

Darker than dark: Searching for hidden particles with tiny couplings

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PRISMA Colloquium
JGU Mainz
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Including results from

arXiv:1809.04849 with Saniya Heeba and Patrick Stöcker

arXiv:1908.09834 with Saniya Heeba

arXiv:1910.02091 with Matthias Geilhufe and Martin Winkler



Outline

- The dark matter puzzle
- Dark matter in the early Universe
 - Weakly Interacting Massive Particles (WIMPs)
 - Feebly Interacting Massive Particles (FIMPs)
- Finding FIMPs
 - Part 1: Decaying dark matter
 - Part 2: New force carriers
 - Part 3: New detector concepts

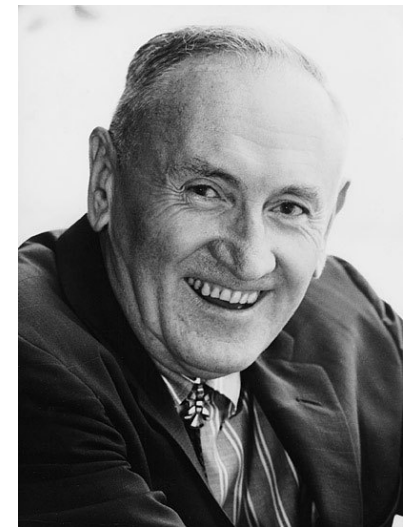
How to find missing (dark) matter

- Map out the **distribution of visible matter**
- Determine its **gravitational potential**
- Calculate the **motion of visible objects** in this potential
- Compare to **observations**

- First done in 1933 by Fritz Zwicky
 - Apply the virial theorem to the Coma cluster
 - Compare gravitational potential and velocity distribution

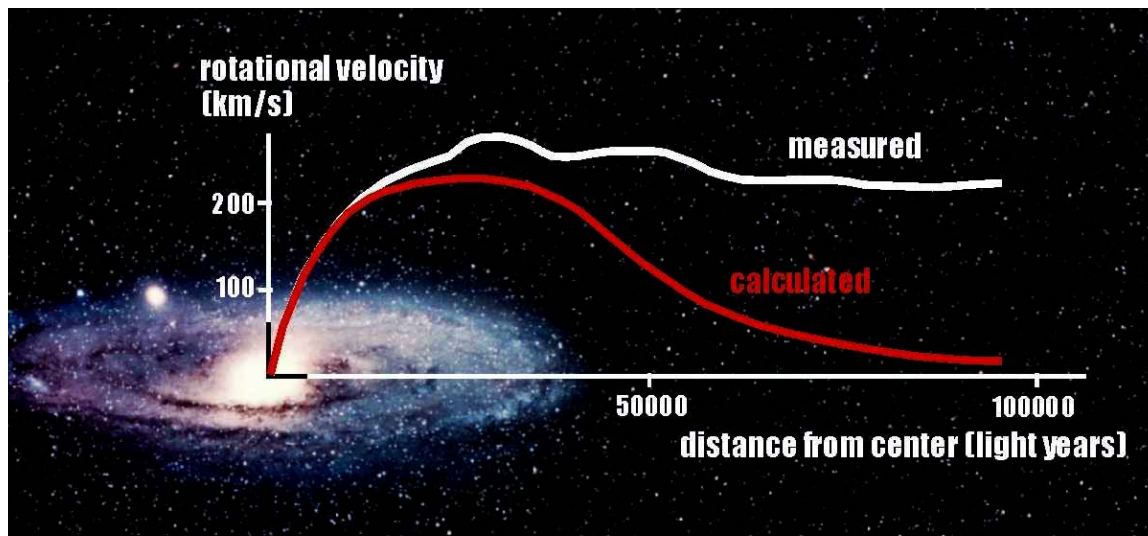
Rotverschiebung extragalaktischer Nebel. 125

Um, wie beobachtet, einen mittleren Dopplereffekt von 1000 km/sek oder mehr zu erhalten, müsste also die mittlere Dichte im Comasystem mindestens 400 mal grösser sein als die auf Grund von Beobachtungen an leuchtender Materie abgeleitete¹⁾. Falls sich dies bewahrheiten sollte, würde sich also das überraschende Resultat ergeben, dass dunkle Materie in sehr viel grösserer Dichte vorhanden ist als leuchtende Materie.



Galaxy scales: Rotation curves

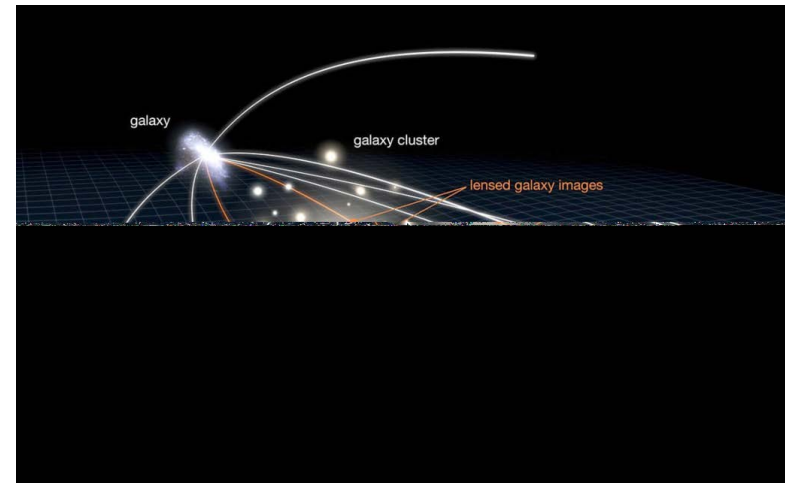
- In the 1970s Vera Rubin applied a similar approach to galaxies
 - Measure the **rotational velocities** of stars and gas at different distances from the centre
 - Infer the **gravitational potential** needed to maintain these velocities



- Beyond the visible disk we expect velocities to decrease
- But velocities stay constant up to very large distances
- An **additional contribution from invisible matter** is necessary to keep these objects bound

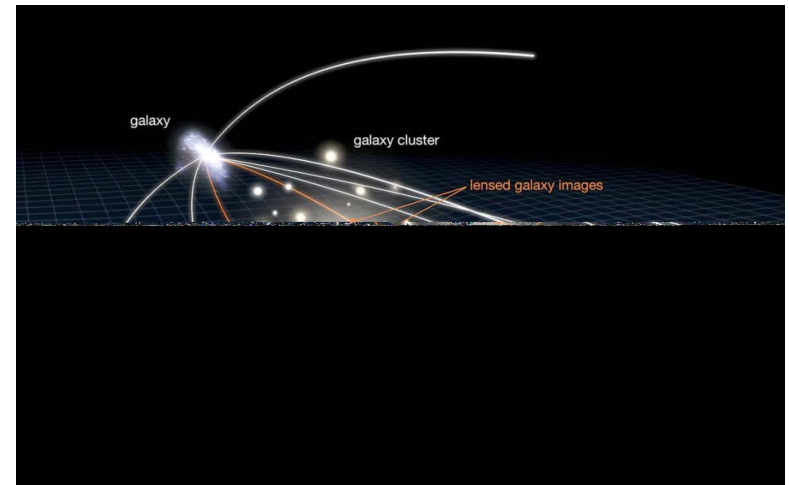
Gravitational lensing & the Bullet Cluster

- To search for dark matter on even larger scales, we can make use of **gravitational lensing**
- Distortions of the shape of far-away objects allow to infer amount of matter along the way
- We can **map out the distribution** of dark matter!



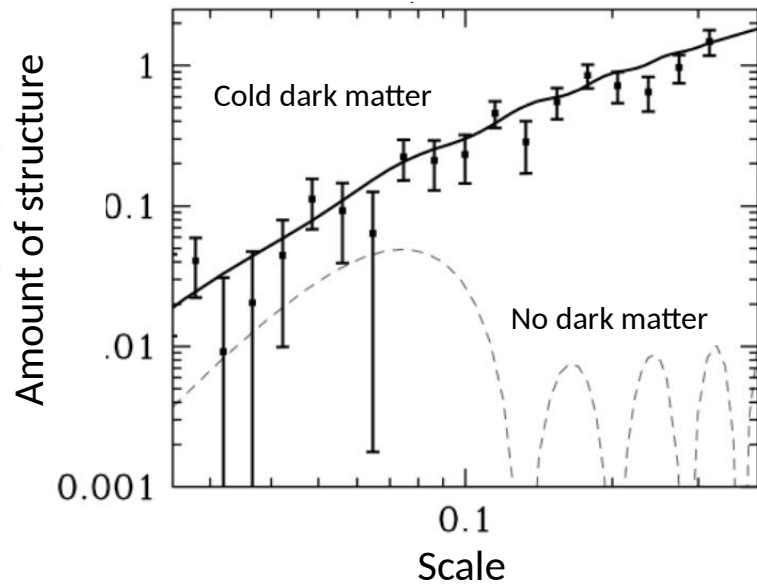
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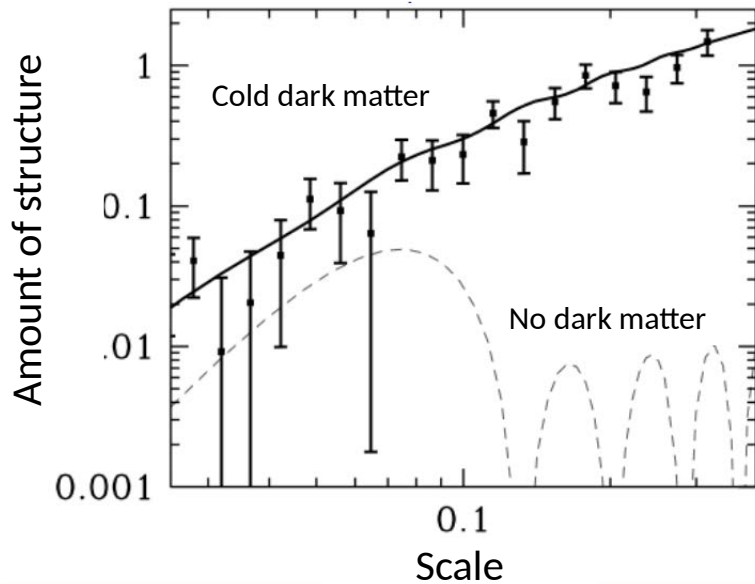
- The Bullet Cluster is a **collision of two galaxy clusters**
- The dominant contribution to gravitational lensing does not coincide with the light emission
- Explanation: **separation of dark and visible matter** due to the collision!

Cosmological scales: Structure formation



- We can even infer the presence of dark matter in the **very early Universe**
- Dark matter is not affected by the high density of energetic photons after the Big Bang
 - Gravitational collapse for dark matter is faster than for visible matter
 - Dark matter **forms structures much earlier** than visible matter
- Dark matter is necessary to explain observed **amounts of structure** in the present Universe

Cosmological scales: Structure formation

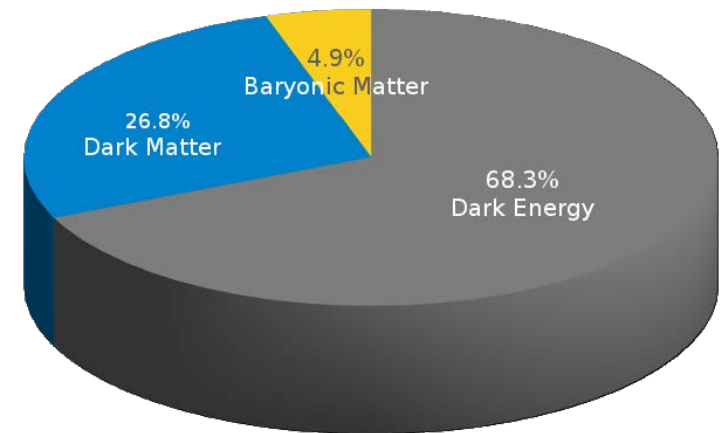


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- Cosmological observations enable us to determine the dark matter contribution to the total energy density

Planck: $\Omega h^2 = 0.1199 \pm 0.0027$

Impressive precision!



What is dark matter?

- Astrophysical observations clearly **confirm the existence** of dark matter (DM) in the Universe, but they give **almost no indications** concerning its nature
- No known particle (within the Standard Model of particle physics) has the **required properties** to be DM
- Need to postulate the **existence of a new particle** with unknown properties
- The only thing we know about it is **its abundance** in the Universe:

$$\Omega h^2 = 0.1199 \pm 0.0027$$

- Any model of dark matter must provide a mechanism to **explain this number**



by Saniya Heeba

Interaction rates in the early Universe

- To determine which **processes are important** in the early Universe, we calculate

$$\Gamma \equiv \langle \sigma v \rangle n^{\text{eq}}$$

with

- σ : cross section for the process of interest
- v : velocity of particles in the initial state
- n^{eq} : number density of particles in the initial state
- $\langle \rangle$: thermal average

- We then compare Γ to the **Hubble expansion rate** $H = a^{-1} da/dt$

– $\Gamma(T) > H(T)$: Interactions are fast

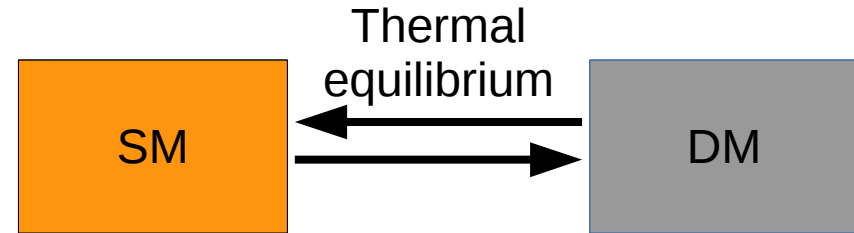


– $\Gamma(T) < H(T)$: Interactions are slow



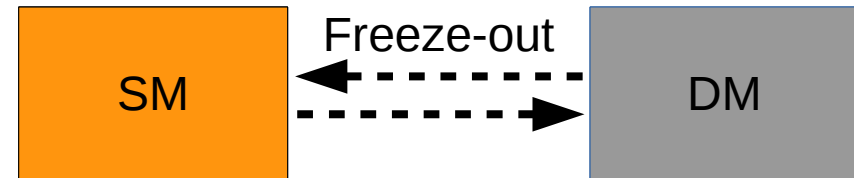
Thermal decoupling: The standard picture

- **High temperatures** ($T \gg m_{\text{DM}}$)
 - DM **annihilation and production** processes are in equilibrium



Universe expands
and cools down


- **Low temperatures** ($T \ll m_{\text{DM}}$)
 - DM **particles decouple** from equilibrium



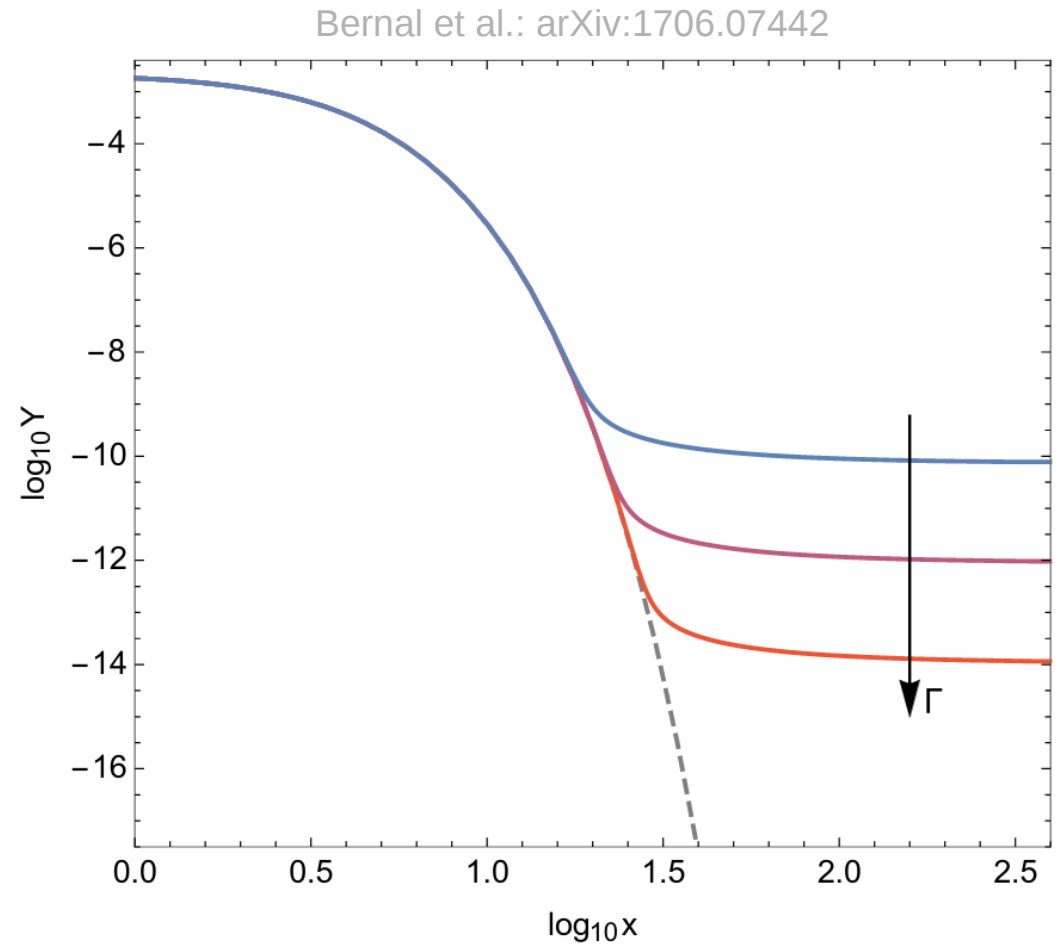
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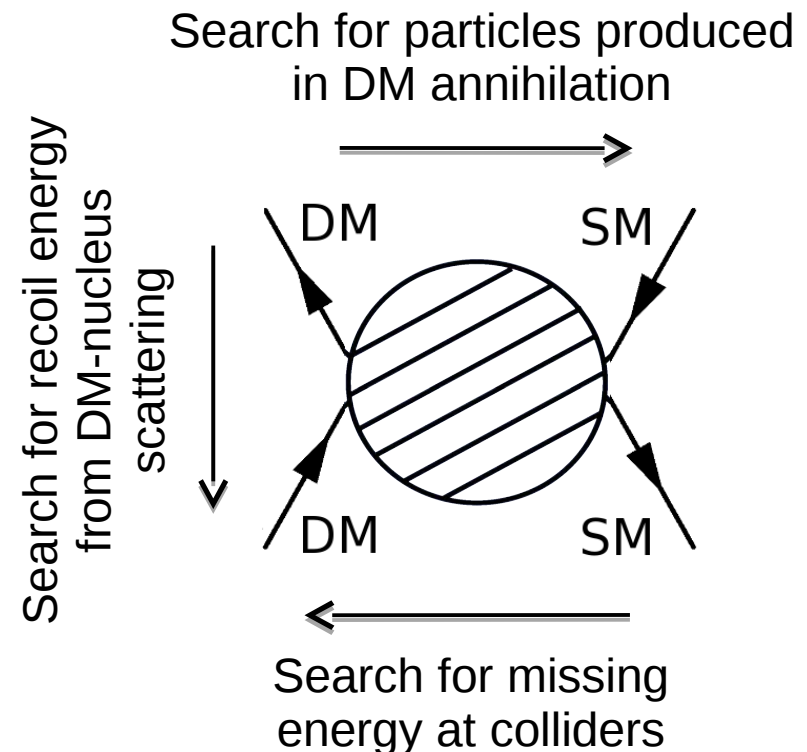
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 - DM **particles decouple** from equilibrium



Qualitative solution: Stronger interactions \leftrightarrow smaller abundance

Weakly Interacting Massive Particles (WIMPs)

- Particles that obtain their relic abundance through **thermal freeze-out** are called WIMPs
- If these particles have similar interactions as known particles but are slightly heavier, thermal freeze-out leads to the **correct relic density**
- We can **hope to observe WIMPs** in the laboratory!



Where Is My Particle?

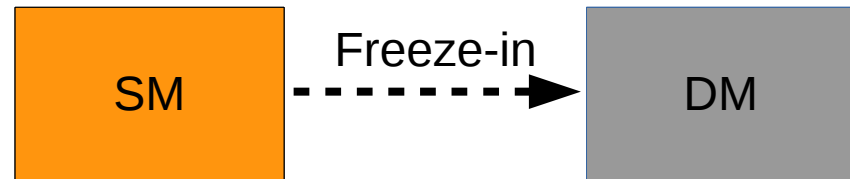
- **Parameter space for WIMPs is getting tight!**
- **Most WIMP models are still viable**, but the non-observation of dark matter signals mounts **substantial pressure** on the WIMP idea
- Well-motivated to **question underlying assumptions** and consider alternative dark matter models



The freeze-in mechanism

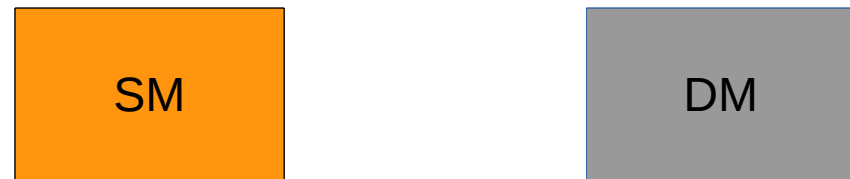
- Assume that initial dark matter number density is negligible and that interactions between DM and the SM are extremely weak

- **High temperatures** ($T \gg m_{\text{DM}}$)
 - Particle production via **“energy leakage”** from the visible sector



Universe expands
and cools down

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 - Interactions between the two sectors **completely negligible**




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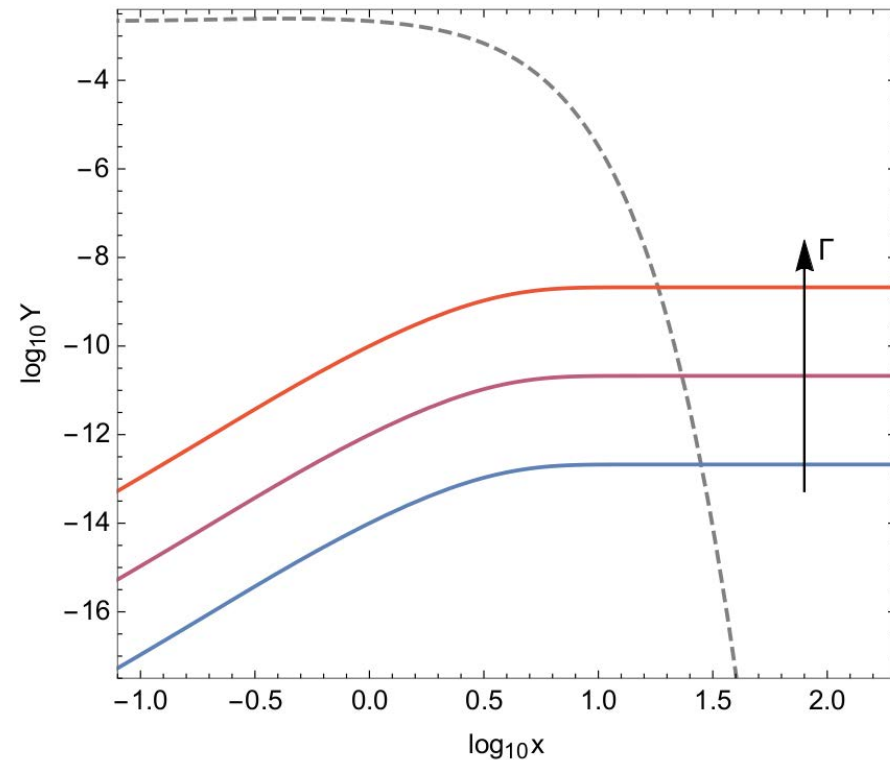
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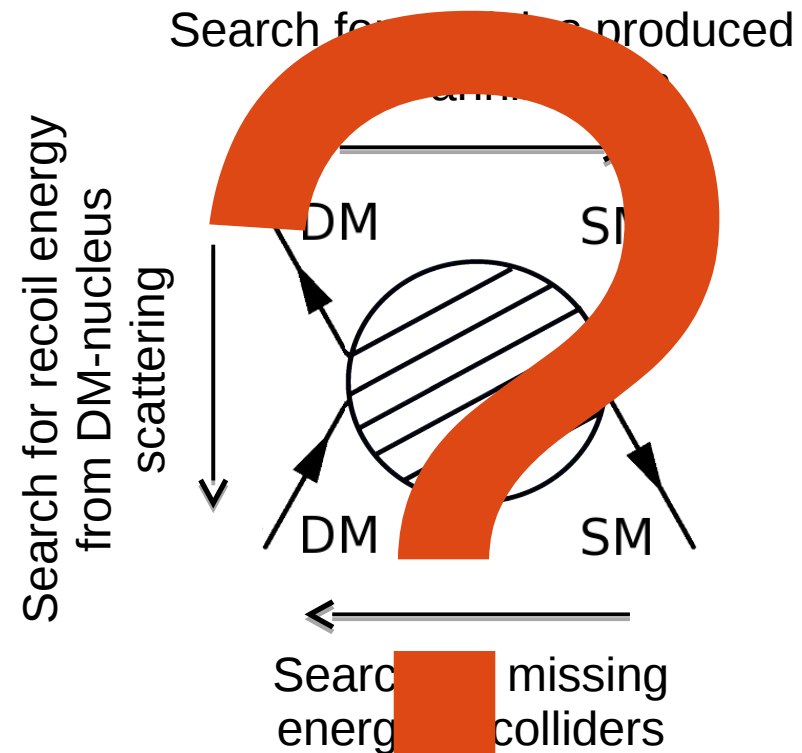
Bernal et al.: arXiv:1706.07442



Qualitative solution: Stronger interactions \leftrightarrow larger abundance

Feebly Interacting Massive Particles (FIMPs)

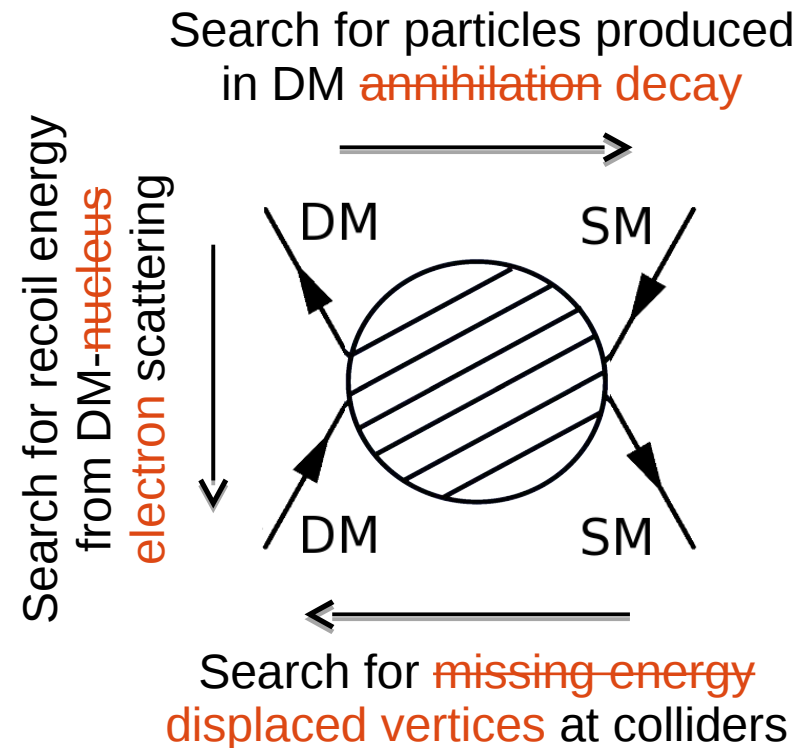
- Particles that obtain their relic abundance via the **freeze-in mechanism** are called FIMPs
- Write interaction rate as $\Gamma = \lambda^2 T$ (with some effective coupling λ)
- The relic density requirement translates to $\lambda \sim 10^{-12}$
- Such tiny couplings are **impossible to probe** with WIMP searches...



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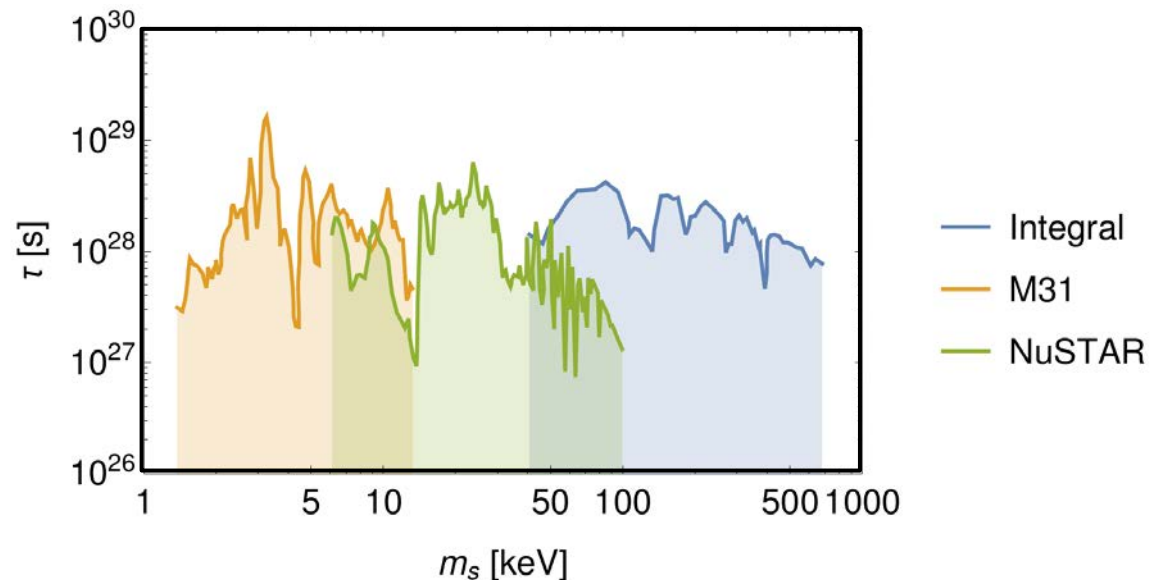
...but they can be discovered with **dedicated search strategies**



Part 1: Decaying dark matter

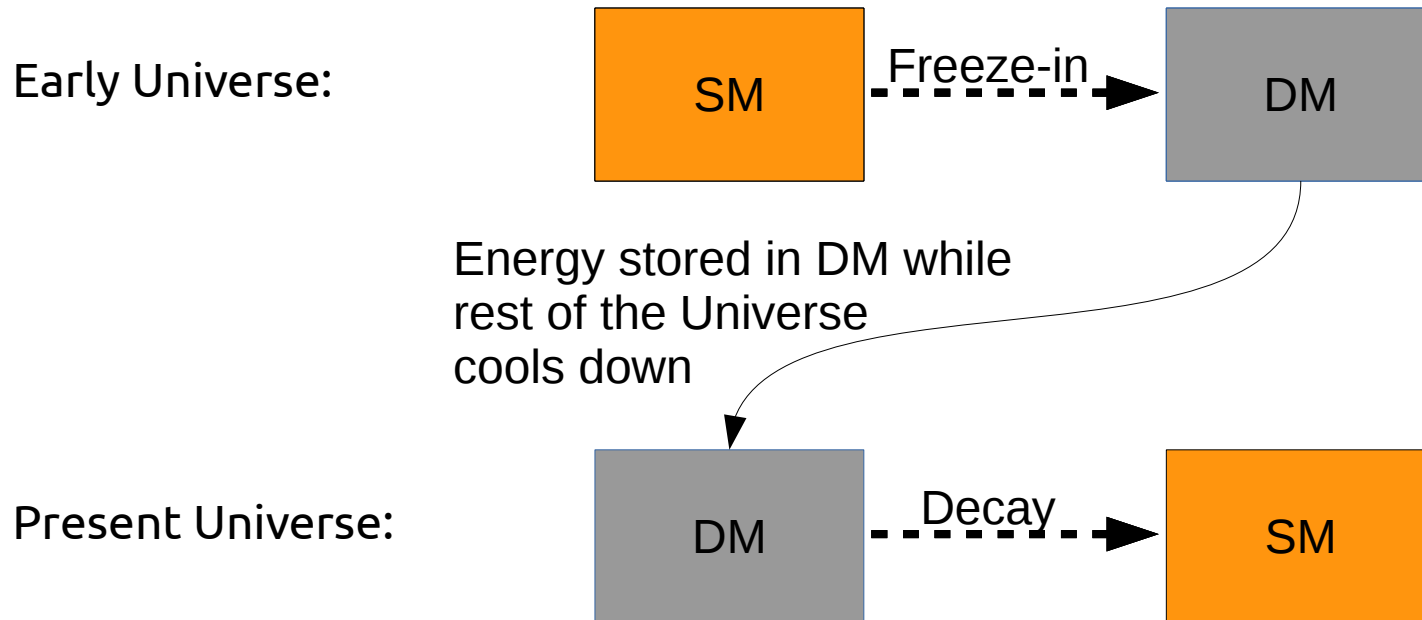
- Challenge the assumption that DM particles are perfectly stable
- Gravitational interactions only tell us that DM survives to the present day, i.e. the particles have a lifetime greater than the age of the Universe ($\sim 10^{17}$ s)

- However, if DM particles decay into SM particles (e.g. photons), observational constraints are much tighter and require $\tau > 10^{28}$ s (for keV-scale DM)



- For a DM particle to have such a long lifetime, it must have tiny couplings to SM particles \rightarrow connection to freeze-in mechanism

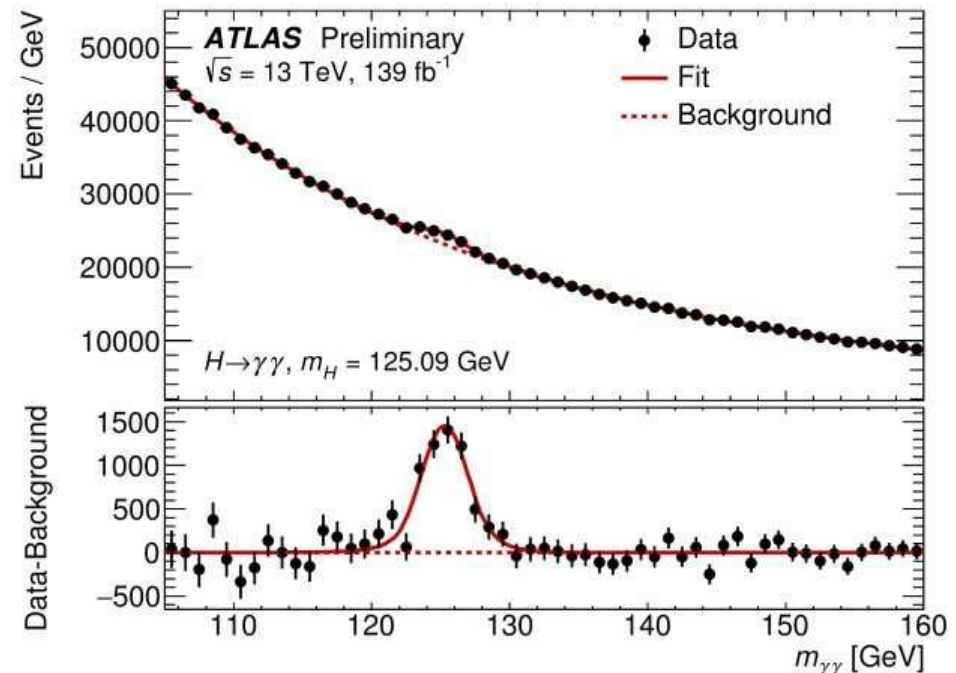
Probing freeze-in with indirect detection



If the same interaction is responsible for DM production and decay, we can probe the freeze-in mechanism by looking for the products of DM decay

Example: Higgs portal DM

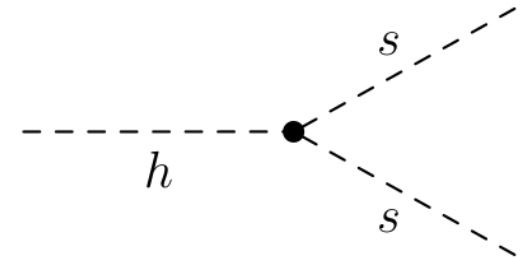
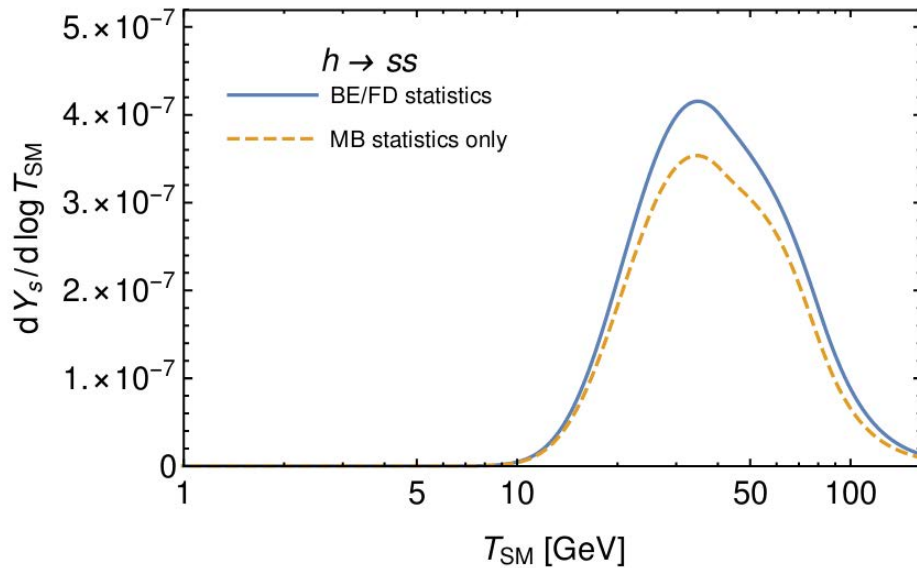
- The Higgs boson h , which arises from the spontaneous breaking of electroweak symmetry, is the only known elementary particle with spin 0
- What if the DM particle also has spin 0? What if it also arises from the spontaneous breaking of a new symmetry?
- If both particles have the same quantum numbers, they can mix with each other, such that mass eigenstates (h and s) are different from interaction eigenstates:



$$\begin{pmatrix} h_{\text{SM}} \\ h_s \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}$$

Consequences of Higgs mixing

- **Consequence 1:** Higgs decays produce DM particles

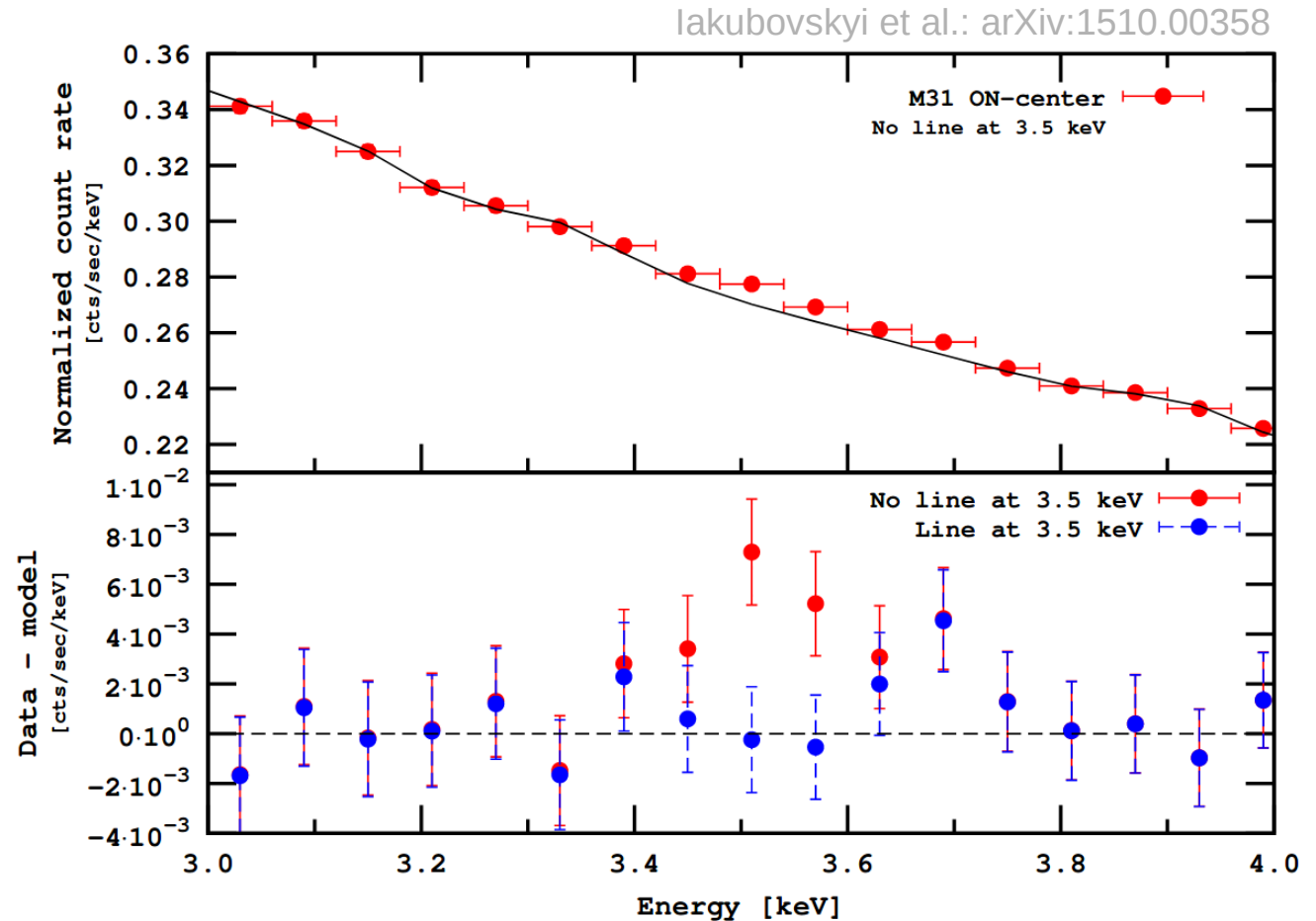


- For $T \sim m_h$ the early Universe is full of Higgs bosons
- Mixing of $\theta \sim 10^{-12}$ sufficient to match observed relic abundance

- **Consequence 2:** DM particles inherit Higgs decay modes
- Phenomenologically most interesting: decay into two photons

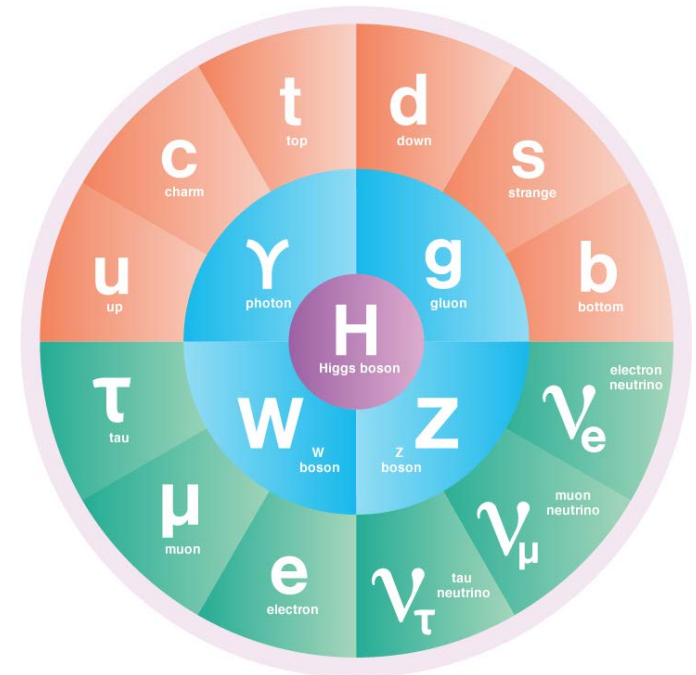
Intriguing coincidences

- For DM masses in the keV range, searches for DM decay products are sensitive to mixing angles $\theta \sim 10^{-12}$, as predicted from cosmological measurements
- Moreover, for several years a number of observations have provided hints for an unexplained x-ray line at 3.5 keV
- If not due to astrophysics, this could be evidence for decays of a spin-0 DM particle with $m_s \sim 7$ keV



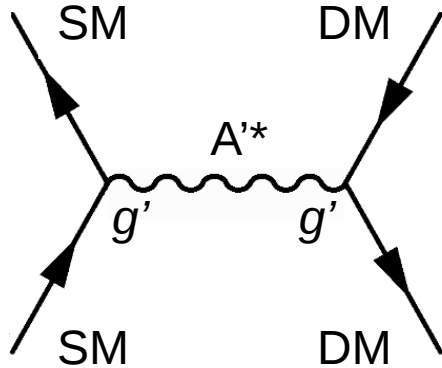
Part 2: New force carriers

- Challenge the assumption that DM particles couple directly to the SM
- Given the **complexity** of the visible sector (making up only 5% of the Universe), it is hardly plausible that the dark sector should be much simpler
- In addition to new stable particles, there may also be **new force carriers**
- Simplest example: a dark fermion x coupled to a dark photon A' (like QED but with much smaller couplings)



Probing freeze-in with accelerator experiments

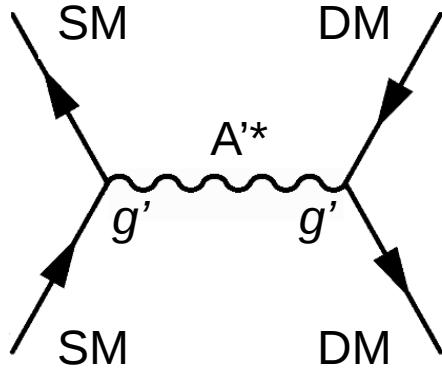
Early Universe:



- For $2 m_{\text{DM}} > m_{A'}$ DM particles can only be produced from virtual dark photons (A'^*)
- Cross section proportional to g'^4/m_{DM}^2
- Observed relic abundance requires $g' \sim 10^{-6}$

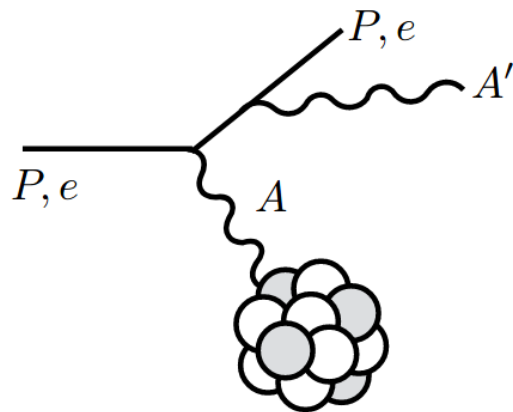
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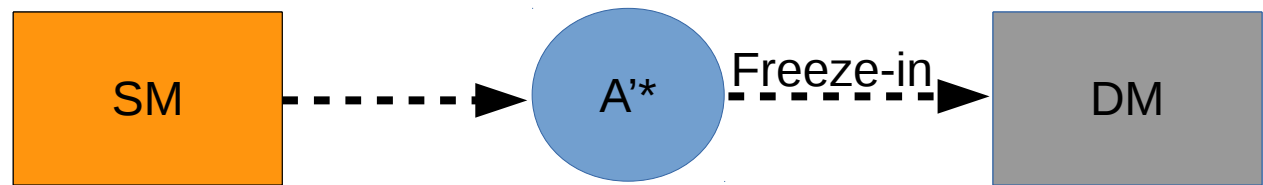
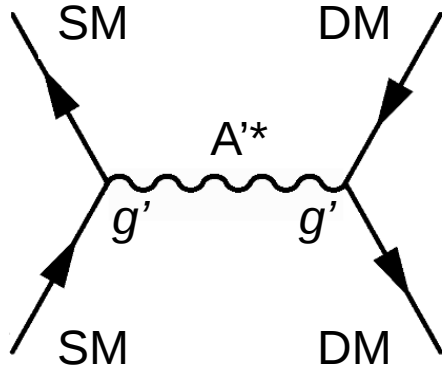
Laboratory:



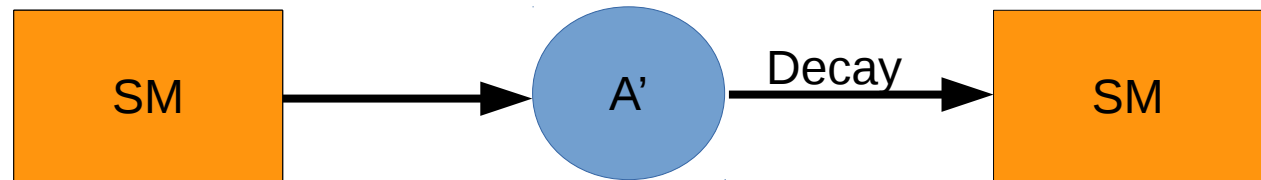
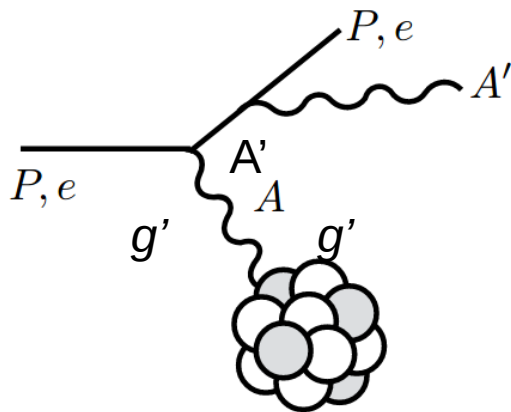
- In the laboratory one can produce real (i.e. on-shell) dark photons by shooting charged particles at a fixed target → dark Bremsstrahlung
- Cross section proportional to $g'^2/m_{A'}^2$

Probing freeze-in with accelerator experiments

Early Universe:

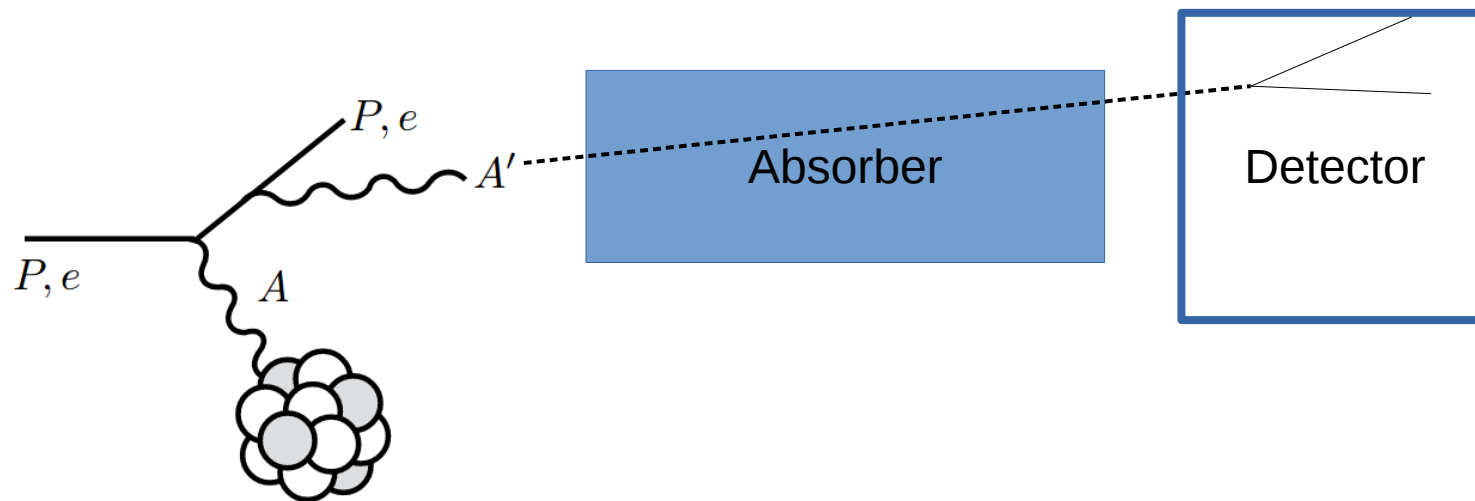


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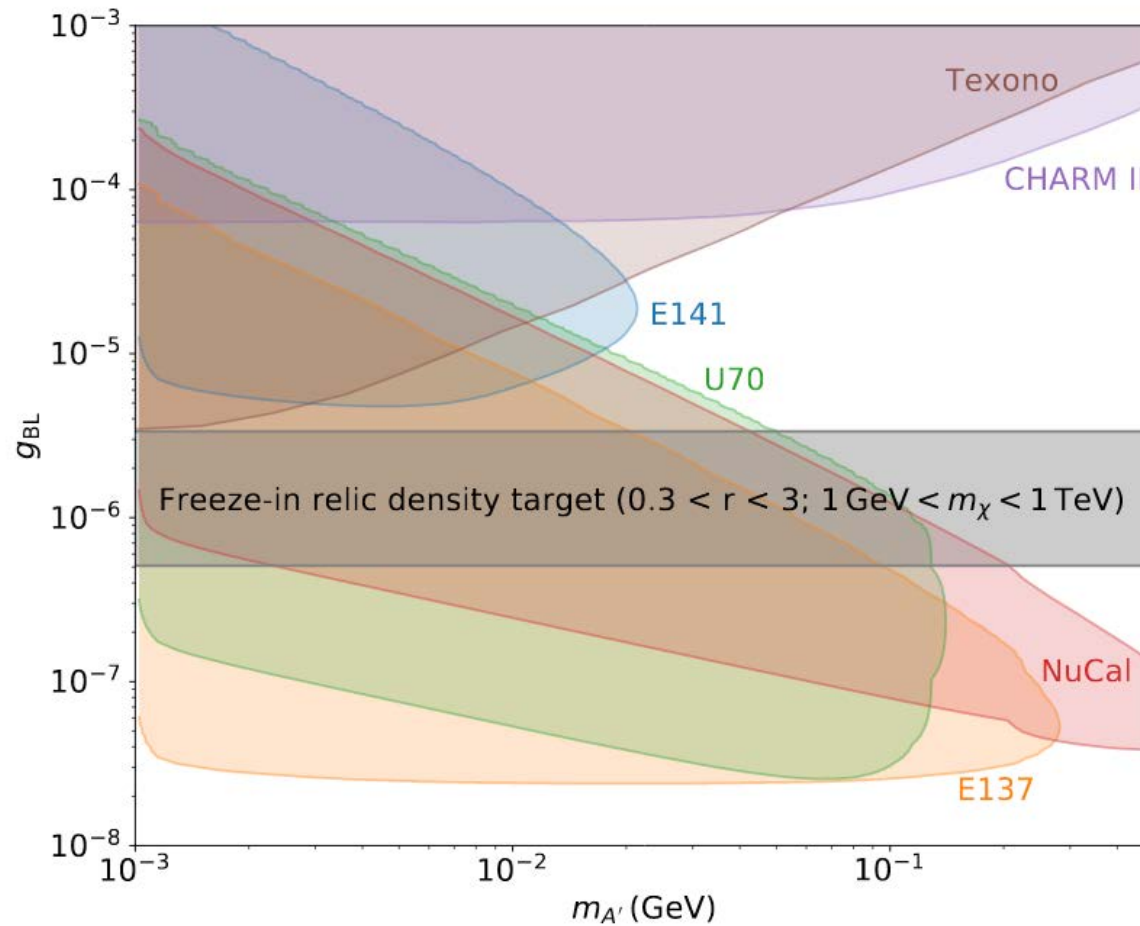


Probing freeze-in with accelerator experiments

- Dark photons are so weakly coupled that they **easily travel through the target and absorber**
- For $m_{A'} > 2 m_e$ they are however **unstable** against the decay into two electrons
- Example: $m_{A'} \sim 10 \text{ MeV}$ and $g' \sim 10^{-6}$ → decay length $\sim 1 \text{ m}$
- We can search for dark photon decays in a **decay volume** behind the absorber



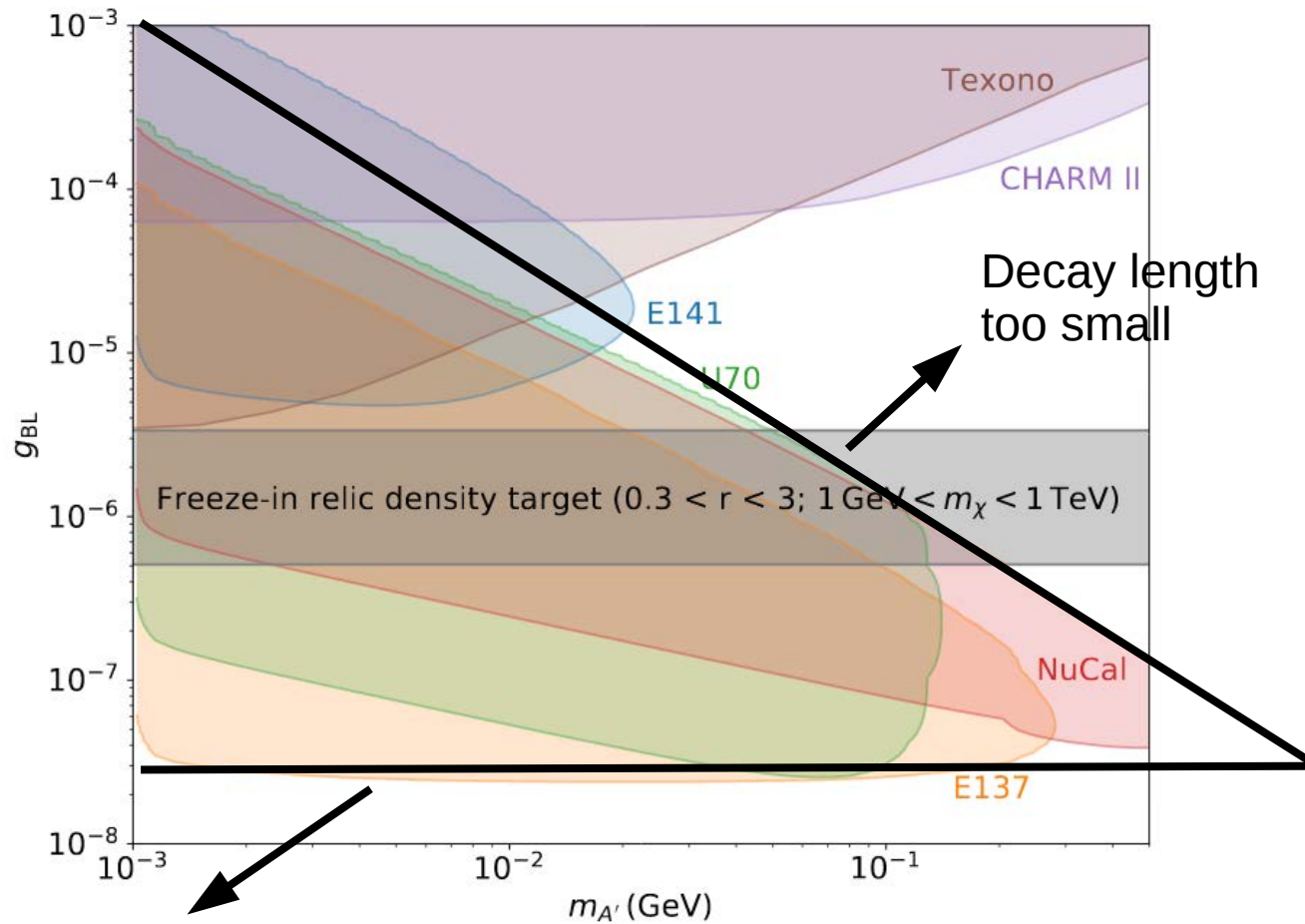
Results



Freeze-in target band for different DM masses and couplings

Constraints from fixed-target experiments

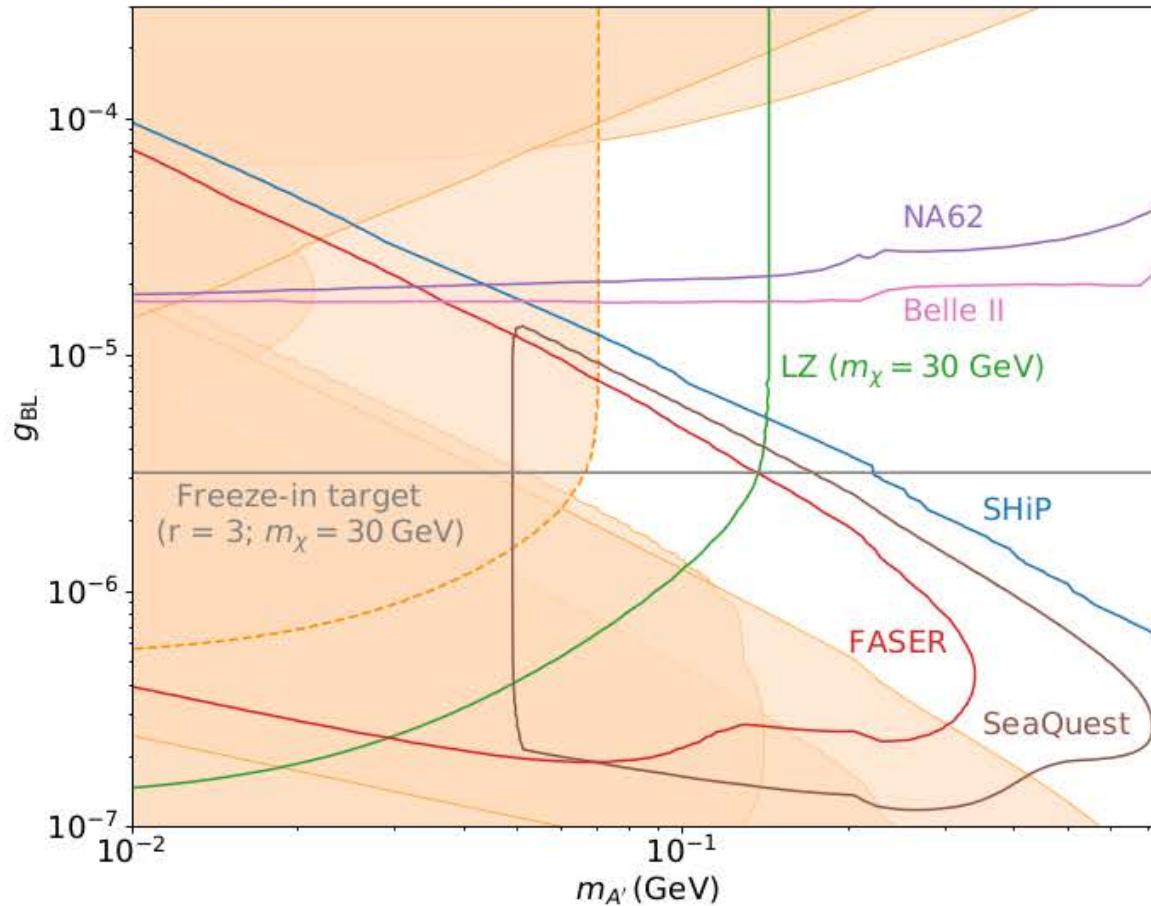
Results



Strong constraints,
but also plenty of
unexplored
parameter space!

Production rate
too small

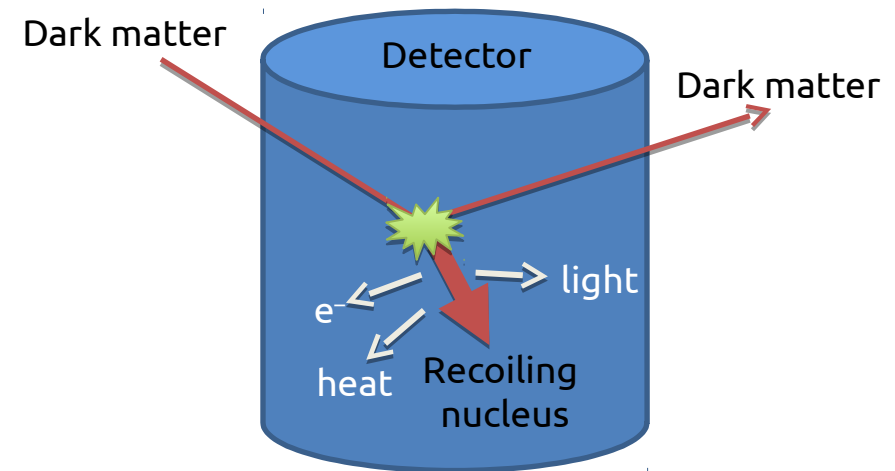
Projections



Many plans to improve sensitivity with fixed-target experiments and e^+e^- colliders!

Part 3: New detector concepts

- Challenge the existing design of dark matter detectors and **explore new approaches**
- Direct detection experiments looking for the scattering of GeV-scale WIMPs typically have an energy threshold of order keV



- DM particles in the Milky Way halo have $v \sim 10^{-3}$ and therefore $E_{\text{kin}} \sim 10^{-6} m_{\text{DM}}$
- When scattering on a heavy nucleus, only a fraction of this energy can be deposited
- Conventional direct detection experiments are **insensitive to keV-scale DM**
- Solution: Search for **DM-electron scattering** in crystals with **small band gap**

Dirac materials

- Materials in which elementary excitations can be described by **Dirac equation**

- Energy-momentum relation:
$$E_{\mathbf{k}}^{\pm} = \pm \sqrt{v_{\text{F}}^2 \mathbf{k}^2 + \Delta^2}$$

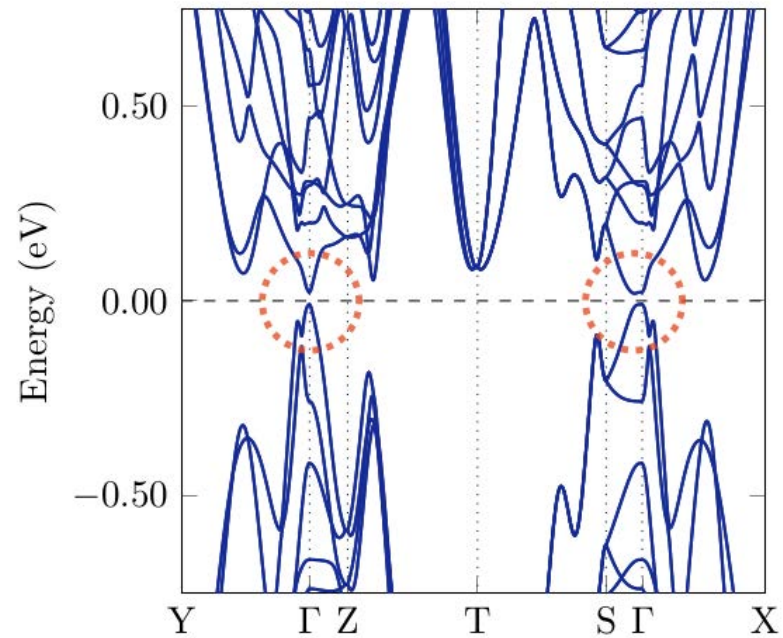
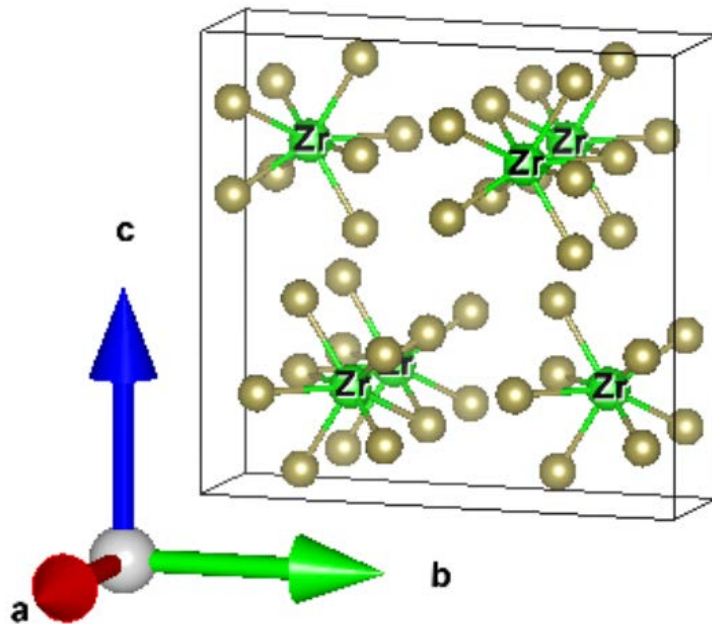
k : lattice momentum

v_{F} : Fermi velocity (replacing speed of light)

2Δ : Band gap (replacing rest mass)

- For $k \gg \Delta$ dispersion relation becomes linear \rightarrow electrons behave like free relativistic fermions
- **Crucial advantage:** Δ can be as small as 10 meV

Example: ZrTe_5

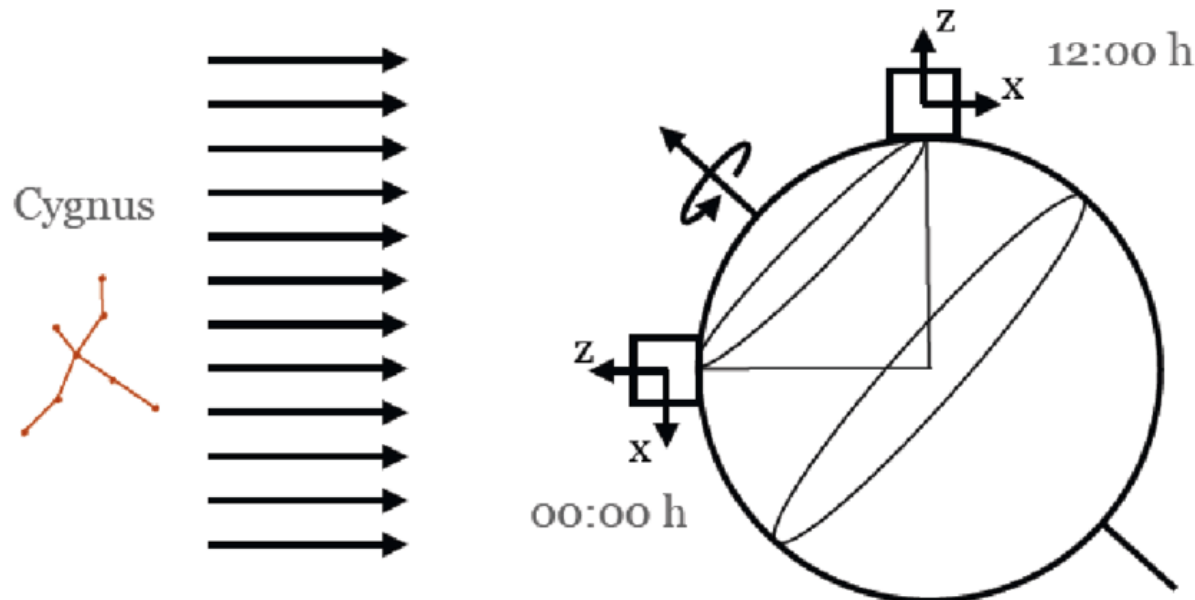


- Band structure calculated with density functional theory + structural optimisation

| | V_{Fx} | V_{Fy} | V_{Fz} | Δ [meV] |
|-------------------|----------|----------|----------|----------------|
| Theory | 1.1e-3 | 9.1e-4 | 4.4e-4 | 15.6 |
| Experiment | 1.3e-3 | 1.6e-3 | 6.5e-4 | 11.75 |

Identifying a dark matter signal

- **Problem:** background from thermal excitations of electrons
- Need to look for **characteristic properties** of DM signal
- Crucial observation: DM flux is **not isotropic** in the laboratory frame (“WIMP wind”)

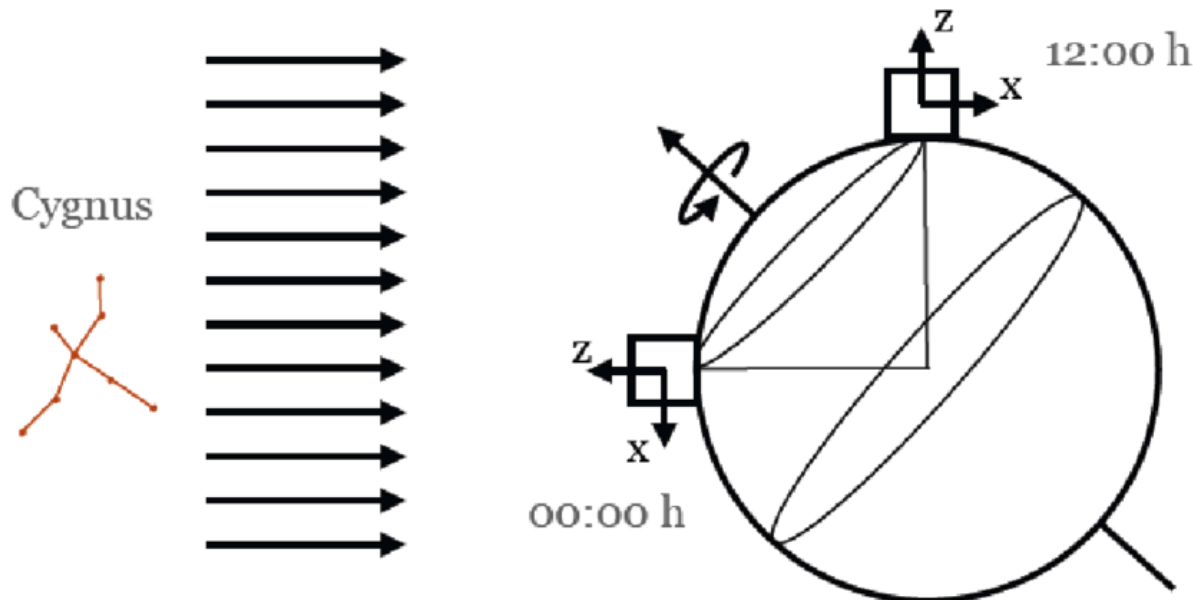


- Direction of the WIMP wind **changes by almost 90 degrees** over the course of 12 hours

Directional detection from anisotropies

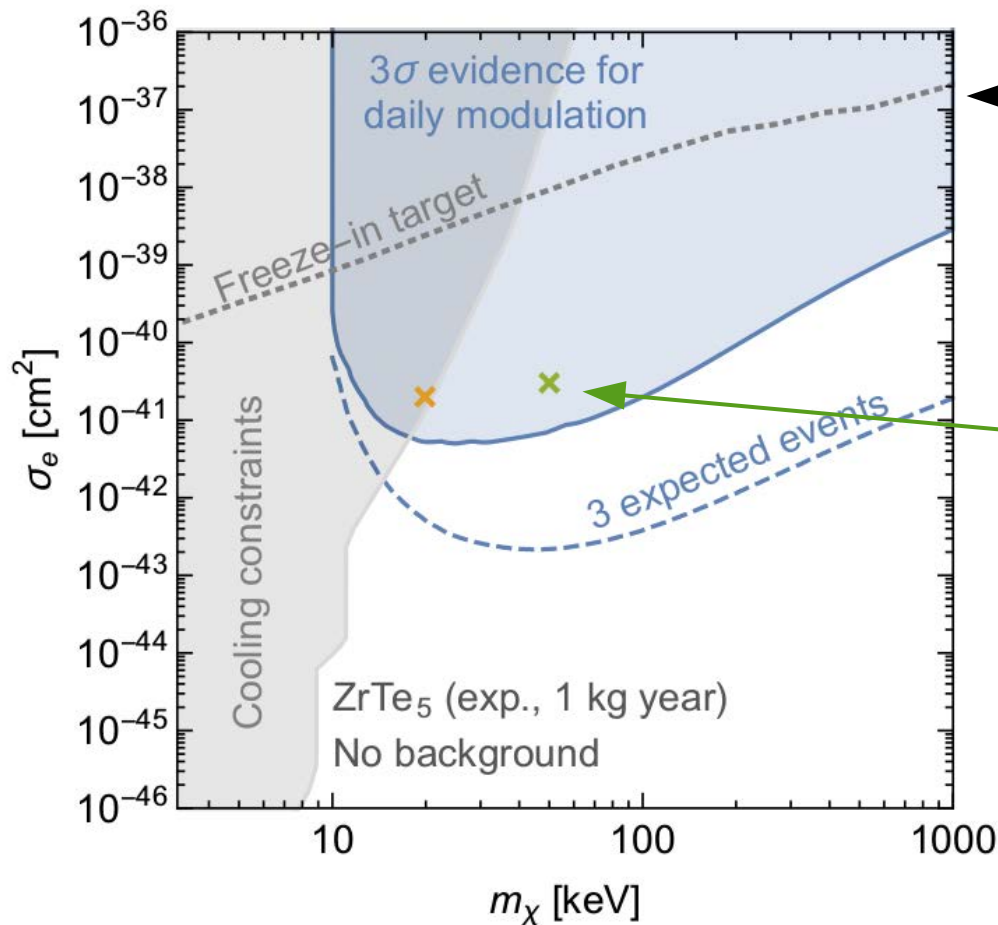
- Dirac materials can have significant anisotropies!

| | V_{Fx} | V_{Fy} | V_{Fz} | Δ [meV] |
|------------|----------|----------|----------|----------------|
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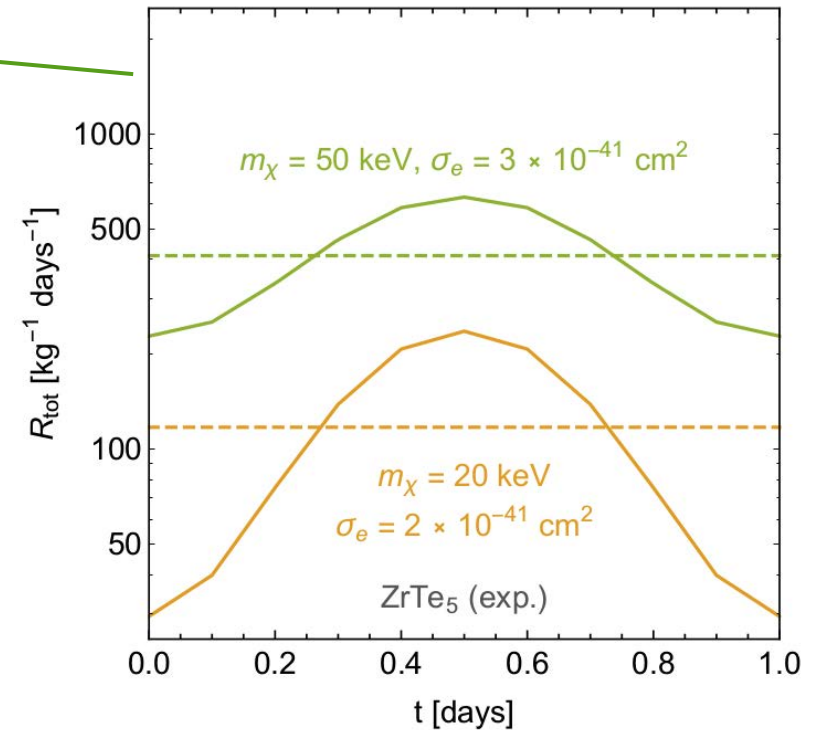


- Scattering is **suppressed** when the Fermi velocity pointing towards the WIMP wind is large

Sensitivity estimates



Dirac materials have **enormous potential** for testing the freeze-in mechanism for keV-scale DM



- Also promising: Organic Dirac materials like BNQ-TTF

Conclusions

- We have **convincing evidence** for the existence of dark matter, but **no experimental information** revealing its nature
- The **non-observation of evidence for WIMPs** suggests that dark matter particles may have never been in thermal equilibrium in the early Universe (so-called **FIMPs**)
- In spite of their **tiny couplings**, FIMPs can be probed in a number of different ways
 - If the same interaction is responsible for dark matter **production and decay**, we can connect the relic abundance to **observable x-ray signals**
 - If a **new force carrier** (e.g. a dark photon) is responsible for dark matter production, we can search for it at **accelerator experiments**
 - If we develop new dark matter detectors based on **quantum materials with tiny energy threshold**, we can probe otherwise inaccessible regions of parameter space
- Let's find FIMPs!

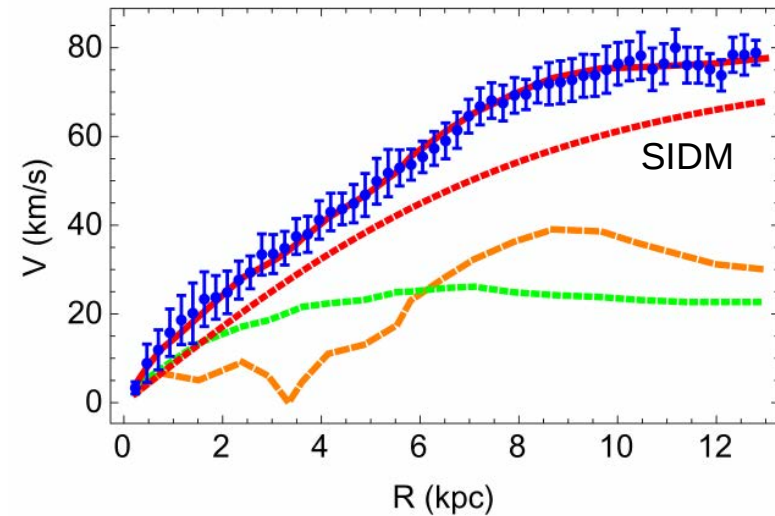
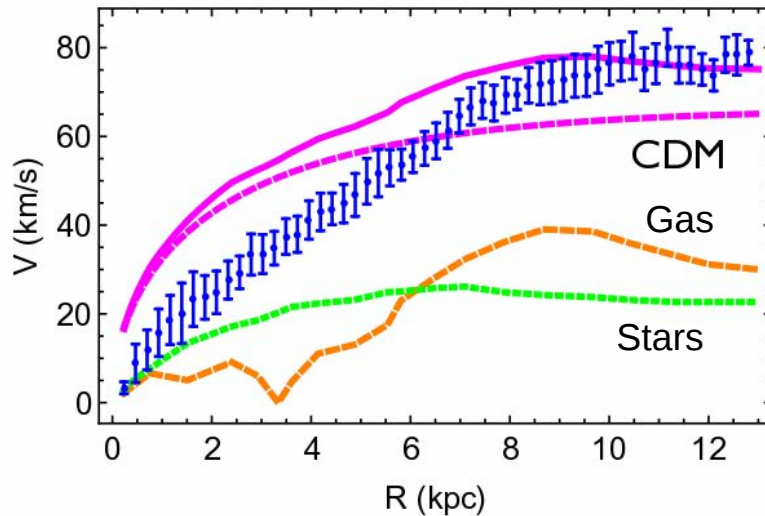
Part 1: Technical points

- For a precise prediction of the expected signal strength, we need accurate calculations of the DM relic abundance:
 - Thermal masses of SM particles in the initial state
 - Quantum statistics for relativistic particles in the plasma
 - Accurate treatment of electroweak phase transition
 - Inclusion of all possible processes
- Although DM particles couple very weakly to the SM, they may still couple strongly to each other
 - Number-changing processes in the Early Universe
 - DM self-interactions in astrophysical systems

| | | | |
|---------------------------------------|---|---|---|
| $W^+, b \rightarrow t, \rho$ | $\tau^+, \tau^- \rightarrow \gamma, \rho$ | $W^-, W^+ \rightarrow h, \rho$ | $W^+, e^- \rightarrow \nu_e, \rho$ |
| $Z, W^+ \rightarrow W^+, \rho$ | $\bar{c}, c \rightarrow \gamma, \rho$ | $\bar{\nu}_e, \nu_e \rightarrow Z, \rho$ | $W^-, \nu_e \rightarrow e^-, \rho$ |
| $g, t \rightarrow t, \rho$ | $\gamma, b \rightarrow b, \rho$ | $\bar{\nu}_m, \nu_m \rightarrow Z, \rho$ | $W^-, \nu_m \rightarrow \mu^-, \rho$ |
| $W^-, t \rightarrow b, \rho$ | $\bar{b}, b \rightarrow \gamma, \rho$ | $\bar{\nu}_t, \nu_t \rightarrow Z, \rho$ | $W^-, \nu_t \rightarrow \tau^-, \rho$ |
| $W^-, c \rightarrow s, \rho$ | $\gamma, \mu^- \rightarrow \mu^-, \rho$ | $Z, \tau^- \rightarrow \tau^-, \rho$ | $Z, t \rightarrow t, \rho$ |
| $W^-, u \rightarrow d, \rho$ | $W^-, W^+ \rightarrow \rho, \rho$ | $Z, e^- \rightarrow e^-, \rho$ | $g, b \rightarrow b, \rho$ |
| $W^+, d \rightarrow u, \rho$ | $\mu^+, \mu^- \rightarrow \gamma, \rho$ | $Z, \mu^- \rightarrow \mu^-, \rho$ | $W^-, W^+ \rightarrow \gamma, \rho$ |
| $W^+, s \rightarrow c, \rho$ | $\bar{c}, c \rightarrow h, \rho$ | $\bar{d}, d \rightarrow g, \rho$ | $\bar{b}, b \rightarrow \rho, \rho$ |
| $W^-, W^+ \rightarrow Z, \rho$ | $\gamma, s \rightarrow s, \rho$ | $\bar{t}, t \rightarrow h, \rho$ | $\bar{\nu}_e, e^- \rightarrow W^-, \rho$ |
| $\bar{u}, d \rightarrow W^-, \rho$ | $Z, Z \rightarrow \rho, \rho$ | $Z, h \rightarrow Z, \rho$ | $\bar{\nu}_m, \mu^- \rightarrow W^-, \rho$ |
| $\bar{c}, s \rightarrow W^-, \rho$ | $h, h \rightarrow \rho, \rho$ | $Z, Z \rightarrow h, \rho$ | $\bar{\nu}_t, \tau^- \rightarrow W^-, \rho$ |
| $\bar{t}, t \rightarrow g, \rho$ | $\bar{s}, s \rightarrow \gamma, \rho$ | $g, u \rightarrow u, \rho$ | $\bar{t}, b \rightarrow W^-, \rho$ |
| $Z, b \rightarrow b, \rho$ | $\bar{s}, s \rightarrow \rho, \rho$ | $\bar{u}, u \rightarrow g, \rho$ | $g, s \rightarrow s, \rho$ |
| $Z, d \rightarrow d, \rho$ | $\tau^+, \tau^- \rightarrow h, \rho$ | $e^+, e^- \rightarrow Z, \rho$ | $\bar{b}, b \rightarrow g, \rho$ |
| $Z, s \rightarrow s, \rho$ | $b, h \rightarrow b, \rho$ | $\mu^+, \mu^- \rightarrow Z, \rho$ | $\gamma, W^+ \rightarrow W^+, \rho$ |
| $Z, c \rightarrow c, \rho$ | $\bar{u}, u \rightarrow \gamma, \rho$ | $\tau^+, \tau^- \rightarrow Z, \rho$ | $\bar{d}, d \rightarrow Z, \rho$ |
| $Z, u \rightarrow u, \rho$ | $\gamma, u \rightarrow u, \rho$ | $W^-, c \rightarrow d, \rho$ | $\bar{s}, s \rightarrow Z, \rho$ |
| $W^+, \tau^- \rightarrow \nu_t, \rho$ | $\mu^+, \mu^- \rightarrow \rho, \rho$ | $W^+, s \rightarrow u, \rho$ | $\bar{b}, b \rightarrow Z, \rho$ |
| $W^+, \mu^- \rightarrow \nu_m, \rho$ | $\gamma, d \rightarrow d, \rho$ | $W^-, u \rightarrow s, \rho$ | $Z, \nu_e \rightarrow \nu_e, \rho$ |
| $\mu^-, h \rightarrow \mu^-, \rho$ | $\bar{d}, d \rightarrow \gamma, \rho$ | $W^+, d \rightarrow c, \rho$ | $Z, \nu_m \rightarrow \nu_m, \rho$ |
| $\bar{d}, d \rightarrow h, \rho$ | $\bar{t}, t \rightarrow \rho, \rho$ | $\bar{u}, s \rightarrow W^-, \rho$ | $Z, \nu_t \rightarrow \nu_t, \rho$ |
| $e^+, e^- \rightarrow \rho, \rho$ | $e^+, e^- \rightarrow \gamma, \rho$ | $\bar{c}, d \rightarrow W^-, \rho$ | $\bar{u}, u \rightarrow Z, \rho$ |
| $\bar{u}, u \rightarrow h, \rho$ | $c, h \rightarrow c, \rho$ | $\gamma, t \rightarrow t, \rho$ | $\bar{c}, c \rightarrow Z, \rho$ |
| $d, h \rightarrow d, \rho$ | $\gamma, e^- \rightarrow e^-, \rho$ | $\bar{c}, c \rightarrow \rho, \rho$ | $\bar{s}, s \rightarrow g, \rho$ |
| $u, h \rightarrow u, \rho$ | $\tau^-, h \rightarrow \tau^-, \rho$ | $\bar{t}, t \rightarrow \gamma, \rho$ | $W^+, h \rightarrow W^+, \rho$ |
| $e^+, e^- \rightarrow h, \rho$ | $\bar{s}, s \rightarrow h, \rho$ | $\tau^+, \tau^- \rightarrow \rho, \rho$ | $t, h \rightarrow t, \rho$ |
| $e^-, h \rightarrow e^-, \rho$ | $\mu^+, \mu^- \rightarrow h, \rho$ | $h, h \rightarrow h, \rho$ | $g, c \rightarrow c, \rho$ |
| | $\bar{d}, d \rightarrow \rho, \rho$ | $\bar{b}, b \rightarrow h, \rho$ | $\bar{t}, t \rightarrow Z, \rho$ |
| | $\bar{u}, u \rightarrow \rho, \rho$ | $\gamma, c \rightarrow c, \rho$ | $\bar{c}, c \rightarrow g, \rho$ |
| | $s, h \rightarrow s, \rho$ | $\gamma, \tau^- \rightarrow \tau^-, \rho$ | $g, d \rightarrow d, \rho$ |

Quick detour: Dark matter self-interactions

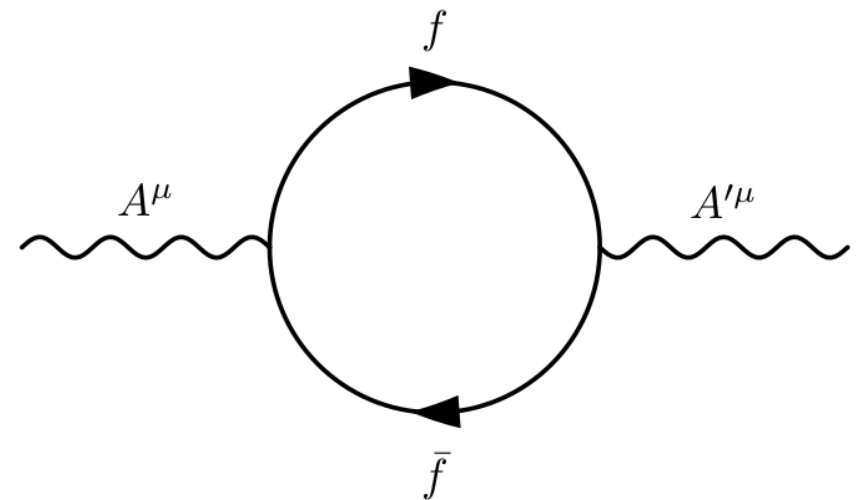
- Bullet Cluster: The dominant form of matter in galaxy clusters behaves very **differently from baryonic gas**
- Observations require $\sigma / m_x \lesssim 1.5 \text{ cm}^2/\text{g}$ (= 3 barn/GeV)
- A self-interaction cross section close to this bound may give a **better fit to observations** than cold DM (cusp-core problem)



Tulin & Yu: arXiv:1705.02358

Part 2: Technical points

- Freeze-in production and laboratory constraints depend on the **specific coupling structure** of the dark photon
- Interesting example: Equal but opposite coupling to **baryons and leptons** ($g' = g_{B-L}$)
- DM charge is a **free parameter**, but should be similar in size to the other couplings
- **Important subtlety:** At finite temperatures the SM photon acquires a **plasma mass**, leading to mixing between the dark and the visible photon
- **Plasmon decays** give important contribution to DM relic density



Part 3: Technical points

- Various ingredients are necessary to calculate scattering rate

$$R_{\mathbf{k} \rightarrow \mathbf{k}'} = \frac{\rho_\chi}{m_\chi} \frac{\bar{\sigma}_e}{8\pi\mu_{\chi e}^2} \int d^3q |F_{\text{DM}}(q)|^2 |\mathcal{F}_{\text{med}}(q)|^2 |f_{\mathbf{k} \rightarrow \mathbf{k}'}(q)|^2 \frac{\tilde{g}(v_{\text{min}}, \psi)}{|\mathbf{q}|}$$

Part 3: Technical points

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$$R_{\mathbf{k} \rightarrow \mathbf{k}'} = \frac{\rho_\chi}{m_\chi} \frac{\bar{\sigma}_e}{8\pi\mu_{\chi e}^2} \int d^3q \underbrace{|F_{\text{DM}}(q)|^2}_{\text{Particle physics}} \underbrace{|\mathcal{F}_{\text{med}}(q)|^2}_{\text{In-medium effects}} \underbrace{|f_{\mathbf{k} \rightarrow \mathbf{k}'}(q)|^2}_{\text{Transition probability}} \underbrace{\frac{\tilde{g}(v_{\text{min}}, \psi)}{|\mathbf{q}|}}_{\text{Astrophysics}}$$

Requires knowledge of dielectric tensor and polarisation tensor

$$\mathcal{F}_{\text{med}}(q) = \frac{1}{1 + \left(q_x^2 \frac{v_{\text{F},z}^2}{\kappa_{xx}} + q_y^2 \frac{v_{\text{F},y}^2}{\kappa_{yy}} + q_z^2 \frac{v_{\text{F},z}^2}{\kappa_{zz}} \right) \frac{\pi(\tilde{q}^2)}{q^2}}$$

$$\pi(\tilde{q}^2) = -\frac{g e^2}{24\pi^2 v_{\text{F},x} v_{\text{F},y} v_{\text{F},z}} \left(\log \left| \frac{4\tilde{\Lambda}^2}{\tilde{q}^2} \right| + i\pi\Theta(\tilde{q}^2) \right)$$

