Meson transition form factors and their applications

Pablo Sánchez Puertas
sanchezp@kph.uni-mainz.de

Johannes Gutenberg-Universität Mainz
In collaboration with P. Masjuan

1. Dezember 2014, Kernphysik Seminar

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

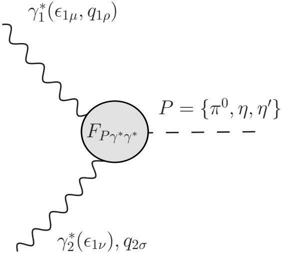
Outline

1. Pseudoscalar transition form factors
2. Our proposal: Padé Approximants
3. Parameters extraction
4. Selected applications
    - η Dalitz decay
    - $(g-2)_\mu$: Hadronic light-by-light
    - $P \to \bar{\ell}\ell$ decays
    - $\eta - \eta'$ mixing
5. Summary & outlook

Section 1

Pseudoscalar transition form factors

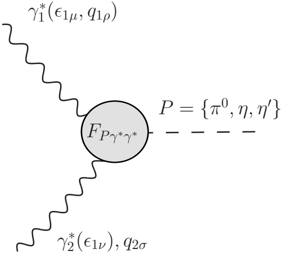
The Transition Form Factor



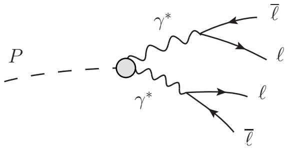
- Governs $P\gamma^*\gamma^*$ interactions
- Vertex constrained by Bose symmetry and Lorentz and gauge invariance
- The TFF depends on γ virtualities, q_1^2, q_2^2
- Bose symmetry $\rightarrow F_{P\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{P\gamma^*\gamma^*}(q_2^2, q_1^2)$

$$i\mathcal{M} = ie^2\varepsilon^{\mu\nu\rho\sigma}\epsilon_{1\mu}\epsilon_{2\nu}q_{1\rho}q_{2\sigma}F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$$

The Transition Form Factor



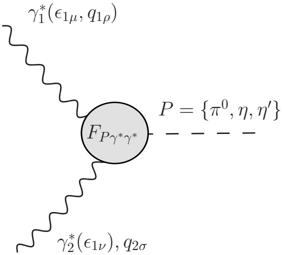
Relevant in many process

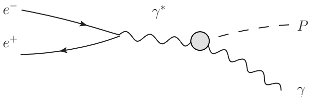


$$i\mathcal{M} = ie^2 \varepsilon^{\mu\nu\rho\sigma} \epsilon_{1\mu} \epsilon_{2\nu} q_{1\rho} q_{2\sigma} F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$$

The Transition Form Factor

Relevant in many process

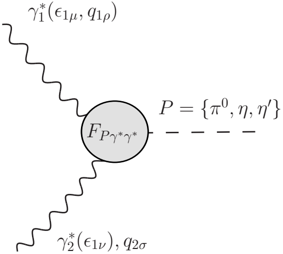


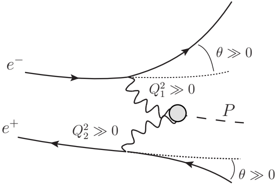


$$i\mathcal{M} = ie^2\varepsilon^{\mu\nu\rho\sigma}\epsilon_{1\mu}\epsilon_{2\nu}q_{1\rho}q_{2\sigma}F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$$

The Transition Form Factor

Relevant in many process
For us, particularly relevant the space-like

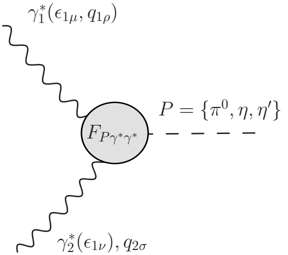




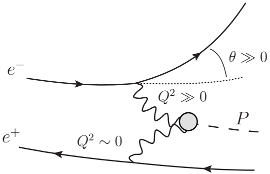
However, double-virtual not available

$$i\mathcal{M} = ie^2 \varepsilon^{\mu\nu\rho\sigma} \epsilon_{1\mu} \epsilon_{2\nu} q_{1\rho} q_{2\sigma} F_{P\gamma^*\gamma}(-Q^2, 0)$$

The Transition Form Factor



Use single-virtual measurements instead



Focus on $F_{P\gamma^*\gamma}(Q^2) \equiv F_{P\gamma^*\gamma}(-q^2, 0)$

$$i\mathcal{M} = ie^2 \varepsilon^{\mu\nu\rho\sigma} \epsilon_{1\mu} \epsilon_{2\nu} q_{1\rho} q_{2\sigma} F_{P\gamma^*\gamma}(-Q^2, 0)$$

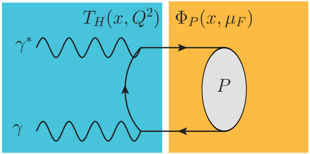
The Transition Form Factor

HIGH ENERGIES (pQCD)

- Space-like (SL) $F_{P\gamma^*\gamma}(Q^2,0)$

$$F_{P\gamma^*\gamma}(Q^2) = \int dx \; T_H(x,Q^2,\mu)\Phi_P(x,\mu_F);$$

- $T_H(x,Q^2)$ perturbative in $\alpha_s(Q^2)$
- $\phi_P(x,\mu_F)$ non-pert. $\rightarrow$ **MODELLED!**



$F_{\pi\gamma^*\gamma}(\infty) = 2F_\pi Q^{-2}$
$F_{\pi\gamma^*\gamma^*}(\infty,\infty) = (2/3)F_\pi Q^{-2}$

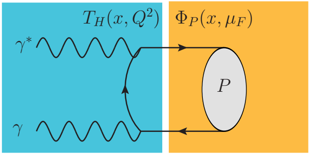
The Transition Form Factor

HIGH ENERGIES (pQCD)

- Space-like (SL) $F_{P\gamma^*\gamma}(Q^2,0)$

$$F_{P\gamma^*\gamma}(Q^2) = \int dx\ T_H(x,Q^2,\mu)\Phi_P(x,\mu_F);$$

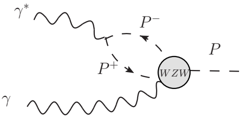
- $T_H(x,Q^2)$ perturbative in $\alpha_s(Q^2)$
- $\phi_P(x,\mu_F)$ non-pert. $\rightarrow$ **MODELLED!**



$F_{\pi\gamma^*\gamma}(\infty) = 2F_\pi Q^{-2}$
$F_{\pi\gamma^*\gamma^*}(\infty,\infty) = (2/3)F_\pi Q^{-2}$

LOW ENERGIES (χPT)

- ABJ anomaly prediction $F_{P\gamma\gamma}(0,0)$
- Extensions for $Q^2 \simeq 0$ poor (vectors)
- **MODEL** the vectors (i.e.: $R\chi PT$)



$F_{\pi\gamma\gamma}(0,0) = (4\pi^2 F_\pi)^{-1}$

How to do physics involving $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ in between χPT and $pQCD$ realms?

- Different models (assumptions) $\rightarrow$ different values.
- Introduces uncontrolled model-dependency
- Big drawback when going for precision physics!

Look for an alternative approach which represents a good (improvable) approximation of reality while avoiding any assumptions $\rightarrow$ data-based

Section 2

Our proposal: Padé Approximants

Padé Approximants: Introduction to the method

$$F_{P\gamma^*\gamma}(Q^2) = F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + c_P Q^4 + ...)$$

Padé Approximants: Introduction to the method

$$F_{P\gamma^*\gamma}(Q^2) = F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + c_P Q^4 + ...)$$

Then, Taylor expand!

Model independent from convergence
Systematic from known error $\mathcal{O}(Q^{2n})$

Padé Approximants: Introduction to the method

$$Q^2 F_{P\gamma^*\gamma}(Q^2) = Q^2 F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + c_P Q^4 + \ldots)$$

Then, Taylor expand!

Model independent from convergence
Systematic from known error $\mathcal{O}(Q^{2n})$

Padé Approximants: Introduction to the method

$$Q^2 F_{P\gamma^*\gamma}(Q^2) = Q^2 F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + c_P Q^4 + \ldots)$$

Then, Taylor expand!

Model independent from convergence
Systematic from known error $\mathcal{O}(Q^{2n})$

Singularities cuts $\longrightarrow$ Poor radius of convergence

Padé Approximants: Introduction to the method

$$Q^2 F_{P\gamma^*\gamma}(Q^2) = Q^2 F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + c_P Q^4 + \ldots)$$

Then, Taylor expand!

Model independent from convergence
Systematic from known error $\mathcal{O}(Q^{2n})$

Singularities cuts $\longrightarrow$ Poor radius of convergence

Padé Approximants have better convergence prop.

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \ldots + \mathcal{O}(Q^2)^{N+M+1}$$

$$P^1_1 = \frac{F_{P\gamma^*\gamma}(0) Q^2}{1 - b_P Q^2} = Q^2 F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + \mathcal{O}(Q^4)) \checkmark$$

Padé Approximants: Introduction to the method

$$Q^2 F_{P\gamma^*\gamma}(Q^2) = Q^2 F_{P\gamma^*\gamma}(0)(1 + b_P Q^2 + c_P Q^4 + \ldots)$$

Then, Taylor expand!

Model independent from convergence
Systematic from known error $\mathcal{O}(Q^{2n})$

Singularities cuts $\longrightarrow$ Poor radius of convergence

Padé Approximants have better convergence prop.

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \ldots + \mathcal{O}(Q^2)^{N+M+1}$$

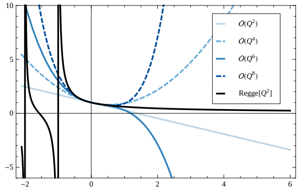
Model independent from convergence theorems
Systematic from error estimation ability

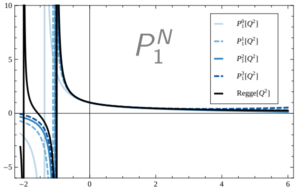
Padé Approximants: Convergence properties

Convergence known for meromorphic (large-N_c) and Stieltjes (DR)

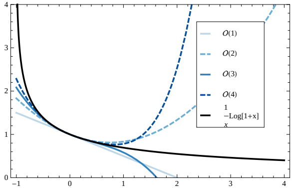
for the last $\lim_{N\to\infty} P^N_{N+1}(x) \leq f(x) \leq P^N_N(x)$

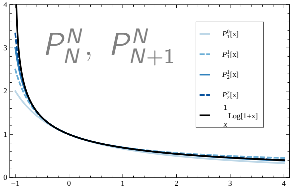
$$— \quad F_{P\gamma^*\gamma}(Q^2) = \frac{F_{P\gamma\gamma}(0)}{Q^2} \frac{a}{\psi^{(1)}\left(\frac{M^2}{a}\right)} \left(\psi^{(0)}\left(\frac{M^2+Q^2}{a}\right) - \psi^{(0)}\left(\frac{M^2}{a}\right)\right) \quad —$$





TAYLOR —— $f(x) = \frac{1}{x} Log(1+x)$ —— PADÉ

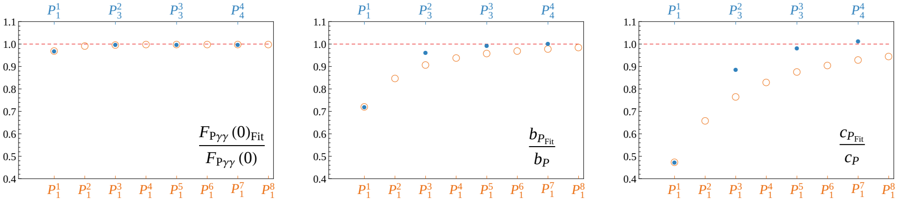




Padé Approximants: Convergence Properties

Obtaining the coefficients? PAs as fitting functions

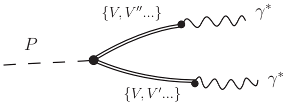
- Generate **space-like** pseudo-data from our toy model
- First pole-dominance → P_1^N sequence: "easy fit"
- Asymptotics and analytic structure → P_N^N: "hard fit"
- Obtain $Q^2 F_{P\gamma^*\gamma}(Q^2) = Q^2 F_{P\gamma^*\gamma}(0)\left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \ldots\right)$



What about the most general $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Based on resonance ideas, factorization is assumed

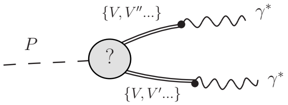
$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma^*\gamma}(Q_1^2, 0) \times F_{P\gamma\gamma^*}(0, Q_2^2)$$



What about the most general $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Based on resonance ideas, factorization is assumed

$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma^*\gamma}(Q_1^2, 0) \times F_{P\gamma\gamma^*}(0, Q_2^2)$$



What about the most general $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Based on resonance ideas, factorization is assumed

$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma^*\gamma}(Q_1^2, 0) \times F_{P\gamma\gamma^*}(0, Q_2^2)$$

This is known to fail at high energies as it falls as Q^{-4} instead of Q^{-2}

What about the most general $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Based on resonance ideas, factorization is assumed

$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma^*\gamma}(Q_1^2, 0) \times F_{P\gamma\gamma^*}(0, Q_2^2)$$

This is known to fail at high energies as it falls as Q^{-4} instead of Q^{-2}

Can we generalize Padé approximants to the bivariate case?

What about the most general $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$?

Based on resonance ideas, factorization is assumed

$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma^*\gamma}(Q_1^2, 0) \times F_{P\gamma\gamma^*}(0, Q_2^2)$$

This is known to fail at high energies as it falls as Q^{-4} instead of Q^{-2}

Can we generalize Padé approximants to the bivariate case?

YES! (Chisholm '73)

— The most simple approximant —

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 - \frac{a_1}{a_0}(Q_1^2 + Q_2^2) + \frac{2a_1^2 - a_0 a_{1,1}}{a_0^2} Q_1^2 Q_2^2} = a_0 + a_1(Q_1^2 + Q_2^2) + a_{1,1} Q_1^2 Q_2^2 + \dots$$

What have we learned so far ?

Systematic easy approximation through PAs in space-like region

Based on well-established mathematical theory

Everything amounts to obtain the derivatives (single and double-virtual)

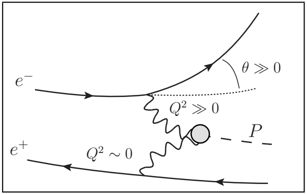
Data driven: forget about (model) prejudices, let data to judge on them

Section 3

Parameters Extraction: data-driven

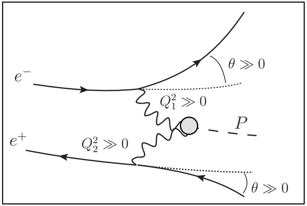
The Transition Form Factor

Single-virtual $F_{P\gamma^*\gamma}(Q^2)$



- Single-tag method: CELLO, CLEO, Belle, BaBar, BES-III*
- Detected $e^\pm \to$ space-like γ^*
- $e^\pm$ along beam axis $\to$ quasireal γ

Double-virtual $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$



- Experimentally challenging
- Not measured yet
- More comments later

Results for the π^0

Take all the available data for $Q^2 F_{\pi\gamma^*\gamma}(Q^2)$

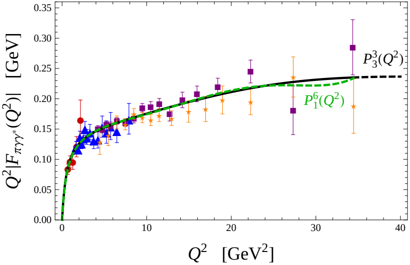
CELLO('91), CLEO('98), BaBar('09), Belle('12)

Our Results

- P_1^N reaches N= 6
- $P_N^N + OPE$ reaches N= 3
- We predict

$b_\pi = 0.0324(12)(19)(m_\pi)^{-2}$

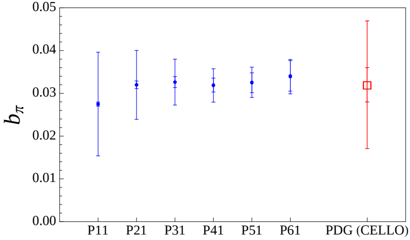
$c_\pi = 1.06(9)(25) \times 10^{-3}(m_\pi)^{-4}$

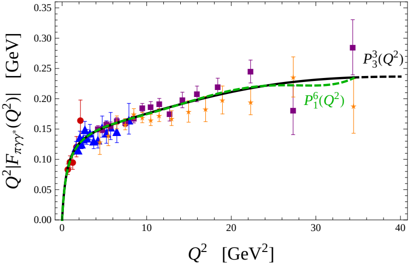


P. Masjuan, Phys.Rev., D86, 2012

Results for the π^0

Take all the available data for $Q^2 F_{\pi\gamma^*\gamma}(Q^2)$ | CELLO('91), CLEO('98), BABAR('09), Belle('12)





P. Masjuan, Phys.Rev., D86, 2012

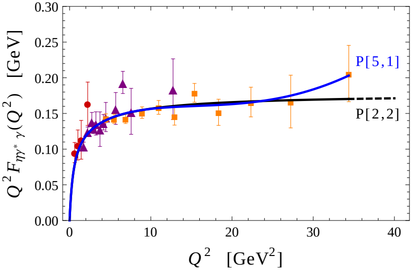
Results for the η

Take all the available data for $Q^2 F_{\eta\gamma^*\gamma}(Q^2)$ | CELLO('91), CLEO('98), BaBar('11)

Our Results

- P_1^N reaches N= 5
- P_N^N reaches N= 2
- We predict

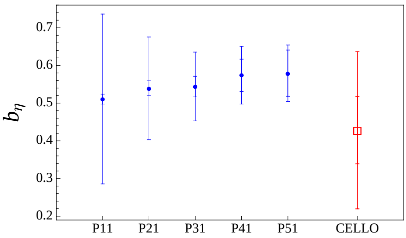
$b_\eta = 0.60(6)(3)(m_\eta)^{-2}$
$c_\eta = 0.37(10)(7)(m_\eta)^{-4}$
Asymptotics $= 0.160(24)$ GeV

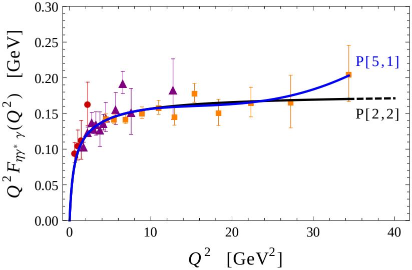


R. Escribano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014

Results for the η

Take all the available data for $Q^2 F_{\eta\gamma^*\gamma}(Q^2)$ | CELLO('91), CLEO('98), BABAR('11)





R. Escribano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014

Results for the η'

Take all the available data for $Q^2 F_{\eta'\gamma^*\gamma}(Q^2)$

CELLO('91), CLEO('98), L3('98)
BaBar('11)

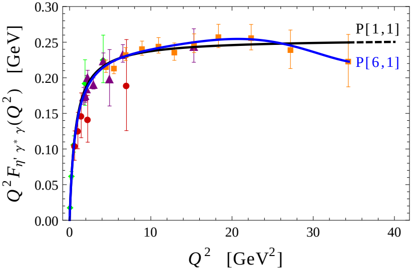
Our Results

- P_1^N reaches N= 6
- P_N^N reaches N= 1
- We predict

$b_{\eta'} = 1.30(15)(7)(m_{\eta'})^{-2}$

$c_{\eta'} = 1.72(47)(34)(m_{\eta'})^{-4}$

Asymptotics = 0.255(4) GeV

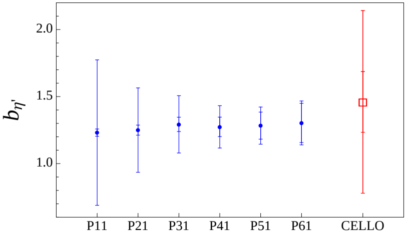


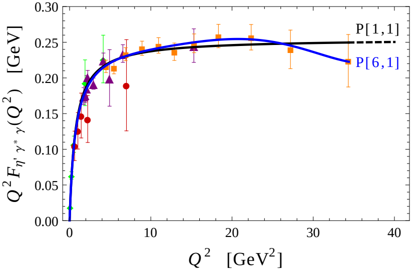
R. Escribano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014

Results for the η'

Take all the available data for $Q^2 F_{\eta'\gamma^*\gamma}(Q^2)$

CELLO('91), CLEO('98), L3('98)
BABAR('11)



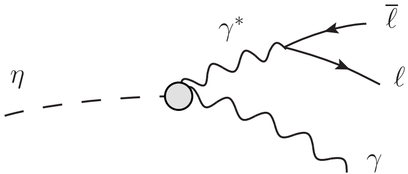


R. Escribano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014

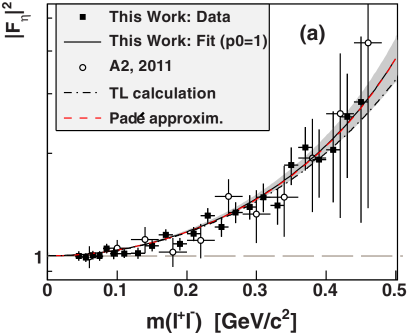
- TFF low energy parameters determined to a high precision (π^0, η, η')
- Inputs ready for TFF reconstruction
- We are ready to do calculations/predictions

Section 4

Predictiveness: $\eta \to \gamma\bar{\ell}\ell$



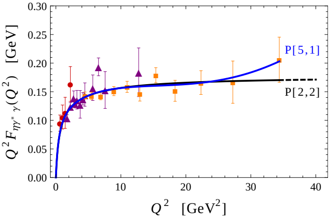
Dalitz decays: $\eta \to \gamma\bar{\ell}\ell$



Compare to A2 Coll. results in Mainz [Phys.Rev. C89 (2014) 044608]
The results are excellent → reasonable to use them in our fit

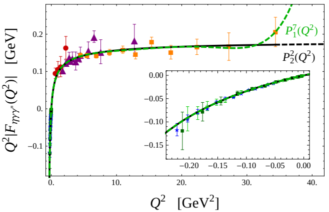
R. Escribano, P. Masjuan, P. Sanchez, In preparation

Dalitz decays: $\eta \to \gamma\bar{\ell}\ell$



Previous Results

$b_\eta = 0.60(6)(3)(m_\eta)^{-2}$
$c_\eta = 0.37(10)(7)(m_\eta)^{-4}$
$d_\eta = -$
Asymptotics $= 0.160(24)$ GeV

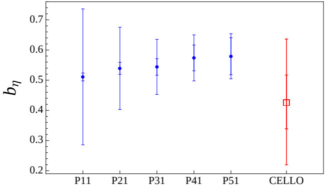


Updated Results

$b_\eta = 0.576(11)(1)(m_\eta)^{-2}$
$c_\eta = 0.339(15)(2)(m_\eta)^{-4}$
$d_\eta = 0.200(14)(10)(m_\eta)^{-6}$
Asymptotics $= 0.177(15)$ GeV

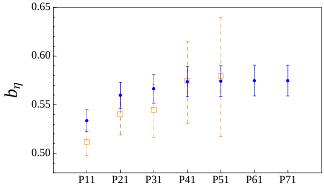
R. Escribano, P. Masjuan, P. Sanchez, In preparation

Dalitz decays: $\eta \to \gamma\bar{\ell}\ell$



Previous Results

$$b_\eta = 0.60(6)(3)(m_\eta)^{-2}$$
$$c_\eta = 0.37(10)(7)(m_\eta)^{-4}$$
$$d_\eta = -$$
$$\text{Asymptotics} = 0.164(21)\ \text{GeV}$$

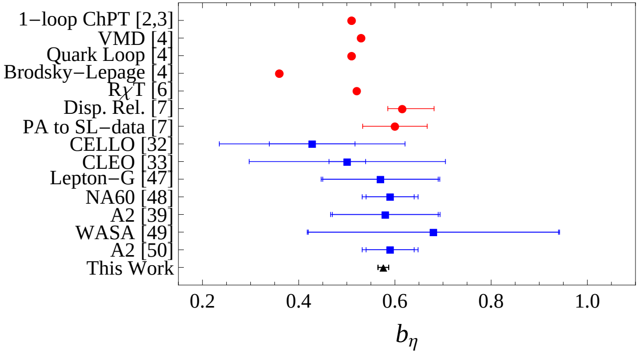


Updated Results

$$b_\eta = 0.576(11)(3)(m_\eta)^{-2}$$
$$c_\eta = 0.339(15)(3)(m_\eta)^{-4}$$
$$d_\eta = 0.200(14)(10)(m_\eta)^{-6}$$
$$\text{Asymptotics} = 0.177(15)\ \text{GeV}$$

R. Escribano, P. Masjuan, P. Sanchez, In preparation

Dalitz decays: $\eta \to \gamma\bar{\ell}\ell$



For Theory, Exp (fit to VMD) and our latest result.

R. Escribano, P. Masjuan, P. Sanchez, In preparation

Dalitz decays: $\eta \to \gamma\bar{\ell}\ell$

Method is very predictive

Convergence is excellent

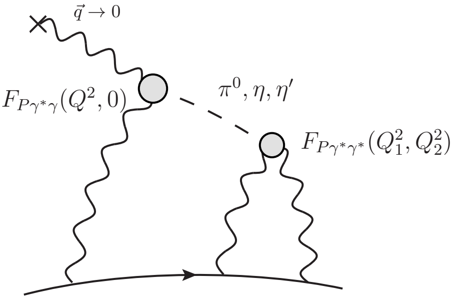
Systematically improves with data

Space-like observables even better: high confidence

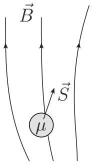
R. Escribano, P. Masjuan, P. Sanchez, In preparation

Section 5

$(g-2)_\mu$: Hadronic light-by-light



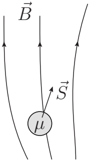
$(g-2)_\mu$: very brief introduction



What is g ?

- Magnetic dipole $\vec{\mu}$ in a $\vec{B}$ field: $H = -\vec{\mu} \cdot \vec{B}$
- Where $\vec{\mu} = g \frac{Q}{2m} \vec{S}$
- In classical physics $g = 1$ expected

$(g-2)_\mu$: **very brief introduction**

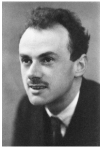


What is g ?

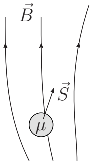
- Magnetic dipole $\vec{\mu}$ in a $\vec{B}$ field: $H = -\vec{\mu} \cdot \vec{B}$
- Where $\vec{\mu} = g \frac{Q}{2m} \vec{S}$
- In classical physics $g = 1$ expected

What about QM?

- In QM, fermions with $S = \frac{1}{2}$ follow Dirac Eqn.
- It follows, for fundamental fermions $g = 2$



$(g-2)_\mu$: very brief introduction

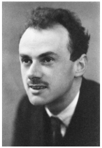


What is g ?

- Magnetic dipole $\vec{\mu}$ in a $\vec{B}$ field: $H = -\vec{\mu}\cdot\vec{B}$
- Where $\vec{\mu} = g\frac{Q}{2m}\vec{S}$
- In classical physics $g = 1$ expected

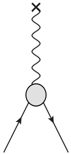
What about QM?

- In QM, fermions with $S = \frac{1}{2}$ follow Dirac Eqn.
- It follows, for fundamental fermions $g = 2$

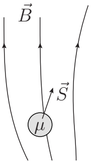


What about QED?

- Radiative corrections change Dirac's value.
- Schwinger, QED @ NLO: $a_f = \dfrac{(g-2)_f}{2} = \dfrac{\alpha}{2\pi}$



$(g-2)_\mu$: very brief introduction



What is g ?

- Magnetic dipole $\vec{\mu}$ in a $\vec{B}$ field: $H = -\vec{\mu}\cdot\vec{B}$
- Where $\vec{\mu} = g\frac{Q}{2m}\vec{S}$
- In classical physics $g = 1$ expected

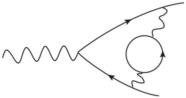
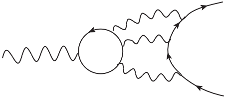
What about the SM?

- $SU(3)\otimes SU(2)\otimes U(1)$: all gauge sectors entering at some order

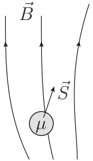
$$a_\mu^{SM} = 11659181.3(5.8)\times 10^{-10} \quad [QED, QCD, EW]$$
$$a_\mu^{Exp.} = 11659208.0(6.3)\times 10^{-10}$$

$$a_\mu^{Exp.} - a_\mu^{SM} = 26.7(8.6)\times 10^{-10} \sim 3\sigma$$

- Uncertainty coming fully from the hadronic contributions

$(g-2)_\mu$: very brief introduction



What is g ?

- Magnetic dipole $\vec{\mu}$ in a $\vec{B}$ field: $H = -\vec{\mu}\cdot\vec{B}$
- Where $\vec{\mu} = g\frac{Q}{2m}\vec{S}$
- In classical physics $g = 1$ expected

What about the SM?

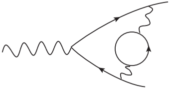
- $SU(3)\otimes SU(2)\otimes U(1)$: all gauge sectors entering at some order

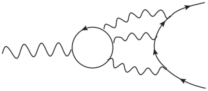
$$a_\mu^{SM} = 11659181.3(5.8)\times 10^{-10} \quad [QED, QCD, EW]$$

$$a_\mu^{Exp.Future} = 11659208.0(1.6)\times 10^{-10}$$

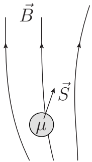
$$a_\mu^{Exp.} - a_\mu^{SM} = 26.7(8.6)\times 10^{-10} \sim 3\sigma$$

- Uncertainty coming fully from the hadronic contributions





$(g-2)_\mu$: very brief introduction



What is g ?

- Magnetic dipole $\vec{\mu}$ in a $\vec{B}$ field: $H = -\vec{\mu}\cdot\vec{B}$
- Where $\vec{\mu} = g\frac{Q}{2m}\vec{S}$
- In classical physics $g = 1$ expected

What about the SM?

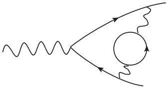
- $SU(3)\otimes SU(2)\otimes U(1)$: all gauge sectors entering at some order

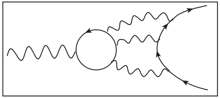
$$a_\mu^{SM} = 11659181.3(5.8)\times 10^{-10} \quad [QED, QCD, EW]$$

$$a_\mu^{Exp.Future} = 11659208.0(1.6)\times 10^{-10}$$

$$a_\mu^{Exp.} - a_\mu^{SM} = 26.7(8.6)\times 10^{-10} \sim 3\sigma$$

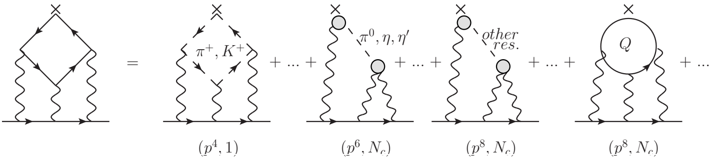
- Uncertainty coming fully from the hadronic contributions





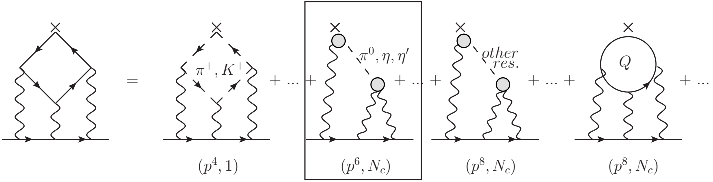
$(g-2)_\mu$: hadronic light-by-light

Eduardo de Rafael ('94): large-N_c + χPT counting



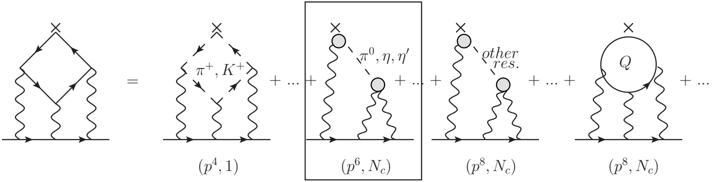
$(g-2)_\mu$: hadronic light-by-light

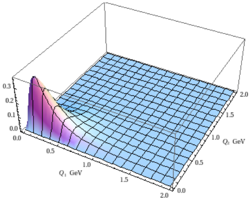
Eduardo de Rafael ('94): large-N_c + χPT counting



$(g-2)_\mu$: hadronic light-by-light

Eduardo de Rafael ('94): large-N_c + χPT counting





Knecht & Nyffeler ('02): π^0, η, η'-exchange

- Expressed as an integral involving $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2)$
- SL low energy regime → our PAs are good
- Multiscale, also high energies relevant → 2-point PAs
- Two virtualities → bivariate PAs

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P^0_1(Q_1^2,Q_2^2)=\frac{F_{P\gamma\gamma}(0)}{1+\frac{b_P}{m_P^2}(Q_1^2+Q_2^2)+\left(2b_P^2-\frac{a_{1,1}}{F_{P\gamma\gamma}(0)}\right)\frac{Q_1^2Q_2^2}{m_P^4}}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0)=2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0)=b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0)=0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2,Q_2^2)=\frac{F_{P\gamma\gamma}(0)}{1+\frac{b_P}{m_P^2}(Q_1^2+Q_2^2)+\left(2b_P^2-\frac{a_{1,1}}{F_{P\gamma\gamma}(0)}\right)\frac{Q_1^2Q_2^2}{m_P^4}}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0)=2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0)=b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0)=0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0)}{1 + \frac{b_P}{m_P^2}(Q_1^2 + Q_2^2)}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0) = 2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = 0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0)}{1 + \frac{b_P}{m_P^2}(Q_1^2 + Q_2^2) + (2b_P^2 - \frac{a_{1,1}}{F_{P\gamma\gamma}(0)})\frac{Q_1^2 Q_2^2}{m_P^4}}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0) = 2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = 0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2,Q_2^2)=\frac{F_{P\gamma\gamma}(0)}{1+\frac{b_P}{m_P^2}(Q_1^2+Q_2^2)+b_P^2\frac{Q_1^2Q_2^2}{m_P^4}}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0)=2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0)=b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0)=0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0)}{1 + \frac{b_P}{m_P^2} m_P^2 (Q_1^2 + Q_2^2) + (2b_P^2 - \frac{a_{1,1}}{a_0}) \frac{Q_1^2 Q_2^2}{m_P^4}}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0) = 2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = 0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0)}{1 + \frac{b_P}{m_P^2}(Q_1^2 + Q_2^2) + 2b_P^2 \frac{Q_1^2 Q_2^2}{m_P^4}}$$

HE: $a_{1,1}/F_{P\gamma\gamma}(0) = 2b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = b_P^2$; LE: $a_{1,1}/F_{P\gamma\gamma}(0) = 0$

$(g-2)_\mu$: hadronic light-by-light

However, no experimental data is available

— Use the simplest approximant —

$$P_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0)}{1 + \frac{b_P}{m_P^2} m_P^2 (Q_1^2 + Q_2^2) + (2b_P^2 - \frac{a_{1,1}}{F_{P\gamma\gamma}(0)}) \frac{Q_1^2 Q_2^2}{m_P^4}}$$

Take $a_{1,1}/F_{P\gamma\gamma}(0) \in \{0 \div 2b_P^2\}$ as a generous estimate (LE *vs.* HE)

$(g-2)_\mu$: hadronic light-by-light

Our Results from Bivariate Padé Approximants

Units of 10^{-10}	π^0	η	η'	Total
$a_{1,1} = 2b_P^2$	6.64(33)	1.69(6)	1.61(21)	$9.94(40)_{stat}(50)_{sys}$
$a_{1,1} = b_P^2$	5.53(27)	1.30(5)	1.21(12)	$8.04(30)_{stat}(40)_{sys}$
$a_{1,1} = 0$	5.10(23)	1.16(7)	1.07(15)	$7.33(28)_{stat}(37)_{sys}$

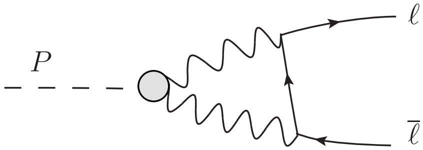
$$a_\mu^{HLbL;P} = (9.94(40)(50) \div 7.33(28)(37)) \times 10^{-10}$$

Big uncertainty from double-virtual term often non-considererd
High-energies *vs.* Low-energies

To be compared with pseudoscalar-pole contributions in the literature
BPP: 8.5(1.3); **HKS**: 8.6(0.6); **KN**: 8.3(1.2)

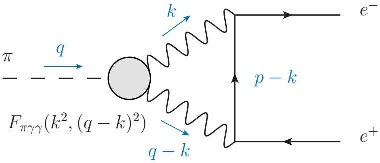
Section 6

$P \to \bar{\ell}\ell$ decays



$P \to \bar{\ell}\ell$ decays: Introduction

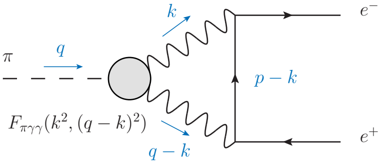
At LO in α_{EM}, this process occurs via 2γ intermediate state.



P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Introduction

At LO in α_{EM}, this process occurs via 2γ intermediate state.

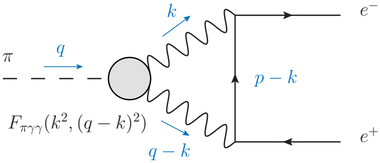


$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2)|\mathcal{A}(m_\pi^2)|^2,$$

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Introduction

At LO in α_{EM}, this process occurs via 2γ intermediate state.



$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2)|\mathcal{A}(m_\pi^2)|^2,$$

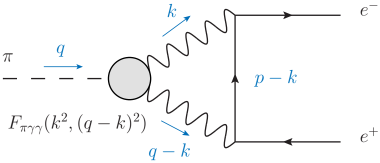
$$\mathcal{A}(q^2) = \frac{2i}{\pi^2 q^2}\int d^4k \; \frac{q^2k^2 - (k\cdot q)^2}{k^2(k-q)^2((p-k)^2 - m_e^2)} F_{\pi\gamma^*\gamma^*}(k^2,(q-k)^2).$$

With normalized $F_{\pi\gamma\gamma}(0,0) = 1$. It diverges if $F_{\pi\gamma\gamma}(k_1^2,k_2^2) = const$.

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Introduction

At LO in α_{EM}, this process occurs via 2γ intermediate state.



$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2)|\mathcal{A}(m_\pi^2)|^2,$$

There has ben a lot of activity since the latest experimental result

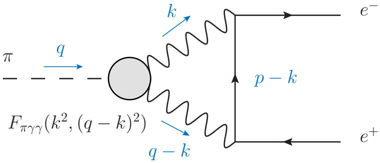
$$BR^{KTeV}(\pi^0 \to e^+e^-) = 7.48(38) \times 10^{-8}$$
$$BR^{Th.}(\pi \to e^+e^-) = 6.23(09) \times 10^{-8}$$

Which represents a **3σ deviation**

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Introduction

At LO in α_{EM}, this process occurs via 2γ intermediate state.



$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2)|\mathcal{A}(m_\pi^2)|^2,$$

Taking into account last radiative corrections results [hep-ph/1405.6927]

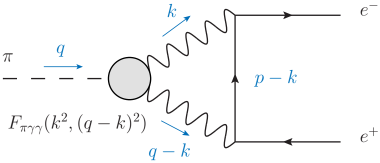
$$BR^{KTeV}(\pi^0 \to e^+e^-) = 6.87(36) \times 10^{-8}$$
$$BR^{Th.}(\pi^0 \to e^+e^-) = 6.23(09) \times 10^{-8}$$

Which represents a **1.7σ deviation**

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Introduction

At LO in α_{EM}, this process occurs via 2γ intermediate state.



$$\frac{BR(\pi^0 \to e^+e^-)}{BR(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e(m_\pi^2)|\mathcal{A}(m_\pi^2)|^2,$$

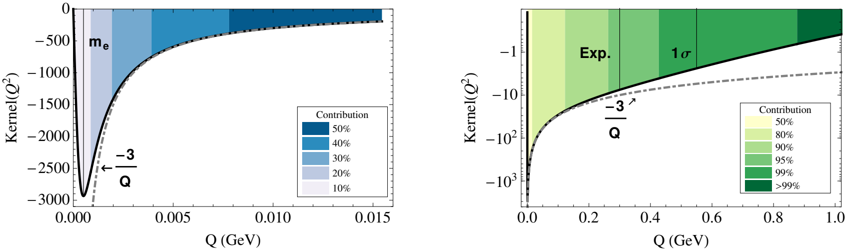
Very interesting since still no model can reproduce such value and $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ is an input to HLbL $\to$ source of uncertainty

Could we help?

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Introduction

$$\mathcal{A}(m_\pi^2) \approx -17.52i + 30.7 + \int_0^\infty dQ \frac{3}{Q}\left(\frac{m_e^2}{m_e^2+Q^2} - F_{\pi\gamma^*\gamma^*}(Q^2,Q^2)\right)$$



- Accurate prediction → accurate low energy description → PA
- Double virtual → bivariate PA within considered $a_{1,1}$ range

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Results

From our (conservative) estimate $a_{1,1} \in \{2b_P^2, 0\}$, we obtain

$$BR(\pi^0 \to e^+e^-) = (6.20 \div 6.41)(5) \times 10^{-8}$$

To be compared to current value: (Dorokhov et.al. PRD75 '07)
$BR(\pi^0 \to e^+e^-) = 6.23(9) \times 10^{-8}$

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Results

From our (conservative) estimate $a_{1,1} \in \{2b_P^2, 0\}$, we obtain

$$BR(\pi^0 \to e^+e^-) = (6.20 \div 6.41)(5) \times 10^{-8}$$

To be compared to current value: (Dorokhov et.al. PRD75 '07)
$BR(\pi^0 \to e^+e^-) = 6.23(9) \times 10^{-8}$

Further improvement if double-virtual data

We obtain for $a_{1,1} = b_P^2$ with 50% error

$$BR(\pi^0 \to e^+e^-) = 6.36(5)_{b_\pi}(4)_{a_{11}}(6)_{sys} \times 10^{-8} \to 6.36(8) \times 10^{-8}$$

Measurement in $(0-1)$GeV range with interval 0.2GeV and 30% error

BES-III measurements

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Results

WHAT IF WE ENFORCE $\pi^0 \to e^+e^-$ TO EXPERIMENT?

Huge impact in *HLBL* (new measurement?)

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Results

WHAT IF WE ENFORCE $\pi^0 \to e^+e^-$ TO EXPERIMENT?

Huge impact in *HLBL* (new measurement?)

	Model	Published Model		Modified Model	
		$\pi^0 \to e^+e^-$ $(\times 10^{-8})$	HLbL $(\times 10^{-10})$	$\pi^0 \to e^+e^-$ $(\times 10^{-8})$	HLbL $(\times 10^{-10})$
JN '09	LMD+V	6.33	6.29	6.47	5.22
Dorokhov '09	VMD	6.34	5.64	6.87	2.44
Our proposal	PA	$(6.20 \div 6.41)$	$(5.10 \div 6.64)$	6.87	2.85

P. Masjuan, P. Sanchez, In preparation

$P \to \bar{\ell}\ell$ decays: Results

What if we enforce $\pi^0 \to e^+e^-$ to Experiment?

Huge impact in *HLBL* (new measurement?)

	Model	Published Model		Modified Model	
		$\pi^0 \to e^+e^-$ ($\times 10^{-8}$)	HLbL ($\times 10^{-10}$)	$\pi^0 \to e^+e^-$ ($\times 10^{-8}$)	HLbL ($\times 10^{-10}$)
JN '09	LMD+V	6.33	6.29	6.47	5.22
Dorokhov '09	VMD	6.34	5.64	6.87	2.44
Our proposal	PA	$(6.20 \div 6.41)$	$(5.10 \div 6.64)$	6.87	2.85

We aimed for theory error around 1.6×10^{-10}, this is 4×10^{-10} ... !

P. Masjuan, P. Sanchez, In preparation

Section 7

$\eta - \eta'$ mixing

$\eta - \eta'$ mixing: Introduction

In an ideal $U(3)$ (large-N_c) chiral world ...

In an ideal $U(3)$ (large-N_c) chiral world ..

$$\eta = \eta_8 \equiv \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right), \quad \eta' = \eta_0 \equiv \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

Physical world: $m_s \neq m_{u,d}$, $\eta_8, \eta_0 \rightarrow$ **mix** into the physical η, η'

R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

$\eta - \eta'$ mixing: Introduction

In an ideal $U(3)$ (large-N_c) chiral world ...

In an ideal $U(3)$ (large-N_c) chiral world ..

$$\eta = \eta_8 \equiv \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right), \quad \eta' = \eta_0 \equiv \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

Physical world: $m_s \neq m_{u,d}$, $\eta_8, \eta_0 \rightarrow$ **mix** into the physical η, η'

—**Using the flavor basis**— $\eta_q = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right), \quad \eta_s = s\bar{s}$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

$\eta - \eta'$ mixing: Introduction

In an ideal $U(3)$ (large-N_c) chiral world ...

In an ideal $U(3)$ (large-N_c) chiral world ..

$$\eta = \eta_8 \equiv \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right), \quad \eta' = \eta_0 \equiv \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

Physical world: $m_s \neq m_{u,d}$, $\eta_8, \eta_0 \rightarrow$ **mix** into the physical η, η'

—Using the flavor basis— $\eta_q = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right), \quad \eta_s = s\bar{s}$

$$\begin{pmatrix} F_\eta^q & F_\eta^s \\ F_{\eta'}^q & F_{\eta'}^s \end{pmatrix} = \begin{pmatrix} F_q \cos(\phi) & -F_s \sin(\phi) \\ F_q \sin(\phi) & F_s \cos(\phi) \end{pmatrix}$$

(only holds (to a good approximation) in flavor basis)

R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

$\eta - \eta'$ mixing: Results

From the mixing, the $pQCD(Q^2 \to \infty)$ and $\chi PT(Q^2 = 0)$ limits read

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\cos\phi - \frac{\hat{c}_s}{F_s}\sin\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q \cos\phi - \hat{c}_s F_s \sin\phi)$$

$$F_{\eta'\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\sin\phi + \frac{\hat{c}_s}{F_s}\cos\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q \sin\phi + \hat{c}_s F_s \cos\phi)$$

We know them all! But only 3 are independent, we take

$$\{F_{\eta\gamma\gamma}(0) = 516(18)keV, F_{\eta'\gamma\gamma}(0) = 4.34(14)keV, \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 0.177(15)GeV\}$$

$$F_q = 1.07(2)F_\pi, \quad F_s = 1.29(16)F_\pi \quad \phi = 38.3(1.7)^\circ$$

R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

$\eta - \eta'$ mixing: Results

From the mixing, the $pQCD(Q^2 \to \infty)$ and $\chi PT(Q^2 = 0)$ limits read

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\cos\phi - \frac{\hat{c}_s}{F_s}\sin\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q \cos\phi - \hat{c}_s F_s \sin\phi)$$

$$F_{\eta'\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\sin\phi + \frac{\hat{c}_s}{F_s}\cos\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q \sin\phi + \hat{c}_s F_s \cos\phi)$$

We know them all! But only 3 are independent, we take

$$\{F_{\eta\gamma\gamma}(0) = 516(18)keV, F_{\eta'\gamma\gamma}(0) = 4.34(14)keV, \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 0.177(15)GeV\}$$

$$F_q = 1.07(2)F_\pi, \quad F_s = 1.29(16)F_\pi \quad \phi = 38.3(1.7)^\circ$$

Then, $\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 0.261(10)\text{GeV}$, we obtained 0.255(4)GeV

R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

$\eta - \eta'$ mixing: Results

From the mixing, the $pQCD(Q^2 \to \infty)$ and $\chi PT(Q^2 = 0)$ limits read

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\cos\phi - \frac{\hat{c}_s}{F_s}\sin\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q \cos\phi - \hat{c}_s F_s \sin\phi)$$

$$F_{\eta'\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}\sin\phi + \frac{\hat{c}_s}{F_s}\cos\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q \sin\phi + \hat{c}_s F_s \cos\phi)$$

We know them all! But only 3 are independent, we take

$$\{F_{\eta\gamma\gamma}(0) = 516(18)keV, F_{\eta'\gamma\gamma}(0) = 4.34(14)keV, \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 0.177(15)GeV\}$$

$$F_q = 1.07(2)F_\pi, \quad F_s = 1.29(16)F_\pi \quad \phi = 38.3(1.7)^\circ$$

EF('05) Update: $F_q = 1.07(1)F_\pi, \quad F_s = 1.63(3)F_\pi \quad \phi = 39.6(0.4)^\circ$
FK('99): $F_q = 1.07(2)F_\pi, \quad F_s = 1.34(6)F_\pi \quad \phi = 39.3(1.0)^\circ$

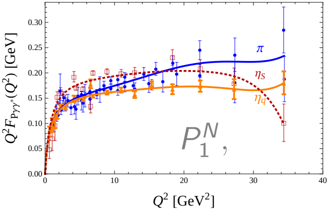
R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

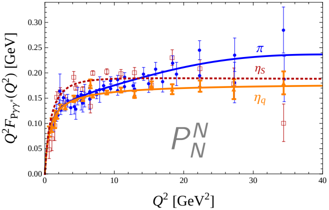
$\eta - \eta'$ mixing: light-quarks TFF

From the mixing we can obtain $F_{\eta_q \gamma^* \gamma}(Q^2), F_{\eta_s \gamma^* \gamma}(Q^2)$ *TFFs*

$$F_{\eta_q \gamma^* \gamma}(Q^2) = F_{\eta \gamma^* \gamma}(Q^2) \cos\phi + F_{\eta' \gamma^* \gamma}(Q^2) \sin\phi$$
$$F_{\eta_s \gamma^* \gamma}(Q^2) = -F_{\eta \gamma^* \gamma}(Q^2) \sin\phi + F_{\eta' \gamma^* \gamma}(Q^2) \cos\phi$$

We expect, up to a charge factor $\hat{c}_q = 5/3$, $F_{\eta_q \gamma^* \gamma}(Q^2) = F_{\pi \gamma^* \gamma}(Q^2)$





Excellent agreement until 6 GeV
Asymptotic for light-quarks reached slower than *s*-quarks (VMD)

R.Escirbano, P. Masjuan, P. Sanchez, Phys.Rev., D89, 2014, In preparation

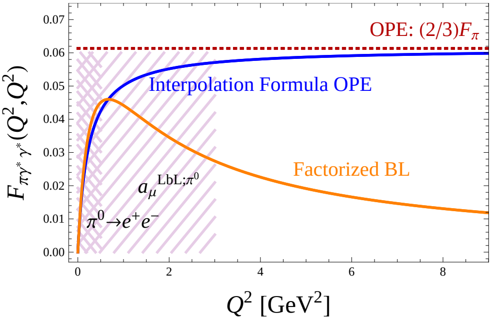
Section 8

Shopping-list

Shopping-list: experiment-oriented summary

Shopping-list: experiment-oriented summary

π^0 DOUBLE VIRTUAL TRANSITION FORM-FACTOR $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

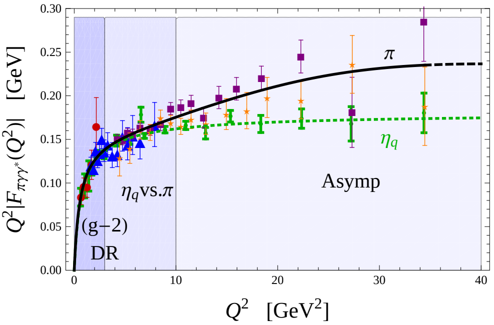


WHY IS IMPORTANT?

- Theory predictions but no exp. results
- Relevant for $\pi^0 \to e^+e^-$, $a_\mu^{LbL;\pi^0}$ (30%)
- What about $\eta, \eta'(\eta_q, \eta_s)$?

Shopping-list: experiment-oriented summary

π^0 SINGLE VIRTUAL TRANSITION FORM-FACTOR $F_{\pi^0\gamma^*\gamma}(Q^2)$

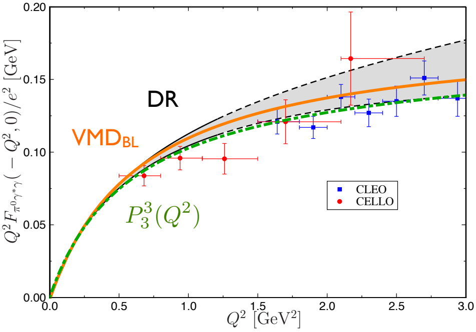


Why is Important?

- No precise data @ low energies
- Mid-energies: π^0 *vs.* η_q (1%)
- $(g-2)^{LbL;\pi^0}$ (2%) Low-Mid
- High-Q^2: Asymptotics (6%)
- Test approachess: i.e.: DR [arXiv:1410.4691] (2%)

Shopping-list: experiment-oriented summary

π^0 SINGLE VIRTUAL TRANSITION FORM-FACTOR $F_{\pi^0\gamma^*\gamma}(Q^2)$

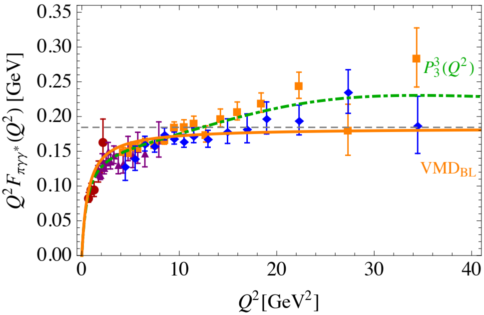


WHY IS IMPORTANT?

- No precise data @ low energies
- Mid-energies: π^0 *vs.* η_q (1%)
- $(g-2)^{LbL;\pi^0}$ (2%) Low-Mid
- High-Q^2: Asymptotics (6%)
- Test approachess: i.e.: DR [arXiv:1410.4691] (2%)

Shopping-list: experiment-oriented summary

π^0 SINGLE VIRTUAL TRANSITION FORM-FACTOR $F_{\pi^0\gamma^*\gamma}(Q^2)$

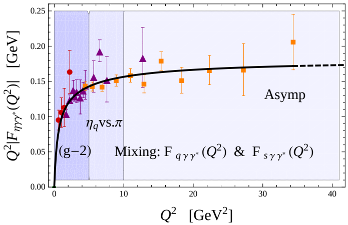


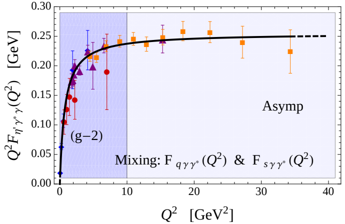
WHY IS IMPORTANT?

- No precise data @ low energies
- Mid-energies: π^0 *vs.* η_q (1%)
- $(g-2)^{LbL;\pi^0}$ (2%) Low-Mid
- High-Q^2: Asymptotics (6%)
- Test approachess: i.e.: DR [arXiv:1410.4691] (2%)

Shopping-list: experiment-oriented summary

$\eta(\eta')$ SINGLE VIRTUAL TRANSITION FORM-FACTORS $F_{\eta(\eta')\gamma^*\gamma}(Q^2)$





WHY ARE IMPORTANT?

- η precise data @ low energies
- Mid-energies: π^0 *vs.* η_q (1%)
- $(g-2)^{LbL;\pi^0}$ (2%) Low-Mid
- High-Q^2: Asymptotics (4%)
- Mixing; $F_{q\gamma^*\gamma}, F_{s\gamma^*\gamma}$
- Test of QCD running

Shopping-list: experiment-oriented summary

η' Dalitz decay

- Check our prediction at low TL energies
- Sensible to the resonance region (ρ,ω); relevant for its $\eta' \to \bar{\ell}\ell$ decays

$\pi^0 \to e^+e^-$

- Persistent discrepancy; great impact in $(g-2)_\mu^{LbL;\pi^0}$ (10%)

$\eta \to \mu^+\mu^-(e^+e^-)$

- Minor discrepancy $BR \sim (4.51 \div 4.70) \times 10^{-6}$ *vs.* $5.8(8) \times 10^{-6}$ Exp.
- (e^+e^-) Never been measured $BR \sim (5.32 \div 5.45) \times 10^{-9}$ *vs.* $< 5.6 \times 10^{-6}$

$\eta' \to \mu^+\mu^-$

- Never measured, no bounds; estimated $BR \sim \mathcal{O}(10^{-7})$

Double Dalitz Decays

- Double virtuality (investigation with S. Gonzalez-Solis and R. Escribano) i.e. $\pi^0 \to 4e$ at (1%)

Conclusions & Outlook

- TFFs lack a genuine QCD description
- Padé Approximants as a model-independent approximation
- Is data driven: better data, better description; easy to apply
- Many physics involved in this work (shopping-list)
- But not restricted mesons, would be applied to nucleon FFs
- Not time: continuum @ charmonium energies productions
- Improve PAs η' description (go TL) for $\bar{\ell}\ell$ decays
- Deeper study of mixing: large-N_c χPT and OZI-violating pars

Thanks for your attention

Section 9

Backup

Backup

- $\pi^0 \to e^+e^-$ RC and NP
- Syst error for $(g-2)$ or $\pi^0 \to e^+e^-$
- Obtaining $V\gamma P$ FFs as Chisholm Residues
- Mixing: degeneracy Eqn. & pQCD & OZI-violation
- $c\overline{c}$ continuum production ?
- $g_{VP\gamma}$ couplings
- QCD matching?
- DPuble Dalitz? Sergi?

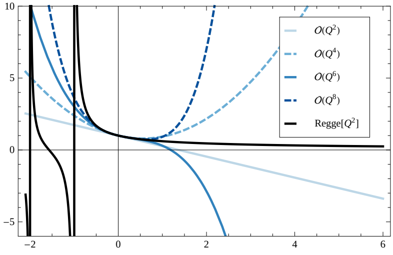
Padé Approximants: Convergence properties

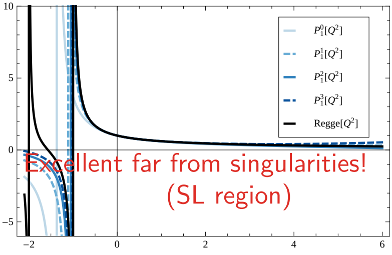
I. Meromorphic functions

- Given a meromorphic function $f(x)$ with M poles, $P^N_M(x)$ converges on the complex-plane $\mathbb{C}$ as $N \to \infty$ (Montessus th.)
- In general $P^N_L(x)$ converges up to the $(L+1)$-*th* pole

Large-N_c Regge model

$$F_{P\gamma^*\gamma}(Q^2) = \frac{F_{P\gamma\gamma}(0)}{Q^2} \frac{a}{\psi^{(1)}\left(\frac{M^2}{a}\right)} \left(\psi^{(0)}\left(\frac{M^2+Q^2}{a}\right) - \psi^{(0)}\left(\frac{M^2}{a}\right)\right)$$



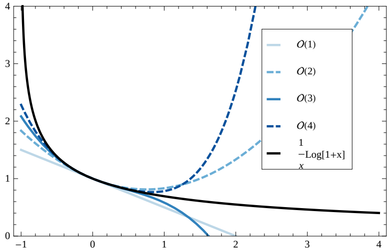


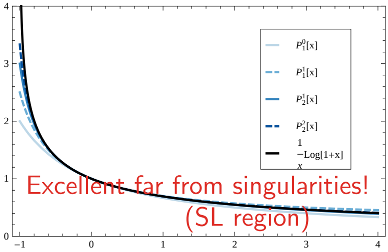
Padé Approximants: Convergence properties

II. Stieljes functions

- Stieljes: $f(x)=\int_0^\infty dt\frac{\rho(t)}{1+xt};\rho(t)\geq 0 \quad \sim \quad f(x)=\frac{1}{\pi}\int_{s_0}^\infty dt\frac{Im(f(t))}{t+x}$
- Given a Stieljes function $f(x)$, $\lim_{N\to\infty} P^N_{N+1}(x)\leq f(x)\leq P^N_N(x)$ on the complex-plane $\mathbb{C}$ (except for the cut) as $N\to\infty$

Log-function $f(x)=\frac{1}{x}Log(1+x)=\frac{1}{\pi}\int_{-1}^{-\infty}dt\frac{Im(f(t))}{(x-t)}=\ldots=\int_0^1\frac{dt}{1+xt}$



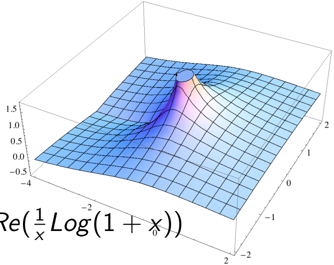


Padé Approximants: Convergence properties

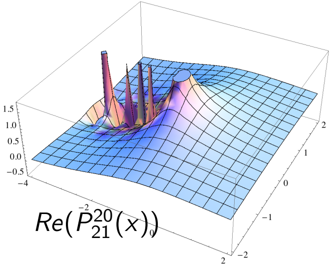
II. Stieljes functions

- Stieljes: $f(x) = \int_0^\infty dt \frac{\rho(t)}{1+xt}; \rho(t) \geq 0 \quad \sim \quad f(x) = \frac{1}{\pi}\int_{s_0}^\infty dt \frac{Im(f(t))}{t+x}$
- Given a Stieljes function $f(x)$, $\lim_{N\to\infty} P^N_{N+1}(x) \leq f(x) \leq P^N_N(x)$ on the complex-plane $\mathbb{C}$ (except for the cut) as $N \to \infty$

Log-function $f(x) = \frac{1}{x} Log(1+x) = \frac{1}{\pi}\int_{-1}^{-\infty} dt \frac{Im(f(t))}{(x-t)} = \ldots = \int_0^1 \frac{dt}{1+xt}$



$Re(\frac{1}{x} Log(1+x))$



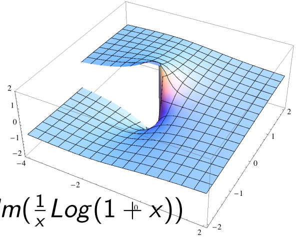
$Re(P^{20}_{21}(x))$

Padé Approximants: Convergence properties

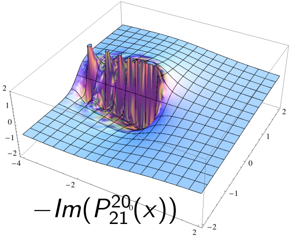
II. Stieljes functions

- Stieljes: $f(x) = \int_0^\infty dt \frac{\rho(t)}{1+xt}; \rho(t) \geq 0 \quad \sim \quad f(x) = \frac{1}{\pi} \int_{s_0}^\infty dt \frac{Im(f(t))}{t+x}$
- Given a Stieljes function $f(x)$, $\lim_{N\to\infty} P^N_{N+1}(x) \leq f(x) \leq P^N_N(x)$ on the complex-plane $\mathbb{C}$ (except for the cut) as $N \to \infty$

Log-function $f(x) = \frac{1}{x} Log(1+x) = \frac{1}{\pi} \int_{-1}^{-\infty} dt \frac{Im(f(t))}{(x-t)} = ... = \int_0^1 \frac{dt}{1+xt}$



$-Im(\frac{1}{x} Log(1+x))$



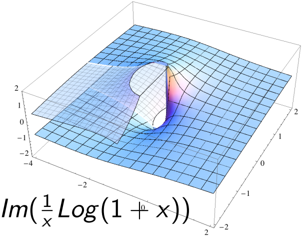
$-Im(P^{20}_{21}(x))$

Padé Approximants: Convergence properties

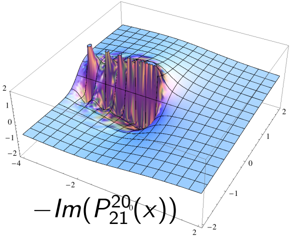
II. Stieljes functions

- Stieljes: $f(x) = \int_0^\infty dt \frac{\rho(t)}{1+xt}; \rho(t) \geq 0 \quad \sim \quad f(x) = \frac{1}{\pi} \int_{s_0}^\infty dt \frac{Im(f(t))}{t+x}$
- Given a Stieljes function $f(x)$, $\lim_{N\to\infty} P^N_{N+1}(x) \leq f(x) \leq P^N_N(x)$ on the complex-plane $\mathbb{C}$ (except for the cut) as $N \to \infty$

Cannot open further Riemann Sheets: No resonance hunting!



$-Im(\frac{1}{x} Log(1+x))$



$-Im(P^{20}_{21}(x))$

Padé Approximants: Convergence Properties

III. Multipoint Padé Approximants

Previous reuslts generalize to N-point expansions, of particular interest:

2-point Padé Approximants

- Well suited if low (i.e. chiral) and asymptotic (i.e. pQCD) expansions are known → matching

N-point Padé Approximants

- This implies Pade Approximants as fitting functions converge!
- Particularly their derivatives at some point converge too!
- This is our method to extract information, lets check convergence

Our proposal: Bivariate Padé Approximants

Use bivariate Padé Approximants (Chisholm '73)

$$P^N_M(x,y) = \frac{\sum_{i,j}^N a_{i,j} x^i y^j}{\sum_{k,l}^M b_{k,j} x^k y^l} \xrightarrow{y\to 0} \frac{\sum_i^N a_i x^i}{\sum_k^M b_k x^k} (P^N_M(x)) \quad a(b)_{i,j} = a(b)_{j,i}$$

Again, coefficients from matching the Taylor series

$$P^N_M(x,y) = \frac{T_N(x,y)}{R_M(x,y)} = a_0 + a_1(x+y) + a_{1,1}xy + \ldots + \mathcal{O}(x^\gamma y^{n+m-\gamma+1(+1)})$$

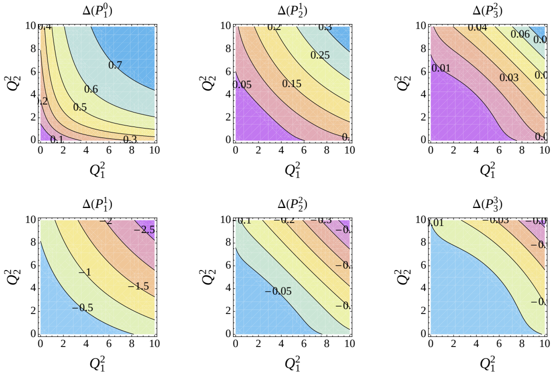
$$P^0_1(x,y) = \frac{a_0}{1 - \frac{a_1}{a_0}(c+y) + \frac{2a_1^2 - a_0 a_{1,1}}{a_0^2} xy} = a_0 + a_1(x+y) + a_{1,1}xy + \ldots$$

Convergence to meromorphic (Stieltjes?) functions (large-N_c limit) is guaranteed

Our proposal: Bivariate Padé Approximants

Lets revisit the Regge Model

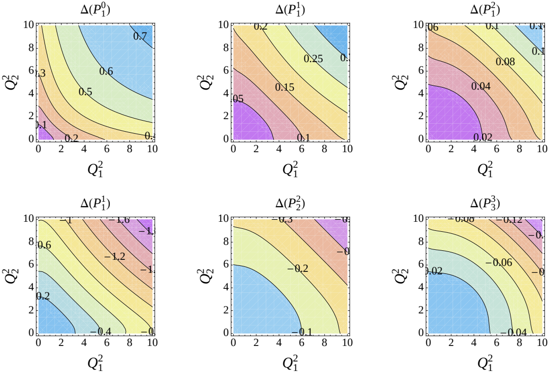
$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{Q_1^2 - Q_2^2} \frac{a}{\psi^{(1)}\left(\frac{M^2}{a}\right)} \left(\psi^{(0)}\left(\frac{M^2+Q_1^2}{a}\right) - \psi^{(0)}\left(\frac{M^2+Q_2^2}{a}\right)\right)$$



Obeys $P^N_{N+1}(x,y) \leq f(x,y) \leq P^N_N(x,y)$ (Stieltjes)

Our proposal: Bivariate Padé Approximants

A bigger challenge: cuts $F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma\gamma}(0,0)\frac{M^2}{Q_1^2 - Q_2^2} ln\left(\frac{M^2+Q_1^2}{M^2+Q_2^2}\right)$



Obeys $P^N_{N+1}(x,y) \le f(x,y) \le P^N_N(x,y)$ (Stieltjes)

$\eta - \eta'$-mixing: Results

We used F_P^0 QCD anomaly-driven running; Without it

$$F_q = 1.07(1)F_\pi, \quad F_s = 1.39(14)F_\pi \quad \phi = 39.3(1.3)^\circ$$

Then $\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 0.321(12)\text{GeV}$ (compare to 0.255(4)GeV)

— The RGE read —

$$\mu\frac{d}{d\mu}F_0 = -N_F\left(\frac{\alpha_s}{\pi}\right)^2 \to F_0(\mu) = F_0(\mu_0)\left(1 + \frac{2N_F}{\pi\beta_0}\left(\alpha_s(\mu) - \alpha_s(\mu_0)\right)\right)$$

Implying the modified equations ($F_0(\infty) = F_0(1GeV)(1+\Delta)$)

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2\left(\hat{c}_q\left(1+(4/5)\Delta\right)F_q\cos\phi - \hat{c}_s\left(1+2\Delta\right)F_s\sin\phi\right)$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2\left(\hat{c}_q\left(1+(4/5)\Delta\right)F_q\sin\phi + \hat{c}_s\left(1+2\Delta\right)F_s\cos\phi\right)$$

$\eta-\eta'$-mixing: Results

The degeneracy equation read

$$F_{\eta\gamma\gamma}\eta_\infty + F_{\eta'\gamma\gamma}\eta'_\infty = \frac{1}{6\pi^2}(9+8\Delta)$$

Where $F_{P\gamma\gamma} = F_{P\gamma\gamma}(0,0)$ and $P_\infty = \lim_{Q^2\to\infty} Q^2 F_{P\gamma^*\gamma}(Q^2)$
Data: $0.9(3)\frac{9}{6\pi^2}$ *vs.* Running: $0.85\frac{9}{6\pi^2}$ $\to$ ~~OZI~~?

— The RGE read —

$$\mu\frac{d}{d\mu}F_0 = -N_F\left(\frac{\alpha_s}{\pi}\right)^2 \to F_0(\mu) = F_0(\mu_0)\left(1+\frac{2N_F}{\pi\beta_0}(\alpha_s(\mu)-\alpha_s(\mu_0))\right)$$

Implying the modified equations ($F_0(\infty) = F_0(1GeV)(1+\Delta)$)

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2\left(\hat{c}_q\left(1+(4/5)\Delta\right)F_q\cos\phi - \hat{c}_s\left(1+2\Delta\right)F_s\sin\phi\right)$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2\left(\hat{c}_q\left(1+(4/5)\Delta\right)F_q\sin\phi + \hat{c}_s\left(1+2\Delta\right)F_s\cos\phi\right)$$

$\eta - \eta'$-mixing: Results

BaBar Coll. obtained deep time-like $q^2 = 112$ GeV^2 data.

At least, as $Q^2 \to \infty$, $q^2|F_{P\gamma^*\gamma}(q^2)| = Q^2|F_{P\gamma^*\gamma}(Q^2)|$

Neglecting q^2 corrections and assuming asymptotic behavior + duality,

$$\lim_{q^2 \to 112\ \mathrm{GeV}^2} q^2|F_{\eta(\eta')\gamma^*\gamma}(q^2)| \ = \ \lim_{Q^2\to\infty} Q^2|F_{\eta(\eta')\gamma^*\gamma}(Q^2)|$$

This way, BaBar obtains

$$\lim_{Q^2\to\infty} Q^2|F_{\eta[\eta']\gamma^*\gamma}(Q^2)| = 0.229(30)(8)\ [0.251(19)(8)]\ \mathrm{GeV}$$

To be compared with our extractions

$$\lim_{Q^2\to\infty} Q^2|F_{\eta[\eta']\gamma^*\gamma}(Q^2)| = 0.177(15)\ [0.255(4)]\ \mathrm{GeV}$$

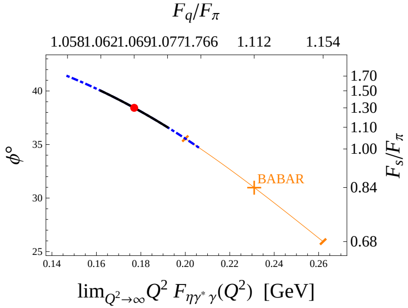
$\eta-\eta'$-mixing: Results

BaBar Coll. obtained deep time-like $q^2 = 112$ GeV^2 data.

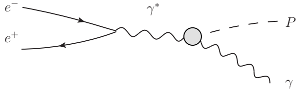
At least, as $Q^2 \to \infty$, $q^2|F_{P\gamma^*\gamma}(q^2)| = Q^2|F_{P\gamma^*\gamma}(Q^2)|$

Neglecting q^2 corrections and assuming asymptotic behavior + duality,

$$\lim_{q^2 \to 112\ \mathrm{GeV}^2} q^2|F_{\eta(\eta')\gamma^*\gamma}(q^2)| \;=\; \lim_{Q^2\to\infty} Q^2|F_{\eta(\eta')\gamma^*\gamma}(Q^2)|$$



$c\bar{c}$ continuum production



- The TL TFF enters this process
- As $q^2 \to \infty$ $F_{P\gamma^*\gamma}(q^2) \equiv F_{P\gamma^*\gamma}(-Q^2)$
- If holds at large but finite but large q^2, use our parametrization
- Test duality ideas

$$\sigma(e^+e^- \to P\gamma) = \frac{2\pi^2\alpha^3}{3}(F_{P\gamma^*\gamma}(s,0))^2\left(1 - \frac{m_P^2}{s}\right)$$

Dominates respect to $J/\Psi \to P\gamma$