Utilizing the causal spectrum of GWs to probe free streaming particles and the cosmological expansion

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arXiv: 2010.03568

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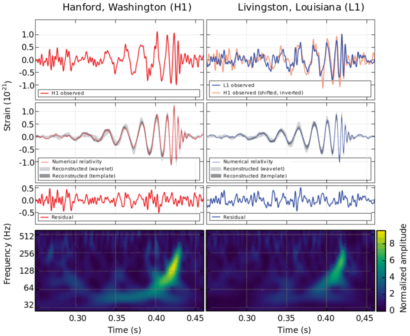
- 2015 marked the beginning of the era of GW astronomy.
- GW150914: first measurement of GWs from a Binary Black Hole merger!

LIGO

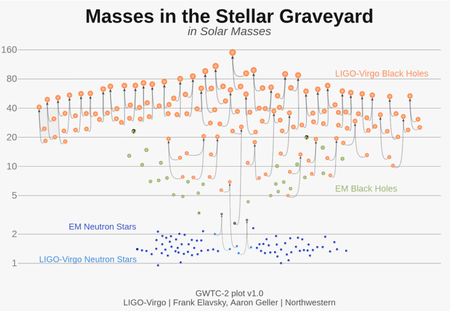
VIRGO



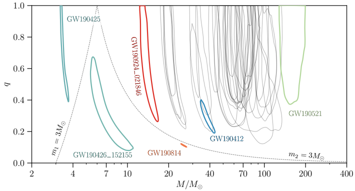
[LIGO, 1602.03837]

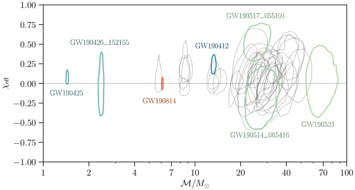
The era of Gravitational Wave astronomy

- The detection of many more mergers of compact objects allows us to study the properties of black holes, neutron stars in an unprecedented way.



[LIGO-VIRGO O3, 2010.14527]

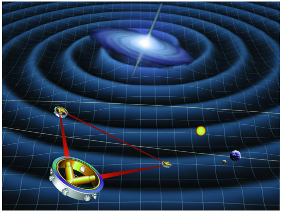


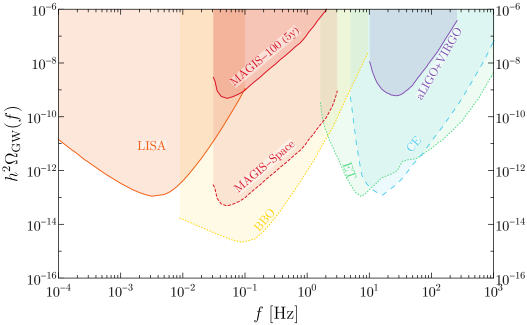


- Exciting prospects regarding astrophysical BHs, binary formation, H_0 measurements, BH superradiance, astro vs primordial BHs, . . .

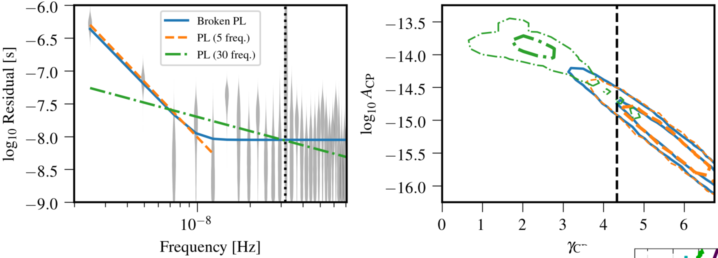
Stochastic Gravitational Wave Background

- One of the next frontiers in this exciting era is the search for a stochastic GW background, analogous to the CMB.
- Present upper limit from LIGO on stochastic GW background: $\Omega_{\mathrm{GW}}(10 - 100\ \mathrm{Hz}) < 1.7 \cdot 10^{-7}$ [1612.02029]
- Future space-based experiments will extend the reach in GW frequencies down to 0.1 mHz.

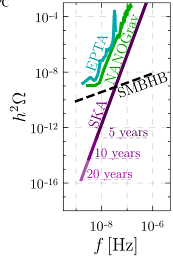




- At present there is a very interesting result from the NANOGrav collaboration (using Pulsar Timing). [NANOGrav 2009.04496]



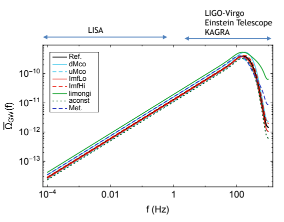
- It's soon to tell the origin of this red-noise process:
  1. Improperly modelled source of systematic noise;
  2. SGWB from the mergers of supermassive BHs;
  3. SGWB from new physics.
- Regardless, we could witness in the near future the discovery of a stochastic background of GWs!



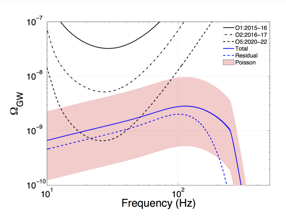
[Breitbach+ '18]

Astrophysical background: binary BH mergers

Unresolved sources lead to a stochastic background.
From stellar-mass BBHs: within reach of LIGO-VIRGO.



[Cusin+ '19]



[LIGO 1602.03847]

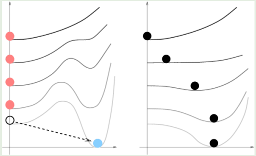
Distinguishing features: tilt $+2/3$ at low f, and peculiar frequency dep. of anisotropy spectrum. [Bartolo+ '19; Cusin+ '19; Hotinli+ '19; for PTs, Geller+ '18]

Primordial GWs from inflation

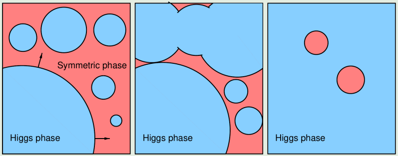
The primordial tensor modes generated by inflation are below $\Omega_{\rm GW} \lesssim 10^{-15}$ for the presently allowed value of r (and flat spectrum) $\Longrightarrow$ unobservable.

Phase transitions

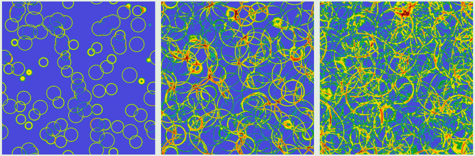
Cosmological phase transitions of the first order source GWs.



1st order 2nd order



[Hindmarsh+ '20]



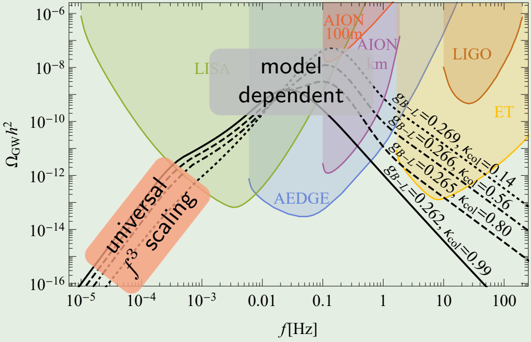
[Hindmarsh+ '15]

In the SM, both EW and QCD phase transitions are of 2nd order, but new physics can generically display 1st order phase transitions.

Phase transitions

GWs can be generated by three sources:

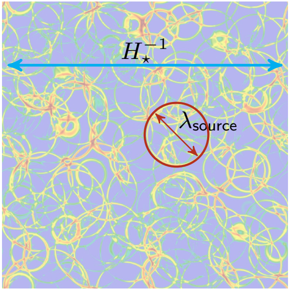
1. bubble collisions (dynamics of the scalar field);
2. sound waves in the plasma;
3. turbulences in the plasma.



[Ellis+ '20]

- The scaling as f^3 at low frequencies is a universal feature (assuming RD). [Caprini+ '09]
- What is the physical origin of this general behaviour?

Causality *prevents local phenomena from being correlated beyond* H^{-1}.



- Source $\Pi_{ij}(x)$ of GWs has a correlation length $\lambda_{\text{source}} \ll H_\star^{-1}$:
$$\langle \Pi(0)\, \Pi(d \gg \lambda_{\text{source}}) \rangle = 0 \;\Rightarrow$$
$$\langle \widetilde{\Pi}(k)\, \widetilde{\Pi}(-k) \rangle \xrightarrow{k \ll \lambda_{\text{source}}^{-1}} \text{constant}$$
- The spectral tilt at low f does not depend on the source.
- Wavelengths which were super-horizon are not sensitive to the details of the generation, but only to the universe expansion and the GW propagation.

Causality-limited GWs $\Longrightarrow$ k^3 scaling

- We consider GWs for which

  wavelength $k^{-1} \gg$ corr. length of the source λ_{source}

  period $f^{-1} \gg$ duration of the phase transition β^{-1}

- Eq. of motion for the GW $h_{ij}^{(+,\times)}(k,\tau) \equiv h$:

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 4\mathcal{H}^2 \Pi(k,\tau) = J_\star \; \delta(\tau - \tau_\star)$$

  approximating the source as instantaneous and constant at small k.

- The solution in a radiation-dominated universe is

$$h(\tau) = \frac{a_\star}{a} \underbrace{\frac{1}{k}}_{\text{specific of RD}} J_\star \sin k(\tau - \tau_\star)$$

- The spectrum of Ω_{GW} at low frequencies is

$$\frac{\mathrm{d}\,\Omega_{\text{GW}}}{\mathrm{d}\ln k} \sim \underbrace{k^3}_{\text{phase space}} \cdot \underbrace{k^2}_{\rho_{\text{GW}} \sim h'^2} \cdot \underbrace{\frac{1}{k^2}}_{\text{for RD}} \cdot \underbrace{P_\Pi(k)}_{k \text{ ind. from causality}} \sim k^3 \,.$$

Utilizing the causality-limited spectrum

- Causality (absence of correlation beyond Hubble for local processes) is precisely what makes the source $J_\star$ independent from k for $k \ll \lambda_{\text{source}}^{-1}$.
- The universality of the spectrum at low frequencies for causality-limited processes makes it an exciting tool to study our universe.
  [Hook, Marques-Tavares, DR 2010.03568]
- Phase transitions are the key example, but also preheating and GWs at 2nd order from peaks in the scalar perturbations are possible causality-limited scenarios.
- What can we extract from it?

1. How can we physically understand the f^3 scaling?
2. What can alter the propagation of GWs and hence their causality-limited spectrum?
3. How is the expansion history of the Universe influencing the causality-limited spectrum?

Causality-limited spectrum: physical understanding

- Let us investigate the physical origin of the k scaling of causality-limited GWs:

$$h(\tau) = \frac{a_\star}{a} \underbrace{\frac{1}{k}}_{\text{specific of RD}} J_\star \sin k(\tau - \tau_\star)$$

- For long period f^{-1} of the GW compared to the duration of the phase transition, it can be treated as an instantaneous impulse at $\tau_\star$:

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 4\mathcal{H}^2 \Pi(k,\tau) = J_\star\,\delta(\tau - \tau_\star)$$

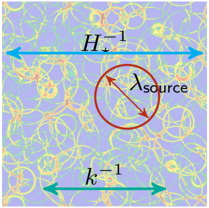
- The system right after $\tau_\star$ is

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 0\,,$$
$$h(\tau_\star + \epsilon) = 0\,,$$
$$\partial_\tau h(\tau_\star + \epsilon) = J_\star\,.$$

- The sudden beat given to the oscillator imprints a velocity and zero displacement to the wave, similarly to a hammer hitting on a string.

Causality-limited spectrum: sub-horizon modes

Sub-horizon modes $\lambda_{\text{source}}^{-1} \gg k \gg \mathcal{H}_\star$



- Modes which are sub-horizon at generation (but still beyond the correlation length of the source) are under-damped

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 0$$

- The solution is a frictionless oscillation, whose amplitude red-shifts as $1/a$:

$$h(\tau) \approx \frac{a_\star}{a} \underbrace{\frac{1}{k}}_{\text{sub-hor.}} J_\star \sin k(\tau - \tau_\star)$$

- Apart from the redshift, the eq. of state w of the universe does not enter.
- Sub-horizon modes are insensitive to the expansion rate.
- The corresponding Ω_{GW} is

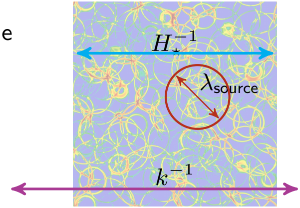
$$\frac{d\,\Omega_{\text{GW}}}{d\ln k} \sim \underbrace{k^3}_{\text{phase space}} \cdot \underbrace{k^2}_{\rho_{\text{GW}} \sim h'^2} \cdot \underbrace{\frac{1}{k^2}}_{\text{sub-hor.}} \cdot P_\Pi(k) \sim k^3\,.$$

Super-horizon modes $k \ll \mathcal{H}_\star$

- Super-horizon modes are over-damped: Hubble friction prevails.

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 0$$

$$\begin{cases} h = 0 \\ h' = J_\star \end{cases} \longrightarrow \begin{cases} h = \frac{J_\star}{\mathcal{H}_\star} \\ h' = 0 \end{cases} \longrightarrow \text{frozen}$$



- The dependence on $\mathcal{H}$ along the whole super-horizon phase explains why they are a tool to study the Universe expansion.
- *Suppression* of $k/\mathcal{H}_\star$ due to the excitation of an over-damped oscillator:

$$h \approx \frac{J_\star}{k} \sin \ \text{(sub-hor.)} \quad \text{vs.} \quad h \approx \frac{J_\star}{\mathcal{H}_\star} \ \text{(super-hor.)}$$

- While super-hor., h doesn't redshift $\rightarrow$ *boost* compared to sub-hor. modes.
- After Hubble crossing at $\mathcal{H}(\tau_k) = k$, h starts redshifting and oscillating:

$$h \approx \frac{a(\tau_k)}{a} \frac{J_\star}{\mathcal{H}_\star} \sin k\tau$$

Radiation domination

- Super-horizon modes: $\mathcal{H} \sim \frac{1}{\tau} \sim \frac{1}{a}$ and they enter the horizon at $\mathcal{H}(\tau_k) = k$, so

$$h \approx \frac{a(\tau_k)}{a} \frac{J_\star}{\mathcal{H}_\star} \sin k\tau = \frac{a_\star}{a} \frac{J_\star}{k} \sin k\tau$$

- They match precisely the sub-horizon solution!
- The reason is that two competing effects precisely cancel during RD:
  1. Suppression $\dfrac{k}{\mathcal{H}_\star}$ due to exciting over-damped mode;
  2. Boost of $\dfrac{a(\tau_k)}{a_\star}$ due to mode being frozen while super-horizon $\overset{\text{RD}}{\to} \dfrac{\mathcal{H}_\star}{k}$.
- As a result, for the standard case of a phase transition during RD, there are no features around $k \sim \mathcal{H}_\star$.
- All modes have an amplitude $\dfrac{1}{k}$, and $\Omega_{\text{GW}} \sim k^3$.

Generic equation of state $a \sim \tau^n$

- Generic equation of state: $a \sim \tau^n$ where $n = \frac{2}{1+3w}$ is 1 for RD, 2 for MD.
- Super-horizon modes: $\mathcal{H} \sim \dfrac{1}{\tau} \sim \dfrac{1}{a^{1/n}}$ so

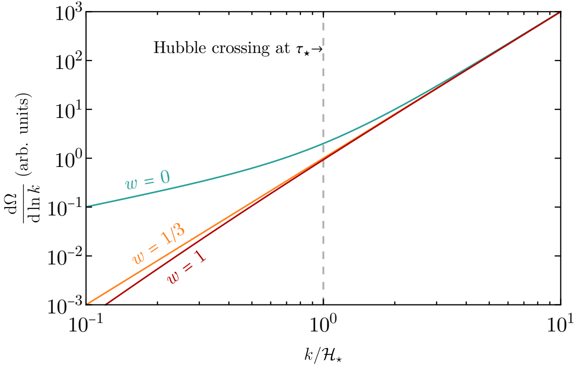
$$h \approx \frac{a(\tau_k)}{a} \frac{J_\star}{\mathcal{H}_\star} \sin k\tau = \frac{a_\star}{a} \left(\frac{\mathcal{H}_\star}{k}\right)^{n-1} \frac{J_\star}{k} \sin k\tau$$

- For $n \neq 1$ the scaling is not $1/k$ like sub-horizon modes.
- Physically, the boost in amplitude due to the mode being frozen is $\left(\frac{\mathcal{H}_\star}{k}\right)^n$, which for MD is larger than the suppression $\frac{k}{\mathcal{H}_\star}$ due to over-damping.
- The conformal time before horizon-entry is the same (from $\mathcal{H}_\star$ to k), but the expansion of a during that time is different.
- The spectral tilt is then

$$\Omega_{\text{GW}} \sim k^3 \text{ (sub-horizon)}$$

$$\Omega_{\text{GW}} \sim k^{5-2n} = \begin{cases} k^3 & \text{RD} \\ k & \text{MD} \end{cases} \text{ (super-horizon)}$$

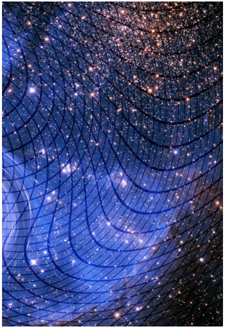
- To confirm these estimates, we solve the full eq. of motion, getting Bessel functions $j_{n-1}(k\tau)$, $y_{n-1}(k\tau)$.
- Notice the change in slope appearing at $k = \mathcal{H}_\star$, the conformal Hubble at the phase transition.



Propagation of GWs: effect of relativistic free-streaming particles

2. What can alter the propagation of GWs and hence their causality-limited spectrum?

- An important effect for the GW spectrum, also known as Weinberg damping [Weinberg '04], concerns the impact of free-streaming (FS) particles on the GW propagation.



- GWs are sourced by the anisotropic component π_{ij} of the stress tensor:

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 4\mathcal{H}^2 \pi_{ij}$$

- FS particles travel distances $\sim H^{-1}$ along geodesics, and are affected by passing GWs.
- In turn, FS particles react on the GWs by acting as a small friction term: $\pi_{ij} \propto h'_{ij}$.
- The effect is active as soon as $h' \neq 0$ and decreases in time as the particles' momenta redshift.
- Only relativistic FS particles have an impact, $T_{ij} \sim p_i p_j$.

Weinberg damping in the SM

- In the SM, the only FS species are neutrinos after their decoupling around $T \sim$ MeV, and they contribute with $f_\nu = \frac{\rho_\nu}{\rho_{\text{tot}}} = 0.4$.
- In the case of primordial GWs, they were frozen $(h' = 0)$ until horizon-entry. The damping is effective as the mode crosses the horizon and starts oscillating.
- The eq. of motion is [Weinberg '04]

$$h'' + 2\mathcal{H}h' + k^2 h = -24\, f_\nu\, \mathcal{H}^2 \int_{\tau_0}^{\tau} K\Big(k(\tau - \tilde{\tau})\Big)\, h'(\tilde{\tau})\, \mathrm{d}\tilde{\tau}$$

$$K(s) = \frac{3\sin s}{s^5} - \frac{3\cos s}{s^4} - \frac{\sin s}{s^3}$$

- In the SM, this effect is frequency independent and reduces the GW amplitude by 0.8:

$$\Omega_{\text{GW}}(k) \longrightarrow 0.64\, \Omega_{\text{GW}}(k)$$

Weinberg damping for phase transitions

- In the case of primordial waves, the effect only occurs at Hubble crossing because h is frozen before.
- In the super-horizon limit $k \to 0$, the eq. of motion simplifies: $K(s) \to \frac{1}{15}$, and the integral is solved to

$$h'' + 2\mathcal{H}h' + k^2 h_{ij} = -\frac{8 f_{\mathsf{FS}}}{5}\mathcal{H}^2 \left(h(\tau) - h(\tau_0)\right)$$

- For phase transitions (fast GW source), there is a further effect *at generation*, if some new FS particles are present at early times.
- The damping occurs for modes which are super-horizon at generation $\Rightarrow$ feature at $k = \mathcal{H}_\star$.
- Initial condition for fast sources: $\begin{cases} h = 0 \\ h' = J_\star \end{cases}$

$$h'' + \frac{2}{\tau}h' + \left(k^2 + \frac{8 f_{\mathsf{FS}}}{5\tau^2}\right) h = 0$$

Weinberg damping for phase transitions

- Sub-horizon modes $k \gg \mathcal{H}_\star$ at generation: both Hubble friction and Weinberg damping are negligible.

$$h'' + 2\mathcal{H}h' + \left(k^2 + \frac{8 f_{\mathsf{FS}}}{5} \mathcal{H}^2 \right) h = 0$$

These modes are unaffected.

- Super-horizon modes $k \ll \mathcal{H}_\star$: these modes are damped by Hubble friction, and the Weinberg term determines whether they are over- or under-damped.

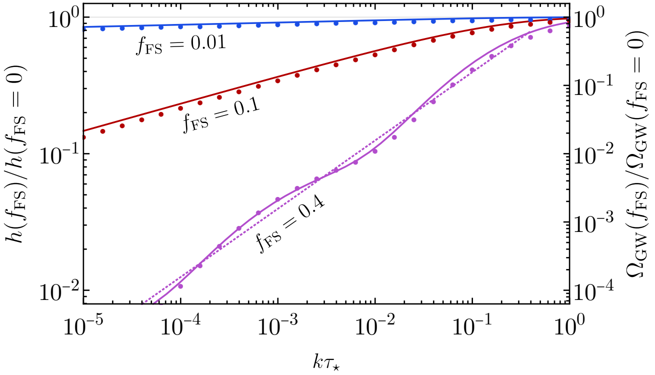
$$h'' + \underbrace{2\mathcal{H}h'}_{\text{friction}} + \underbrace{\left(k^2 + \frac{8 f_{\mathsf{FS}}}{5} \mathcal{H}^2 \right)}_{\text{mass term}} h = 0$$

- **Over-damped**: $(\text{friction})^2 \gg (\text{mass})^2$, or $f_{\mathsf{FS}} \ll 1$. The mode does not oscillate while super-horizon, and its amplitude is dampened compared to the case $f_{\mathsf{FS}} = 0$.
- **Under-damped**: $(\text{friction})^2 < (\text{mass})^2$, or $f_{\mathsf{FS}} > 16\%$. The Weinberg term is so large to induce oscillations while super-horizon. On top of the damping, oscillations appear in the spectrum.

- In presence of rel. FS particles at the phase transition, the GW spectrum changes tilt below $k < \mathcal{H}_\star$:

$$\frac{\Omega_{\mathsf{GW}}^{(f_{\mathsf{FS}})}}{\Omega_{\mathsf{GW}}} \sim \begin{cases} k^{\frac{16 f_{\mathsf{FS}}}{5}} & f_{\mathsf{FS}} < \frac{5}{32} \\ k\left[c_1 + c_2 \sin\left(\sqrt{\frac{32}{5} f_{\mathsf{FS}} - 1}\, \ln(k\tau_\star) + c_3\right)\right] & f_{\mathsf{FS}} > \frac{5}{32} \end{cases}$$

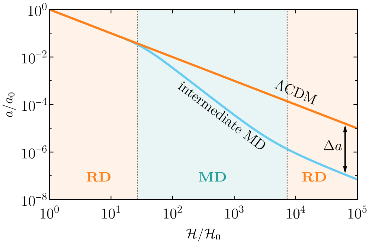
- Current bounds $\Delta N_{\mathsf{eff}} < 0.3$ allow for $f_{\mathsf{FS}} \sim 9\%$ at early times.



Probing the cosmological expansion with causal GWs

3. How is the expansion history of the Universe influencing the causality-limited spectrum?

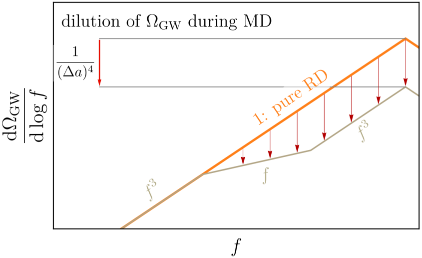
- Alternative expansion histories imply two modifications:
  1. Change the shape of the GW spectrum for modes which are super-horizon at generation;
  2. Change the rescaling between comoving modes k and physical frequencies $f = k/a$.

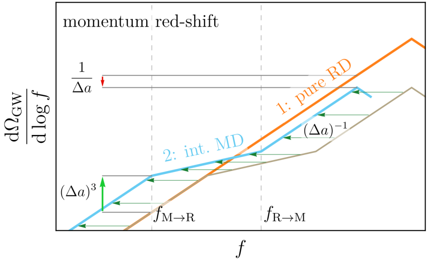


- We consider an intermediate MD era:
  - case 1: only RD
  - case 2: RD → intermediate MD → RD.
- The transition from RD to MD happens due to some non rel. species taking over.
- This species later decays into radiation.
- The scale factor has an overall difference

$$\Delta a = \left(\frac{T_{\mathrm{R}\to\mathrm{M}}}{T_{\mathrm{M}\to\mathrm{R}}} \right)^{1/3} > 1$$

Intermediate phase of MD



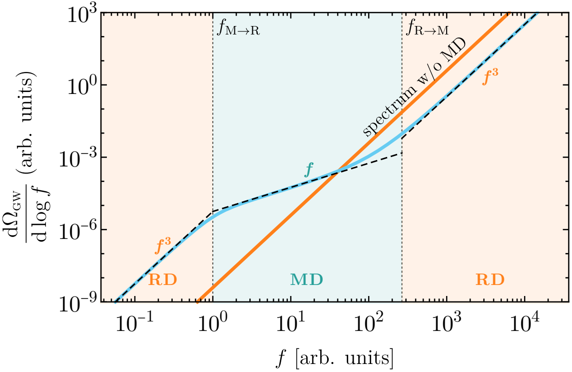


- GW modes that were sub-horizon during MD redshift more in case 2: a expands more.
- $\Rightarrow$ suppression $(\Delta a)^{-4}$ at high f.
- Modes which enter after M→R have the same evolution in the two cases.
- The intermediate range interpolates between the two, with $\Omega_{\rm GW} \sim k$.

- The physical frequency $f = k/a$ is moved to lower values in case 2, because of the larger redshift: $f \to f/(\Delta a)$.
- Given the tilt f^3, this implies that **low frequency modes have an overall boost** of $(\Delta a)^3$.
- The net effect for high frequencies is a suppression $(\Delta a)^{-1}$.

Intermediate phase of MD

- The numerical solution confirms these scalings.
- The low-frequency range (which could be the only one potentially accessible for GWs from reheating) is made more visible by a MD phase.

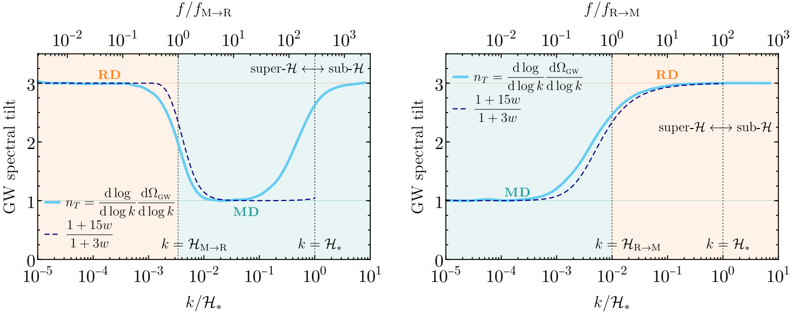


- The GW spectrum for super-horizon modes and generic eq. of state is

$$\Omega_{\mathsf{GW}}(k) \sim k^{5-2n} = k^{\frac{1+15w}{1+3w}}$$

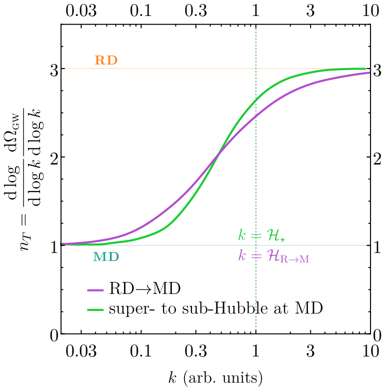
- This is valid for constant w. For generic $w(\tau)$, there is no exact solution.
- If we identify w for each mode k with its value at Hubble crossing, we can approximate (as long as $w'(\tau) \ll \mathcal{H}$)

$$\text{GW tilt } = \frac{\mathrm{d}\log \Omega_{\mathsf{GW}}(k)}{\mathrm{d}\log k} \approx \frac{1+15w(\tau)}{1+3w(\tau)} \,.$$



Measuring $w(\tau)$?

- The agreement ends up being quite good, although approximate.
- Could we distinguish between two kinds of $k^3 \to k$ transitions?
  - k^3 of super-hor. modes entering in RD $\to$ k of super-hor. modes during MD
  - k^3 of sub-hor. modes $\to$ k of super-hor. modes entering in MD
- Despite possible in principle, it seems infeasible in practice.



Conclusions

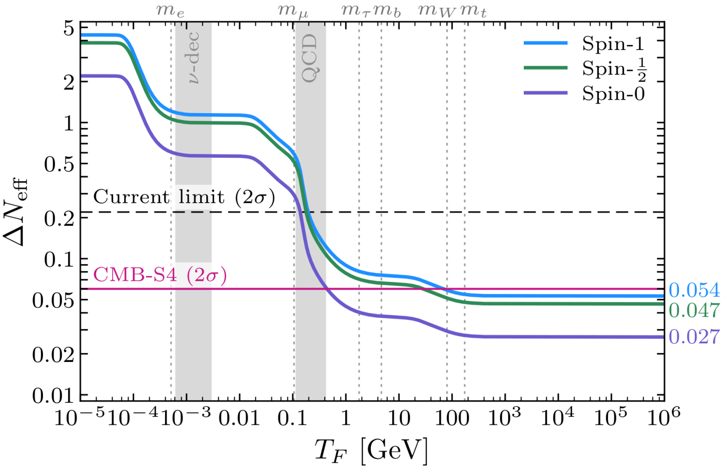
- Gravity waves generated by causal phenomena (uncorrelated beyond the Hubble radius), such as a phase transition, are insensitive to the details of the generation.
- The universal behaviour of that part of the spectrum makes it an attracting tool to explore the cosmology of the early universe.
- Deviations from the standard prediction of f^3 would signal new physics in a robust way.
- Their physics can be understood in simple physical terms, which highlight the impact of modifications of the cosmological model.

- The presence of extra free-streaming species could be read off from the GW spectrum, and cross-checked with measurements of ΔN_{eff}.
- Intermediate phases of MD, which can arise in modifications of ΛCDM, amplify the GW signal at low frequencies.
- Various phenomena could imprint a change of tilt around $k = \mathcal{H}_\star$, potentially allowing to measure the conformal Hubble rate around the phase transition.


Thanks for your attention!

1. BACKUP SLIDES

Measurement of ΔN_{eff}



[CMB-S4 Science report 1907.04473]

Schematic derivation of Weinberg damping

[Weinberg '04; Watanabe, Komatsu '06]

- GWs are sourced by the anisotropic component π_{ij} of the stress tensor:

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 4\mathcal{H}^2\pi_{ij}$$

$$T_{ij} = p\, g_{ij} + a^2\pi_{ij}\,, \qquad T^{(\nu)}_{ij} = \frac{1}{\sqrt{-g}}\int \frac{\mathrm{d}^3 q}{q^0} q_i q_j F^{(\nu)}(q)$$

- The ν phase space distribution $F(x,p)$ is obtained from the collisionless Boltzmann (i. e. Vlasov) equation.
- By decomposing $F(x,p) = \overline{F}(p) + \delta F(x,p)$ where $\overline{F}(p)$ is the equilibrium distribution, and keeping 1st order terms in perturbation theory:

$$0 = \frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial \tau} + \frac{\mathrm{d}x^i}{\mathrm{d}t}\frac{\partial F}{\partial x^i} + \frac{\mathrm{d}p^0}{\mathrm{d}t}\frac{\partial F}{\partial p^0}$$

- The last term is obtained from the geodesic equation:

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\lambda} = -\Gamma^\mu_{\alpha\beta}p^\alpha p^\beta \implies \frac{1}{p^0}\frac{\mathrm{d}p^0}{\mathrm{d}t} = -H - \frac{1}{2}\frac{\partial h_{ij}}{\partial t}\frac{p^i p^j}{(p^0)^2}$$

  As ν's propagate in a FRW universe with GWs, they lose (or gain) energy depending on the sign of h'.
- δF is computing by integrating the Boltzmann eq. over time, and the result is

$$h'' + 2\mathcal{H}h' + k^2 h = -24 f_\nu \mathcal{H}^2 \int_{\tau_0}^{\tau} \mathrm{d}\tau' \frac{j_2[k(\tau-\tau')]}{k^2(\tau-\tau')^2} h'(\tau')$$