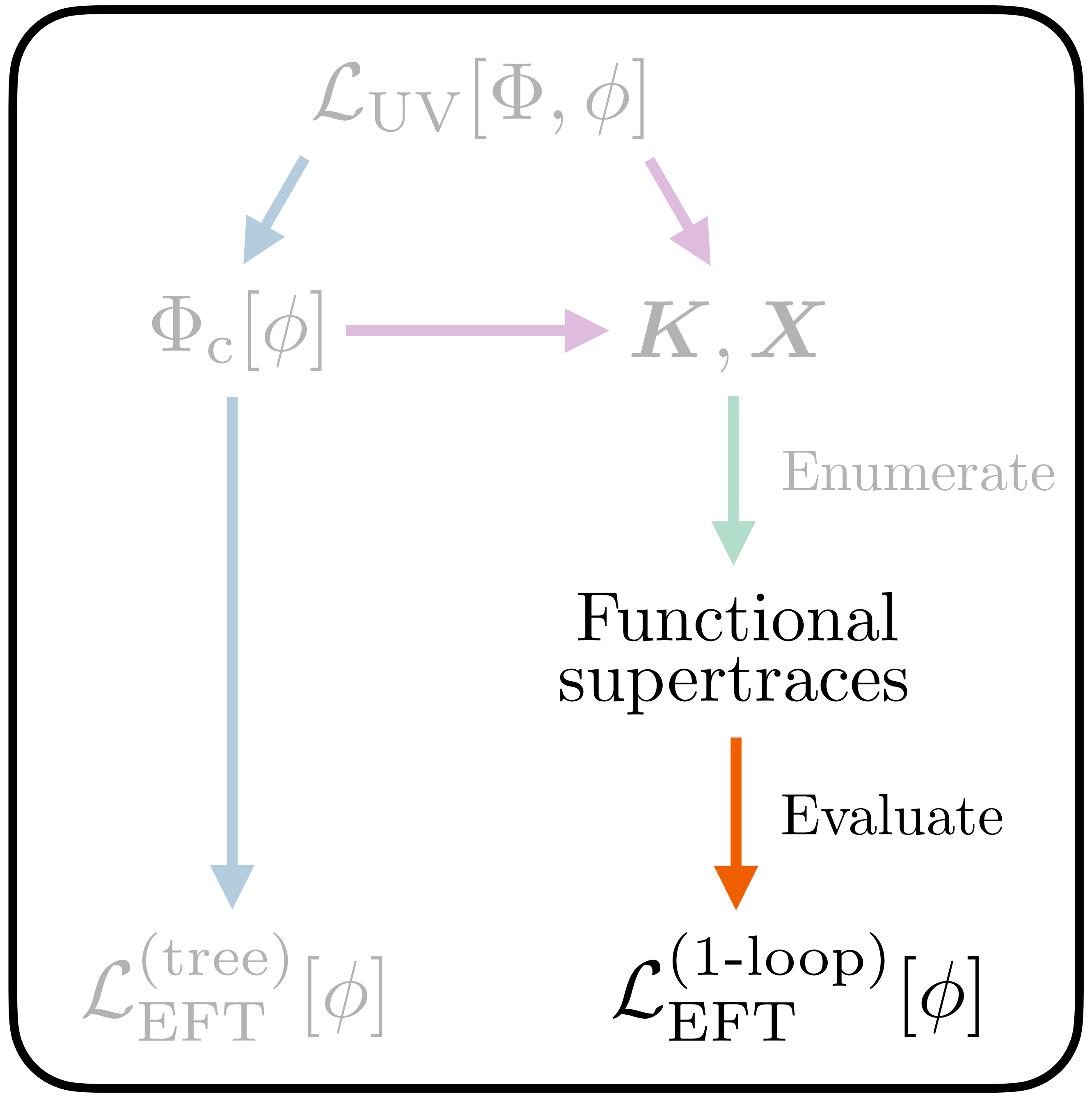
generic functional of ϕ with operator dimension 2

$$= \int \mathrm{d}^d x\, \frac{1}{16\pi^2}\,\mathrm{tr}\left[M^2\left(1-\log\frac{M^2}{\mu^2}\right)\boxed{U_1} + \frac{1}{12M^2}F_{\mu\nu}F^{\mu\nu}\boxed{U_1}\right].$$

$$U_{SS} = \kappa\,|H|^2 + \mu_S\, S_c + \frac{1}{2}\lambda_S\, S_c^2$$

0

$$\Rightarrow \mathscr{L}_{\text{EFT}}^{(1-\text{loop})} \supset \frac{1}{16\pi^2}\frac{1}{2}\left(1-\log\frac{M^2}{\mu^2}\right)\Bigg\{(\kappa M^2-\mu_S A)|H|^2 + \left[\frac{\lambda_S A^2}{2M^2}+\frac{\mu_S A}{M^2}\left(\kappa-\frac{\mu_S A}{2M^2}\right)\right]|H|^4$$
$$-\frac{1}{M^2}\left[\frac{\lambda_S A^2}{M^2}\left(\kappa-\frac{2\mu_S A}{3M^2}\right)+\frac{\mu_S A}{M^2}\left(\kappa-\frac{\mu_S A}{M^2}\right)\left(\kappa-\frac{\mu_S A}{2M^2}\right)\right]|H|^6$$
$$-\frac{1}{M^2}\left[\frac{\lambda_S A^2}{M^2}+\frac{\mu_S A}{M^2}\left(\kappa-\frac{\mu_S A}{M^2}\right)\right]|H|^2(\partial^2|H|^2)\Bigg\},$$



- (*) If desired, post-process the results into a non-redundant operator basis.

Step 4: evaluate supertraces

- Full results in agreement with existing results obtained using amplitude matching.
- Nontrivial test of our functional prescription.

Operator	Coefficient $\times\, 16\pi^2$
$\lvert H\rvert^2$	$\left[\frac{1}{2}(\kappa M^2-\mu_S A)+A^2\left(1+\frac{m^2}{M^2}+\frac{m^4}{M^4}\right)\right]\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^4$	$\frac{\kappa^2}{4}\left(-\log\frac{M^2}{\mu^2}\right)+\frac{\mu_S A}{M^2}\left(\frac{\kappa}{2}-\frac{\mu_S A}{4M^2}+\frac{A^2}{M^2}\right) +\frac{A^2}{M^2}\left[\left(\frac{\lambda_S}{4}+3\lambda_H\right)\left(1-\log\frac{M^2}{\mu^2}\right)-2\left(\kappa+\frac{A^2}{M^2}\right)\left(\frac{3}{2}-\log\frac{M^2}{\mu^2}\right)\right] +\frac{m^2}{M^2}\frac{A^2}{M^2}\left[6\lambda_H\left(1-\log\frac{M^2}{\mu^2}\right)-3\left(\kappa+\frac{2A^2}{M^2}\right)\left(\frac{4}{3}-\log\frac{M^2}{\mu^2}\right)+\frac{\mu_S A}{M^2}\left(2-\log\frac{M^2}{\mu^2}\right)\right]$
$\lvert D_\mu H\rvert^2$	$\frac{A^2}{2M^2}+\frac{A^2m^2}{M^4}\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$

Table 2. Corrections to renormalizable operators.

Operator	Coefficient $\times\, 16\pi^2$
$\lvert H\rvert^6$	$\frac{1}{M^2}\left(-\frac{\kappa^3}{12}-\frac{\kappa^2\mu_S A}{4M^2}+\frac{\kappa\mu_S^2A^2}{2M^4}-\frac{\lambda_S A^4}{2M^4}-\frac{\mu_S^3A^3}{6M^6}+\frac{\mu_S^2A^4}{M^6}\right) +\frac{\kappa A^2}{M^4}\left[3\kappa\left(\frac{11}{6}-\log\frac{M^2}{\mu^2}\right)-\frac{\lambda_S}{4}\left(2-\log\frac{M^2}{\mu^2}\right)\right] +\frac{9\lambda_H A^2}{M^4}\left[-\kappa\left(\frac{4}{3}-\log\frac{M^2}{\mu^2}\right)+\lambda_H\left(1-\log\frac{M^2}{\mu^2}\right)\right] +\frac{\mu_S A^3}{M^6}\left[-\kappa\left(5-\log\frac{M^2}{\mu^2}\right)+\frac{\lambda_S}{12}\left(4-\log\frac{M^2}{\mu^2}\right)+3\lambda_H\left(2-\log\frac{M^2}{\mu^2}\right)\right] +\frac{A^4}{M^6}\left[\frac{21\kappa}{2}\left(\frac{37}{21}-\log\frac{M^2}{\mu^2}\right)-18\lambda_H\left(\frac{4}{3}-\log\frac{M^2}{\mu^2}\right)\right] -\frac{7\mu_S A^5}{2M^8}\left(\frac{15}{7}-\log\frac{M^2}{\mu^2}\right)+\frac{9A^6}{M^8}\left(\frac{43}{27}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\partial^2\lvert H\rvert^2)$	$-\frac{\kappa^2}{24M^2}-\frac{5\kappa\mu_S A}{12M^4} +\frac{A^2}{M^4}\left[2\kappa\left(\frac{17}{12}-\log\frac{M^2}{\mu^2}\right)-\frac{\lambda_S}{2}\left(1-\log\frac{M^2}{\mu^2}\right)-\frac{\lambda_H}{2}\left(\frac{9}{2}-\log\frac{M^2}{\mu^2}\right)\right] +\frac{11\mu_S^2A^2}{24M^6}-\frac{4\mu_S A^3}{3M^6}+\frac{3A^4}{2M^6}\left(\frac{20}{9}-\log\frac{M^2}{\mu^2}\right)-\frac{3g_2^2A^2}{8M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\lvert D_\mu H\rvert^2$	$\frac{A^2}{M^4}\left[\left(\lambda_H-\frac{A^2}{M^2}\right)\left(\frac{9}{2}-\log\frac{M^2}{\mu^2}\right)-\frac{3\kappa}{2}+\frac{\mu_S A}{2M^2}\right]-\frac{3g_2^2A^2}{2M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$
$\frac{1}{2}(H^\dagger\overleftrightarrow{D}^\mu H)^2$	$\frac{3g_1^2A^2}{4M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$
$\lvert D^2H\rvert^2$	$\frac{A^2}{6M^4}$

Operator	Coefficient $\times\, 16\pi^2$
$\frac{ig_2}{2}(H^\dagger\sigma^I\overleftrightarrow{D}^\mu H)(D^\nu W^I_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3}-\log\frac{M^2}{\mu^2}\right)$
$\frac{ig_1}{2}(H^\dagger\overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3}-\log\frac{M^2}{\mu^2}\right)$
$ig_2(D^\mu H)^\dagger\sigma^I(D^\nu H)\,W^I_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$ig_1(D^\mu H)^\dagger(D^\nu H)\,B_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$\lvert H\rvert^2W^I_{\mu\nu}W^{I\mu\nu}$	$\frac{g_2^2A^2}{16M^4}$
$\lvert H\rvert^2B_{\mu\nu}B^{\mu\nu}$	$\frac{g_1^2A^2}{16M^4}$
$H^\dagger\sigma^I HW^I_{\mu\nu}B^{\mu\nu}$	$\frac{g_1g_2A^2}{8M^4}$

Table 3. Dimension six bosonic operators.

Operator	Coefficient $\times\, 16\pi^2$
$(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H)(\bar{q}\sigma^I\gamma^\mu q)$	$\frac{A^2}{8M^4}(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger+\boldsymbol{y}_d\boldsymbol{y}_d^\dagger)\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$	$-\frac{A^2}{8M^4}(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger-\boldsymbol{y}_d\boldsymbol{y}_d^\dagger)\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_u^\dagger\boldsymbol{y}_u\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$	$-\frac{A^2}{4M^4}\,\boldsymbol{y}_d^\dagger\boldsymbol{y}_d\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H)(\bar{l}\sigma^I\gamma^\mu l)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}\gamma^\mu l)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$	$-\frac{A^2}{4M^4}\,\boldsymbol{y}_e^\dagger\boldsymbol{y}_e\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(\widetilde{H}^\dagger i(D_\mu H))(\bar{u}\gamma^\mu d)$ (+h.c.)	$-\frac{A^2}{2M^4}\,\boldsymbol{y}_u^\dagger\boldsymbol{y}_d\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I H)(\bar{q}\,\sigma^I i\overleftrightarrow{\not{D}}\,q)$	$-\frac{A^2}{8M^4}(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger-\boldsymbol{y}_d\boldsymbol{y}_d^\dagger)\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{q}\,i\overleftrightarrow{\not{D}}\,q)$	$\frac{A^2}{8M^4}(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger+\boldsymbol{y}_d\boldsymbol{y}_d^\dagger)\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{u}\,i\overleftrightarrow{\not{D}}\,u)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_u^\dagger\boldsymbol{y}_u\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{d}\,i\overleftrightarrow{\not{D}}\,d)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_d^\dagger\boldsymbol{y}_d\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I H)(\bar{l}\,\sigma^I i\overleftrightarrow{\not{D}}\,l)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{l}\,i\overleftrightarrow{\not{D}}\,l)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{e}\,i\overleftrightarrow{\not{D}}\,e)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_e^\dagger\boldsymbol{y}_e\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{q}\,u\,\widetilde{H}$ (+h.c.)	$\frac{A^2}{M^4}\,\boldsymbol{y}_u\boldsymbol{y}_u^\dagger\boldsymbol{y}_u\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{q}\,d\,H$ (+h.c.)	$\frac{A^2}{M^4}\,\boldsymbol{y}_d\boldsymbol{y}_d^\dagger\boldsymbol{y}_d\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{l}\,e\,H$ (+h.c.)	$\frac{A^2}{M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\boldsymbol{y}_e\left(1-\log\frac{M^2}{\mu^2}\right)$

Table 4. Dimension six operators with fermions.

Outline

- What is functional matching, and what is new?
- The prescription.
- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.
- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

Evaluating functional supertraces with CDE

- The goal is to compute the two types of functional supertraces:

$$\int \mathrm{d}^d x\, \mathcal{L}_{\mathrm{EFT}}^{(1\text{-loop})}[\phi] = \frac{i}{2}\,\mathrm{STr}\log \boldsymbol{K}\Big|_{\mathrm{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\Big[\big(\boldsymbol{K}^{-1}\boldsymbol{X}\big)^n\Big]\Big|_{\mathrm{hard}}\,.$$

- Log-type can be converted to power-type by differentiation:

$$\frac{\partial}{\partial m_\Phi^2}\Big[i\,\mathrm{STr}\log\left(P^2 - m_\Phi^2\right)\Big] = -i\,\mathrm{STr}\left[\frac{1}{P^2 - m_\Phi^2}\right] \qquad \frac{\partial}{\partial m_\Phi}\Big[i\,\mathrm{STr}\log\left(\not{P} - m_\Phi\right)\Big] = -i\,\mathrm{STr}\left[\frac{1}{\not{P} - m_\Phi}\right]$$

- So we only need to deal with supertraces of the form:

$$-\,i\,\mathrm{STr}\Big[\,f\left(P_\mu\,,\,\{U_k\}\right)\Big]\Big|_{\mathrm{hard}} \qquad f = \Big[\,\cdots\,\left(P_{\mu_1}\ldots P_{\mu_n}\right)\left(\Delta_i \;\text{ or }\; \Lambda_i\right)\left(P_{\nu_1}\ldots P_{\nu_m}\right)U_k\;\cdots\,\Big]$$

where $\Delta_i \equiv \dfrac{1}{P^2 - m_i^2}\,, \qquad \Lambda_i \equiv \dfrac{1}{\not{P} - m_i}\,.$

Evaluating functional supertraces with CDE

- First, address the "super" part: $-i\,\mathrm{STr}\Big[f\big(P_\mu,\{U_k\}\big)\Big] = \boxed{\pm}\Big\{-i\,\mathrm{Tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\Big\}$

determined by the first propagator block

- Next, the "functional" part.
  - Conveniently, work in the quantum mechanics language:

$$
\begin{aligned}
-i\,\mathrm{Tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big] &= -i\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\Big\langle q\Big|\,\mathrm{tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\Big|q\Big\rangle \\
&= -i\int\mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\langle q|x\rangle\Big\langle x\Big|\,\mathrm{tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\Big|q\Big\rangle \\
&= -i\int\mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,e^{iq\cdot x}\,\mathrm{tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\,e^{-iq\cdot x}\,. \\
&= -i\int\mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\Big[f\big(P_\mu-q_\mu,\{U_k\}\big)\Big]\,.
\end{aligned}
$$

$$
\begin{aligned}
e^{iq\cdot x}\,P_\mu\,e^{-iq\cdot x} &= P_\mu + q_\mu\,, \\
e^{iq\cdot x}\,U_k\,e^{-iq\cdot x} &= U_k\,;
\end{aligned}
$$

Note: $P_\mu(\hat{x},\hat{q}) = \hat{q}_\mu + g_a\,G^a_\mu(\hat{x})\,T^a$

Evaluating functional supertraces with CDE

$$-i\,\mathrm{Tr}\left[f\big(P_\mu, \{U_k\}\big)\right] = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\;\mathrm{tr}\left[f\big(P_\mu - q_\mu, \{U_k\}\big)\right].$$

- Different ways to proceed.
  - "Simplified CDE."
    - Directly expand $f\big(P_\mu - q_\mu, \{U_k\}\big)$ in powers of P and U.
    - Produce non-gauge-invariant terms (e.g. when a $P_\mu(\hat{x}, \hat{q}) = \hat{q}_\mu + g_a\, G^a_\mu(\hat{x})\, T^a$ appears on the rightmost).
    - All such terms cancel in the final result, as they must.
    - Simplifying trick: use gauge invariance as a constraint to reduce the number of terms to compute.

B. Henning, X. Lu, H. Murayama, 1604.01019.
J. Fuentes-Martin, J. Portoles, P. Ruiz-Femenia, 1607.02142.
ZZ, 1610.00710.

Evaluating functional supertraces with CDE

$$-\,i\,\mathrm{Tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big] = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\Big[f\big(P_\mu - q_\mu,\{U_k\}\big)\Big]\,.$$

- Different ways to proceed.
  - "Simplified CDE."
    - Directly expand $f\big(P_\mu - q_\mu,\{U_k\}\big)$ in powers of P and U.
  - "Original CDE."
    - First apply a transformation: $-\,i\,\mathrm{Tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big] = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,e^{P\cdot\frac{\partial}{\partial q}}\,\mathrm{tr}\Big[f\big(P_\mu - q_\mu,\{U_k\}\big)\Big]\,e^{-P\cdot\frac{\partial}{\partial q}}$
    - This makes all covariant derivatives "closed."

M. K. Gaillard, Nucl. Phys. B 268 (1986) 669.
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B. Henning, X. Lu, H. Murayama, 1412.1837.

$$P_\mu^{\mathrm{CDE}} \equiv e^{P\cdot\frac{\partial}{\partial q}}\left(P_\mu - q_\mu\right)e^{-P\cdot\frac{\partial}{\partial q}} = -q_\mu + G_{\mu\nu}^{\mathrm{CDE}}\underline{\partial^\nu}\,,\qquad G_{\mu\nu}^{\mathrm{CDE}} \equiv -i\underline{\sum_{n=0}^{\infty}}\frac{n+1}{(n+2)!}\underline{\left(P_{\alpha_1}\cdots P_{\alpha_n}F_{\mu\nu}\right)}\,\underline{\partial^{\alpha_1}\cdots\partial^{\alpha_n}}$$

$$U_k^{\mathrm{CDE}} \equiv e^{P\cdot\frac{\partial}{\partial q}}\,U_k\,e^{-P\cdot\frac{\partial}{\partial q}} = \underline{\sum_{n=0}^{\infty}}\frac{1}{n!}\underline{\left(P_{\alpha_1}\cdots P_{\alpha_n}U_k\right)}\,\underline{\partial^{\alpha_1}\cdots\partial^{\alpha_n}}\qquad \partial^\alpha \equiv \frac{\partial}{\partial q_\alpha}$$

- Automatically guarantees gauge-invariance, at the cost of an additional sum + q differentiation.

Evaluating functional supertraces with CDE

$$-i\,\mathrm{Tr}\left[f\left(P_\mu,\{U_k\}\right)\right]=-i\int \mathrm{d}^dx\int\frac{\mathrm{d}^dq}{(2\pi)^d}\,\mathrm{tr}\left[f\left(P_\mu-q_\mu,\{U_k\}\right)\right].$$

- Different ways to proceed.
  - "Simplified CDE."
    - Directly expand $f\left(P_\mu-q_\mu,\{U_k\}\right)$ in powers of *P* and *U*.
  - "Original CDE."
    - First apply a transformation: $-i\,\mathrm{Tr}\left[f\left(P_\mu,\{U_k\}\right)\right]=-i\int \mathrm{d}^dx\int\frac{\mathrm{d}^dq}{(2\pi)^d}\,e^{P\cdot\frac{\partial}{\partial q}}\,\mathrm{tr}\left[f\left(P_\mu-q_\mu,\{U_k\}\right)\right]e^{-P\cdot\frac{\partial}{\partial q}}$
- We chose to automated the original CDE.
  - Result is a series of terms involving loop integrals of the form $-i\int\frac{\mathrm{d}^dq}{(2\pi)^d}\frac{(q^2)^r}{\left(q^2-m_1^2\right)^{n_1}\cdots\left(q^2-m_k^2\right)^{n_k}}$

    $\times$ gauge-invariant operators built from $(P_{\alpha_1}\cdots P_{\alpha_n}F_{\mu\nu})$ and $(P_{\alpha_1}\cdots P_{\alpha_n}U_k)$.

STrEAM (SuperTrace Evaluation Automated for Matching)

T. Cohen, X. Lu, ZZ, 2012.07851.

A Simple Example

```
In[•]:= SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]
```

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\right]\Big|_{\text{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\{$$

$$-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2 \qquad (U_1) \qquad (\text{dim-}2)$$

$$\frac{1}{12\,m_1^2} \qquad (F_{\mu1,\mu2})\,(F_{\mu1,\mu2})\,(U_1) \qquad (\text{dim-}6)$$

$$\}$$

Out[•]= $\left\{\left\{\left\{\left\{-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2\right\}\right\},\{\{U_1\}\},2\right\},\left\{\left\{\left\{\frac{1}{12\,m_1^2}\right\}\right\},\left\{\{F_{\mu1,\mu2}\},\{F_{\mu1,\mu2}\},\{U_1\}\right\},6\right\}\right\}$

STrEAM (SuperTrace Evaluation Automated for Matching)

T. Cohen, X. Lu, ZZ, 2012.07851.

Additional Examples

In[∘]:= `SuperTrace[6, {Δ1}, display → True];`

$$-i\mathrm{STr}\left[\frac{1}{P^2 - m_1^2}\right]\Big|_{\mathrm{hard}} = \int d^4x \, \frac{1}{16\pi^2} \, \mathrm{tr}\{$$

$\frac{1}{12\, m_1^2}$ $(F_{\mu 1,\mu 2})(F_{\mu 1,\mu 2})$ (dim-4)

$\frac{i}{90\, m_1^4}$ $(F_{\mu 1,\mu 2})(F_{\mu 1,\mu 3})(F_{\mu 2,\mu 3})$ (dim-6)

$\frac{1}{60\, m_1^4}$ $(P_{\mu 1} P_{\mu 2} F_{\mu 2,\mu 3})(F_{\mu 1,\mu 3})$ (dim-6)

}

In[∘]:= `SuperTrace[6, {Δ1}, NoγinU → True, display → True];`

$$-i\mathrm{STr}\left[\frac{1}{\mathrm{Pslash} - m_1}\right]\Big|_{\mathrm{hard}} = \int d^4x \, \frac{1}{16\pi^2} \, \mathrm{tr}\{$$

$\frac{1}{6\, m_1}$ $(F_{\mu 1,\mu 2})(F_{\mu 1,\mu 2})$ (dim-4)

$-\frac{i}{90\, m_1^3}$ $(F_{\mu 1,\mu 2})(F_{\mu 1,\mu 3})(F_{\mu 2,\mu 3})$ (dim-6)

$\frac{1}{15\, m_1^3}$ $(P_{\mu 1} P_{\mu 2} F_{\mu 2,\mu 3})(F_{\mu 1,\mu 3})$ (dim-6)

}

reproduce log-type supertraces upon integration over m :

$$\frac{\partial}{\partial m_\Phi^2}\left[i\,\mathrm{STr}\log\left(P^2 - m_\Phi^2\right)\right] = -i\,\mathrm{STr}\left[\frac{1}{P^2 - m_\Phi^2}\right]$$

$$\frac{\partial}{\partial m_\Phi}\left[i\,\mathrm{STr}\log\left(\not{P} - m_\Phi\right)\right] = -i\,\mathrm{STr}\left[\frac{1}{\not{P} - m_\Phi}\right]$$

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Additional Examples

```
In[•]:= SuperTrace[6, {Δ1, U1, Δ2, Pν, Zν, Δ0, U3, Δ2, U4}, Udimlist → {1, 1, 2, 1}, display → True];
```

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\frac{1}{P^2-m_2^2}P_\nu Z_\nu\frac{1}{P^2}U_3\frac{1}{P^2-m_2^2}U_4\right]\Big|_{\mathrm{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\{$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad(U_1)\,(Z_{\mu_1})\,(P_{\mu_1}U_3)\,(U_4)\qquad(\mathrm{dim}\text{-}6)$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{2\,m_1^4}\quad(U_1)\,(P_{\mu_1}Z_{\mu_1})\,(U_3)\,(U_4)\qquad(\mathrm{dim}\text{-}6)$$

$$\frac{5-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad(P_{\mu_1}U_1)\,(Z_{\mu_1})\,(U_3)\,(U_4)\qquad(\mathrm{dim}\text{-}6)$$

$$\}$$

derivative interaction

By default:
1 = heavy mass
2, 3, … = light masses
0 = zero mass
Otherwise the user should specify which masses are heavy.

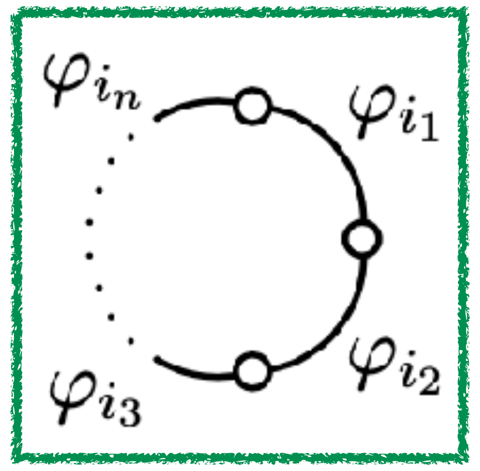
Summary & outlook

- We have devised a prescription for EFT matching up to one loop that is functional (pun intended).
- STrEAM automates the most tedious part (CDE) in this STrEAMlined prescription.

$\boxed{\text{EOM}}$

$$\begin{tikzcd} & \mathcal{L}_{\mathrm{UV}}[\Phi,\phi] \arrow[dl] \arrow[dr] & \\ \Phi_{\mathrm{c}}[\phi] \arrow[rr] \arrow[dd] & & \boldsymbol{K}, \boldsymbol{X} \arrow[d, "\text{Enumerate}"] \\ & & \text{Functional supertraces} \arrow[d, "\text{Evaluate}"] \\ \mathcal{L}^{(\mathrm{tree})}_{\mathrm{EFT}}[\phi] & & \mathcal{L}^{(\text{1-loop})}_{\mathrm{EFT}}[\phi] \end{tikzcd}$$

$$\boxed{-\left.\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\right|_{\Phi=\Phi_{\mathrm{c}}[\phi]} = \mathbf{K} - \mathbf{X}}$$



$\boxed{\text{(automated) CDE + matrix algebra}}$

Summary & outlook

- We have devised a prescription for EFT matching up to one loop that is functional (pun intended).
- STrEAM automates the most tedious part (CDE) in this STrEAMlined prescription.

- Many pheno applications.

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Summary & outlook

- We have devised a prescription for EFT matching up to one loop that is functional (pun intended).
- STrEAM automates the most tedious part (CDE) in this STrEAMlined prescription.

- Many pheno applications.
- Beyond relativistic EFT?
- Beyond one loop?
- Beyond matching: also efficient RG calculation.
- New insights on other aspects of EFTs?
- ??

T. Cohen, M. Freytsis, X. Lu, 1912.08814 (HQET).

THANK YOU