

Chiral Symmetry Breaking and Confinement: separating the scales

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Work with Kostas Rigatos

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Introduction

Gauge theories are the only renormalizable theories of force we know.

Perturbative theories have UV poles (caveat the possibility of asymptotic safety with scalars)

Asymptotically free theories have strong coupling in the IR but are well defined

We still know relatively little about the “periodic table” of SCGTs...

Are they mostly like QCD: entry to strong coupling, chiral symmetry breaking and confinement all happen together at one scale.

Are these scales generically or occasionally separated?

Strongly Coupled BSM (with Erdmenger & Porod)

The ideas I present emerged from BSM logic and studying SCGT that underly composite higgs models using holography. Eg. $Sp(4)$ gauge theory with 4 fundamentals and 6 sextet quarks....

	AdS/ $Sp(4)$ no decouple	AdS/ $Sp(4)$ A2 decouple	AdS/ $Sp(4)$ quench	lattice [78] quench	lattice [79] unquench
$f_{\pi A_2}$	0.120	0.120	0.103	0.1453(12)	
$f_{\pi F}$	0.0569	0.0701	0.0756	0.1079(52)	0.1018(83)
M_{VA_2}	1*	1*	1*	1.000(32)	
f_{VA_2}	0.517	0.517	0.518	0.508(18)	
M_{VF}	0.61	0.814	0.962	0.83(19)	0.83(27)
f_{VF}	0.271	0.364	0.428	0.411(58)	0.430(86)
M_{AA_2}	1.35	1.35	1.28	1.75 (13)	
f_{AA_2}	0.520	0.520	0.524	0.794(70)	
M_{AF}	0.938	1.19	1.36	1.32(18)	1.34(14)
f_{AF}	0.303	0.399	0.462	0.54(11)	0.559(76)
M_{SA_2}	0.375	0.375	1.14	1.65(15)	
M_{SF}	0.325	0.902	1.25	1.52 (11)	1.40(19)
M_{BA_2}	1.85	1.85	1.86		
M_{BF}	1.13	1.53	1.79		

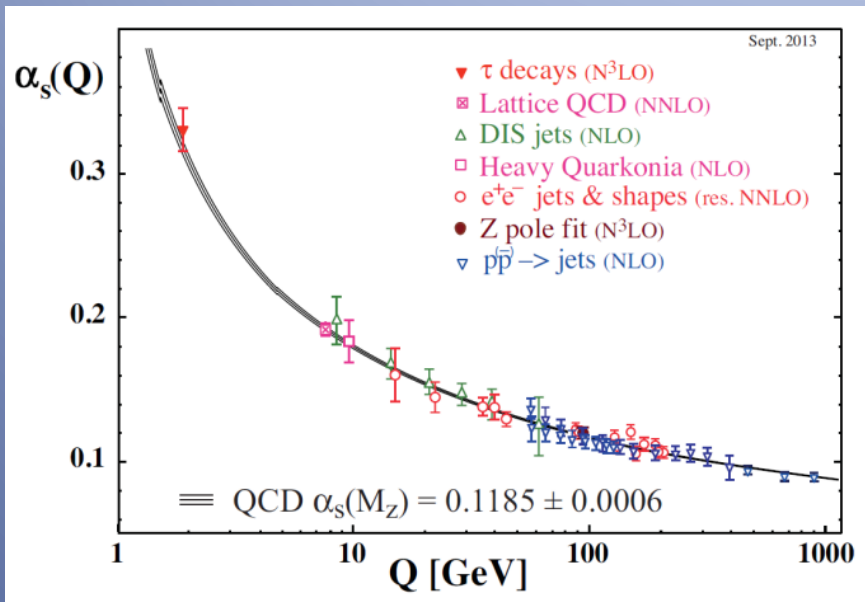
Has a gap
between the
F and A_2
sectors..

Holographic
predictions
depend on a
number of
guesses...

Here I want to present our questions/speculations mostly divorced from holography.

I don't really know the answer to any of the questions – but maybe the lattice can eventually tell us the clear answers – I'm trying to highlight the best places to look for those answers.

Asymptotic Freedom in QCD:



Quarks on top of each other are free...

As they separate coupling strength grows and perturbation theory breaks down...

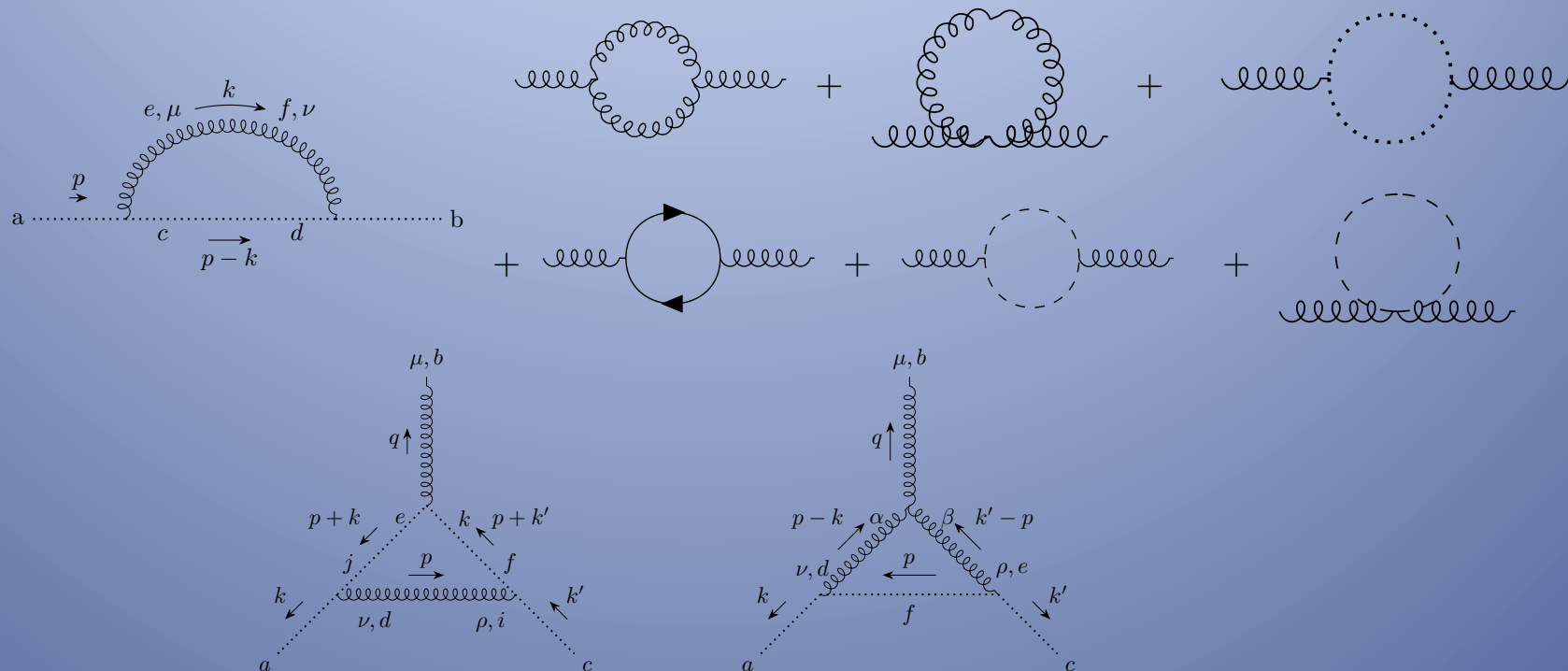
- Confinement
- Chiral symmetry breaking

Quantum Field Theory III

Easter Problem

Non-Abelian Gauge and $N = 1, 2, 4$ SYM beta functions

Nakorn Thongyoi



$$\frac{\partial \alpha_R}{\partial \ln \mu^2} = -\frac{b \alpha_R^2}{4\pi}$$

$$\alpha_R = \frac{\alpha_0}{1 + \frac{b \alpha_0}{2\pi} \ln \left(\frac{\mu}{\mu_0} \right)}$$

$$b = \frac{11C_2(G)}{3} - \frac{4N_f C(r)}{3} - \frac{N_s C(r)}{3}$$

Chiral Symmetry Breaking

The u and d quarks are basically massless

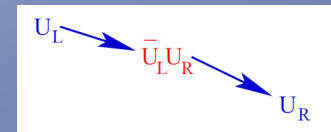
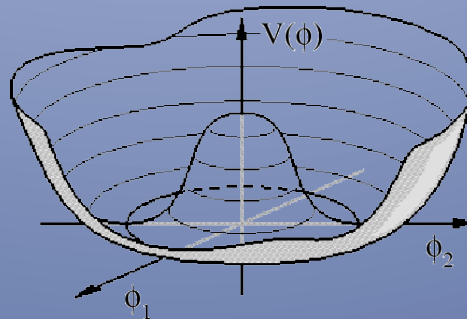
$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + h.c.)$$

$$\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$$

Evidence: lack of parity doubling, proton mass, Goldstone pions

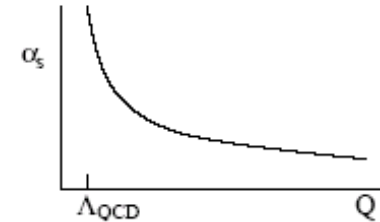
$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c. \rangle \neq 0$$



Confinement

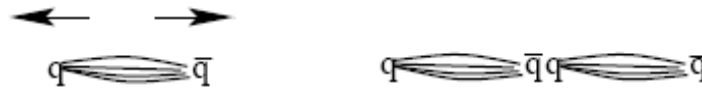
Coulomb law
vs linearity

The force is asymptotically
free (Wilczek, Gross, Politzer)



Confinement:

Quarks can not be liberated from hadrons.

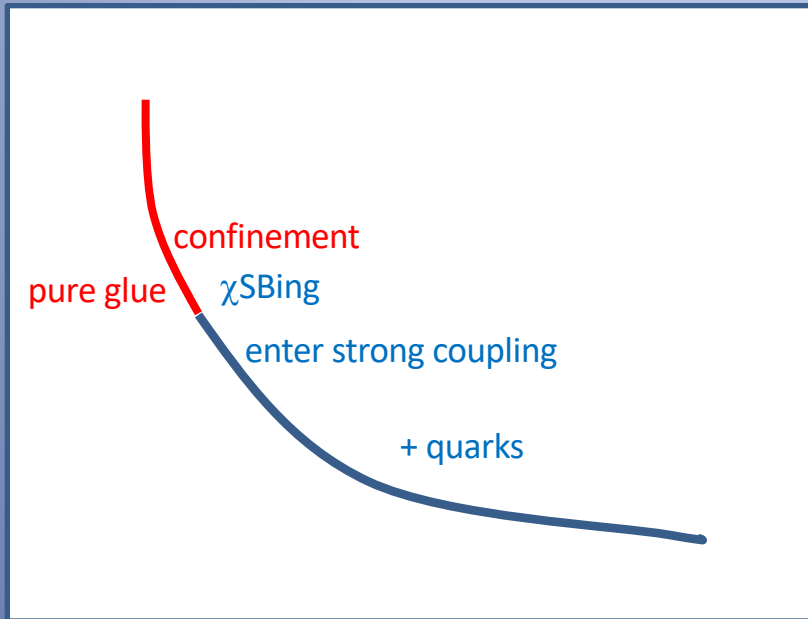


Localized, magnetically charged, scalar
gauge configurations form and
condense leading to a dual Meissner
effect (?).

• t'Hooft speculation

Seiberg Witten $N=2$
SYM realization

Compressed scales in QCD:



In QCD all scales are very close
– the thermal transition is a cross over with no distinction between the phenomena.

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Frithjof Karsch

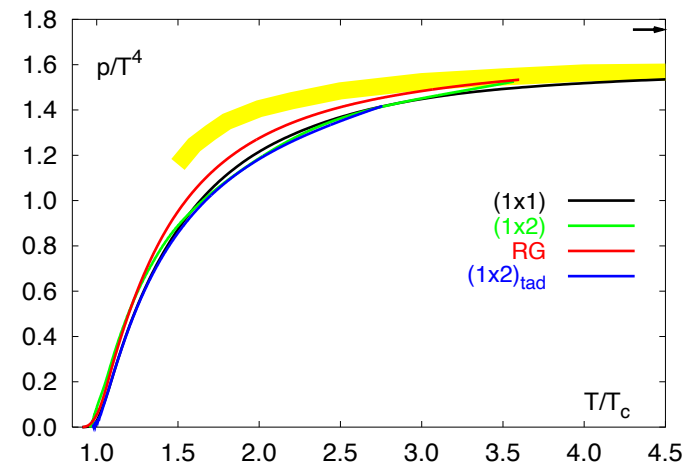


Fig. 10. Pressure of the SU(3) gauge theory calculated on lattices with different temporal extent and extrapolated to the continuum limit. Shown are results from calculations with the standard Wilson (1×1) -action [36] and several improved actions [38,39], which are defined in the Appendix. The broad band shows the approximately self-consistent HTL calculation of [41].

Walking – separating the entry to strong coupling and the IR mass gap (Holdom)

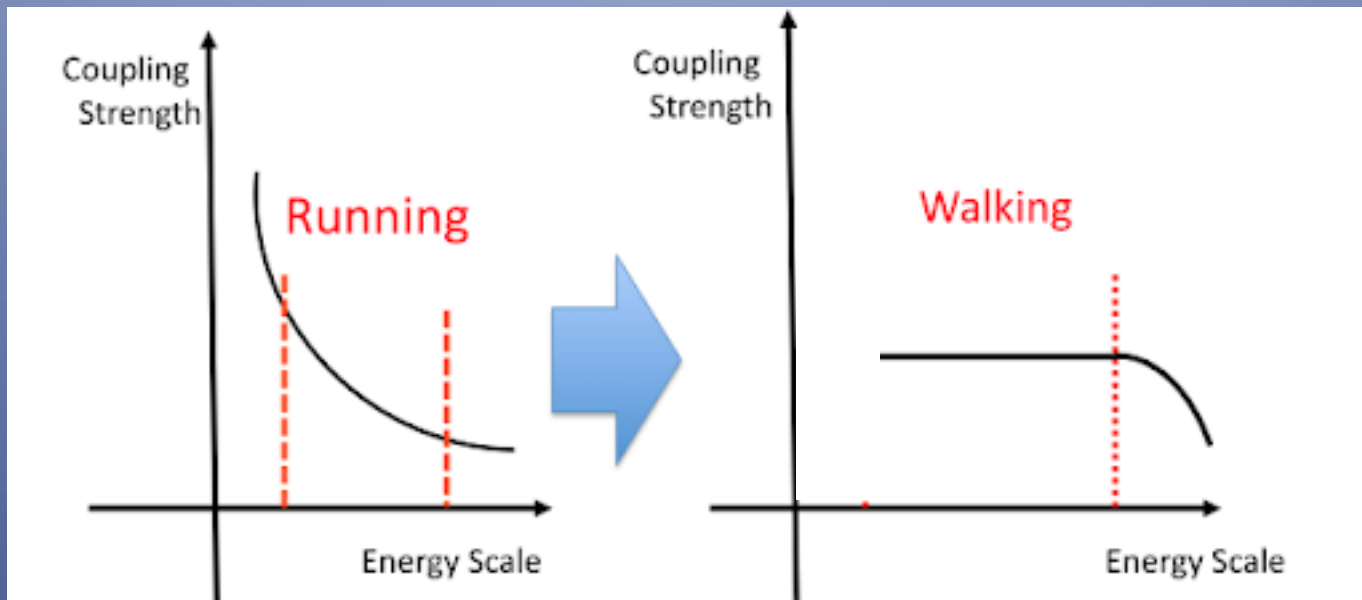
$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2 - b_1 \alpha^3 ,$$

$$b_1 = \frac{1}{24\pi^2} \left(34C_2^2(G) - \sum_R (20C_2(G) + 12C_2(R))T(R)N(R) \right) .$$

The two loop beta function has IR fixed points for some $N_f N_c$..

Near the edge of the AF region this is a perturbative fixed point (at large $N_c N_f$) – Banks Zak FP

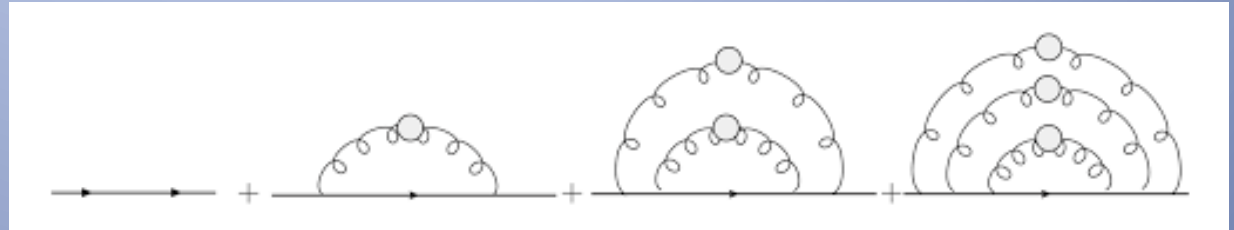
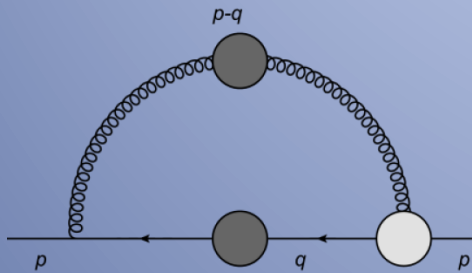
It grows as N_f decreases



Presumably at some point, lowering N_f , the coupling at the fixed point is sufficient to trigger chiral symmetry breaking and confinement.... Appelquist, Terning and Wijewardhana tried to quantify this first... hep-ph/9602385

1980s Gap Equations:

Cohen and Georgi, Nucl. Phys. B 314 (1989) 7



The computations showed a mass forms if γ (anomalous dimension of $\bar{q}q$) > 1 .

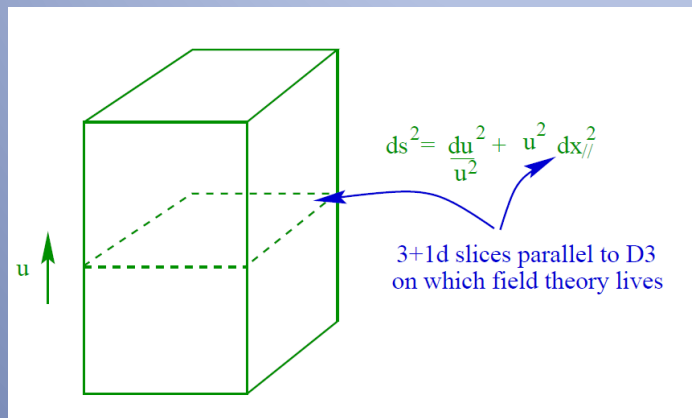
$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + h.c.)$$

$$\text{At one loop: } \gamma = \frac{3 C_2(R)}{2\pi} \alpha$$

$C_2(R)$ is the quadratic Casimir of the representation.

Holography Supports This Criteria:

Matti Jarvinen, Elias Kiritsis. 1112.1261. Raul Alvares, Nick Evans, Keun-Young Kim 1204.2474



A scalar in AdS represents the dimension 3 quark condensate...

Dilatations in conformal $\mathcal{N} = 4$ SYM:

$$\int d^4x \partial^\mu \phi \partial_\mu \phi, \quad x \rightarrow e^{-\alpha} x, \quad \phi \rightarrow e^\alpha \phi$$

Become spacetime symmetry of AdS

$$u \rightarrow e^\alpha u$$

u is a continuous mass dimension

→ RG Scale

A field for the mass/condensate:

$$S = \int d^4x \int d\rho \frac{1}{2} \rho^3 (\partial_\rho L)^2$$

$$\partial_\rho [\rho^3 \partial_\rho L] = 0$$

$$L = m + \frac{c}{\rho^2}$$

Holography Supports This Criteria:

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$$\partial_\rho[\rho^3 \partial_\rho L] - \rho \Delta m^2 L = 0 .$$

$$L = \frac{m_{FP}}{\rho^\gamma} + \frac{c_{FP}}{\rho^{2-\gamma}}, \quad \gamma(\gamma - 2) = \Delta m^2$$

A scalar in AdS represents the dimension 3 quark condensate...

$$\gamma(\gamma - 2) = \Delta m^2$$

If the dimension falls to 2 the BF bound is violated in AdS. ie $\gamma = 1$.

There's a pesky factor of 2 that Appelquist and Terning used to reduce the criteria to $\frac{1}{2}$ We'll keep their convention...

$$\Delta m^2 = -2\gamma + \dots$$

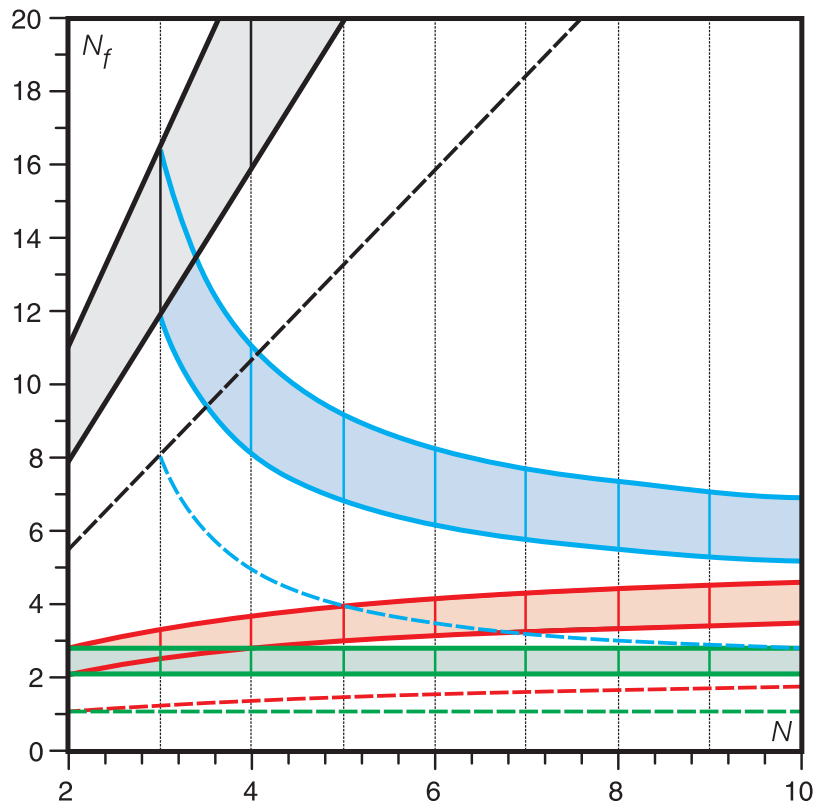


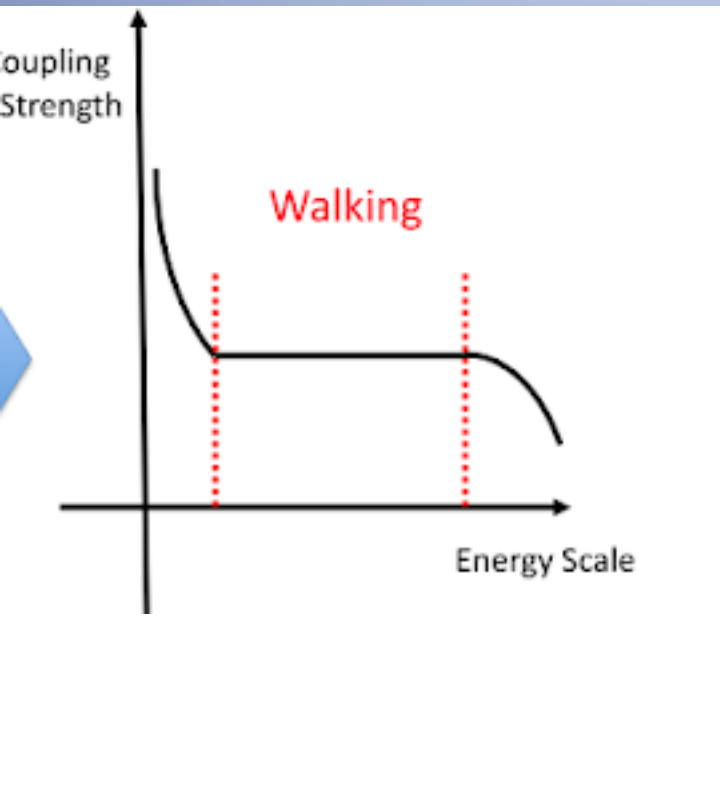
FIG. 1: Phase diagram for theories with fermions in the (from top to bottom in the plot; colour online): i) fundamental representation (grey), ii) two-index antisymmetric (blue), iii) two-index symmetric (red), iv) adjoint representation (green) as a function of the number of flavours and the number of colours. The shaded areas depict the corresponding conformal windows. The upper solid curve represents $N_f^I[R(N)]$ (loss of asymptotic freedom), the lower $N_f^{II}[R(N)]$ (loss of chiral symmetry breaking). The dashed curves show $N_f^{III}[R(N)]$ (existence of a Banks–Zaks fixed point). Note how consistently the various representations merge into each other when, for a specific value of N , they are actually the same representation.

Dietrich & Sannino mapped these conformal window bands for different representations.... Compute fixed point at two loops.. Compute γ .. See where it crosses $1/2$.

EG for SU(3) with fundamentals the edge of the window for Appelquist and Terning is $N_f=12$ But one can play with γ to get different values...

This is now lore and lattice simulations seek the boundary

$$10 < N_f < 12$$



It's a tricky game on the lattice because the coupling is slowly running...

You have to exclude the UV perturbative region if your lattice spacing isn't small enough...

You might miss the chiral symmetry breaking in the IR if your lattice is too small...

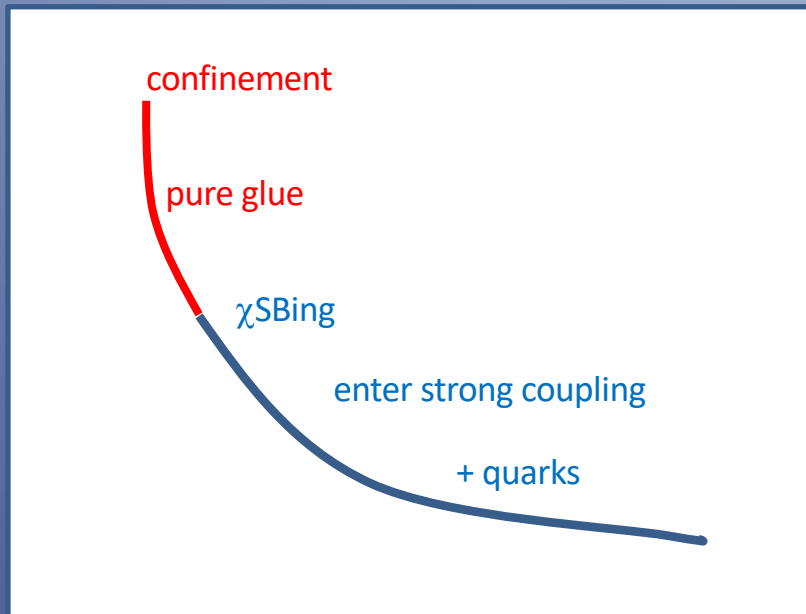
You don't have as much money as the QCD guys....

Lot's of contentious fun to be had...

Separating Chiral Symmetry Breaking and Confinement

B fields induce chiral symmetry breaking even at weak coupling – no confinement

Gauge theory + NJL interactions are believed to separate the two phenomena
(analytic, holographic, lattice computations agree)

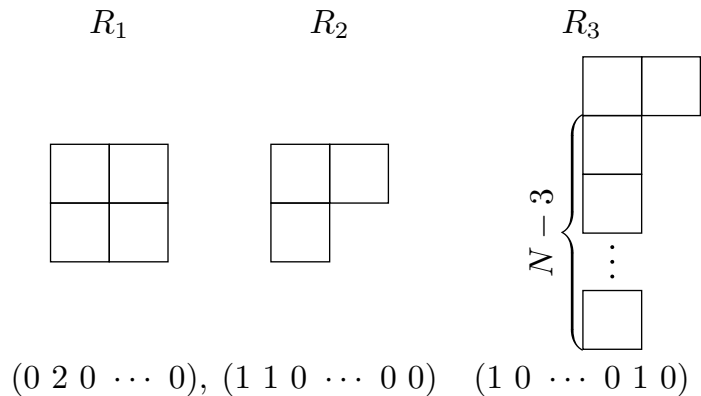
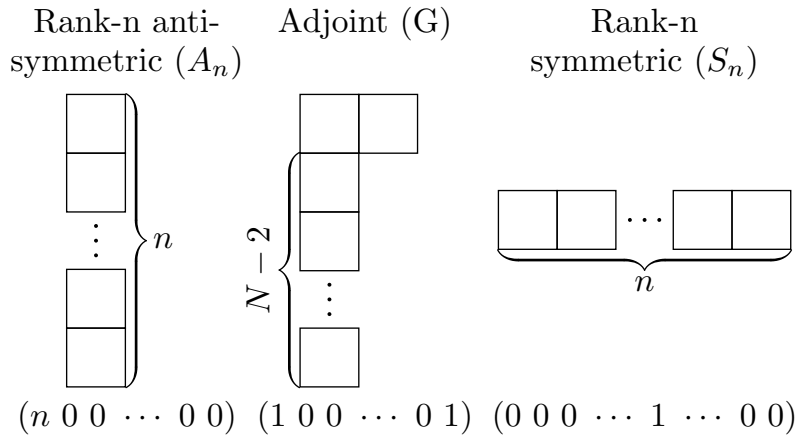


Could this (old gap equation story) be the true pattern of events... ??

Condensing monopoles is harder than condensing quarks...

If so at one loop what is the maximum gap between passing the critical γ and reaching the pole?

Dynkin indices of the singlet is just $(0\ 0\ 0\ \cdots\ 0\ 0)$ and the Young diagram is \bullet . The fundamental representation is $F = (1\ 0\ 0\ \cdots\ 0\ 0)$ and the Young tableaux \square . The remaining representations we consider are:



$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2,$$

$$b_0 = \frac{1}{2\pi} \left(\frac{11}{3} C_2(G) - \frac{4}{3} \sum_R T(R) N_f(R) \right)$$

$$\alpha_c = \frac{\pi}{3\ C_2(R)}$$

$$\mathcal{R}(R) = \frac{\Lambda_{\chi^{SB}}}{\Lambda_{\text{pole}}} = \exp \left(\frac{9}{11} \frac{N^2 - 1}{N} \frac{T(R)}{d(R)} \right)$$

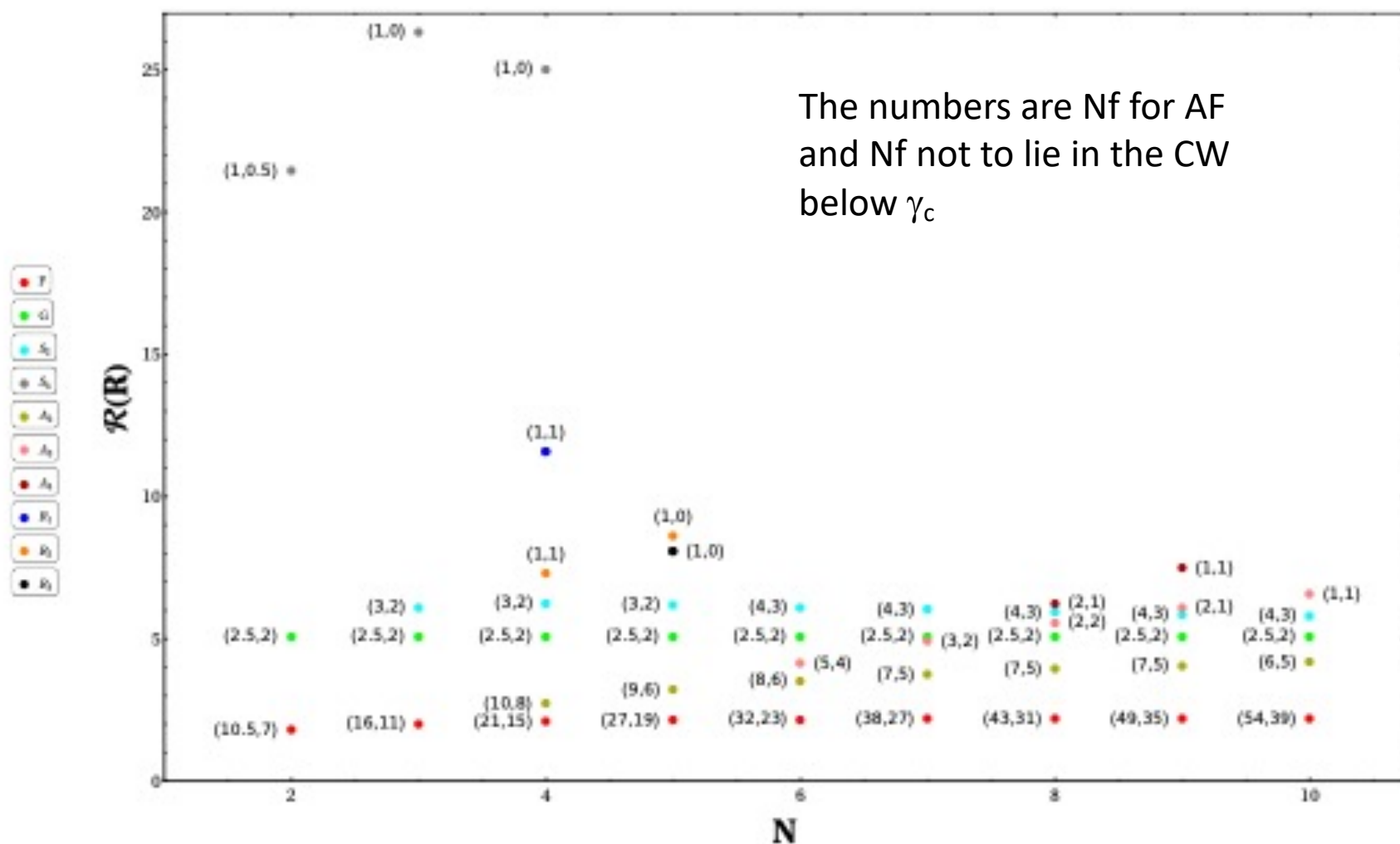


FIG. 1. Plot of $\mathcal{R}(R) = \Lambda_{NSB}/\Lambda_{pole}$ against N_c for various single representation theories: we have used red for the fundamental, green for the adjoint, cyan for the rank-2 symmetric, gray for three-rank symmetric, gold for the rank-2 antisymmetric, pink for the A_3 , maroon for the A_4 , and blue, orange, black for the $R_{1,2,3}$ respectively. The points are marked by the maximum number of flavours for which the theory is asymptotically free and the lower number of flavours that marks the last chiral symmetry breaking theory before the conformal window begins.

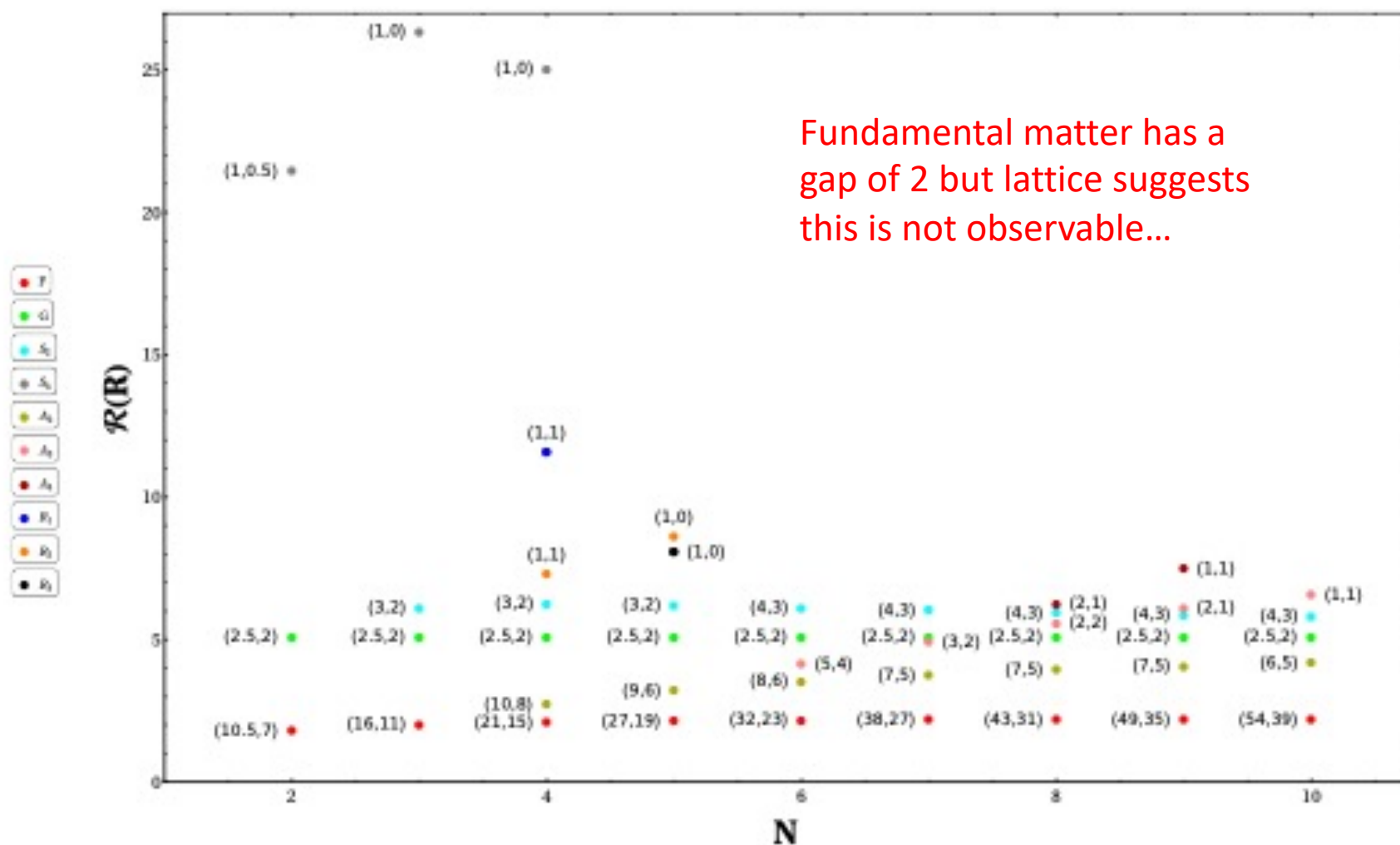


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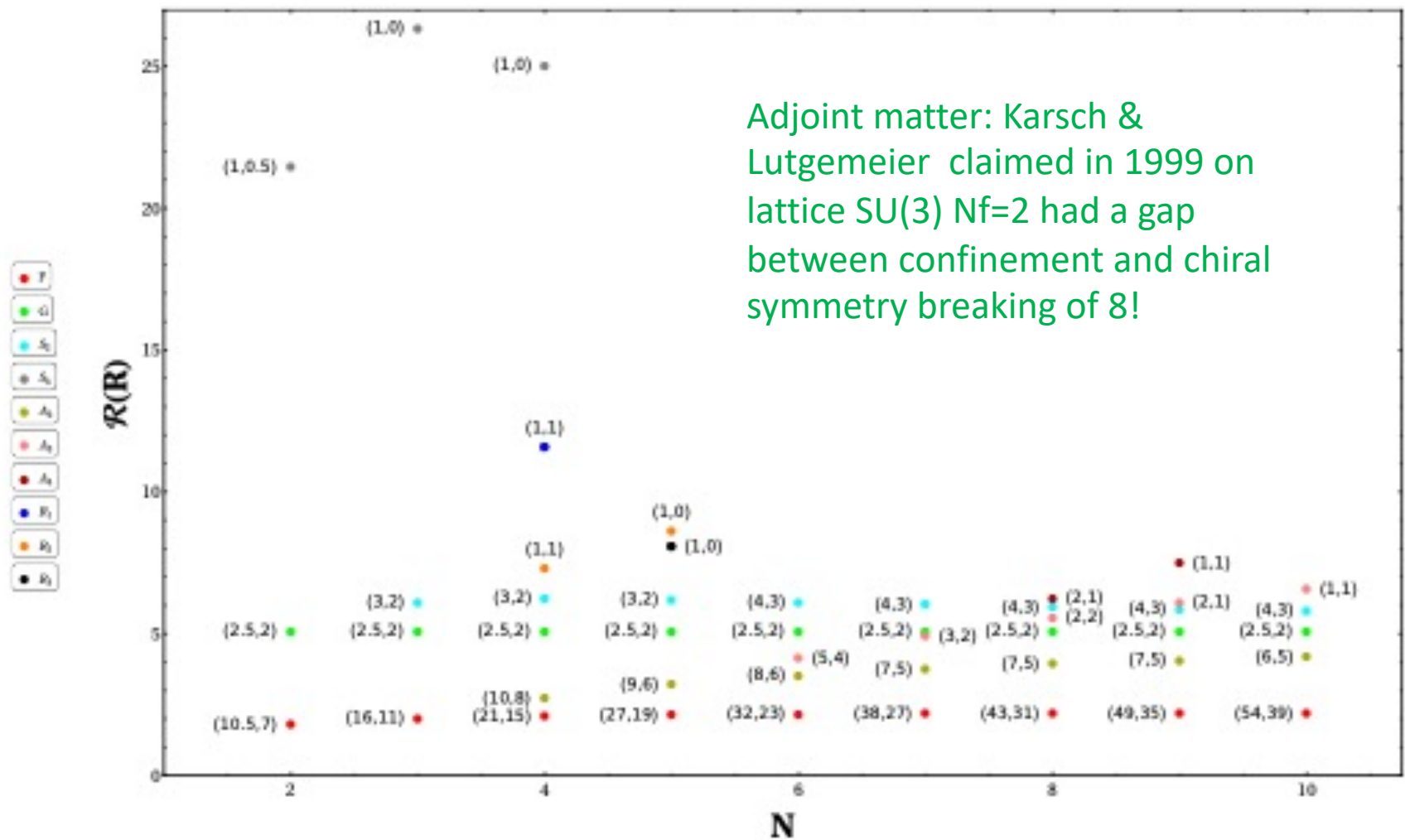


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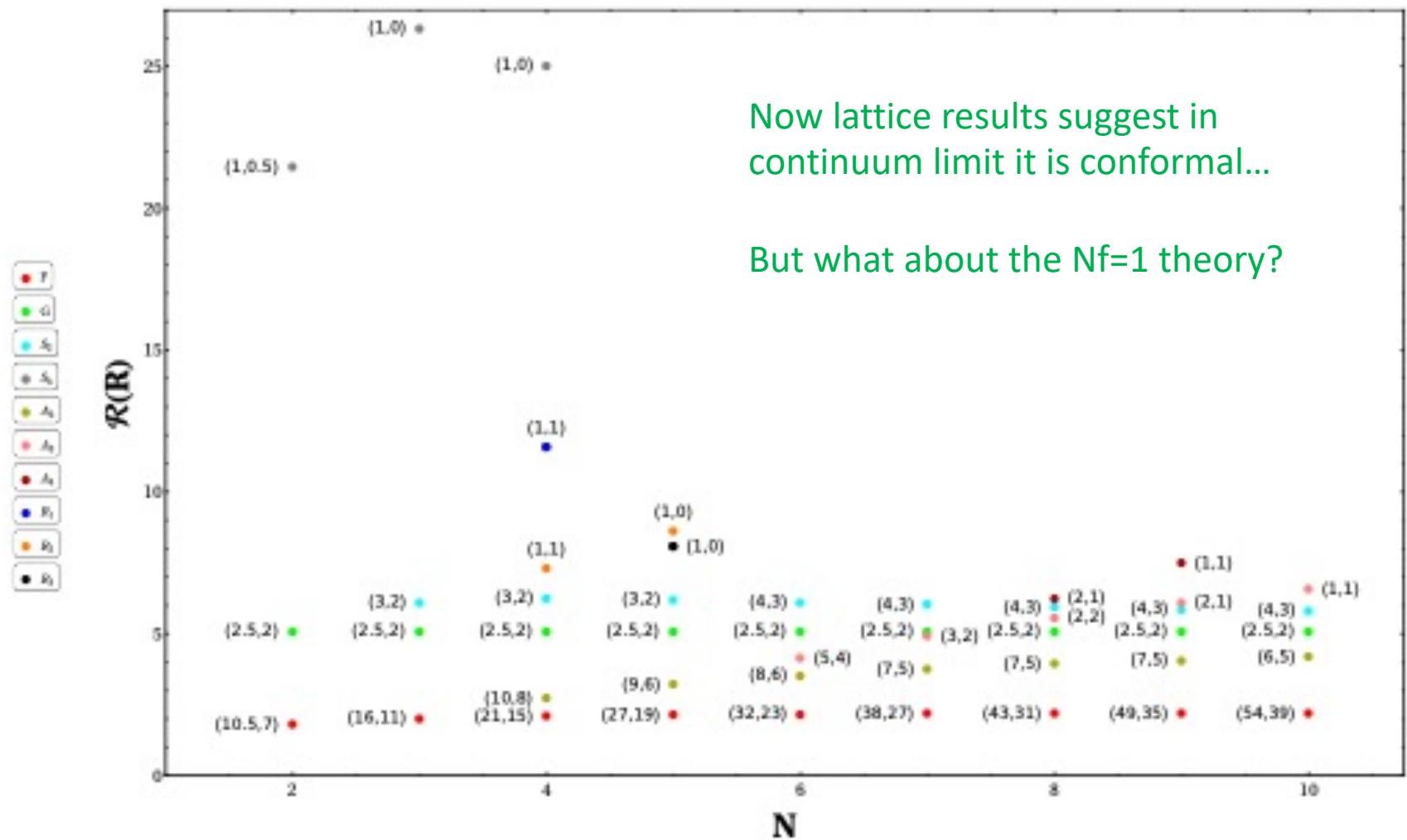


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Anomaly Alert!!!

$N_f=1$ theories have anomalous chiral symmetries... so how can you look for chiral symmetry breaking?

The anomaly's size is determined by the vev of

$$\text{Tr} F \tilde{F}$$

Which might also be of order the confinement scale... if there's a big gap then the symmetry may effectively be restored...

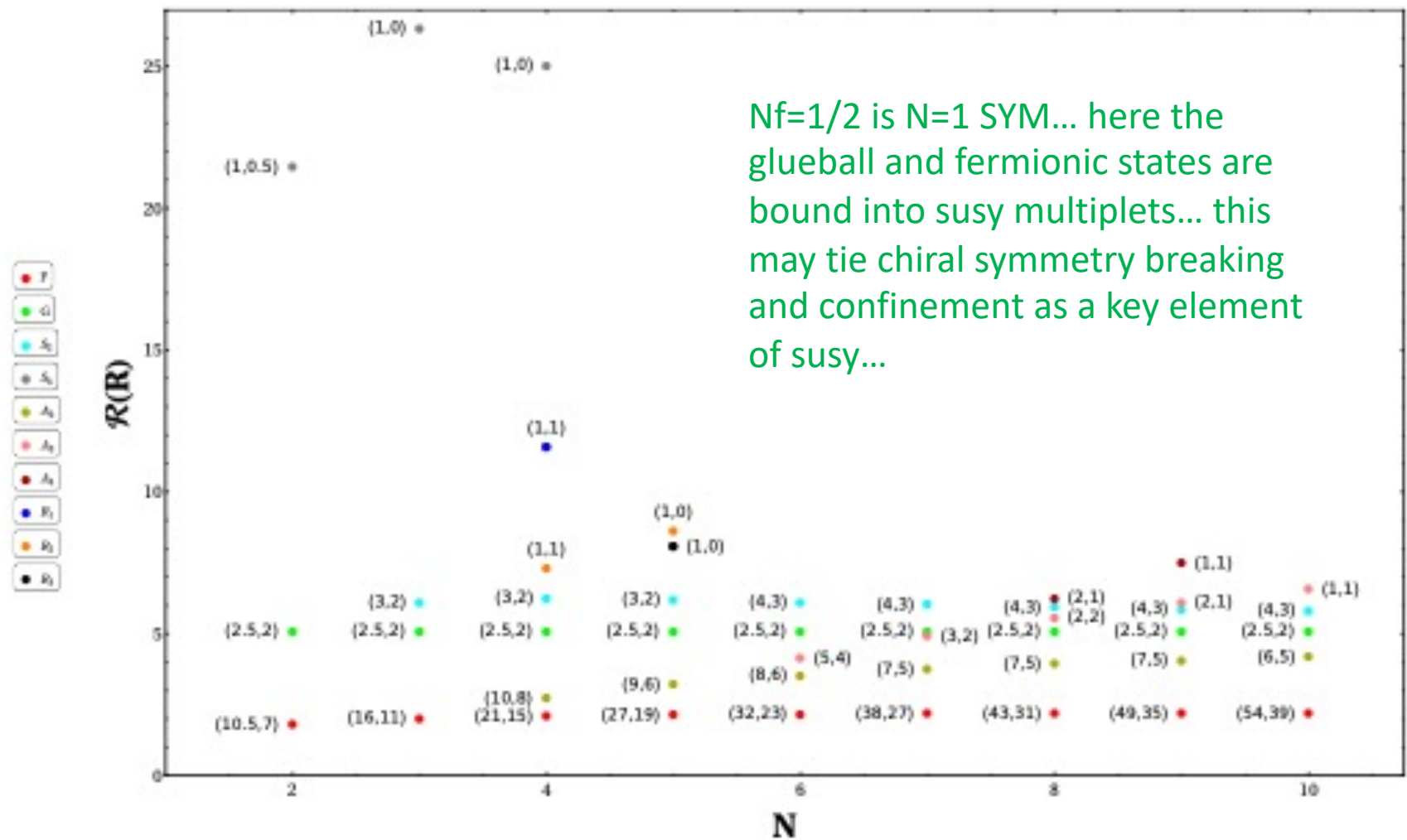


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Chiral Fermion Alert!!!

$N_f=1/2$ theories are chiral and potentially have a sign problem on the lattice (you can't prove they don't).

The group of Montvay studied a Majorana adjoint with a mass on the lattice and tested whether there was a sign problem as you reduce the mass....

Georg Bergner & Stefano Piemonte are pursuing this approach currently: they say that by careful massaging of how one takes the the chiral and continuum limits the sign problem may be avoided...

Note that to study the chiral transition you only need the mass to be less than the transition value not strictly zero...

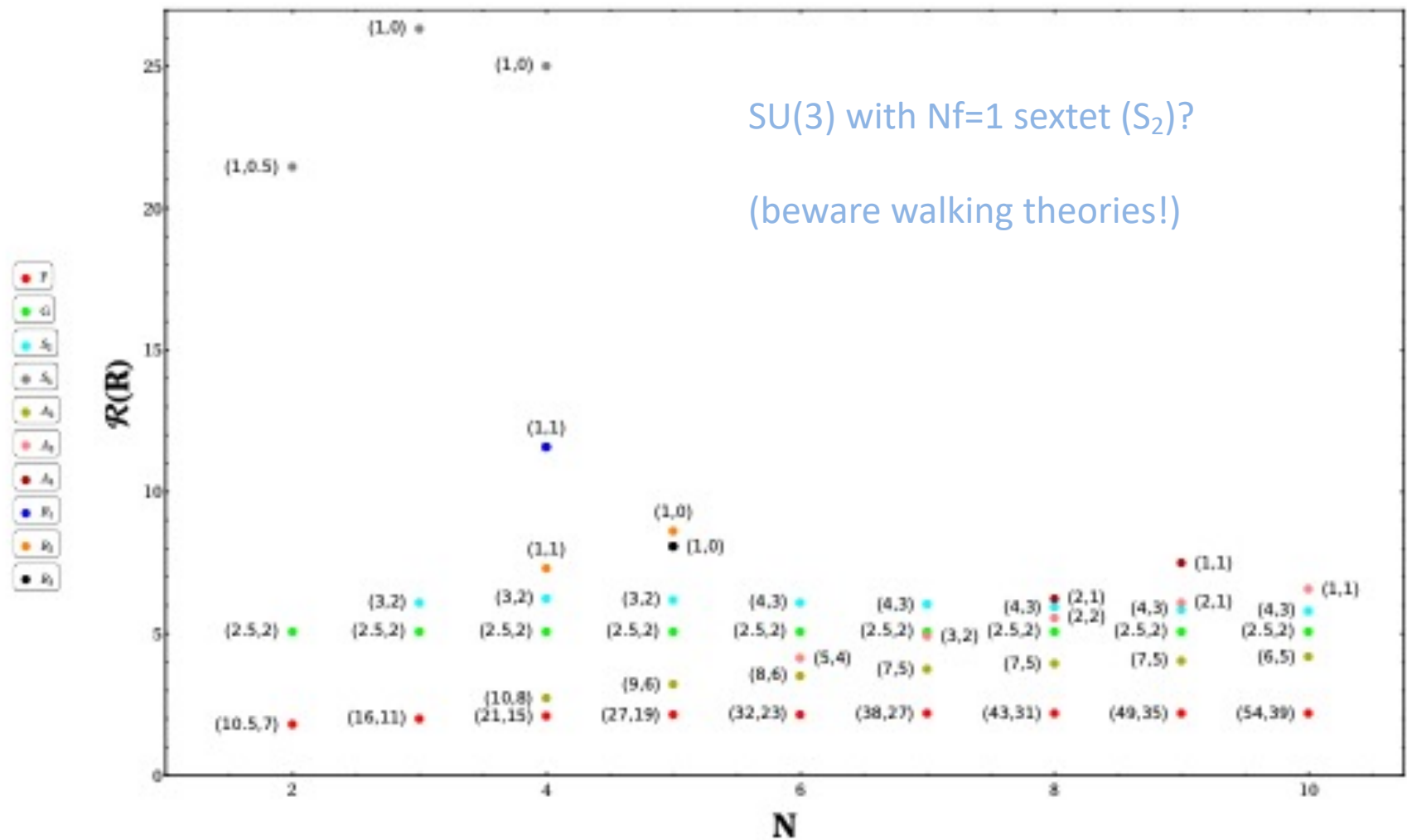


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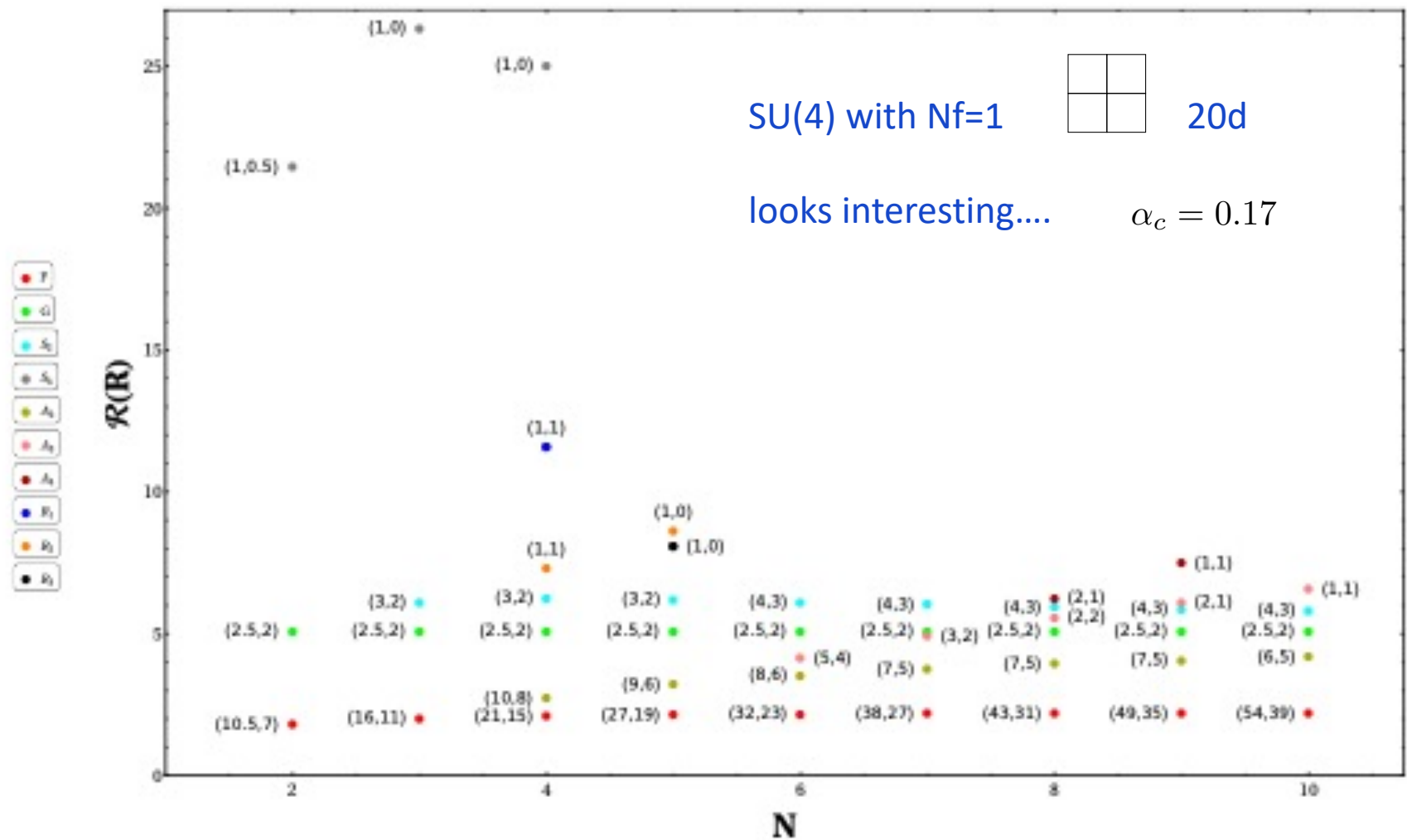


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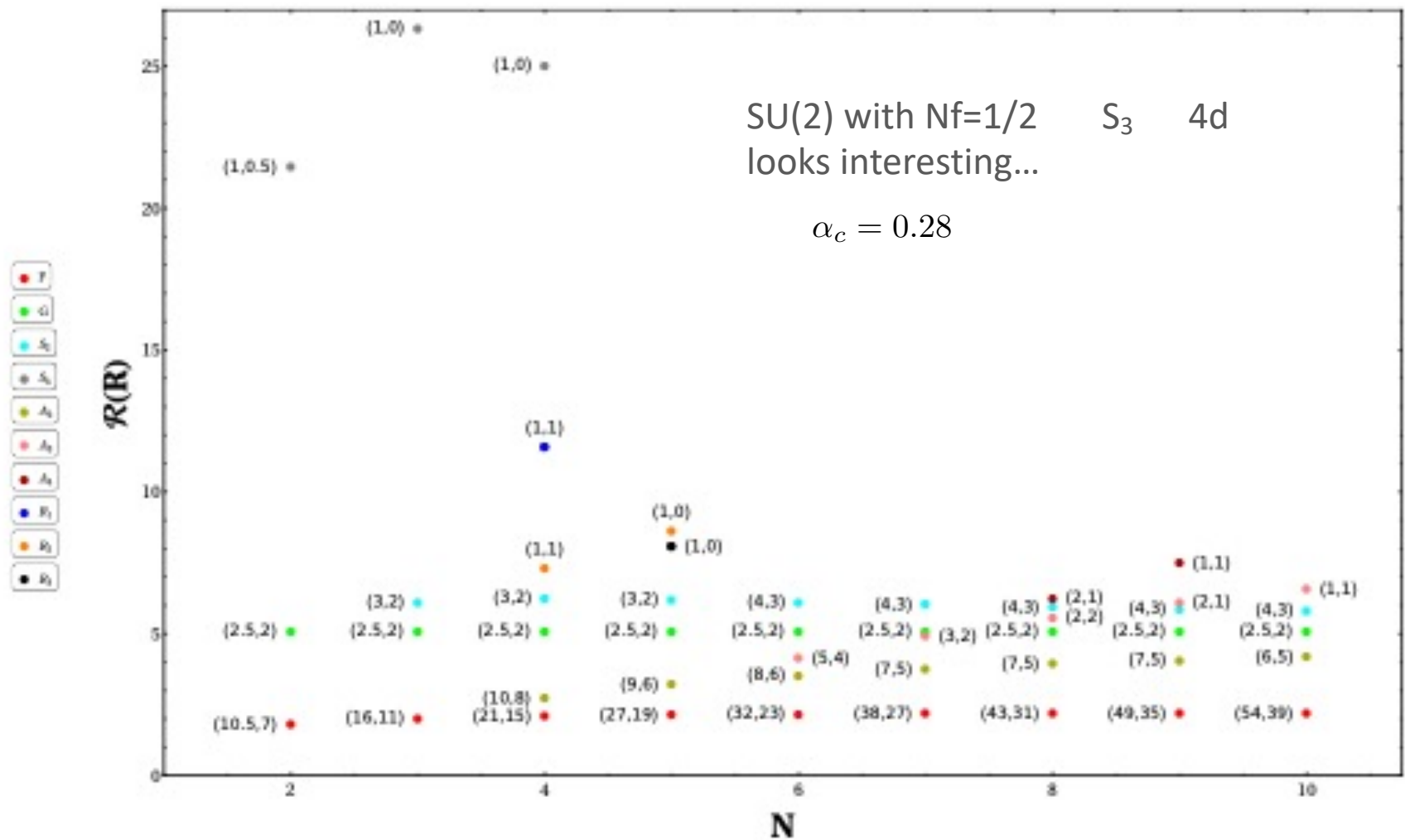


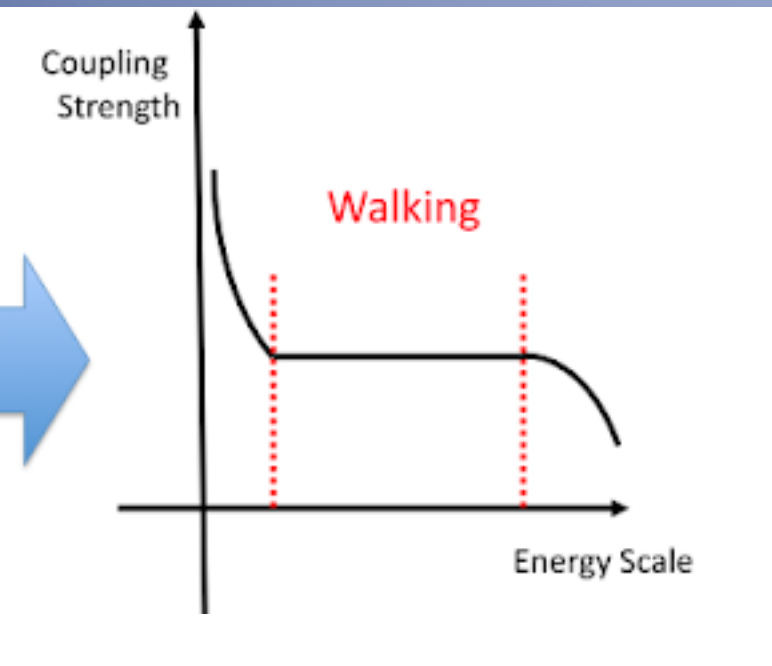
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Do I believe these simple models have a gap? Really not sure... the operators that cause confinement and chiral symmetry breaking will interact at strong coupling and may trigger each other to condense...

But the very largest gap theories may be weakly enough coupled at the chiral symmetry breaking scale to avoid this...

So...

Two Representation Theories



Lattice studies already propose a 20% difference in the gaps in multi-rep theories... can we use the walking nature of fermions between the two scales to delay confinement?

Now the red lines are the scales at which the two different representations condense...

Confinement is lower yet...

I'm not the first....

W. J. Marciano, Phys. Rev. D **21**, 2425 (1980).

Proposed that if a gap of 100 could be generated then a higher dimension rep of QCD could condense to break the electroweak symmetry...

QCD = technicolour!

(I wish this was true!)

Consider $SU(N_c)$ with $N_f^R = 1$, $\frac{1}{2}$ of some higher rep + N_f^F fundamentals....

We use the 2-loop results to run from $\gamma_c(R)$ to $\gamma_c(F)$

$$Q(R) = \frac{\Lambda_{\chi SB \ R}}{\Lambda_{\chi SB \ F}}$$

We tune N_f^F to maximize the gap whilst keeping the full theory out of the conformal window...

Note one could argue the gap to confinement is $Q(R) \cdot R(R)$

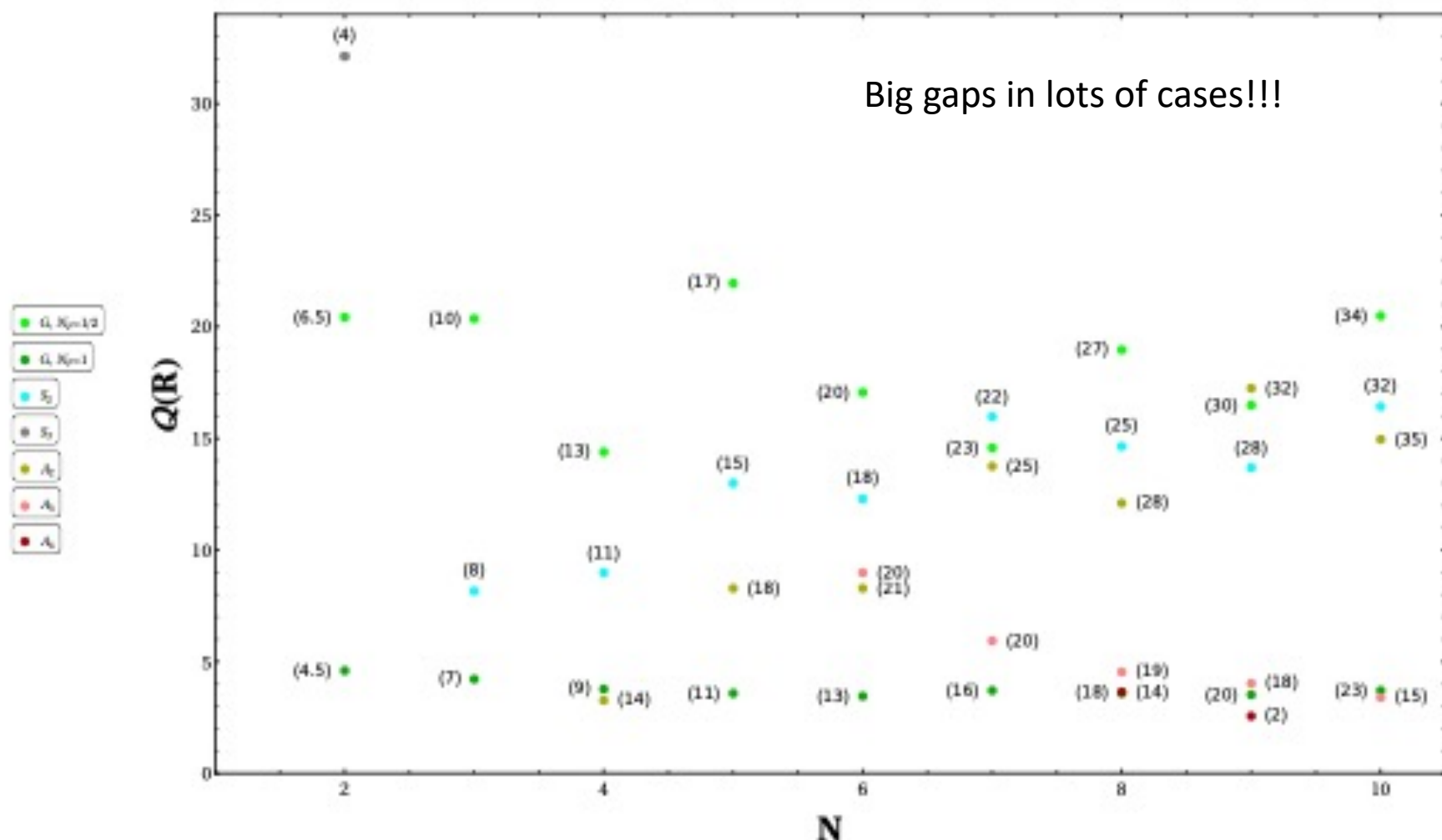


FIG. 2. Plot of $Q(R) = \Lambda_{XSB\ R} / \Lambda_{XSB\ F}$ against N_c in theories with the minimal number of fermions in the higher dimensional representation (either 1 or 1/2 for real representations) and N_f^F in the fundamental representation. N_f^F has been tuned to maximize $Q(R)$ and its value is next to the point. We have used red for the fundamental, green for the adjoint, cyan for the rank-2 symmetric, gray for three-rank symmetric, gold for the rank-2 antisymmetric, pink for the A_3 , maroon for the A_4 , and blue, orange, black for the $R_{1,2,3}$ respectively.

But a lot of these theories
are very walking above the
highest scale...

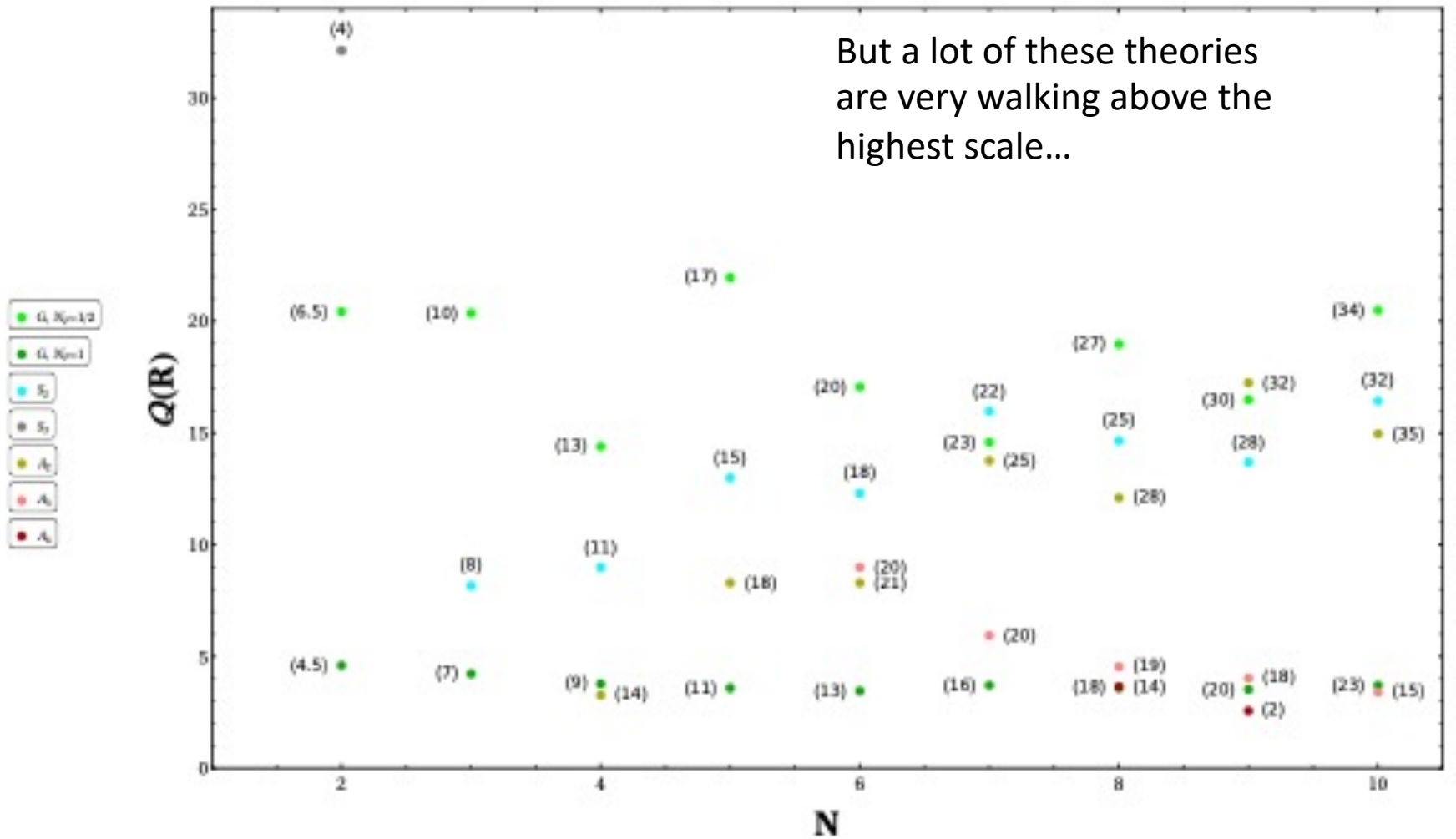


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EG SU(3) N=1 SQCD plus susy breaking fundamentals....

$$N_f^F=0 \quad b_0=1.43 \quad \alpha_c=0.35 \quad \alpha_*= \infty$$

$$N_f^F=4 \quad b_0=1.01 \quad \alpha_c=0.35 \quad \alpha_*= \infty \quad \frac{\Lambda_{\chi^{SB} \ R}}{\Lambda_{\chi^{SB} \ F}} = 2.6$$

$$N_f^F=8 \quad b_0=0.58 \quad \alpha_c=0.35 \quad \alpha_*=0.97 \quad \frac{\Lambda_{\chi^{SB} \ R}}{\Lambda_{\chi^{SB} \ F}} = 5.8$$

$$N_f^F=10 \quad b_0=0.37 \quad \alpha_c=0.35 \quad \alpha_*=0.40 \quad \frac{\Lambda_{\chi^{SB} \ R}}{\Lambda_{\chi^{SB} \ F}} = 20.3$$

Nf = 4,8 look easier to study than 10...

EG SU(3) $N_f=1$ sextet plus fundamentals....

$$N_f^F=0 \quad b_0 = 1.220 \quad \alpha_c = 0.314 \quad \alpha_* = \text{infinity}$$

$$N_f^F=4 \quad b_0 = 0.796 \quad \alpha_c = 0.314 \quad \alpha_* = 1.96 \quad Q(R) = 3.15$$

$$N_f^F=8 \quad b_0 = 0.371 \quad \alpha_c = 0.314 \quad \alpha_* = 0.354 \quad Q(R) = 8.2$$

An alternative measure of how walking the UV theory is

$$\mathcal{S} = \frac{\Lambda_{\alpha_c/2}}{\Lambda_{\alpha_c}}$$

	Nc = 2	Nc = 3
Nff = 10		$\gamma^* = 0.05$
Nff = 9		$\gamma^* = 0.17$
Nff = 8		$\gamma^* = 0.31$
Nff = 7		$Q = 4.3 \quad S = 1 \times 10^{17}$
Nff = 6	$\gamma^* = 0.17$	$Q = 3.5 \quad S = 72,000$
Nff = 5	$\gamma^* = 0.42$	$Q = 3.0 \quad S = 1160$
Nff = 4	$Q = 3.9 \quad S = 51,500$	$Q = 2.6 \quad S = 180$
Nff = 3	$Q = 3.1 \quad S = 365$	$Q = 2.4 \quad S = 61$
Nff = 2	$Q = 2.6 \quad S = 59$	$Q = 2.2 \quad S = 30$
Nff = 1	$Q = 2.3 \quad S = 23$	$Q = 2.1 \quad S = 18$

1 adjoint + Nff fundamentals

Finite T signal:

Introduce finite T and there should be a deconfined massive quark phase...



It's possible that all these theories have such a phase at the right T, mu...

Decoupling at Strong Coupling

In this analysis we have assumed that quarks decouple at IR scales below their (dynamical) mass.

Holography hints that decoupling at strong coupling may be less sharp... if the effect of the quark loops persist further into the IR then gaps may grow...

Help from the lattice needed – again the size of the gaps in the theories above will help dis-entangle this issue...

Conclusion

We've made some very naïve arguments here based on perturbative results extended beyond their regime of true validity... the dynamics may close these gaps (interesting!)...

But the questions are very pertinent and interesting (and influence our view of QCD)... And we may have shown the lattice where to look to resolve these issues...

Studies are under way: Lucini SU(3) with 1 adjoint.

Rummukainen, Kari. SU(2) with 1 adjoint

Bergner & Piemonte: $\frac{1}{2}$ adjoint + fundamental

We simply seek to encourage these studies – they are very interesting!!

And maybe QCD at finite μ has a deconfined massive quark phase... quark cores of neutron stars... LIGO/Virgo signals... see NEs holographic work...

Strongly Coupled Gauge Theory Forum

Curator: [Nick Evans](#) (University of Southampton),...

Asymptotically free, non-abelian gauge theories with fermionic matter underly our understanding of quantum theories of force. The strong nuclear force is described by an $SU(3)$ gauge theory; the weak nuclear force by an $SU(2)$ gauge theory. The catagorisation of these theories is likely to be important for understanding physics beyond the Standard Model but is in anycase an interesting field theoretic problem in itself. There are potentially theories which behave rather differently than QCD and understanding these theories will help test our understanding of QCD. Here we provide links that overview the state of knowledge in the field.

Classification Tables for
 $SU(N_c)$ with Fundamentals

Classification Tables for
 $SU(N_c)$ with Adjoints

<https://www.southampton.ac.uk/~evans/SCGT/>

Dynamic AdS/YM

Timo Alho, NE, Kimmo Tuominen
1307.4896

$$S = \int d^4x \, d\rho \, \text{Tr} \, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 \right]$$

$$X = L(\rho) \, e^{2i\pi^a T^a}.$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2,$$

$|X| = L$ is now the dynamical field **whose solution will determine the condensate** as a function of m - the phase is the pion.

We use the top-down IR boundary condition **on mass-shell**: $X'(\rho=X) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate – no hard wall

The gauge DYNAMICS is input through a guess for Δm

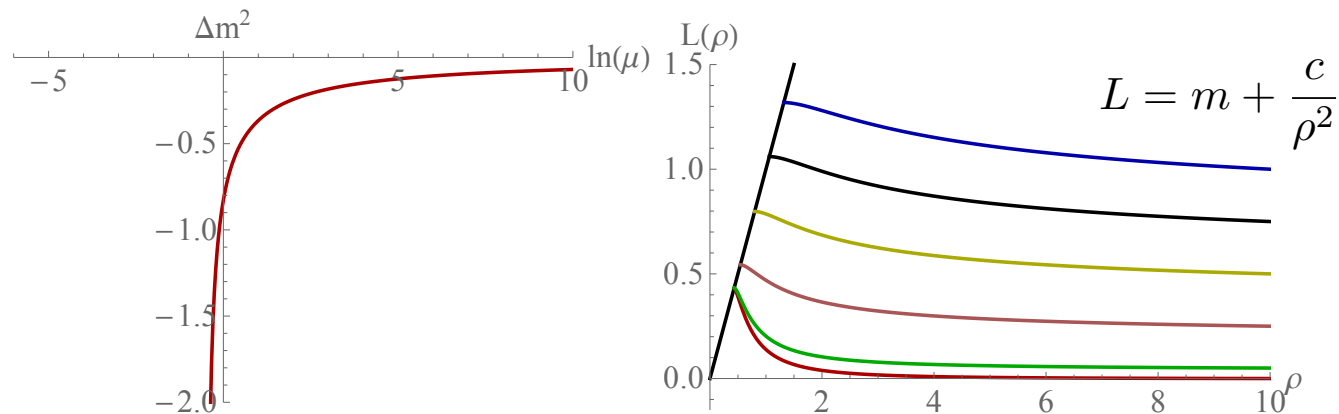
$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

The only free parameters are N_c, N_f, m, Λ

Formation of the Chiral Condensate

We solve for the vacuum configuration of L

$$\partial_\rho[\rho^3 \partial_\rho L] - \rho \Delta m^2 L = 0.$$



Read off m
and qq in
the UV...

Δm^2 from QCD

Shoot out with

$$L'(\rho = L) = 0$$

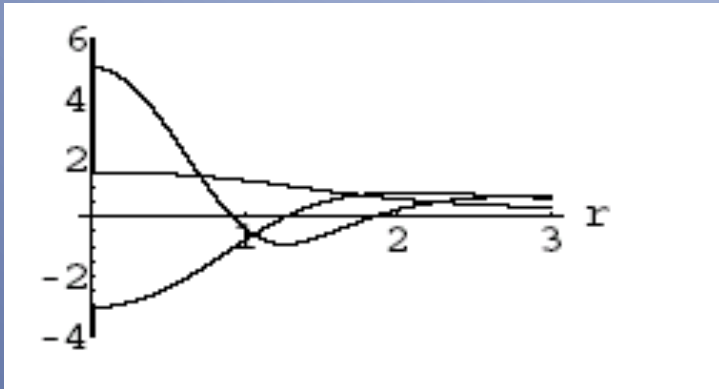
Meson Fluctuations

$$S = \int d^4x d\rho \text{Tr} \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$L = L_0 + \delta(\rho) e^{ikx} \quad k^2 = -M^2$$

$$\partial_\rho(\rho^3 \delta') - \Delta m^2 \rho \delta - \rho L_0 \delta \frac{\partial \Delta m^2}{\partial L} \Big|_{L_0}$$

$$+ M^2 R^4 \frac{\rho^3}{(L_0^2 + \rho^2)^2} \delta = 0.$$



The source free solutions pick out particular mass states... the σ and its radial excited states...

The gauge fields let us also study the operators and states

$$\bar{q} \gamma^\mu q \rightarrow \rho \text{ meson}$$

$$\bar{q} \gamma^\mu \gamma^5 q \rightarrow \text{a meson}$$

	AdS/ $Sp(4)$ no decouple	AdS/ $Sp(4)$ A2 decouple	AdS/ $Sp(4)$ quench	lattice [78] quench	lattice [79] unquench
$f_{\pi A_2}$	0.120	0.120	0.103	0.1453(12)	0.1018(83)
$f_{\pi F}$	0.0569	0.0701	0.0756	0.1079(52)	
M_{VA_2}	1*	1*	1*	1.000(32)	
f_{VA_2}	0.517	0.517	0.518	0.508(18)	
M_{VF}	0.61	0.814	0.962	0.83(19)	0.83(27)
f_{VF}	0.271	0.364	0.428	0.411(58)	0.430(86)
M_{AA_2}	1.35	1.35	1.28	1.75 (13)	1.34(14)
f_{AA_2}	0.520	0.520	0.524	0.794(70)	
M_{AF}	0.938	1.19	1.36	1.32(18)	
f_{AF}	0.303	0.399	0.462	0.54(11)	
M_{SA_2}	0.375	0.375	1.14	1.65(15)	1.40(19)
M_{SF}	0.325	0.902	1.25	1.52 (11)	
M_{BA_2}	1.85	1.85	1.86		
M_{BF}	1.13	1.53	1.79		

Sp(4) 4 F 6 A₂ Consider our quenched model against the lattice quenched results of

- [78] E. Bennett, D. K. Hong, J.-W. Lee, C.-J. D. Lin, B. Lucini, M. Mesiti, M. Piai, J. Rantaharju, and D. Vadamchino, “Sp(4) gauge theories on the lattice: quenched fundamental and antisymmetric fermions,” [arXiv:1912.06505](https://arxiv.org/abs/1912.06505) [hep-lat].

SU(4) 3 F 3 \bar{F} 5 A_2

G. Ferretti, “UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group,” [JHEP 06 \(2014\) 142](#), [arXiv:1404.7137 \[hep-ph\]](#).

In this model the A_2 symmetry breaking generates the SM higgs and the Fs are to give F A_2 F top partners

V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, “Spectroscopy of SU(4) composite Higgs theory with two distinct fermion representations,” [Phys. Rev. D 97 no. 7, \(2018\) 074505](#), [arXiv:1710.00806 \[hep-lat\]](#).

The lattice has simulated (unquenched) SU(4) 2 F 2 \bar{F} 4 A_2

	Lattice [80] 4A ₂ , 2F, 2 \bar{F} unquench	AdS/SU(4) 4A ₂ , 2F, 2 \bar{F} no decouple	AdS/SU(4) 4A ₂ , 2F, 2 \bar{F} decouple	AdS/SU(4) 5A ₂ , 3F, 3 \bar{F} no decouple	AdS/SU(4) 5A ₂ , 3F, 3 \bar{F} decouple	AdS/SU(4) 5A ₂ , 3F, 3 \bar{F} quench
$f_{\pi A_2}$	0.15(4)	0.0997	0.0997	0.111	0.111	0.102
$f_{\pi F}$	0.11(2)	0.0949	0.0953	0.0844	0.109	0.892
M_{VA_2}	1.00(4)	1*	1*	1*	1*	1*
f_{VA_2}	0.68(5)	0.489	0.489	0.516	0.516	0.517
M_{VF}	0.93(7)	0.933	0.939	0.890	0.904	0.976
f_{VF}	0.49(7)	0.458	0.461	0.437	0.491	0.479
M_{AA_2}		1.37	1.37	1.32	1.32	1.28
f_{AA_2}		0.505	0.505	0.521	0.521	0.522
M_{AF}		1.37	1.37	1.21	1.23	1.28
f_{AF}		0.501	0.504	0.453	0.509	0.492
M_{SA_2}		0.873	0.873	0.684	0.684	1.18
M_{SF}		1.03	1.02	0.811	0.798	1.25
M_{JA_2}	3.9(3)	2.21	2.21	2.21	2.21	2.22
M_{JF}	2.0(2)	2.07	2.08	1.97	2.00	2.17
M_{BA_2}	1.4(1)	1.85	1.85	1.85	1.85	1.86
M_{BF}	1.4(1)	1.74	1.75	1.65	1.68	1.81

The pattern is right...

The A₂-F gap is very well described...

Adding extra flavours is not a huge change...

Scalar masses get lighter as add extra flavours