Magnetic gliding in the early universe

Gustavo Marques-Tavares, †
(University of Maryland & Johns Hopkins University)

1912.08817
20xx.xxxxx } with A. Hook and Y. Tsai

Motivation

Coherent bosonic fields might play a central part in the early universe

Their phenomenology has been significantly less explored than "weak scale" BSM. There are still opportunities to discover surprising new dynamics that can help address questions/features in fundamental physics.

Important Examples

> QCD axion

solves the strong CP problem

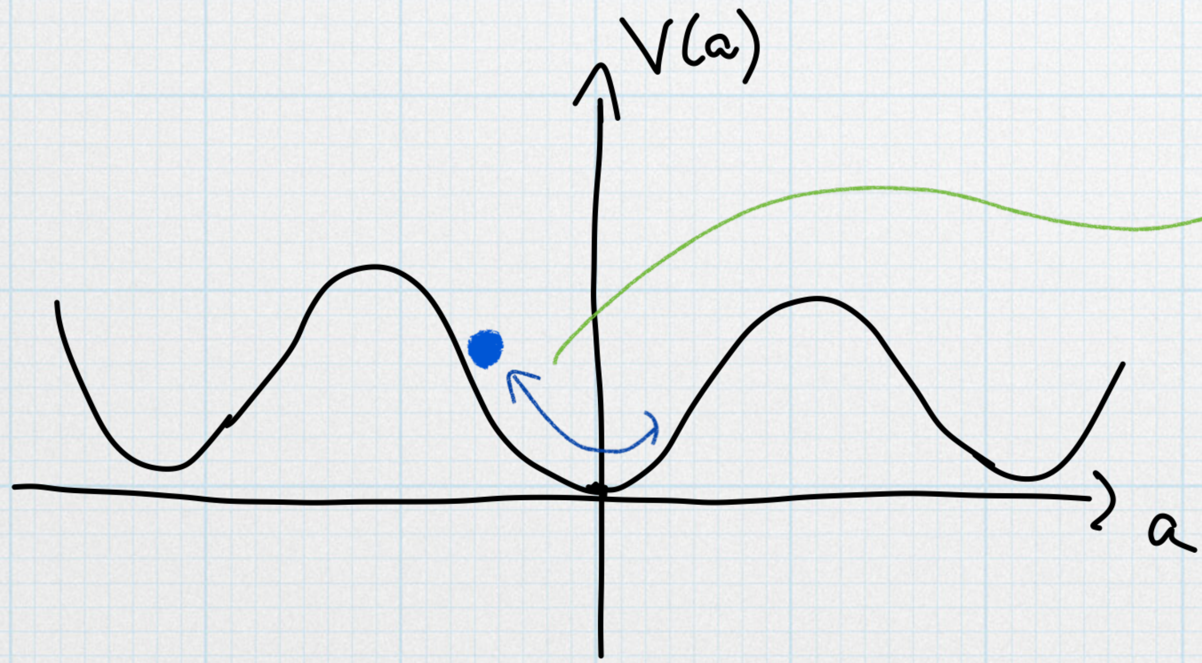
can account for all of dark matter

$$\theta G\tilde{G} \longrightarrow \frac{a(x)}{f} G\tilde{G}$$

Important Examples

> QCD axion → solves the strong CP problem
> → can account for all of dark matter

$$\theta G\tilde{G} \longrightarrow \frac{a(x)}{f} G\tilde{G}$$



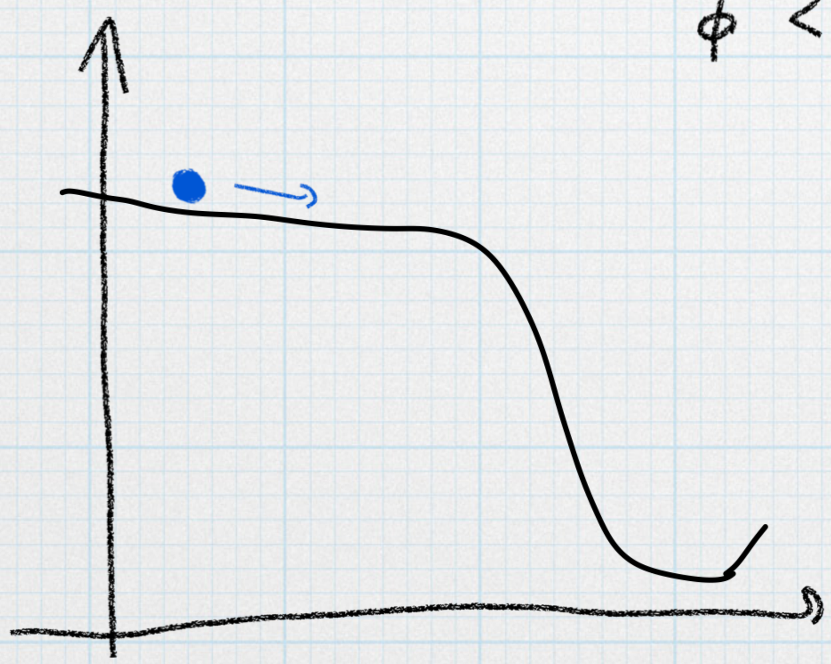
$a \to 0$

$\Omega_a \sim 1$ if $m_a \sim \mu eV$

Important Examples

- Inflaton :
  - scalar field on very flat potential can sustain long period of inflation
  - It also generates the primordial fluctuations for structure formation & CMB

$\dot{\phi}^2 << V(\phi)$



However to fit the data

V probably is very fine tuned

Important Examples

- Also baryogenesis, relaxion(s), EDE, moduli...

New tools for modifying scalar field cosmology are very useful!

"Gliding"

- Single derivative mixing of massive bosonic fields
- Under the right conditions (large mixing)
  - → Slows down the dynamics of lightest field
  - → Decrease the impact of Hubble friction in the energy density

Magnetic gliding

Today will focus exclusively:

$$\frac{\phi}{f} F \tilde{F}_D$$

Magnetic gliding

Today will focus exclusively:

$$\frac{\phi}{f} F \tilde{F}_D$$

In the presence of magnetic field

* Will assume magnetic fields are large in the early universe.

Possible origin: phase-transitions or Inflation

Magnetic gliding

$$\mathcal{L} \supset -\frac{1}{2} m^2 \phi^2 - \frac{1}{2} M^2 A_d^\mu A_{d\mu} + \frac{\phi}{2f} F_d \tilde{F}$$

→ Eqs of motion (homogeneous)

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + m^2\phi = \dfrac{B}{af} \dot{A}_d \\ \ddot{A}_d + H\dot{A}_d + M^2 A_d = -\dfrac{aB}{f} \dot{\phi} \end{cases}$$

$$\vec{A}_d = A_d \hat{B}$$

Simpler case

→ Consider first static case : $a(t) = \text{const} = 1$

$B(t) = \text{const} = B$

$$\begin{cases} \ddot{\phi} - \frac{B}{f}\dot{A}_d + m^2\phi = 0 \\ \ddot{A}_d + \frac{B}{f}\dot{\phi} + M^2 A_d = 0 \end{cases} \longrightarrow \begin{pmatrix} \phi \\ A_d \end{pmatrix} = e^{i\omega t}\begin{pmatrix} \phi_0 \\ A_{d_0} \end{pmatrix}$$

Simpler case

→ Consider first static case : $a(t) = \text{const} = 1$

$B(t) = \text{const} = B$

$$\begin{cases} \ddot{\phi} - \frac{B}{f}\dot{A}_d + m^2\phi = 0 \\ \ddot{A}_d + \frac{B}{f}\dot{\phi} + M^2 A_d = 0 \end{cases} \longrightarrow \begin{pmatrix} \phi \\ A_d \end{pmatrix} = e^{i\omega t}\begin{pmatrix} \phi_0 \\ A_{d_0} \end{pmatrix}$$

$$\Rightarrow \quad \omega^2_{\mp} = \frac{\left(\frac{B^2}{f^2} + m^2 + M^2\right) \mp \sqrt{\left(\frac{B^2}{f^2} + m^2 + M^2\right)^2 - 4m^2M^2}}{2}$$

Large $\frac{B}{f}$ limit: $\quad \omega_-^2 \approx \dfrac{m^2M^2}{\frac{B^2}{f^2} + m^2 + M^2} \quad ; \quad \omega_+^2 \approx \dfrac{B^2}{f^2} + m^2 + M^2$

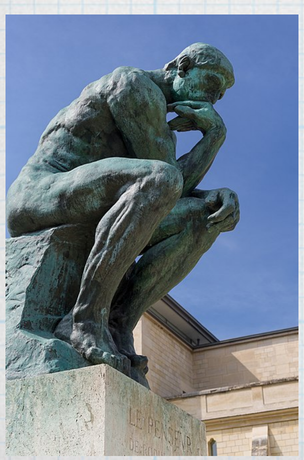
Simpler case

$$\omega_-^2 \approx \frac{m^2M^2}{\frac{B^2}{f^2}+m^2+M^2} \quad ; \quad \omega_+^2 \approx \frac{B^2}{f^2}+m^2+M^2$$

$$m^2 << M^2 << B^2/f$$

$$A_d \approx -i\,\frac{m}{M}\,\frac{B/f}{\sqrt{\frac{B^2}{f^2}+M^2+m^2}}\,\phi \quad \Rightarrow \quad A_d \approx -i\,\frac{m}{M}\,\phi$$

$\omega_- << m$ & M ; $|A_d| << |\phi|$



contrast with other types of mixing

Mass mixing

$$V = \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 + \Lambda^2\phi_1\phi_2$$

$$M^2 = \begin{pmatrix} m_1^2 & \Lambda^2 \\ \Lambda^2 & m_2^2 \end{pmatrix}$$

If $\Lambda^2 \gg m_i^2$:

$$M^2 = \begin{pmatrix} \bar{m}^2 & \Lambda^2 \\ \Lambda^2 & \bar{m}^2 \end{pmatrix} + \begin{pmatrix} \delta m^2 & 0 \\ 0 & -\delta m^2 \end{pmatrix}$$

$$\omega^2 \approx \pm\Lambda^2 + \bar{m}^2 + \mathcal{O}\left(\frac{\delta m^4}{\Lambda^2}\right)$$

$$\phi_2 \approx \pm\phi_1$$

Mass mixing

$$V = \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 + \Lambda^2 \phi_1 \phi_2$$

$$M^2 = \begin{pmatrix} m_1^2 & \Lambda^2 \\ \Lambda^2 & m_2^2 \end{pmatrix}$$

If $\Lambda^2 \gg m_i^2$:

$$M^2 = \begin{pmatrix} \bar{m}^2 & \Lambda^2 \\ \Lambda^2 & \bar{m}^2 \end{pmatrix} + \begin{pmatrix} \delta m^2 & 0 \\ 0 & -\delta m^2 \end{pmatrix}$$

$$\omega^2 \approx \pm \Lambda^2 + \bar{m}^2 + \mathcal{O}\left(\frac{\delta m^4}{\Lambda^2}\right)$$

$$\phi_2 \approx \pm \phi_1$$

Kinetic mixing

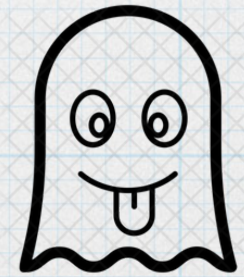
$$\frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \lambda \partial_\mu \phi_1 \partial^\mu \phi_2$$

$$- V(\phi_1, \phi_2)$$

$\Rightarrow$ Diag. kinetic term

$$\begin{pmatrix} 1 & \lambda \\ \lambda & 1 \end{pmatrix} \to \begin{pmatrix} 1+\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}$$

$\Rightarrow$ large kinetic mixing:



Friction

$$\omega_-^2 \approx \frac{m^2 M^2}{\frac{B^2}{f^2} + m^2 + M^2} \longrightarrow \omega_- \approx m \frac{M}{B/f} \ll m$$

If this picture holds in an FRW universe Hubble friction should be less important: $H\dot{\phi} \sim \omega_- \phi H$

Friction

$$\omega_-^2 \approx \frac{m^2M^2}{\frac{B^2}{f^2}+m^2+M^2} \qquad \omega_- \approx m\,\frac{M}{B/f} \ll m$$

If this picture holds in an FRW universe Hubble friction should be less important: $H\dot{\phi} \sim \omega_- \phi H$

Another way to see this is through how energy is stored:

$$A_d \approx -i\frac{m}{M}\phi \quad \Rightarrow \quad \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{A}_d^2 \sim \frac{1}{2}\omega_-^2\phi_0^2\left(\cos^2\omega_- t + \frac{m^2}{M^2}\sin^2\omega_- t\right)$$

$$\frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2A_d^2 \sim \frac{1}{2}m^2\phi_0^2\cos^2\omega t + \frac{1}{2}M^2\left(\frac{m}{M}\phi_0\right)^2\sin^2\omega_- t$$

Gliding in an expanding universe

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + m^2\phi = \frac{B}{af}\dot{A}_d \\ \ddot{A}_d + H\dot{A}_d + M^2 A_d = -\frac{aB}{f}\dot{\phi} \end{cases}$$

* will focus on $m \ll M$

and $B = B_0 \left(\frac{a_0}{a}\right)^n$

@ early times: $\phi = \phi_0$; $A_d \approx 0$ & $\phi \approx$ const until $H \sim m$

Assuming $\frac{B}{f} \gg m$ when $H \sim m$

$M^2 A_d \gg \ddot{A}_d$ & $H\dot{A}_d \Rightarrow A_d \approx -\frac{aB}{M^2 f}\dot{\phi}$

Gliding in an expanding universe

$$\begin{cases} \ddot{\phi} + 3H\dot{\phi} + m^2\phi = \frac{B}{af}\dot{A}_d \\ \ddot{A}_d + H\dot{A}_d + M^2 A_d = -\frac{aB}{f}\dot{\phi} \end{cases}$$

* will focus on $m \ll M$

and $B = B_0 \left(\frac{a_0}{a}\right)^n$

@ early times: $\phi = \phi_0$; $A_d \approx 0$ & $\phi \approx$ const until $H \sim m$

Assuming $\frac{B}{f} \gg m$ when $H \sim m$

$M^2 A_d \gg \ddot{A}_d$ & $H\dot{A}_d \Rightarrow A_d \approx -\frac{aB}{M^2 f}\dot{\phi}$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = -\frac{B^2}{f^2M^2}\ddot{\phi} - \frac{HB^2}{M^2f^2}\dot{\phi} - \frac{B}{M^2f^2}\dot{\phi}\dot{B}$$

Gliding in an expanding universe

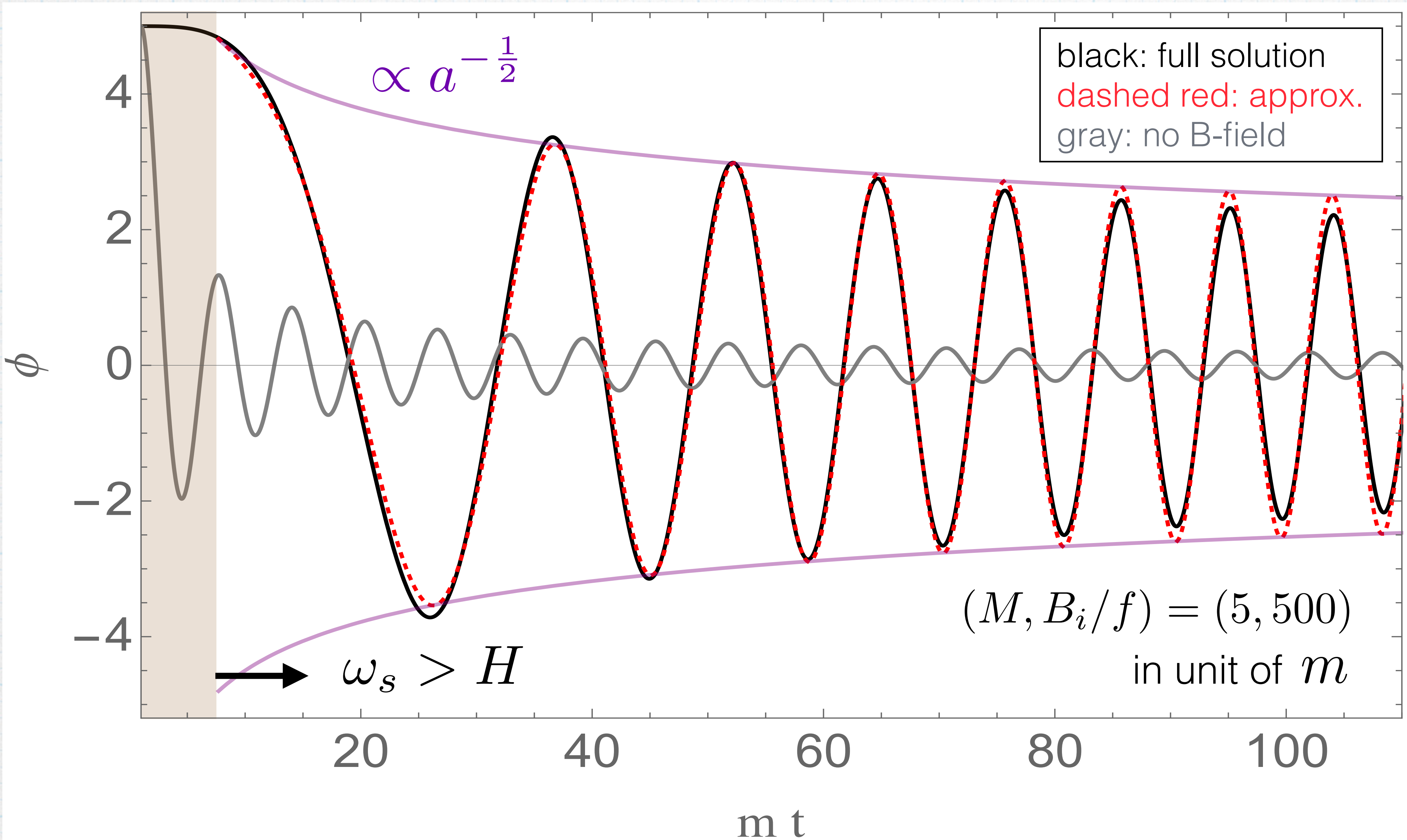
$$\ddot{\phi} + \left[\left(3 - \frac{2B^2/f^2}{M^2 + B^2/f^2}\right)H + \frac{B^2/f^2}{M^2 + B^2/f^2}\frac{\dot{B}}{B}\right]\dot{\phi} = -\frac{m^2 M^2}{M^2 + B^2/f^2}\phi$$

Gliding in an expanding universe

$$\ddot{\phi} + \left[\left(3 - \frac{2B^2/f^2}{M^2 + B^2/f^2}\right)H + \frac{B^2/f^2}{M^2 + B^2/f^2}\,\frac{\dot{B}}{B}\right]\dot{\phi} = -\frac{m^2 M^2}{M^2 + B^2/f^2}\,\phi$$

$$\boxed{\phi(t) = \varphi(t)\, e^{i\int_{t_0}^{t} dt'\, \omega_-(t)}} \quad ; \quad \boxed{\varphi(t) = \varphi_0\, e^{-\frac{1}{2}\int^{t} dt'\, H\left[1 + \frac{2M^2}{M^2 + B^2/f^2}\right]}}$$

Numerical vs Analytical



$$\phi \approx \phi_0 \sqrt{\frac{a_{osc}}{a}}\, e^{i\int dt'\,\omega_-(t')}$$

QCD axion cosmology

Quick review: $\frac{\phi}{f} G\tilde{G} \longrightarrow \Lambda^4(1-\cos \phi/f)$

$@\ T=0 \qquad \Lambda^4 \approx m_\pi^2 f_\pi^2 \sim (100\ \text{MeV})^4$

$T > T_c \sim 200\ \text{MeV} \quad \longrightarrow \quad \Lambda^4 \sim (100\ \text{MeV})^4 \left(\frac{T_c}{T}\right)^8$

$$\Rightarrow m_\phi(T) \approx \begin{cases} \frac{(100\ \text{MeV})^2}{f}\left(\frac{T_c}{T}\right)^4, & T > T_c \\ \frac{(100\ \text{MeV})^2}{f}, & T < T_c \end{cases}$$

$$m_\phi(T_{osc}) \approx \frac{T_{osc}^2}{M_p}$$

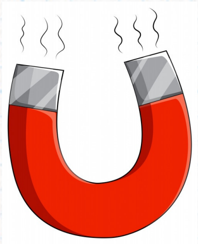
QCD axion cosmology

$$m_a(T) \approx \begin{cases} \frac{(100\ \mathrm{MeV})^2}{f_a}\left(\frac{T_c}{T}\right)^4, & T > T_c \\ \frac{(100\ \mathrm{MeV})^2}{f_a}, & T < T_c \end{cases}$$

After T_{osc} $\quad n_\phi \simeq m_\phi(T)\phi^2$ is "conserved" : $n_\phi \simeq n_\phi(T_{osc})\left(\frac{a_{osc}}{a}\right)^3$

$$\Rightarrow \rho(T < T_c) = m_\phi(0)\, m_\phi(T_{osc})\, \theta_0^2 f^2 \left(\frac{T}{T_{osc}}\right)^3$$

Gliding axion

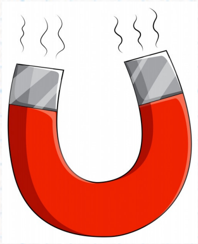


$$\mathcal{L} \supset \frac{\phi}{f_d} F_D \tilde{F} + \frac{M^2}{2} A_d^{\mu} A_{d\,\mu}$$

$M > m_\phi (T=0)$

$\frac{B}{f_d} (T \sim \mathrm{GeV}) \gg M$

Gliding axion



$$\mathcal{L} \supset \frac{\phi}{f_d} F_D \tilde{F} + \frac{M^2}{2} A_d^{\mu} A_{d\,\mu}$$

$M > m_\phi(T=0)$

$\frac{B}{f_d}(T \sim \mathrm{GeV}) \gg M$

2 effects:

- $\omega_-(T_{osc}) \approx \frac{T_{osc}^2}{M_p} \approx m_\phi(T_{osc}) \frac{M}{B/f}$ delayed oscillation

- ϕ scaling is non-trivial $T_B < T < T_{osc}$

$$\phi \approx \phi_0 \sqrt{\frac{m_\phi(T_{osc})}{m_\phi(T)}} \left(\frac{T}{T_{osc}}\right)^{1/2} e^{i\int dt'\,\omega_-(t')}$$

Gliding axion

$$Y \simeq \frac{n_\phi}{s_\gamma} \sim \frac{m_\phi \, \phi_0^2 \, T_c^4}{T_b^2 \, T_{osc}^5}$$

$\ast$ assuming $B \approx B_c \left(\frac{T}{T_0}\right)^2$

$$\frac{\rho_B}{\rho_{B=0}} \sim \left(\frac{T_{osc}(B=0)}{T_{osc}}\right)^{13} \lesssim \left(\frac{M_p f^2}{f_d^3}\right)^{13/24}$$

Gliding axion

$$Y \simeq \frac{n_\phi}{s_\gamma} \sim \frac{m_\phi \phi_0^2 T_c^4}{T_b^2 T_{osc}^5}$$

$*$ assuming

$$B = B_c \left(\frac{T}{T_0}\right)^2$$

$$\vdots$$

$$\frac{\rho_B}{\rho_{B=0}} \sim \left(\frac{T_{osc}(B=0)}{T_{osc}}\right)^{13} \lesssim \left(\frac{M_p f^2}{f_d^3}\right)^{13/24}$$

Enough to have $\Omega_\phi \sim 1$

for $f \gtrsim 10^8$ GeV !!

Ongoing effort

So far focused only on $m < M$ scenario

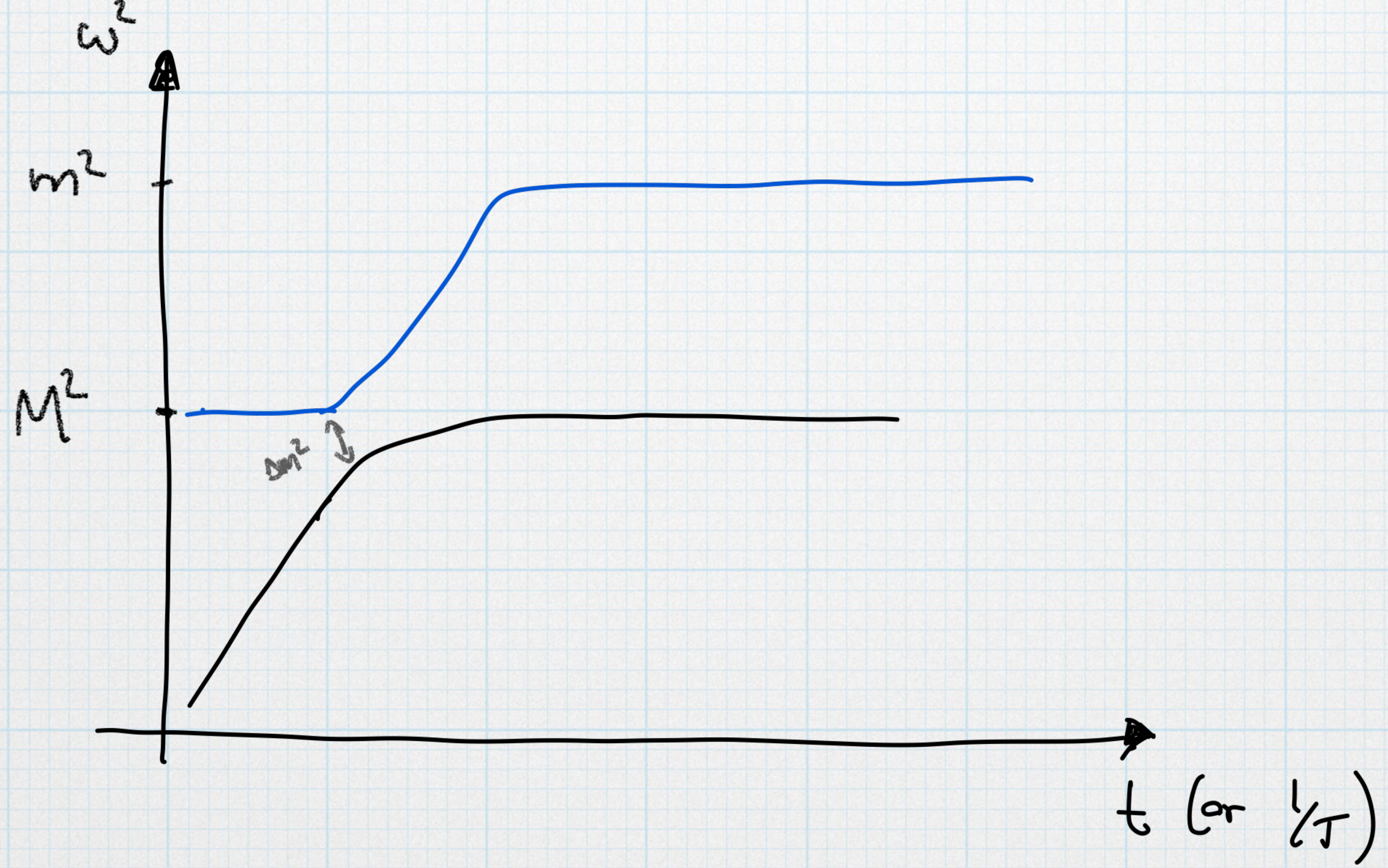
$M < m$ is also very interesting!

$\phi \longrightarrow A_d$ conversion

2 "problems" can be linked:
- $f > 10^{12}$ GeV Ω_ϕ too large
- Difficult to produce light vector dark matter abundance

Axion ⟷ Dark photon

0^{th} order picture ("ignore gliding")



As long as ω^2 evolves adiabatically → 100% conversion.

Resonant conversion with gliding

- For $T \gg T_c$ $\quad m(T) \ll M$, so slow mode: $A_d \simeq -i\frac{m(T)}{M}\phi$

- If $\frac{B}{f} \gg m \,\&\, M$ $\Rightarrow$ $A_d \simeq -i\frac{m}{\tilde{M}}\phi$ $\Rightarrow$ slow mode goes from being pure scalar to pure vector

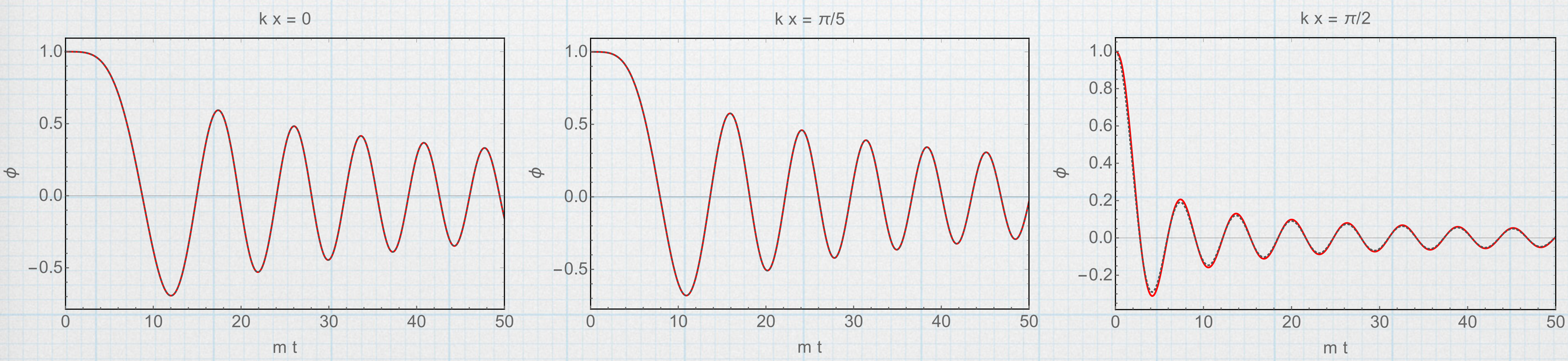
- At the conversion time $m(T_*) = M$ $\quad n_\phi \to n_{A_d}$

  $\Rightarrow A_d$ are produced cold

Inhomogeneous B

- General case very complicated. Focus on single harmonic: $\vec{B} = \hat{n} B(t) \cos(\vec{k} \cdot \vec{x})$

- Simplest case $k \ll m, M \Rightarrow$ locally the same as homogeneous case.



Inhomogeneous B

• For large $k \gg m$ & M fourier transf. in x.

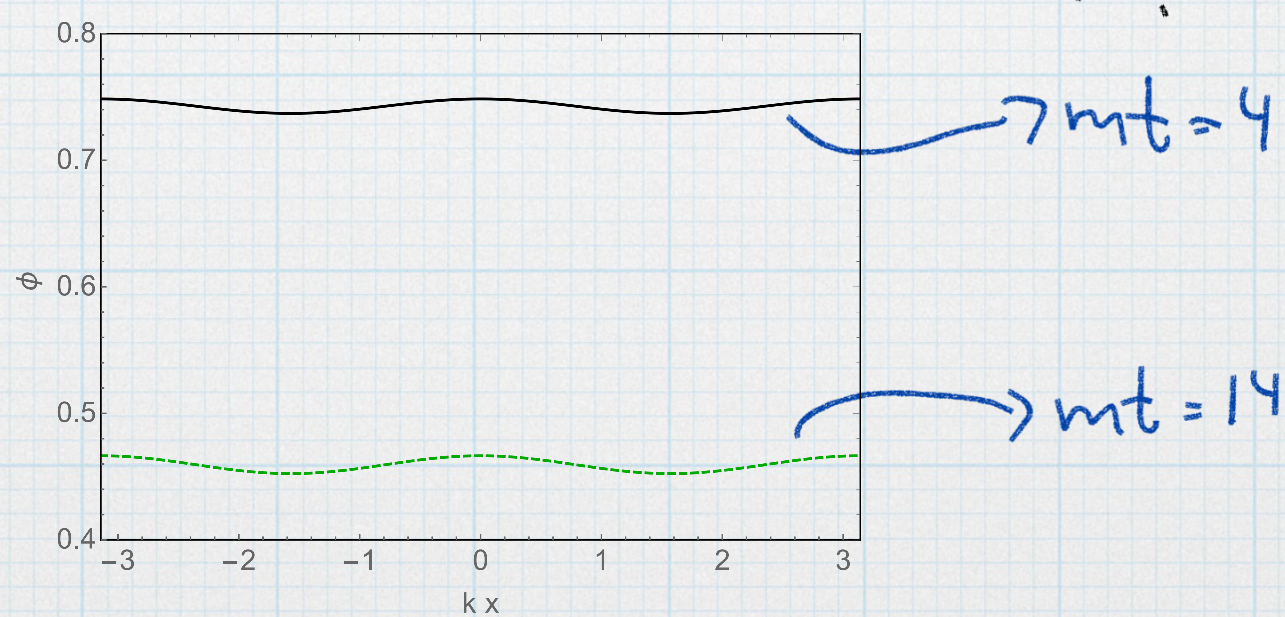
→ Assuming homogeneous initial conditions $\phi(t_0, x) = \phi_0$

$A_d(t_0, x) = 0$

Only need to track $\tilde{\phi}(t, nk)$ n even

$\tilde{A}_d(t, nk)$ n odd

Can show that large n modes decouple $\propto \frac{1}{k^n n!}$ and $\omega^2 \approx \frac{m^2 k^2}{B^2/2f^2}$



2-state approximation

- Can treat the system approximately as coupling only $\tilde{\phi}(t, q=0)$ and $\tilde{A}_d(t, q=k)$ $*(k \gg m, M)$

- Because energy in $\tilde{A}_d$ is mostly gradient $\rightarrow$ redshifts faster

$$\rho_{\phi+A_d} \propto a^{-2} \quad \text{while} \quad \frac{k}{a} > M$$

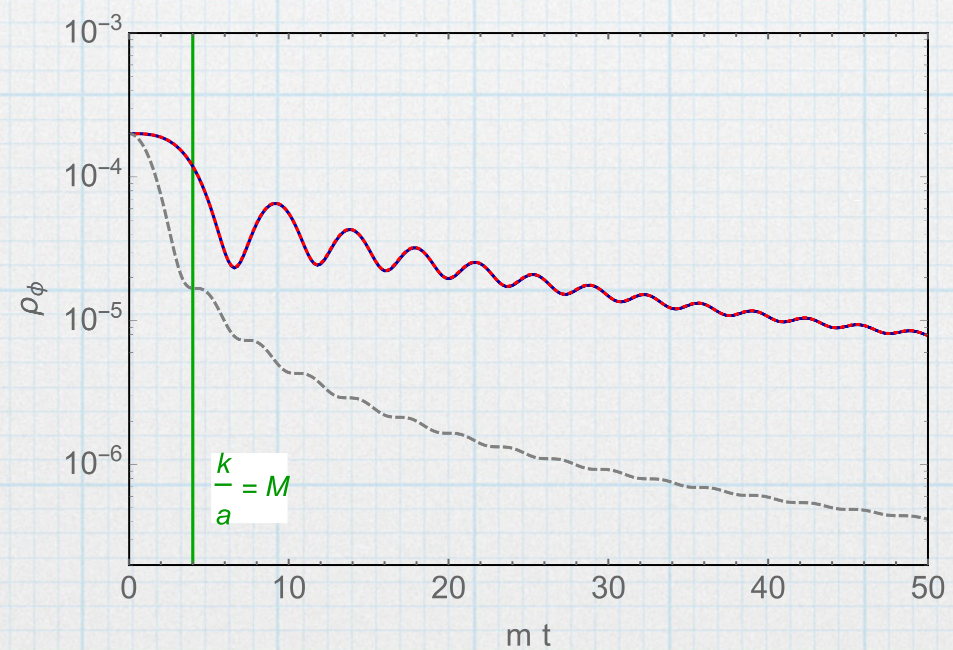
- Dark photons produced through resonant conversion $p \sim \frac{k}{a_*}$

2-state approximation

- Can treat the system approximately as coupling only $\tilde{\phi}(t, q=0)$ and $\tilde{A}_d(t, q=k)$ $*(k \gg m, M)$
- Because energy in $\tilde{A}_d$ is mostly gradient $\rightarrow$ redshifts faster

$$\rho_{\phi + A_d} \propto a^{-2} \quad \text{while} \quad \frac{k}{a} > M$$

- Dark photons produced through resonant conversion $p \sim \frac{k}{a_*}$



Conclusions

- Coupling massive fields through single $\frac{d}{dt}$ lead to novel dynamical regime
- Slow down the dynamics of the field and in consequence decreases the effect of H friction
- Can enhance axion abundance so axions are 100 % of DM even for $f < 10^{11}$ GeV
- New production mechanism for light vector DM
- ... new applications?