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Self-Organised Localisation

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Based on 2105.08617 G. Giudice, M. McCullough, TY

Outline

Motivation

- Criticality
- Quantum phase transitions (QPT)

• Fokker-Planck Volume (FPV) equation

• FPV dynamics

• FPV + QPT = SOL

- Discontinuity
- Flux conservation

SOL solutions

- Metastability
- Higgs mass
- Cosmological constant

Conclusion

• Measure problem

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• 1) Metastability



• Living on critical boundary of two phases coexisting



• 2) Higgs mass



• Tuned close to boundary between ordered and disordered phase

• 3) Cosmological constant



• Tuned close to boundary between implosion and explosion

- Higgs potential **metastability**
- Higgs mass
- Cosmological constant
- Why do we appear to live at a **special point** close to criticality?
 - **Metastability**: heavy new physics restores stability?
 - **Higgs mass**: new symmetries?
 - **Cosmological constant**: anthropics?
- Alternatively, hints for a **new principle** at play?

Self-Organised Criticality

• Many systems in nature **self-tuned** to live near criticality



• Need a mechanism for self-organisation of fundamental parameters

e.g. Self-Organized Criticality in eternal inflation landscape: J. Khoury et al 1907.07693, 1912.06706, 2003.12594

 Cosmological quantum phase transitions localise fluctuating scalar fields during inflation at critical points: self-organised localisation (SOL)

Phase Transitions (PT)

• Classical PT: varying background temperature



$$V = \frac{\lambda}{4} \left(\psi^2 - \rho^2\right)^2 + \kappa \phi \psi$$



$$V(H)$$

$$T=T_{c}$$

$$T

$$T=0$$

$$H$$$$

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Fokker-Planck Volume (FPV) equation

- Langevin equation: classical slow-roll + Hubble quantum fluctuations $\phi(t + \Delta t) = \phi(t) - \frac{V'}{3H}\Delta t + \eta_{\Delta t}(t)$
- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial \phi} \begin{bmatrix} \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V'P}{3H} \end{bmatrix} + 3HP = \frac{\partial P}{\partial t} \qquad H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$
Quantum diffusion term
$$\begin{array}{c} \text{Classical drift} \\ \text{term} \end{array} \quad \text{Volume term} \end{array}$$

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- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial \phi} \left[\frac{\hbar}{8\pi^2} \frac{\partial (H^{2+\xi} P)}{\partial \phi} + \frac{V' P}{3H^{2-\xi}} \right] + 3H^{\xi} P = H_0^{\xi-1} \frac{\partial P}{\partial t_{\xi}} \qquad H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

• **Ambiguity** in choosing time "gauge" $dt_{\xi}/dt = (H/H_0)^{1-\xi}$

- ϕ is *not* the inflaton; **apeiron** field scanning parameters
- Restrict to **EFT** field range f $\varphi \equiv \frac{\phi}{f}$ $V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi)$, $\omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!} \varphi^n$
- Assume sub-dominant energy density
- Expand around constant inflationary background H_0 $H(\varphi) \simeq H_0 \left(1 + \frac{\epsilon^2 f^4 \omega(\varphi)}{6M_n^2 H_0^2}\right)$

• FPV becomes
$$\frac{\alpha}{2} \frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial(\omega' P)}{\partial \varphi} + \beta \omega P = \frac{\partial P}{\partial T}$$
$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2} \frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

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Quantum
diffusion term
Classical drift
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• Maximum number of e-folds for non-eternal inflation: $N_{e-folds} < S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$

• Stationary FPV distributions $P(\varphi, T) = \sum_{\lambda} e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta \omega - \lambda) p = 0$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2} \frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

- Largest eigenvalue $\lambda = \lambda_{max}$ inflates most
- Eigenvalue determines peak location
- Note: **boundary conditions** necessary input for solution

• Stationary FPV distributions $P(\varphi, T) = \sum_{\lambda} e^{\lambda T} p(\varphi, \lambda)$ $\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta \omega - \lambda) p = 0 \Rightarrow Discriminant D>0 for positive solution:$ $D = \omega'^2 + 2\alpha(\lambda - \beta \omega - \omega'')$ $\alpha \equiv \frac{3\hbar H_0^4}{12^3 2\pi^4}, \quad \beta \equiv \frac{3}{2} \frac{\xi f^2}{15^2}, \quad T \equiv \frac{t}{12}, \quad t_R \equiv \frac{3H_0}{2\pi^2} = \frac{\alpha\beta S_{ds}}{2\pi^4}, \quad S_L = \frac{8\pi^2 M_p^2}{2\pi^4}$

$$\alpha \equiv \frac{3\hbar H_0^2}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{5}{2} \frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha \beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

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$$\frac{\alpha}{2}p'' + \omega'p' + (\omega'' + \beta\omega - \lambda)p = 0 \qquad \Longrightarrow \qquad \lambda = \beta\omega(\bar{\varphi}) + \omega''(\bar{\varphi}) - \frac{\alpha}{2\sigma^2}$$

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• C regime: $\alpha\beta \ll 1$. Peak is located as far down the potential as allowed by boundary condition.

- QV regime: $\alpha\beta \gg 1$, $\alpha^2\beta \ll 1$. Peak is a distance $1/(\alpha\beta)$ from the top with width $\sigma \simeq 1/\sqrt{\beta}$.
- $Q^2 V$ regime: $\alpha^2 \beta \gg 1$. Peak as close to the top as possible, with a distance comparable to the width $\sigma \simeq (\alpha/\beta)^{1/3}$.

FPV dynamics

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2\epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$$

 $S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$



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• ϕ triggers 1st order **quantum phase transition** at ϕ_c

- **Discontinuity** in V' leads to discontinuous P'
- Requiring continuity of FPV across the critical point gives a junction condition to satisfy



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• **Coexistence** of branches of different phases, require continuity of P_V and $P_V + P_h$ in FPV at ϕ_T : **flux conservation** junction conditions

$$P_h(\phi_T) = 0 \qquad \Delta P'_v = -P'_v(\phi_T) \qquad \Delta P_v = 0$$

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Higgs metastability

$$V(\varphi,h) = \frac{M^4}{g_*^2} \,\omega(\varphi) + \frac{\lambda(\varphi,h)}{4} \left(h^2 - v^2\right)^2$$

$$\lambda(\varphi, M/g_*) = -g_*^2 \varphi$$



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Higgs mass naturalness

$$V(\varphi,h) = \frac{M^4}{g_*^2} \,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h) h^4}{4}$$

$$\frac{V(\varphi, \langle h \rangle)}{M^4} = \begin{cases} \kappa_{\rm EW} \varphi + \kappa_2 \varphi^2 + \dots & \text{for } \varphi < 0 & (\text{unbroken EW: } \langle h \rangle = 0) \\ \kappa_{\rm EW} \varphi + \kappa_{\rm IR} \varphi^2 + \dots & \text{for } 0 < \varphi < \varphi_+ & (\text{IR phase: } \langle h \rangle = v) \\ -\kappa_0 + \kappa_{\rm UV} \varphi + \kappa_2 \varphi^2 + \dots & \text{for any } \varphi & (\text{UV phase: } \langle h \rangle = c_{\rm UV} M) \end{cases}$$
$$\kappa_{\rm EW} = \frac{\omega'(0)}{g_*^2} , \quad \kappa_2 = \frac{\omega''(0)}{2g_*^2} , \quad \kappa_{\rm IR} = \kappa_2 - \Delta \kappa , \quad \kappa_0 = \frac{-\lambda_{\rm UV} c_{\rm UV}^4}{4} , \quad \kappa_{\rm UV} = \kappa_{\rm EW} - \frac{c_{\rm UV}^2}{2} \end{cases}$$



- Use broken IR to broken UV phase transition

- Need lower instability scale Λ_I : ~TeV through VL fermions

 - (Naturalness motivation: scalars and vectors heavy, only VL fermions at TeV scale)



Cosmological constant

- Hidden phase: vanishing cosmological constant by R-symmetry
- Visible phase: SOL localises at vacuum degeneracy point



Solution must be in C regime with appropriate boundary conditions

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- Scalar fields undergoing quantum fluctuations during inflation can be localised at the critical points of quantum phase transitions: SOL
- SOL suggests our Universe lives at the critical boundary of coexistence of phases
- Measure problem: ambiguous choice of time parametrisation (recall $\beta \equiv \frac{3}{2} \frac{\xi f^2}{M^2}$)
- Related to regularisation of **infinite reheating surface**
- We have **not specified** the inflaton sector
- Motivates further study in context of SOL