NNLL q_T resummation and **beyond**

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Mainz Theorie Palaver
3.2.2015

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based on:
arXiv:1209.0682, 1401.1222, 1403.6451
with Thomas Gehrmann, Li Lin Yang
arXiv:150x.abcd
with Thomas Becher, Matthias Neubert, Daniel Wilhelm

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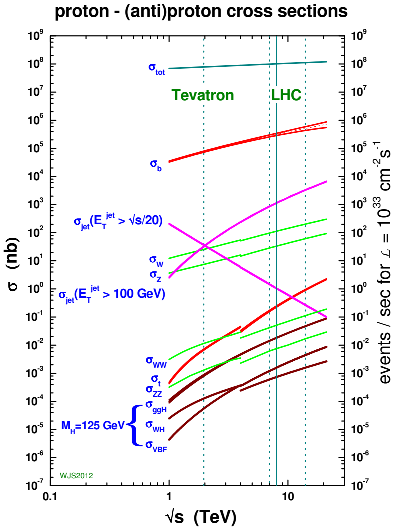
Motivation

- Excellent performance of LHC: high experimental precision.
- Need high theory precision to
  - identify and measure signal (SM, BSM) on top of large (QCD) background,
  - extract SM parameters to high accuracy.

$\Rightarrow$ Need multi-loop calculations. Especially in QCD.

- Preferable: Analytic calculations, since:
  - offer many checks,
  - exact result,
  - elegant and explicit representation,
  - can learn more about theory, structure.

Cross sections

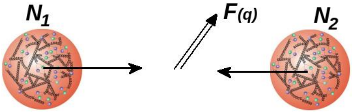


- Many relevant processes produce heavy, color neutral particles.
- Popular: H.
- σ much larger for V production.
- Test SM to high precision.

Observable

Consider:

$N_1 + N_2 \to F(q) + X$



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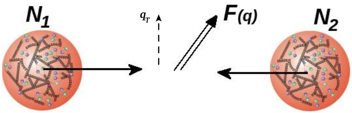
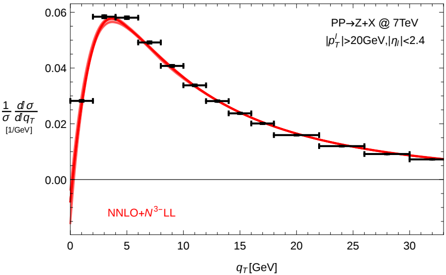


- $F = l^+ l^-, Z, W, H, Z', \ldots$
- Test SM to high precision.
- $d\sigma/dq_T$ in region $q_T^2 \ll q^2$.

$\Rightarrow$ Need to resum large logarithms.

$\Rightarrow$ Transverse PDFs (TPDFs) (Beam functions).



[Becher, TL, Neubert, Wilhelm]

Outline

0. Motivation

1. k_T factorization
   - Large logarithms & Concepts
   - Transverse PDFs
   - Rapidity divergences
2. Perturbative calculation
   - Warming up
   - NNLO
   - Results & Checks
3. Phenomenology
   - Resummation
   - Data, CuTe and pert. input
   - Conclusions

Perturbation theory

- perturbative expansion:

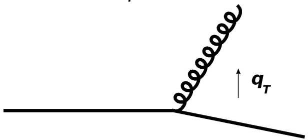
$$\frac{d\sigma}{dq_T} = \sum_{n=0}^{\infty} \alpha_s^n \frac{d\sigma}{dq_T}^{(n)}$$

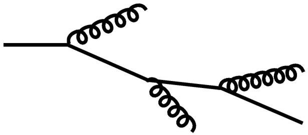
fixed order: stop at e.g. $n = 2$ (NNLO).

- For each power in α_s IR divergences, cancel between real and virtual corrections by KLN theorem.
- However, large logarithms of scale ratios can remain, which
  - spoil convergence,
  - need to be resummed to all orders in α_s.

Large logarithms

Momenta of emitted particle soft, collinear
$\Rightarrow$ up to $\log^2 \frac{q_T^2}{q^2}$ for each power α_s.





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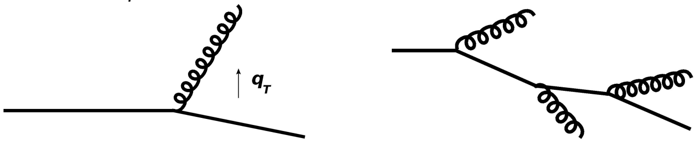


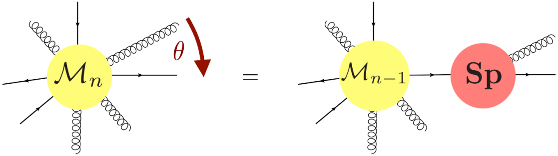
Due to specific structure, factorize, can account for them to all orders in α_s (resum).

In expansion $\frac{d\sigma}{dq_T} = \sum_{n=0}^{\infty} \sum_{k=0}^{2n} \alpha_s^n L^k \frac{d\sigma}{dq_T}^{(n,k)} = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \alpha_s^n (\alpha_s L^2)^k C_{N^nLL_e}^{(k)}$.

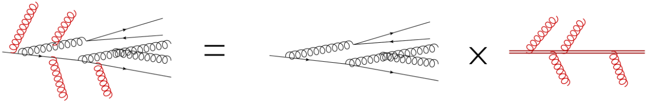
QCD simplified

Soft and collinear emissions: Important contributions to high-energy processes; have characteristic structure.
In both limits interactions simplify:

- Collinear limit, multiple particles move in similar direction



- Soft limit, particles with small energy and momentum are emitted $\Rightarrow$ Eikonal interactions.



Soft-collinear effective theory

EFT of QCD.

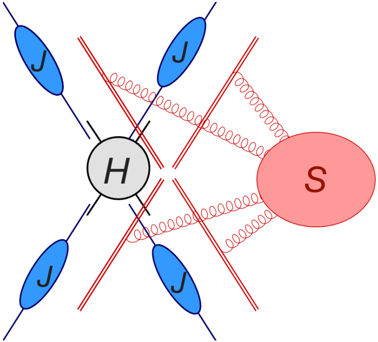
- Structure of soft and collinear interactions implemented on Lagrangian level.

$\Rightarrow$ Soft and collinear fields with definite interactions and power counting.

- Efficient formalism to (re-)derive factorization theorems for multi scale problems.

$+$: Gauge invariant operator definitions for the soft and collinear contributions.

- Resummation via renormalization group.



$$d\sigma \sim H(s_{ij}^2)S(\Lambda_{ij}^2) \otimes \prod_i J_i(M_i^2)$$

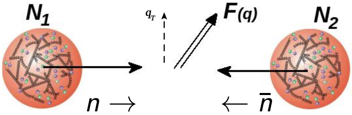
[Bauer, Fleming, Pirjol, Stewart, ...]

Factorization, pictorially

Consider:

$N_1 + N_2 \to F(q) + X$

- $F = l^+ l^-,\ Z,\ W,\ H,\ Z', \ldots$

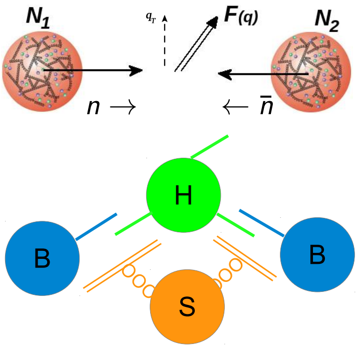


Factorization, pictorially

Consider:

$N_1 + N_2 \to F(q) + X$

- $F = l^+ l^-,\ Z,\ W,\ H,\ Z', \ldots$
- Enhanced contributions from collinear and soft regions.
- Characteristic structure $\Rightarrow$ factorization $d\sigma = H \otimes S \otimes B \otimes B$.



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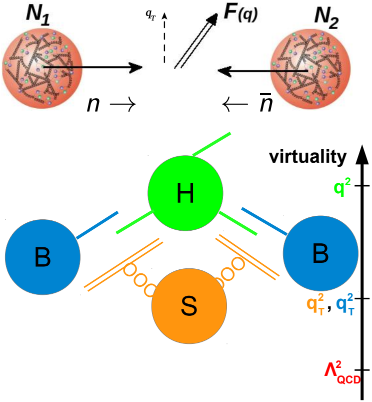
- Here $\lambda^2 = q_T^2 / q^2 \ll 1$.



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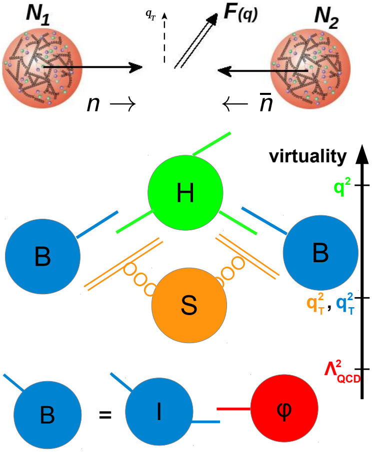
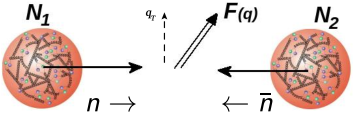


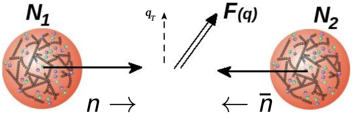
Notation

- LC vectors n, $\bar{n}$,
  $n^2, \bar{n}^2 = 0,\ n \cdot \bar{n} = 2$
- LC basis:



$$v^\mu = (\bar{n}v)\frac{n^\mu}{2} + (nv)\frac{\bar{n}^\mu}{2} + v_\perp^\mu = (v_-, v_+, v_\perp)^\mu$$

Notation



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- Distinguishing fields with different momentum scaling

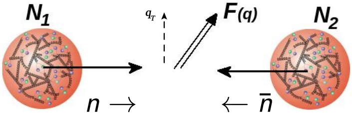
$$\begin{array}{lll} \text{hard } (h): & p_h & \sim \sqrt{q^2}(1,1,1) \\ n\text{−collinear } (n): & p_n & \sim \sqrt{q^2}(1,\lambda^2,\lambda) \\ \bar{n}\text{−collinear } (\bar{n}): & p_{\bar{n}} & \sim \sqrt{q^2}(\lambda^2,1,\lambda) \\ \text{soft } (s): & p_s & \sim \sqrt{q^2}(\lambda,\lambda,\lambda) \end{array}$$

$$p_h^2 \sim q^2$$

$$p^2 \sim q_T^2$$

- For $\lambda^2 = \frac{q_T^2}{q^2}$

Notation



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- Distinguishing fields with different momentum scaling

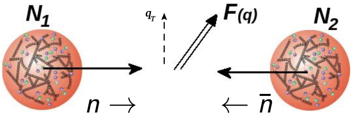
(hard (h):	p_h	$\sim \sqrt{q^2}(1,1,1)$)	$p_h^2 \sim q^2$
$n-$collinear (n):	p_n	$\sim \sqrt{q^2}(1,\lambda^2,\lambda)$	$p^2 \sim q_T^2$
$\bar{n}-$collinear ($\bar{n}$):	$p_{\bar{n}}$	$\sim \sqrt{q^2}(\lambda^2,1,\lambda)$	
soft (s):	p_s	$\sim \sqrt{q^2}(\lambda,\lambda,\lambda)$	

- For $\lambda^2 = \frac{q_T^2}{q^2} \ll 1$:

$$\mathcal{L}_{QCD} \xrightarrow{\int DA_h} \mathcal{L}_{SCET} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_s + \mathcal{O}(\lambda^2)$$

Factorization

Consider e.g.:
$N_1 + N_2 \to V(q) + X$



$$d\sigma \sim \sum_X \delta^4(\sum \hat{p})(-g_{\mu\nu}) \langle N_1 N_2 | J^{\mu\dagger}(0) | X \rangle \langle X | J^{\nu}(0) | N_1 N_2 \rangle \ .$$

Match to SCET. Current:

$$J^\mu(x) = \bar{q}(x)\gamma^\mu q(x) \to \underbrace{C_V(-q^2)}_{\text{hard}} \underbrace{(\bar{\xi}_{\bar{n}} W_{\bar{n}})(x)}_{n\text{-col.}} \gamma_\perp^\mu \underbrace{(S_{\bar{n}}^\dagger S_n)(x)}_{\text{soft}} \underbrace{(W_n^\dagger \xi_n)(x)}_{\bar{n}\text{-col}}$$

Wilson lines: $W_{\bar{n}}(x) = P \exp \left[ig \int_{-\infty}^0 ds \, n \cdot A_{\bar{n}}(x + sn) \right]$,
correspondingly for W_n, $S_{\bar{n}}$, S_n.

$$|X\rangle \to |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_s\rangle \ , \ |N_1 \, N_2\rangle \to |N_1\rangle \otimes |N_2\rangle \otimes |0\rangle \ .$$

Multipole expand in $x = (x_-, x_+, x_\perp)$.

'Factorized' differential cross section

From this obtain [Becher, Neubert]:

$$\frac{d^2\sigma}{dq_T^2 dy} - \mathcal{O}\left(\frac{q_T^2}{q^2}\right) = \frac{\pi\alpha_{ew}^2}{N_c s} \left| C_V(-q^2) \right|^2 \int d^2x_\perp \, e^{-iq_\perp \cdot x_\perp} \sum_q e_q^2$$

$$\times \left[\mathcal{S}(x_T^2) \mathcal{B}_{q/N_1}(z_1, x_T^2) \bar{\mathcal{B}}_{\bar{q}/N_2}(z_2, x_T^2) + (q \leftrightarrow \bar{q}) \right]_{q^2},$$

with

TPDF (quark, n collinear, gauge invariant)

$$\mathcal{B}_{q/N_1}(z, x_T^2) = \frac{1}{2\pi} \int \mathrm{d}t \, e^{-izt\bar{n}\cdot p} \sum_{X_n} \frac{\bar{\not{n}}_{\alpha\beta}}{2}$$

$$\times \langle N_1(p) | (\bar{\xi}_n W_n)_\alpha (t\bar{n} + x_\perp) | X_n \rangle \langle X_n | (W_n^\dagger \xi_n)_\beta (0) | N_1(p) \rangle .$$

- $\bar{n}$-collinear $\bar{\mathcal{B}}_{\bar{q}/N_2}$ correspondingly with $p \sim n \leftrightarrow \bar{n} \sim \bar{p}$ & $q \leftrightarrow \bar{q}$.
- Soft function $\mathcal{S}(x_\perp) = \frac{1}{N_c} \sum_{X_s} Tr \langle 0 | (S_n^\dagger S_{\bar{n}})(x_\perp) | X_s \rangle \langle X_s | (S_{\bar{n}}^\dagger S_n)(0) | 0 \rangle .$

Transverse PDFs

TPDF (quark, n collinear, gauge invariant)

$$\mathcal{B}_{q/N_1}(z, x_T^2) = \frac{1}{2\pi}\int \mathrm{d}t\; e^{-izt\bar{n}\cdot p} \sum_{X_n} \frac{\bar{\not{n}}_{\alpha\beta}}{2} \times \langle N_1(p)|\, \bar{\chi}_\alpha(t\bar{n} + x_\perp)\,|X_n\rangle\, \langle X_n|\, \chi_\beta(0)\,|N_1(p)\rangle\, .$$

- $\chi_n = (\bar{\xi}_n W_n)$
- TPDF: Generalization of usual PDF $\phi_{i/N}(z)$.
- k = momentum of X_n:
  k_- and (after F.T.) $k_\perp$ fixed:
- $\int dte^{-iztp_-}\bar{\chi}(t\bar{n} + x_\perp) \Rightarrow \delta(k_- - (1-z)p_-)\bar{\chi}$,
- $\bar{\chi}(t\bar{n} + x_\perp) \Rightarrow e^{-ik_\perp x_\perp} \overset{\text{F.T.}}{\rightarrow} \delta^{(2)}(k_\perp + q_\perp)\bar{\chi}$.

Gluons

- Similar fact. theorems for other $q\bar{q}$ and gg initiated processes.
- If gg initiated: C, $\mathcal{B}$ & $\bar{\mathcal{B}}$ become Lorentz tensors.

gluon TPDF (n collinear) [Becher, Neubert, Wilhelm]

$$\mathcal{B}^{\mu\nu}_{g/N}(z, x_\perp) = \frac{-z\bar{n}\cdot p}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \sum_X \langle N| \mathcal{A}^{\mu a}_{n,\perp}(t\bar{n} + x_\perp) |X\rangle \langle X| \mathcal{A}^{\nu a}_{n,\perp}(0) |N\rangle$$

- with gauge invariant gluon field $\mathcal{A}^\mu_{n\perp}(x) = \left(W_n^{\mathrm{adj}} A^\mu_{n\perp}\right)(x)\,.$
- Decompose tensor as

$$\mathcal{B}^{\mu\nu}_{g/N}(z, x_\perp) = \frac{g_\perp^{\mu\nu}}{d-2} \mathcal{B}_{g/N}(z, x_T^2) + \left[\frac{g_\perp^{\mu\nu}}{d-2} + \frac{x_\perp^\mu x_\perp^\nu}{x_T^2}\right] \mathcal{B}'_{g/N}(z, x_T^2)\,.$$

- Discussion below for $\mathcal{B}$ only. $\mathcal{B}'$ analogous.

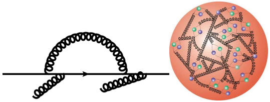
Matching kernel

If $x_T^{-2} \gg \Lambda^2_{\rm QCD}$, refactorize these scales:

Matching kernel $\mathcal{I}_{i/k}$

$$\mathcal{B}_{i/N}(z, x_T^2) = \sum_k \int_z^1 \frac{d\rho}{\rho} \mathcal{I}_{i/k}(\rho, x_T^2) \phi_{k/N}(z/\rho) + \mathcal{O}(\Lambda^2 x_T^2)$$

$$= \sum_k \mathcal{I}_{i/k}(z, x_T^2) \otimes \phi_{k/N}(z) \, .$$

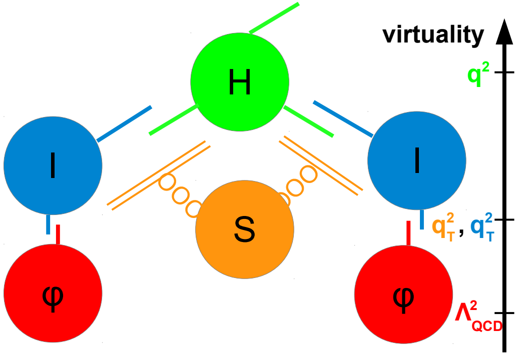


- $\mathcal{I}_{i/k}$ perturbative: Extract from perturbative $\mathcal{B}_{i/j}$ and $\phi_{k/j}$.
- Determine to NNLO: [Gehrmann,TL,Yang].

Factorization, pictorially

$N_1 + N_2 \to F(q) + X$
at $\Lambda^2_{\mathrm{QCD}} \ll q_T^2 \ll q^2$:

$$\frac{d\sigma}{dq_T^2} = H \otimes S \otimes \mathcal{I} \otimes \bar{\mathcal{I}} \otimes \phi \otimes \phi.$$



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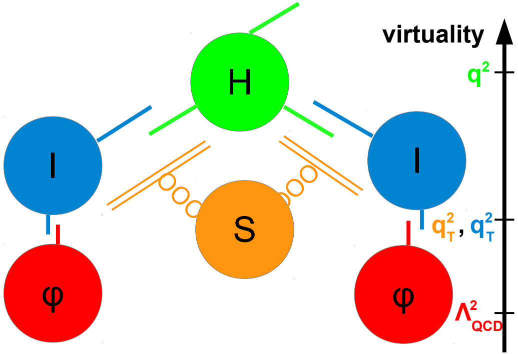
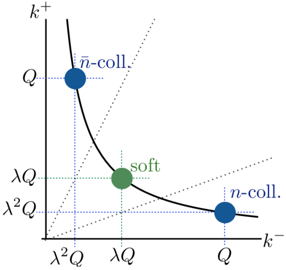


Focus on S, $\mathcal{I}$, $\bar{\mathcal{I}}$.

Same virtuality
$k^2 \sim q_T^2$.

Differ in rapidity
$y(k) = \frac{1}{2} \log \frac{k_+}{k_-}$



Divergences

- Besides usual UV/IR divergences regulated in $d = 4 - 2\epsilon$, encounter 'rapidity' divergences: Unregulated by ϵ; from extreme rapidities $y(k) = \frac{1}{2} \log \frac{k_+}{k_-}$.
- Arise in integrals along LC-direction: $\int d^d k = \frac{1}{2} \int dk_+ dk_- d^{d-2} k_\perp$.
- $\delta^{(d-2)}(k_\perp + q_\perp) \;\Rightarrow\;$ kills $\int d^{d-2} k_\perp$ without putting ϵ to k_+ and k_-.

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$\Rightarrow$ Need additional regulator. Various possibilities:

  - Avoid divergent $k_\pm$ denominators: Use W away from n, $\bar{n}$; Introduce 'LC-mass', ...
  - Regulate $k_\pm$ integrals: Analytic regulator in phase space, ...

- For each various ways.
- Combination $\mathcal{S}_i \mathcal{B}_{i//j} \bar{\mathcal{B}}_{\bar{i}/k}$ must not dependent on this regularization.

Analytic regulator

- We use analytic regulator α [Becher, Bell]:
  $\times\left(\frac{\nu}{n \cdot l_i}\right)^{\alpha}$ for each external parton.
- Same LC vector n for all functions. Breaks symmetry $(n \leftrightarrow \bar{n})$.
- Simple soft function $\mathcal{S} = 1$.
- α poles cancel in product:

Refactorization

$$\left[\mathcal{S}(x_T^2)\mathcal{B}_{i/j}(z_1, x_T^2)\bar{\mathcal{B}}_{\bar{i}/k}(z_2, x_T^2)\right]_{q^2} \overset{\alpha=0}{=} \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{i\bar{i}}^b(x_T^2)} B_{i/j}^b(z_1, x_T^2) B_{\bar{i}/k}^b(z_2, x_T^2)\,,$$

$\nwarrow$ $b_0 = 2e^{-\gamma_e}$

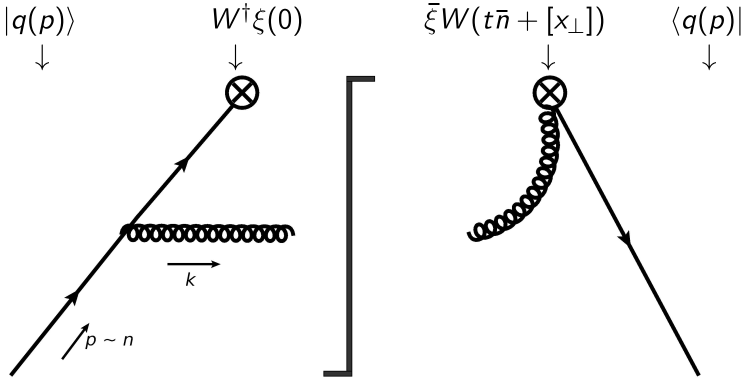
- On refactorized RHS no α and ν dependence left.
- Hard scale q^2 generated.
- 'Collinear anomaly' [Becher, Neubert].
  From RRG [Chiu, Jain, Neill, Rothstein].

Perturbative calculation

Perturbative calculation

- Explicit operator definitions.
  $\mathcal{B}_{q_f/j} = \ldots \langle j(p)| \, (\bar{\xi}_{f,n} W_n)_\alpha (t\bar{n} + x_\perp) \, |X_n\rangle \, \langle X_n| \, (W_n^\dagger \xi_{f,n})_\beta(0) \, |j(p)\rangle_*$
  $W_n(x) = P \exp\left[ig \int_{-\infty}^0 ds \, \bar{n}\cdot A_n(x + s\bar{n}) \right]$.
- Here discuss n-collinear case. Other case via $n \leftrightarrow \bar{n}$.
- Gauge independent due to W_n.
- Calculate perturbatively $\mathcal{B} = \sum_m (\frac{\alpha_s}{4\pi})^m \mathcal{B}^{(m)}$.
- Use dim. reg. $d = 4 - 2\epsilon$, and analytic reg. α.
- n-collinear fields only: Can use QCD Feynman rules.
- General gauge: $W \Rightarrow \quad \bar{n} \cdot (\ldots)$ denominators.
- Special gauge: LC gauge using $\bar{n}$
  $\Rightarrow W = 1$, but LC propagators.
- Do both: LC and Feynman gauge.

Example diagram at NLO



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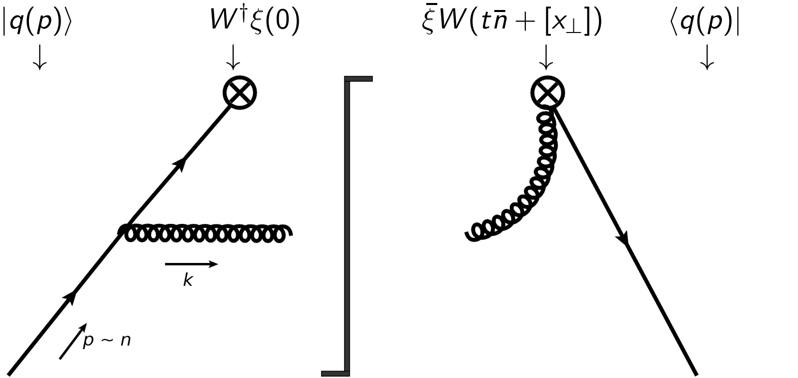
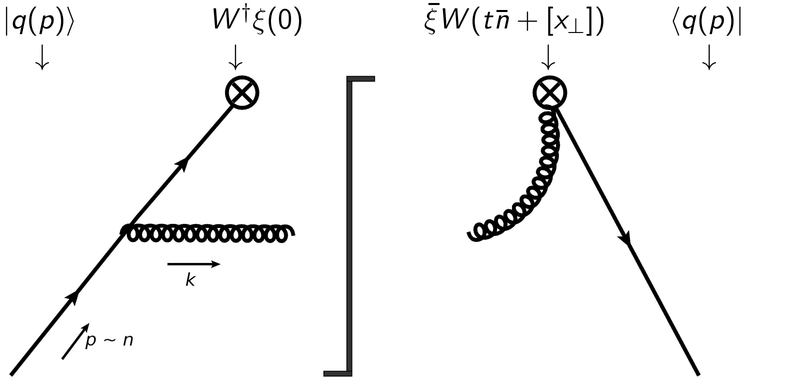


$$\frac{\alpha_s}{4\pi}\left(\mathcal{B}_{qq}^{(1)}, \phi_{qq}^{(1)}\right) = \int \frac{\mathrm{d}^d k}{(2\pi)^d}(2\pi)\delta^+(k^2)\delta(\bar{n}\cdot[k-(1-z)p])\left(e^{-ik_\perp\cdot x_\perp}, 1\right)\mathcal{M}$$

Example diagram at NLO



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Integrals for ϕ and purely virtual contributions vanish (scaleless).

Contributions at NNLO [Gehrmann, TL, Yang]

$$\mathcal{B}_{q/q}(z,x_T^2)=\frac{1}{2\pi}\int dt\ e^{-izt\bar{n}\cdot p}\sum_X \frac{\bar{\not{n}}_{\alpha\beta}}{2}\ \langle q|\,(\bar{\xi}_n W_n)_\alpha(t\bar{n}+x_\perp)\,|X\rangle\,\langle X|\,(W_n^\dagger \xi_n)_\beta(0)\,|q\rangle$$

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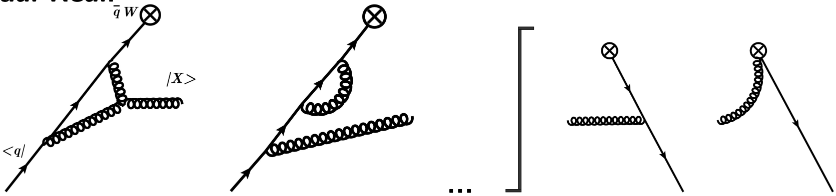
Virtual-Real:



- Can use QCD Feynman rules.
- General gauge: W in $\otimes$ $\Rightarrow$ $\bar{n}\cdot(\ldots)$ denominators.

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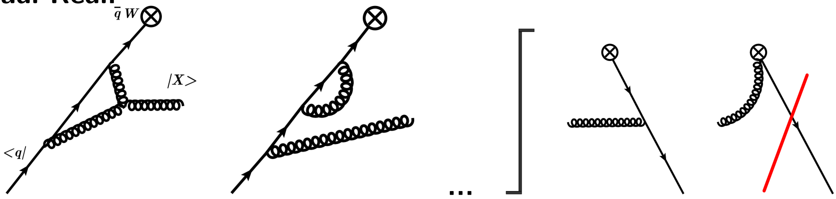

- Can use QCD Feynman rules.
- General gauge: W in $\otimes$ $\Rightarrow$ $\bar{n}\cdot(\ldots)$ denominators.
- Special gauge: LC gauge using $\bar{n}$ $\Rightarrow$ $W = 1$.
  $\bar{n}\cdot(\ldots)$ denominators from gluon propagators.

Contributions at NNLO [Gehrmann, TL, Yang]

$$\mathcal{B}_{q/q}(z,x_T^2)=\frac{1}{2\pi}\int dt\ e^{-izt\bar{n}\cdot p}\sum_X \frac{\bar{\not{n}}_{\alpha\beta}}{2}\ \langle q|\,(\bar{\xi}_n W_n)_\alpha(t\bar{n}+x_\perp)\,|X\rangle\,\langle X|\,(W_n^\dagger \xi_n)_\beta(0)\,|q\rangle$$

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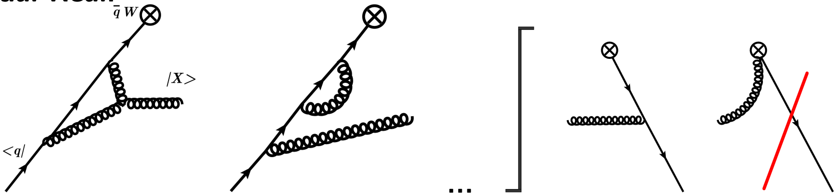
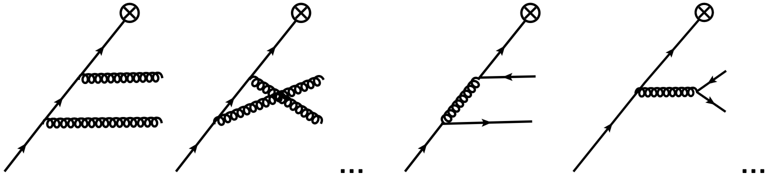


Real-Real:



Integration

- Phase space integral ($\hat{k}_z = \bar{n}\cdot[k-(1-z)p]$):

$$\int d\Pi_{n_r}^{\mathrm{TD}} = \left[\prod_i \int \frac{d^d l_i}{(2\pi)^{d-1}}\,\delta^+(l_i^2)\left(\frac{\nu}{n\cdot l_i}\right)^{\alpha}\right]\int d^d k\;\delta^d\Big(k-\sum_i l_i\Big)\,e^{-ik_\perp\cdot x_\perp}\,\delta\big(\hat{k}_z\big)\,.$$

- LC and full denominators build from p, k, $\{l_i\}$.

[Gehrmann, TL, Yang]

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- LC and full denominators build from p, k, $\{l_i\}$.
- Partly performing & reformulating $\Rightarrow$ typical class:

$$\sim \int_0^1 \mathrm{d}y \int_0^1 \mathrm{d}t\; y^{1-a_2-a_3-a_5}(1-y)^{1-a_1-a_2-a_6-2\epsilon}\left[1-(1-z)(1-y)\right]^{-a_7}$$
$$\times\,(1-t)^{1-2a_2-2\epsilon}\,{}_2F_1(1-a_2-\epsilon,1-a_2-2\epsilon;1-\epsilon;t)\,\Big[t^{-a_3-\epsilon}\{1-y(1-t)\}^b$$
$$+t^{-a_1-\epsilon}\{1-(1-y)(1-t)\}^b\Big]\;,\quad b=-2+a_1+a_2+a_3+2\epsilon\,.$$

exponent	a_1	a_3	a_5
integer part	n_1	n_3	n_5
regulator part	α	α	(-2α)

[Gehrmann, TL, Yang]

Results

- Results of all $I_{i/j}$ and $F_{i\bar{i}}$ up to NNLO ⇒ [Gehrmann,TL,Yang]
- With 1403.6451, full results in electronic form.

Results

- Results of all $I_{i/j}$ and $F_{i\bar{i}}$ up to NNLO $\Rightarrow$ [Gehrmann,TL,Yang]
- With 1403.6451, full results in electronic form.
- E.g. scale independent part of $I^{(2)}_{g/g}$ at $\mu_x = \frac{b_0}{x_T}$. Expressed via HPLs $H_{\{m\}} \equiv H(\{m\}, z)$ and $p_{gg}(z) = \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)$.

$$
\begin{aligned}
I^{(2)}_{g/g}(z, x_T^2, \mu_x) = C_a^2 \Big\{ & \delta(1-z)\left[\frac{25}{4}\zeta_4 - \frac{77}{9}\zeta_3 - \frac{67}{6}\zeta_2 + \frac{1214}{81}\right] + p_{gg}(z)\Big[-4H_{0,0,0} + 8H_{0,1,0} + 8H_{0,1,1} \\
& - 8H_{1,0,0} + 8H_{1,0,1} + 8H_{1,1,0} + 52\zeta_3 - \frac{808}{27}\Big] + p_{gg}(-z)\Big[-16H_{-1,-1,0} + 8H_{-1,0,0} + 16H_{0,-1,0} \\
& - 4H_{0,0,0} - 8H_{0,1,0} - 8H_{-1}\zeta_2 + 4\zeta_3\Big] + \Big[-16(1+z)H_{0,0,0} + \frac{8(1-z)(11-z+11z^2)}{3z}(H_{1,0} + \zeta_2) \\
& + \frac{2(25-11z+44z^2)}{3}H_{0,0} - \frac{2z}{3}H_1 - \frac{(701+149z+536z^2)}{9}H_0 + \frac{4(-196+174z-186z^2+211z^3)}{9z}\Big]\Big\} \\
& + C_a T_f N_f \Big\{ \delta(1-z)\left[\frac{28}{9}\zeta_3 + \frac{10}{3}\zeta_2 - \frac{328}{81}\right] + \frac{224}{27}p_{gg}(z) + \Big[\frac{8(1+z)}{3}H_{0,0} + \frac{4z}{3}H_1 + \frac{4(13+10z)}{9}H_0 \\
& - \frac{4(-65+54z-54z^2+83z^3)}{27z}\Big]\Big\} \\
& + C_f T_f N_f \Big\{ 8(1+z)H_{0,0,0} + 4(3+z)H_{0,0} + 24(1+z)H_0 - \frac{8(1-z)(1-23z+z^2)}{3z}\Big\} .
\end{aligned}
$$

Checks

Results confirmed with many checks:

- ✓ Feynman and LC gauge.
- ✓ α^{-n} cancel in $\mathcal{S}\mathcal{B}\bar{\mathcal{B}}$.
- ✓ Results independent of α & ν, with q^2 refactorized.
- ✓ ϵ^{-n} removed by ren.; ren. factors as implied by RGEs.
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- ✓ $I_{i/j}$ and $F_{i\bar{\imath}}$ obey RGEs.
- ✓ $F_{q\bar{q}}$, F_{gg} agree with [Becher, Neubert].
- ✓ Our results combined with app. hard functions confirm $\mathcal{H}^{DY(2)}_{q\bar{q}\leftarrow i\bar{j}}$ and $\mathcal{H}^{H(2)}_{gg\leftarrow i\bar{j}}$ in [Catani, Cieri, de Florian, Ferrera, Grazzini].

Renormalization

- UV poles removed by renormalization ($\overline{\text{MS}}$).

$$F^{\text{b}}_{i\bar{i}}(x_T^2) = F_{i\bar{i}}(x_T^2,\mu) + Z_i^F(\mu)\,,$$
$$B^{\text{b}}_{i/j}(z,x_T^2) = Z_i^B(x_T^2,\mu)\, B_{i/j}(z,x_T^2,\mu)\,,$$
$$\phi^{\text{b}}_{i/j}(z) = \sum_k Z^{\phi}_{i/k}(z,\mu)\otimes\phi_{k/j}(z,\mu).$$

- Dependence on μ described by RGEs

$$\frac{d}{d\log\mu}F_{i\bar{i}}(x_T^2,\mu) = \ 2\,\Gamma^i(\alpha_s)\,,$$
$$\frac{d}{d\log\mu}B_{i/j}(z,x_T^2,\mu) = \left[\Gamma^i(\alpha_s)\log\frac{x_T^2\mu^2}{b_0^2} - 2\gamma^i(\alpha_s)\right]B_{i/j}(z,x_T^2,\mu)\,,$$
$$\frac{d}{d\log\mu}\phi_{i/j}(z,\mu) = \ 2\sum_k P_{ik}(z,\mu)\otimes\phi_{k/j}(z,\mu)\,.$$

- Imply eqs. for $I_{i/k}$ in $B_{i/j}(z) = \sum_k I_{i/k}(z,x_T^2)\otimes\phi_{k/j}(z)\,.$
- With RGEs: a) Check poles and scale dependent terms.
  b) Resum logarithms.

Phenomenology

Apply CuTe

Rephrased factorization formula

- At $\Lambda^2_{\rm QCD} \ll q_T^2 \ll q^2$:

$$\frac{d^2\sigma}{dq_T^2\,dy} = \frac{\pi\alpha_{ew}^2}{N_c s}\sum_{i,j}\sum_q e_q^2 \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp} \times \left[\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu) + (q\leftrightarrow\bar{q})\right] \otimes \phi_{i/N_1}(z_1,\mu) \otimes \phi_{j/N_2}(z_2,\mu)\,,$$

with perturbative

$$\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu) = \left|C_V(-q^2,\mu)\right|^2 (x_T^2 q^2/b_0^2)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,x_T^2,\mu) I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)\,.$$

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- Determine each factor at appropriate scale μ.
- Solving **RGEs** $\Rightarrow$ **resummation** of $L = \log(x_T^2 q^2/b_0^2)$.

$$\widetilde{C}_{q\bar q\leftarrow ij} = \tilde c(\alpha_s)\exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \ldots\right]$$

- N^{3}LL requires determination of $g_1, \ldots, g_4$.

Towards N^{3}LL, required elements

Numbers refer to power n in expansion $X = \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n X^{(n)}$.

expression	needed to	known to	
Γ^i	4	3	for RGEs
γ_i	3	3	
$P_{i/j}(z)$	3	3	
β	4	4	
$C(q^2)$	2	2	at appr. μ
$F_{i\bar{i}}(x_T^2)$	3	2	
$I_{i/j}(z, x_T^2)$	2	2	

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$I_{i/j}(z, x_T^2)$	2	2	
$I'_{g/j}(z, x_T^2)$	1 (2)*	1 (+)	

*: $B'_{g/j} = \sum_k I'_{g/k} \otimes \phi_{k/j}$. I' starts at α_s^1.

$\Rightarrow$ $n = 1$ sufficient if C_{gg} does not mix I' & I. (E.g. for Higgs.)

CuTe

$$\frac{d^2\sigma}{dq_T\,dy} = \frac{2\pi\alpha_{ew}^2}{N_c q_T s}\sum_{i,j}\sum_q e_q^2 \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}$$
$$\times\left[\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu) + (q\leftrightarrow\bar{q})\right]\otimes\phi_{i/N_1}(z_1,\mu)\otimes\phi_{j/N_2}(z_2,\mu)\,,$$

with perturbative

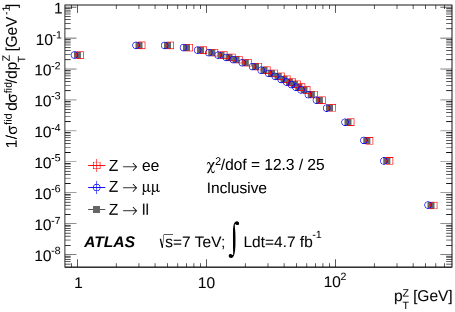
$$\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu) = |C_V(-q^2,\mu)|^2(x_T^2q^2/b_0^2)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,x_T^2,\mu) I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)\,.$$

Evaluate $\frac{d^2\sigma}{dq_T[dy]}$ $\Rightarrow$ **CuTe** (http://cute.hepforge.org) [Becher, Neubert, Wilhelm]:

- Numerically evaluate $\otimes\phi$, $x_\perp$, y **integrals**.
- Evaluate **fixed order**. **Match** both.
- Many **options**: Process, PDFs (LHAPDF), pert. order, scale choices, matching schemes, precision, **cuts**,...

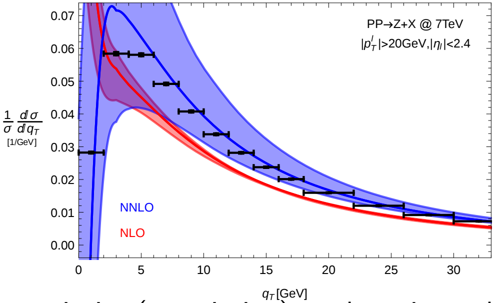
Data and cuts

- Compare to ATLAS data [arXiv:1406.3660]:
- Z/γ^* q_T spectrum, $\sqrt{s} = 7\text{TeV}$, $\int \mathcal{L}dt = 4.7\text{fb}^{-1}$;
- Decay to $ll = ee, \mu\mu$ with $66 < M_{ll}/GeV < 116$.
- **Cuts** for $d\sigma_{\text{fiducial}}/dq_T$:
  $p_{T,l} > 20\mathrm{GeV}$, $|\eta_l| < 2.4$, excluding $1.37 < |\eta_l| < 1.52$.



Fixed order

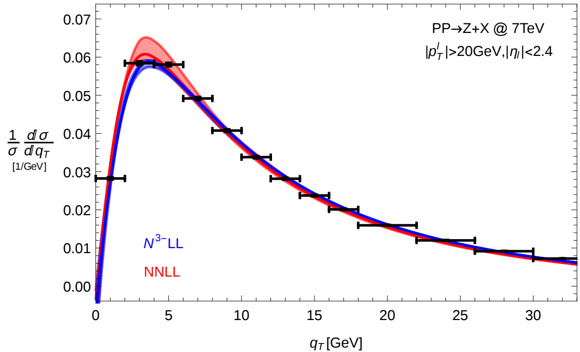
- Cuts: Reweight by CuTe: cutted P.S./full P.S.
- $\mu = q_T + q_*$, $q_* = M_Z / \exp\left(1/2\Gamma_0^q a_s\right)$.
- Z-production: NNLO results from [Gonsalves,Pawlowski,Wai]



At **peak**:

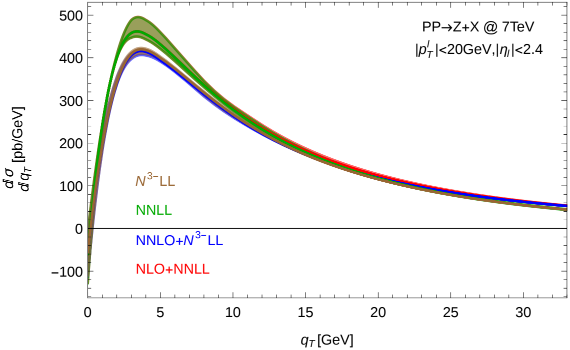
- Large uncertainties (μ variation), underestimated at NLO.
- Divergent at very small q_T.
- Requires resummation.

Pure Resummed



- **Good** description in **peak** region. Already at NNLL.
- Small errors bands (μ variation only).
- Without normalization $N^{3-}LL$ okay. NNLL to high peak.
- For long **tail**, need to **match** to fixed order.

Matched



- Here **un**normalized.
- In **peak** region nearly **identical** to resummed part.
- For long tail: improvement.
- Matching scheme: 'Constant'.

Matching schemes

$$\frac{d\sigma^{\mathrm{matched}}}{dq_T} = \frac{d\sigma^{\mathrm{res}}}{dq_T} + \left.\frac{d\sigma^{\mathrm{MC}}}{dq_T}\right|_{\mathrm{scheme}}$$

with **scheme dependent** MC ($H(\mu,\mu_h) = |C_V(-q^2,\mu,\mu_h)|^2$)

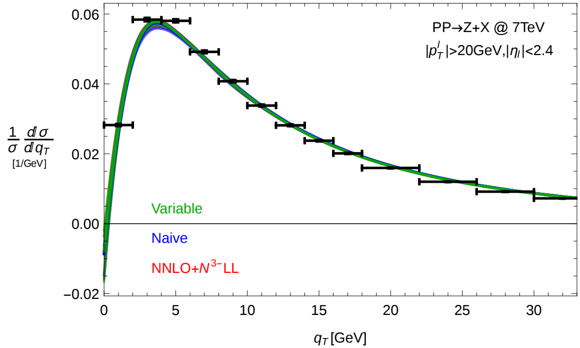
$$\left.\frac{d\sigma^{\mathrm{MC}}}{dq_T}\right|_{\mathrm{Naive}} = \left(\frac{d\sigma^{\mathrm{NNLO}}}{dq_T} - \frac{d\sigma^{\mathrm{N^3LL}}}{dq_T}\right)\Bigg|_{\text{expanded to NNLO}}.$$

with $H(\mu,\mu)$. Taking evolution effects into account:

$$\left.\frac{d\sigma^{\mathrm{MC}}}{dq_T}\right|_{\mathrm{Variable}} = \left(H(\mu,q^2)\cdot H(\mu,\mu)^{-1}\cdot \left.\frac{d\sigma^{\mathrm{MC}}}{dq_T}\right|_{\mathrm{Naive}}\right)\Bigg|_{\text{expanded to NNLO}},$$

$$\left.\frac{d\sigma^{\mathrm{MC}}}{dq_T}\right|_{\mathrm{Constant}} = \left(H(\mu_c,q^2)\cdot H(\mu,\mu)^{-1}\cdot \left.\frac{d\sigma^{\mathrm{MC}}}{dq_T}\right|_{\mathrm{Naive}}\right)\Bigg|_{\text{expanded to NNLO}}.$$

Matching schemes



- Very good agreement.
- Negligible scheme dependence (of considered ones).

Non-perturbative effects

- TPDFs must vanish rapidly at $x_T > r_{\text{proton}}$. Ansatz:

$$B_{i/N}(z, x_T^2, \mu) = f_{\text{hadr}}(x_T \Lambda_{\text{NP}})\, B_{i/N}^{\text{pert}}(z, x_T^2, \mu)\ ,$$

- with

$$f_{\text{hadr}}^{\text{gauss}}(x_T \Lambda_{\text{NP}}) = \exp\left[-\Lambda_{\text{NP}}^2\, x_T^2\right],$$

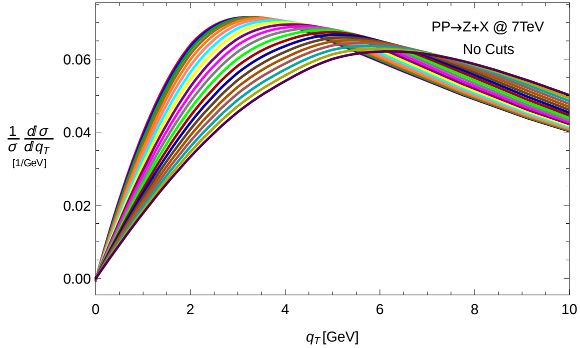
$$f_{\text{hadr}}^{\text{dipole}}(x_T \Lambda_{\text{NP}}) = \frac{1}{1 + \Lambda_{\text{NP}}^2\, x_T^2},$$

$$f_{\text{hadr}}^{\text{enhanced}}(x_T \Lambda_{\text{NP}}) = \exp\left[-\Lambda_{\text{NP}}^2\, x_T^2 \log(x_T^2 M^2 / b_0^2)\right].$$

- Last see [Becher, Bell].
- $\Lambda_{\text{NP}} = 0\text{GeV}$: no correction.

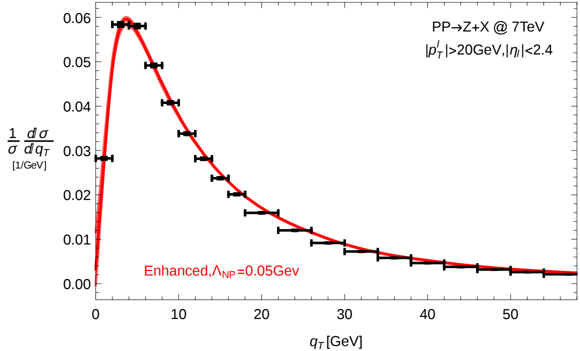
Non-perturbative effects

- Results for Gauss and Dipole basically equivalent.
- Gauss: $\Lambda_{\mathrm{NP}} = 0\,\mathrm{GeV} \longrightarrow 2\,\mathrm{Gev}$: **Shifts right** & damps.



- Enhanced for 'enhanced'. Form different, though.

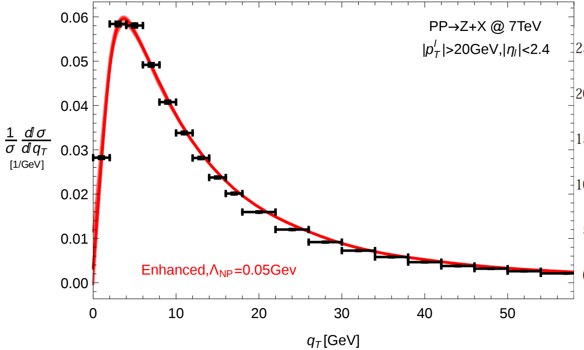
NNLO+N$^{3^-}$LL



[Becher, TL, Neubert, Wilhelm]

- Excellent description of spectrum: 0 to 800 GeV.
- **Tiny** experimental and theory **errors**.
- Correspondingly for Tevatron data.
- Impressive confirmation of SM and soft & col. behavior of QCD.

NNLO+N^{3-}LL



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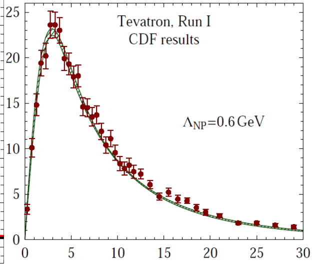

NNLO+N^{3-}LL vs NLO+N^{2}LL

- Excellent description of spectrum: 0 to 800 GeV.
- **Tiny** experimental and theory **errors**.
- Correspondingly for Tevatron data.
- Impressive confirmation of SM and soft & col. behavior of QCD.

Alternatives and extensions

- See also: FEWZ, DYNNLO, Res-Bos, ...
- Our methods can be applied to **all** other processes with color neutral final states: V, H, VV', HH', Z', ...
- Currently available in CuTe: γ, Z, W, H.
- For others to include F.O. part.
- VV' at fixed order see e.g. Mainz/Zürich-group [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Manteuffel, Pozzorini, Rathlev, Tancredi, Torre, Weihs]

Conclusions

- q_T **resummation** requires **T**PDFs.
- Calculated matching kernels perturbatively to **NNLO** using
  - gauge invariant operator definitions,
  - analytic regulator (LC singularities).
- **Generic** ingredients to obtain **N^{2}LO+N^{3}LL** precision of $d\sigma/dq_T$ for large class of processes at hadron colliders.
- **CuTe** determines these for γ, Z, W, H.
- Obtained very accurate description of q_T spectrum.

Appendix

Overview: Perturbative determination

- Aim: NNLO+N^{3}LL for $d\sigma/dq_T$.
- Calculate $\mathcal{B}_{i/j}$ and $\phi_{i/j}$ to NNLO regulated by α & ϵ.
- $\mathcal{B}_{i/j}\bar{\mathcal{B}}_{\bar{\imath}/k} = (\ldots q^2)^{-F_{i\bar{\imath}}} B_{i/j} B_{\bar{\imath}/k}$. $\alpha \to 0$.
- Obtain $I_{i/j}$ from $B_{i/k} = \sum_j I_{i/j} \otimes \phi_{j/k}$.
- Renormalize I and F. $\epsilon \to 0$.
- With $I^{(2)}$ obtained new generic ingredients for N^{3}LL resummation.

Reconstruct scale logarithms

For

$$F_{i\bar{i}}(L_\perp, \alpha_s) = \sum_{n\geq 1}\sum_{l=0}^{n} F_{i\bar{i}}^{(n,l)} \left(\frac{\alpha_s}{4\pi}\right)^n L_\perp^l \,,$$

$$I_{i/j}(z, L_\perp, \alpha_s) = \sum_{n\geq 0}\sum_{l=0}^{2n} I_{i/j}^{(n,l)}(z) \, \left(\frac{\alpha_s}{4\pi}\right)^n L_\perp^l \,,$$

solving RGEs imply recursion relations

$$F_{i\bar{i}}^{(n+1,l+1)} = \frac{1}{l+1}\left[\delta_{l,0}\Gamma_n^i + \sum_{s=0}^{n} s\,\beta_{n-s} F_{i\bar{i}}^{(s,l)}\right],$$

$$I_{i/j}^{(n+1,l+1)}(z) = \frac{1}{l+1}\sum_{s=0}^{n}\left[\frac{1}{2}\Gamma_{n-s}^i I_{i/j}^{(s,l-1)}(z) + \left(s\,\beta_{n-s} - \gamma_{n-s}^i\right) I_{i/j}^{(s,l)}(z) - \sum_k I_{i/k}^{(s,l)}(z) \otimes P_{k/j}^{(n-s)}(z)\right].$$

Relation to framework by Collins, Soper, Sterman

$$\widetilde{C}_{q\bar{q}\leftarrow ij}(z_1,z_2,x_T^2,q^2,\mu)$$

$$= \left|C_V(-q^2,\mu)\right|^2 \left(\frac{x_T^2 q^2}{b_0^2}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} I_{q\leftarrow i}(z_1,x_T^2,\mu)\, I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)\,,$$

compare this to [Collins, Soper, Sterman]:

$$= \exp\left\{ -\int_{\mu_b}^{q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\log\frac{q^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu}))\right]\right\}$$
$$\times\, C_{qi}(z_1,\alpha_s(\mu_b))\, C_{\bar{q}j}(z_2,\alpha_s(\mu_b))\,,$$

x_T dependence via $\mu_b = b_0 x_T{}^{-1}$.

Relations:

- $C_{qi}(z,\alpha_s(\mu_b)) = \left|C_V(-\mu_b^2,\mu_b)\right| I_{q/i}(z,x_T^2,\mu_b)\,,$
- A & B related to F and anomalous dimensions.

Dictionary CSS vs BN

Using $b_0 = 2e^{-\gamma_e}$, $\mu_b = b_0 x_T^{-1}$ and $\bar{x}_T = b_0 \bar{\mu}^{-1}$:

$$C_{qi}(z, \alpha_{s(\mu_b)}) = \left| C_V(-\mu_b^2, \mu_b) \right| I_{q/i}(z, \bar{x}_T^2, \mu_b) \,,$$

$$A(\alpha_{s(\bar{\mu})}) = \Gamma^q(\alpha_s) - \bar{\mu}^2 \frac{dF_{q\bar{q}}(\bar{x}_T, \bar{\mu})}{d\bar{\mu}^2} \,,$$

$$B(\alpha_{s(\bar{\mu})}) = 2\gamma_q(\alpha_s) + F_{q\bar{q}}(\bar{x}_T, \bar{\mu}) - \bar{\mu}^2 \frac{d \log |C_V(-\bar{\mu}^2, \bar{\mu})|^2}{d\bar{\mu}^2} \,.$$

$\Rightarrow$ Apply results in preferred resummation framework.

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$\Rightarrow$ Apply results in preferred resummation framework.

Moreover, can reconstruct $\mathcal{H}^{DY}_{q\bar{q}\leftarrow i\bar{j}}$, $\mathcal{H}^{H}_{gg\leftarrow i\bar{j}}, \ldots$ in [Catani, Cieri, de Florian, Ferrera, Grazzini].
$\Rightarrow$ Apply for q_T subtraction. Recently e.g. in [Höche, Li, Prestel]