Gauge/Gravity Duality:

An introduction

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Outline

I. Foundations

1. String theory
2. Dualities between quantum field theories and gravity theories

   AdS/CFT correspondence
3. Originates from string theory

II. Applications

1. Theory of Strong Interactions (QCD)
2. Quark-gluon plasma and black holes
3. Superfluidity and superconductivity

Well-known fundamental theories:

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- Standard model of elementary particle physics:

  describes strong, weak and electromagnetic forces

  Local quantum field theory (gauge theory), particles point-like

  Confirmed experimentally up to a scale of about $\mathcal{O}(100)$ GeV

  Effective Theory (free parameters, for example quark masses)

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  Effective Theory (free parameters, for example quark masses)

- Einstein's theory of general relativity (Theory of gravitation)

  Classical theory

  Non-renormalizable

  Quantum effects expected at $M_{Planck} \simeq 10^{19} GeV$

Introduction: String theory

Search for a

Unified Theory of fundamental interactions

- Quantum theory of gravitation
- Description of all four interactions in a unified framework
- Reduction of the number of free parameters

Promising candidate: String theory

Introduction: String theory

Quantum theory of gravitation and unification of all four interactions:

Locality is abandoned at very short distances

Natural cut-off: String length

$$l_s \sim \frac{1}{M_{Planck}}, \quad l_s = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} m$$

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Search for indirect experimental evidence

New:

String theory:

New framework for describing strongly coupled systems

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At arbitrary energies

New applications of string theory

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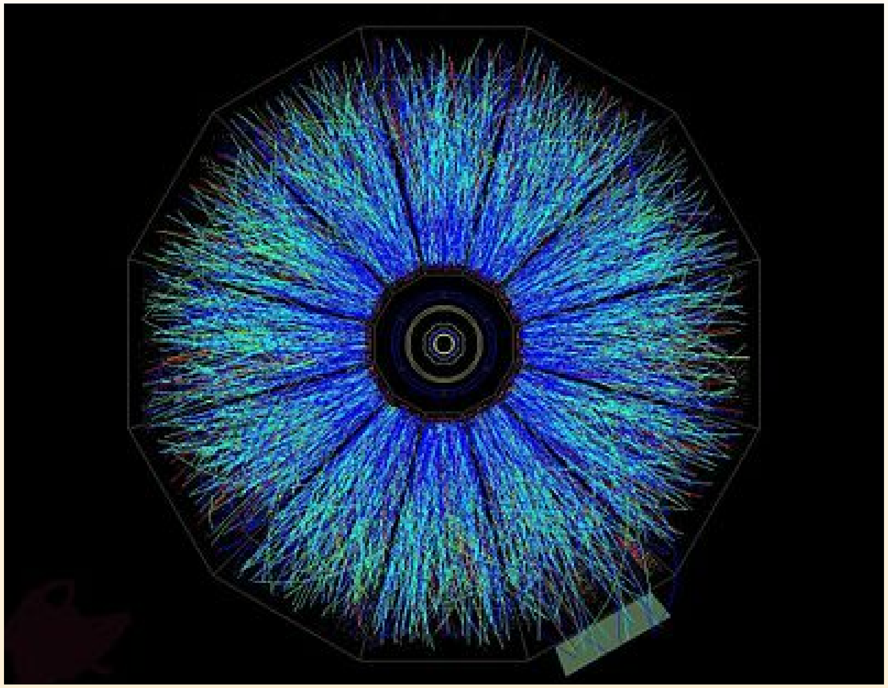
At arbitrary energies

New applications of string theoretical methods

- within elementary particle physics
- within condensed matter theory

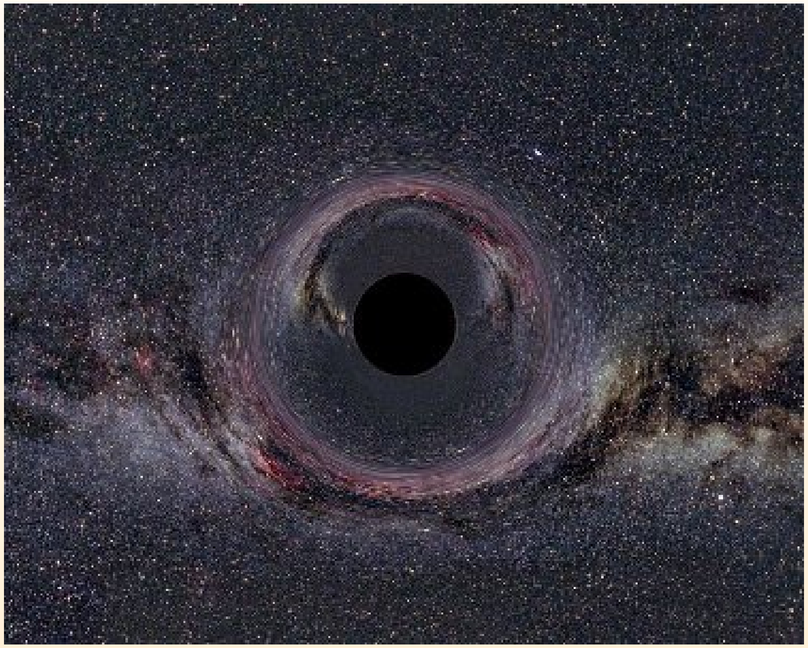
Dualities between quantum field theories and gravitation

Map: Quantum field theory $\Leftrightarrow$ Classical theory of Gravitation



Event at the RHIC accelerator

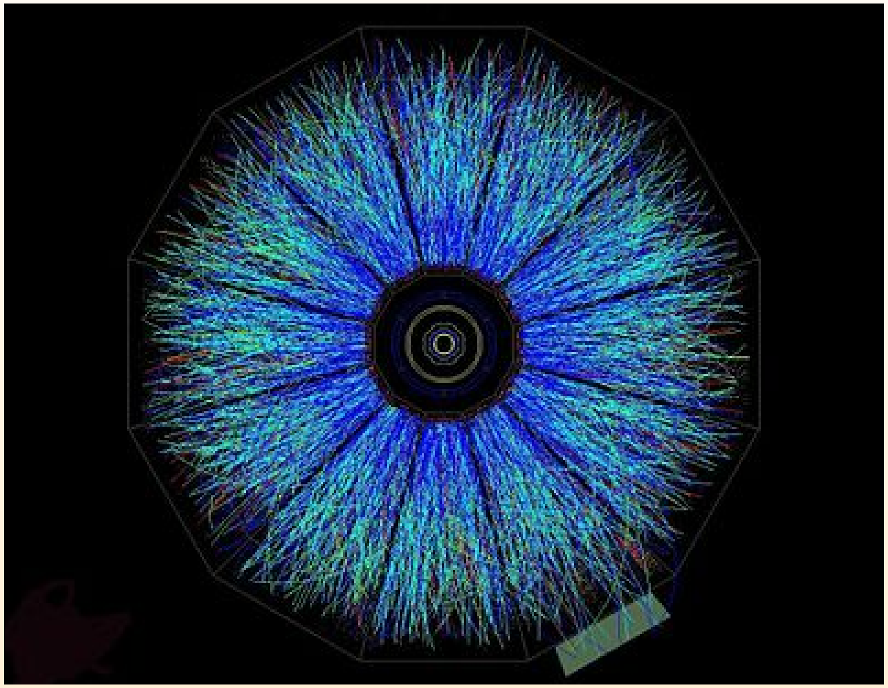
$\Leftrightarrow$



Black hole geometry

Dualities between quantum field theories and gravitation

Map: Quantum field theory ⇔ Classical theory of Gravitation



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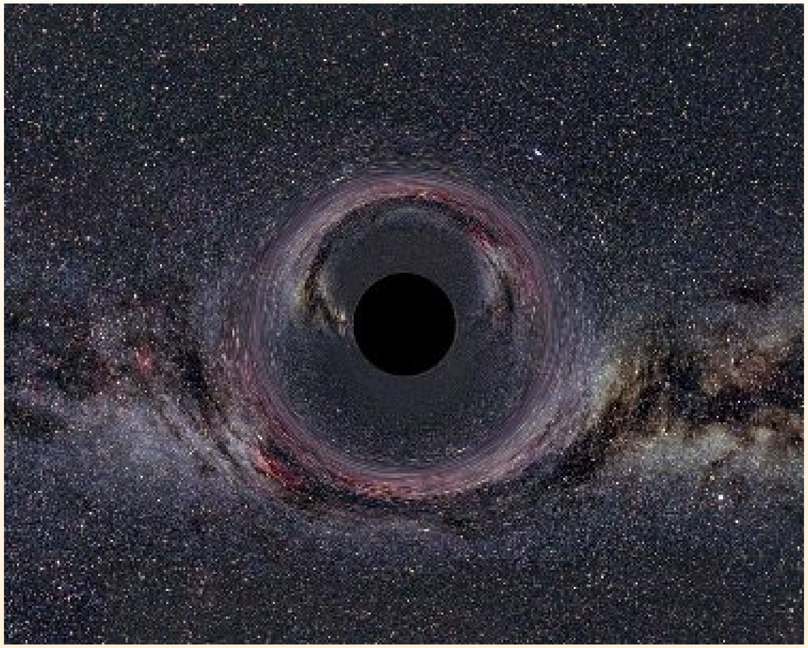
⇔



Black hole geometry

Practical Significance:

- Map: Strongly coupled quantum field theory
  ⇔ Weakly coupled classical theory of gravitation (Solvable)

AdS/CFT Correspondence: Foundations

(Maldacena 1997, AdS: Anti de Sitter space, CFT: conformal field theory)

- Follows from a low-energy limit of string theory

- Duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Theory of gravitation at weak coupling

- Holography:

  Quantum field theory in four dimensions

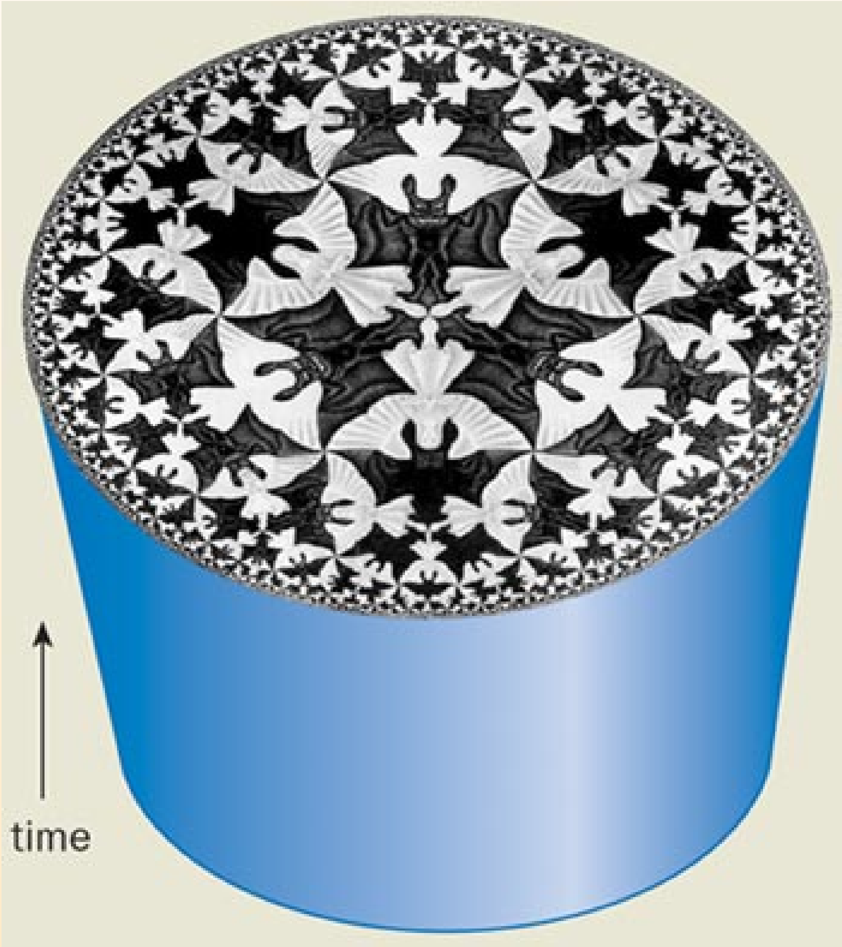
  $\Leftrightarrow$ Gravitational theory in five dimensions

Anti-de Sitter Space

Space of constant negative curvature, has a boundary

$ds^2 = e^{2r/L} dx_\mu dx^\mu + dr^2$

Figure source: Institute of Physics, Copyright: C. Escher



Quantum field theory

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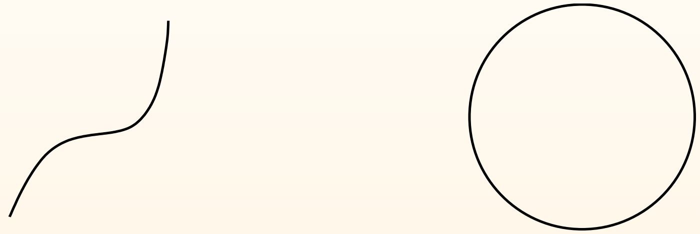
Note: Confinement and conformal symmetry are incompatible!

String theory



Open strings : Gauge degrees of freedom of the Standard Model

Closed Strings: Gravitation

Two types of degrees of freedom: open and closed strings

Higher oscillation modes are excited

Quantization:

Supersymmetric string theory well-defined in $9 + 1$ dimensions

(No tachyons, no anomalies)

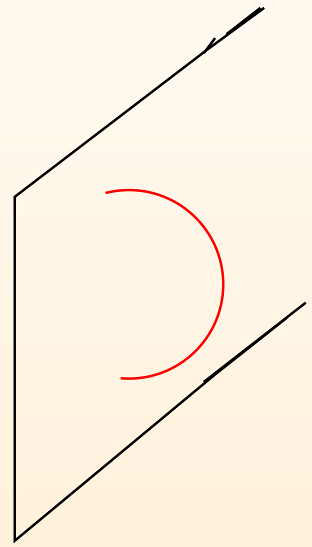
Supersymmetry: Symmetry Bosons $\Leftrightarrow$ Fermions

What is the significance of the additional dimensions?

1. Compactification

2. D-branes

D-branes are hypersurfaces embedded into 9+1 dimensional space



D3-Branes: (3+1)-dimensional hypersurfaces

Open Strings may end on these hypersurfaces $\Leftrightarrow$ Dynamics

D-Branes

Low-energy limit (Strings point-like) $\Rightarrow$

Open Strings $\Leftrightarrow$ Dynamics of gauge fields on the brane

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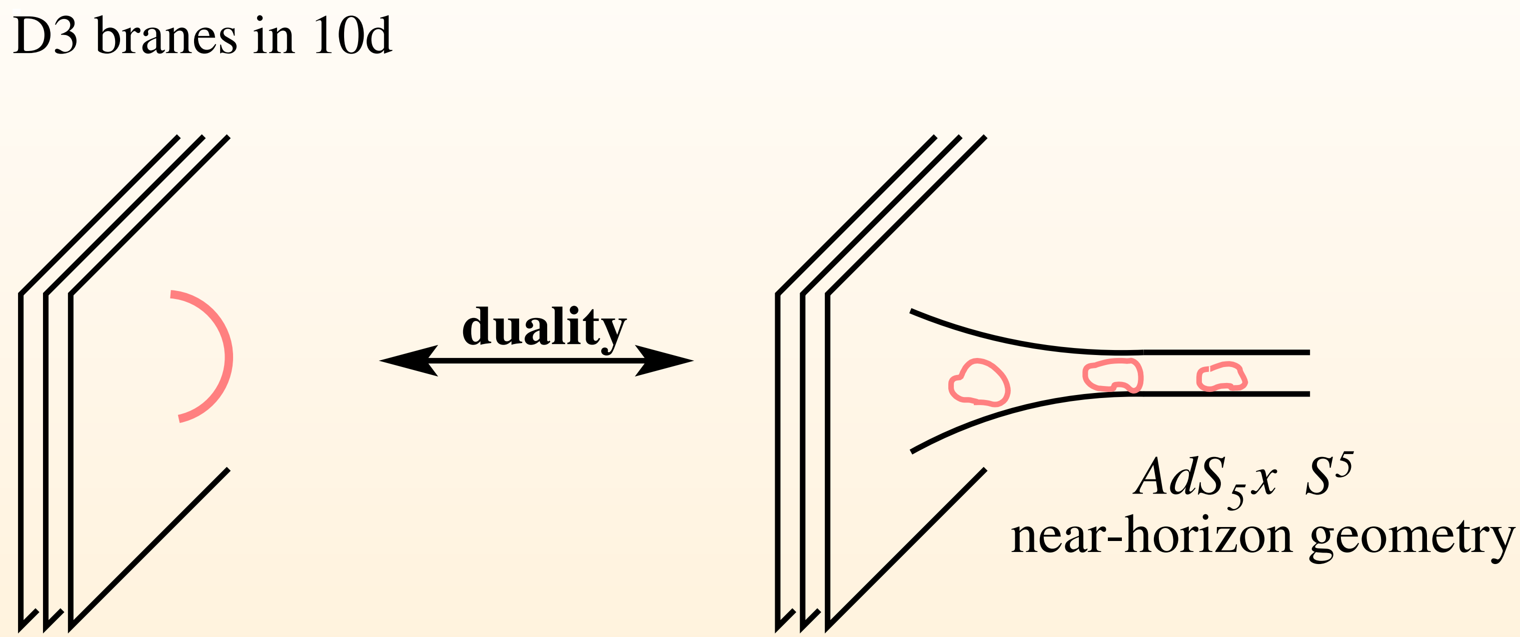
Second interpretation of the D-branes:

Solitonic solutions of ten-dimensional supergravity

Heavy objects which curve the space around them

String theory origin of the AdS/CFT correspondence

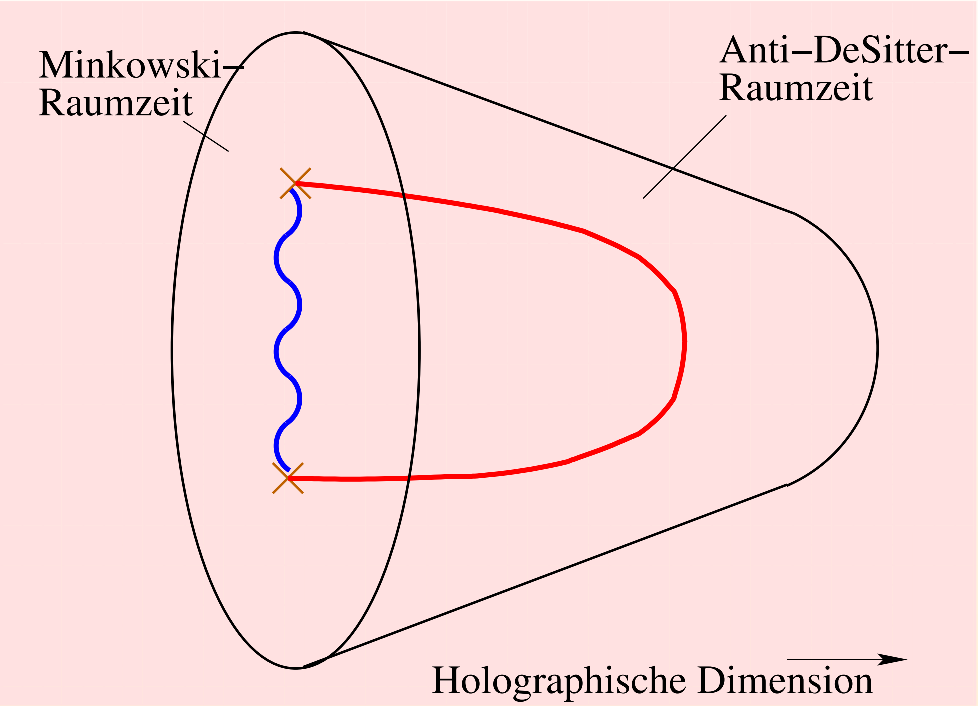


$\Downarrow$ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions ($N \to \infty$)

Supergravity on the space $AdS_5 \times S^5$

AdS/CFT correspondence



'Dictionary' Field theory operators ⇔ Classical fields in gravity theory

Symmetry properties coincide

Test: (e.g.) Calculation of correlation functions

Applications

1. Strong interactions - Quarks and Gluons

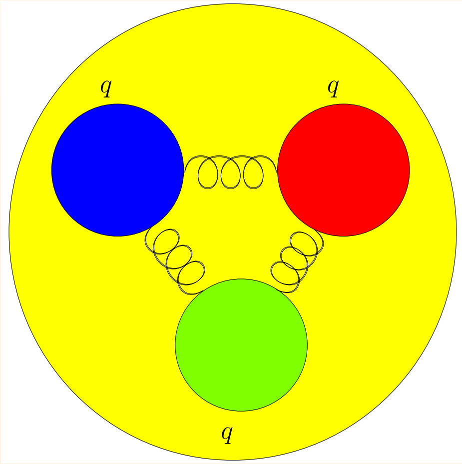
2. Finite temperature: Quark Gluon Plasma

3. Superfluiditiy and Superconductivity

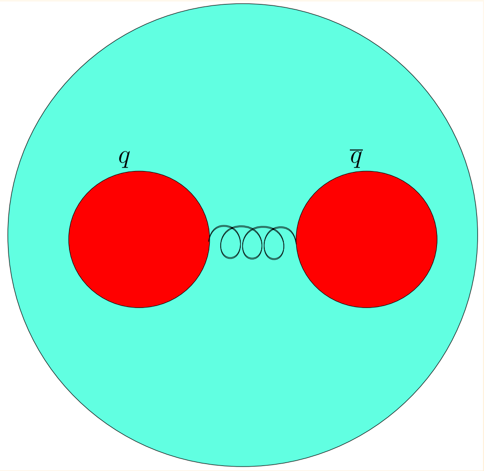
Strong Interactions: Quarks and Gluons

Theory of Strong Interactions: Quantum Chromodynamics (QCD)

Baryon (e.g. Proton)



Meson (e.g. Pion)



Asymptotic freedom and confinement:

Quarks and Gluons are strongly coupled at low energies

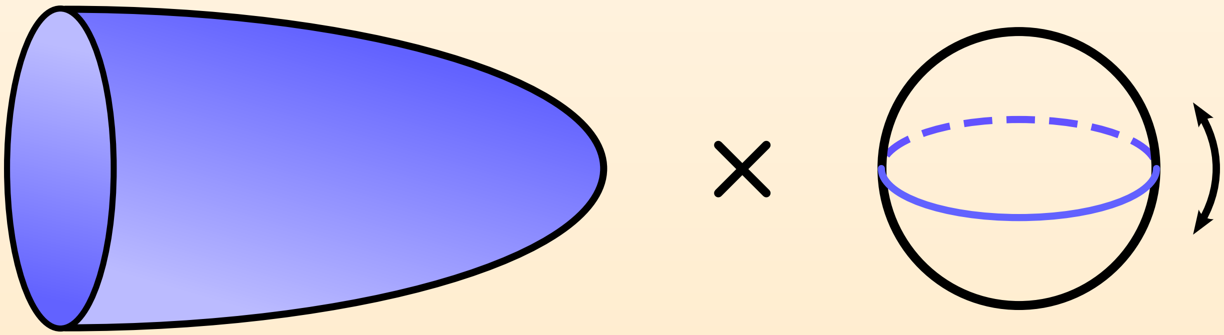
Quarks in the AdS/CFT correspondence

Add D7-Branes (Hypersurfaces)

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
1,2 D7	X	X	X	X	X	X	X	X		

Quarks: Low energy limit of open strings between D3- and D7-Branes

Meson masses from fluctuations of the hypersurface (D7-Brane):



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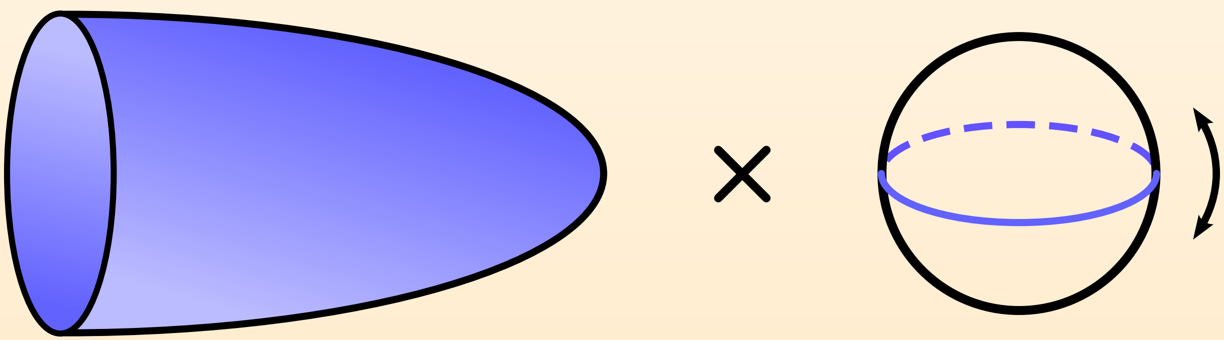
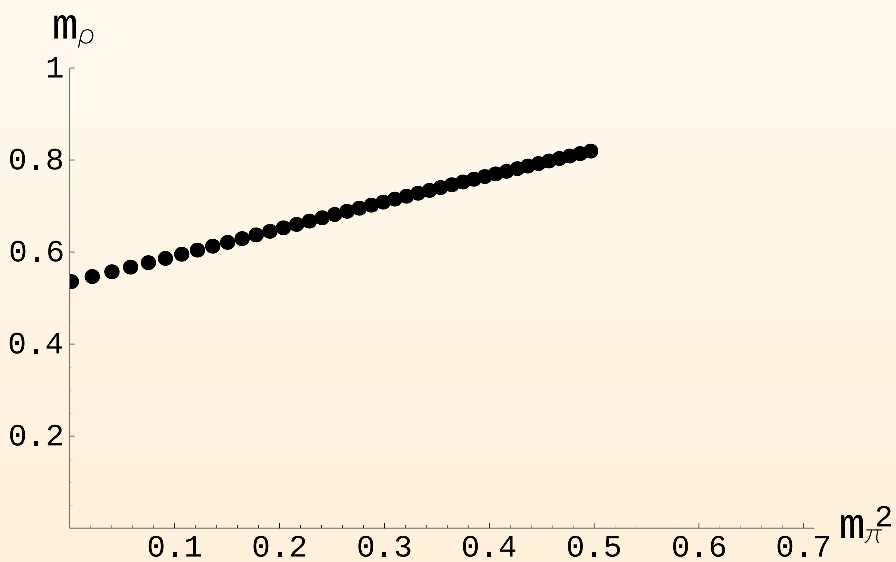
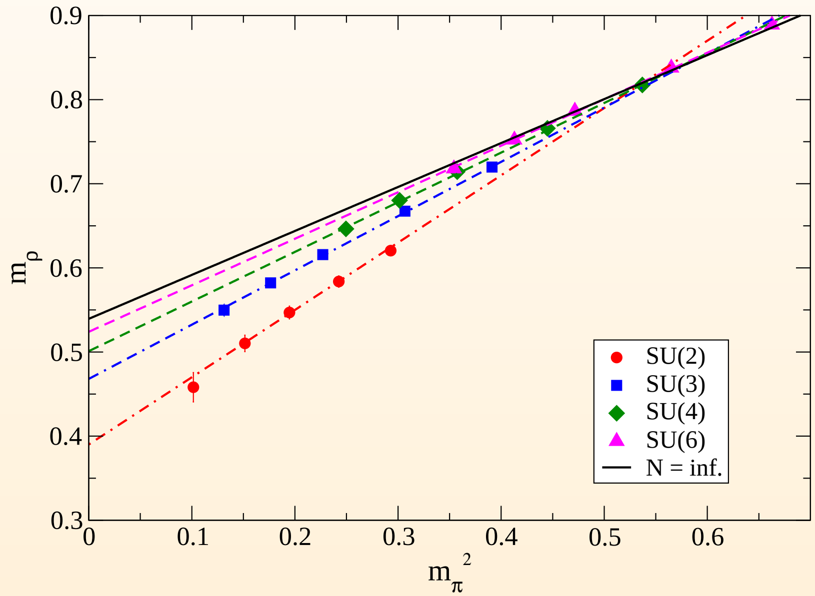


Confinement: AdS$_5 \times S^5$-metric has to be deformed

Comparison to lattice gauge theory

Mass of ρ meson as function of π meson mass^2 (for $N \to \infty$)





J.E., Evans, Kirsch, Threlfall '07, review EPJA

Lattice: Lucini, Del Debbio, Patella, Pica '07

AdS/CFT result:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left(\frac{m_\pi}{m_\rho(0)}\right)^2$$

Lattice result (from Bali, Bursa '08): slope 0.341 ± 0.023

Prime example: Phase diagram of strongly interacting matter

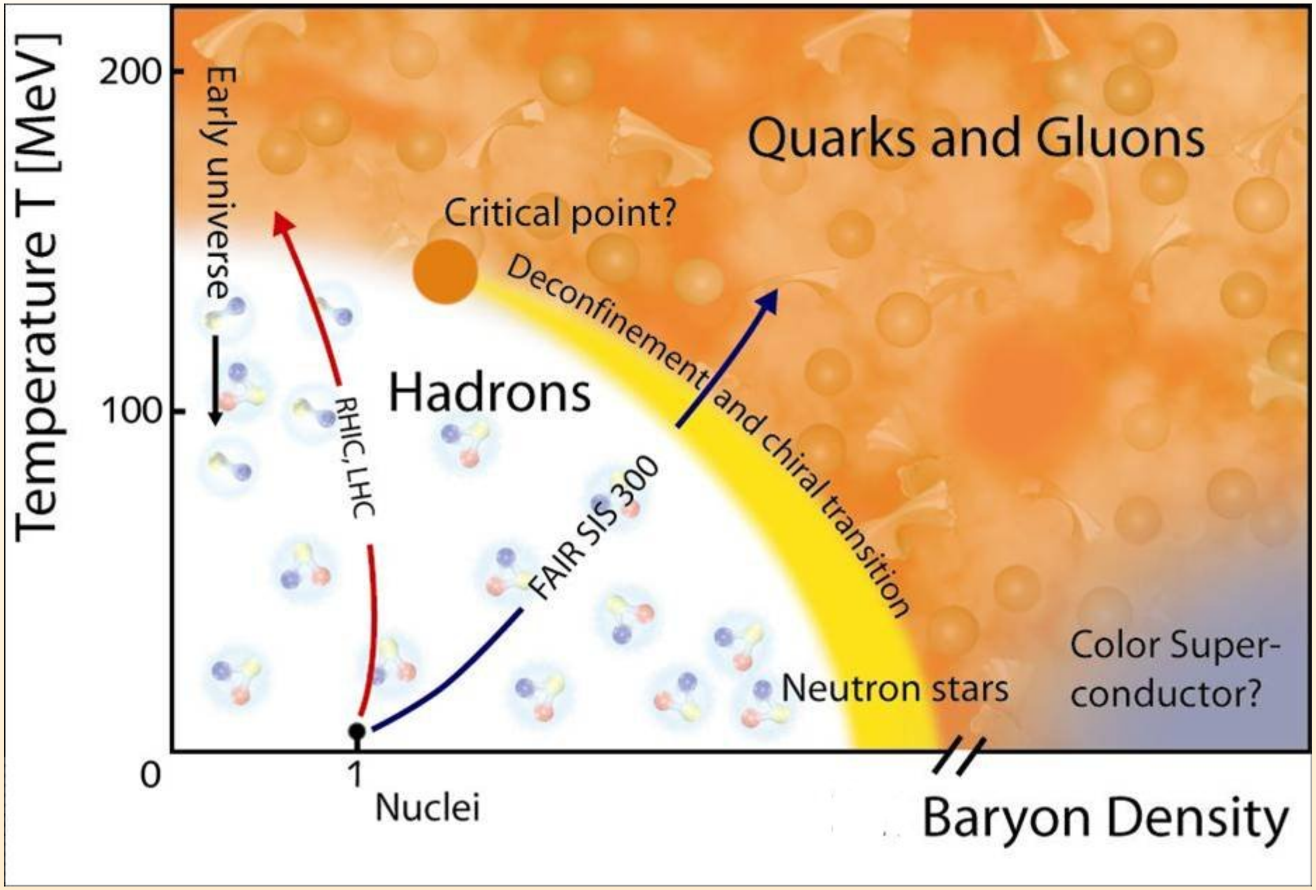


Bild: CBM @ FAIR, GSI

Quark-gluon plasma:

Strongly coupled state of matter above deconfinement temperature T_d

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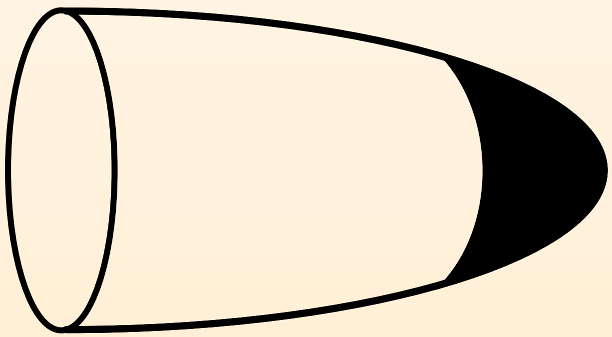
AdS/CFT dual of field theory at finite temperature: AdS black hole



Hawking temperature $\Leftrightarrow$ temperature in the dual quantum field theory

Universal result:

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Ratio of shear viscosity and entropy

$$\frac{\eta}{s} = \frac{\hbar}{k_B}\frac{1}{4\pi}$$

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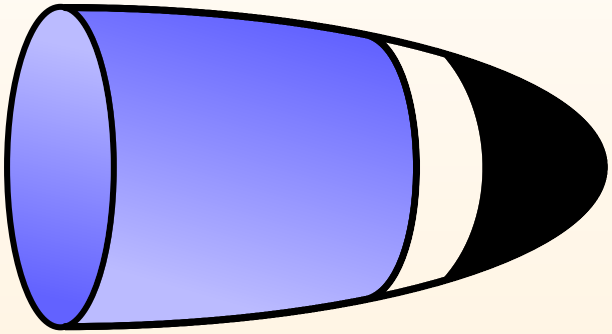
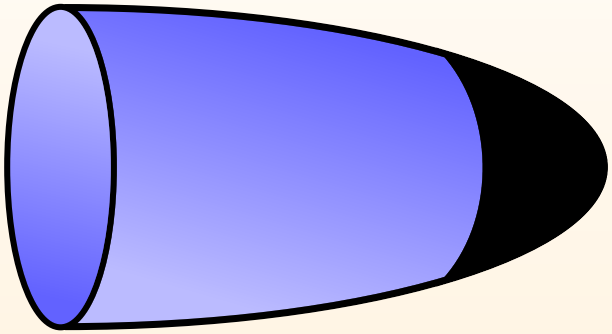
Lower bound: smallest possible value

2. Quarks and mesons at finite temperature

Babington, J.E., Evans, Guralnik, Kirsch 2004; Mateos, Myers, Thomson 2006





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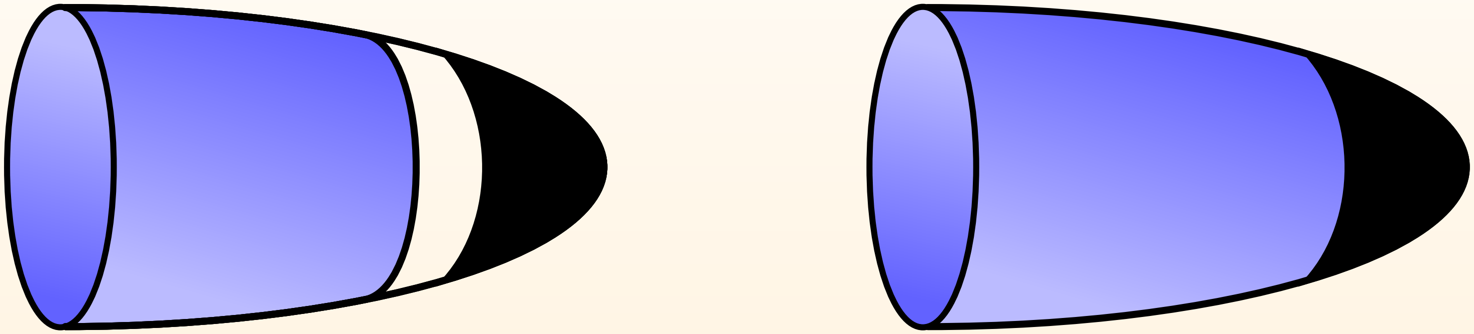


Prediction:

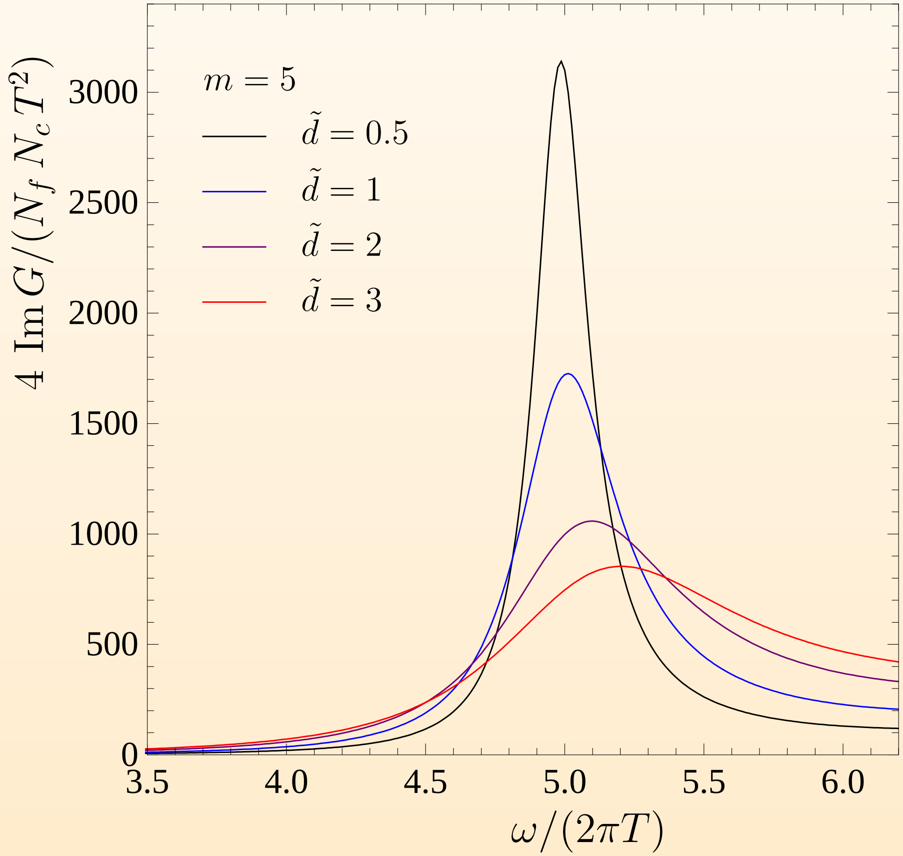
Mesons survive deconfinement if quark mass m_q sufficiently large

1st order fundamental phase transition at $T = T_f$ where mesons dissociate

$T_f \sim 0.12 M_{\text{meson at } T=0}$

Easy to introduce: VEV for time component of gauge field

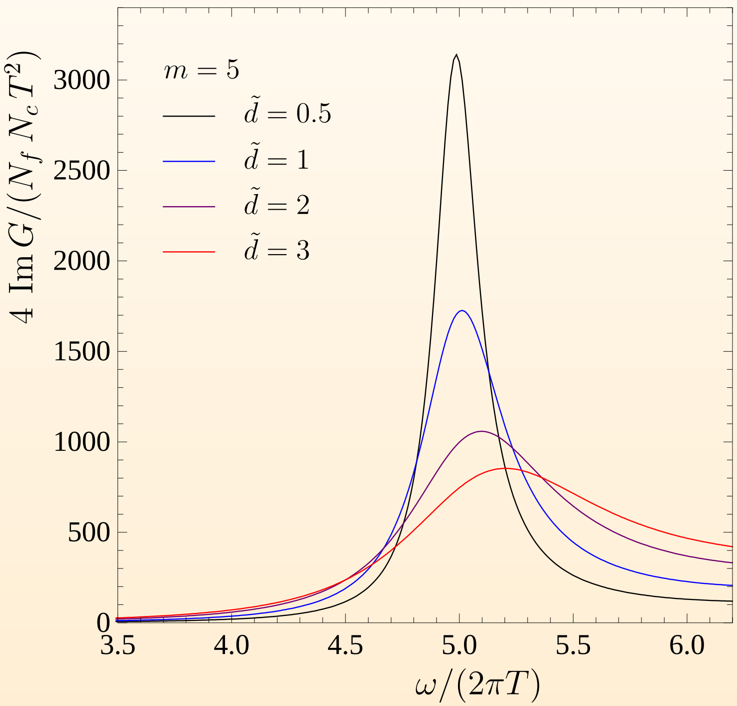
ρ vector meson spectral function in dense hadronic medium



AdS/CFT result (J.E., Kaminski, Kerner, Rust 2008)

Spectral function at finite baryon density

ρ vector meson spectral function in dense hadronic medium



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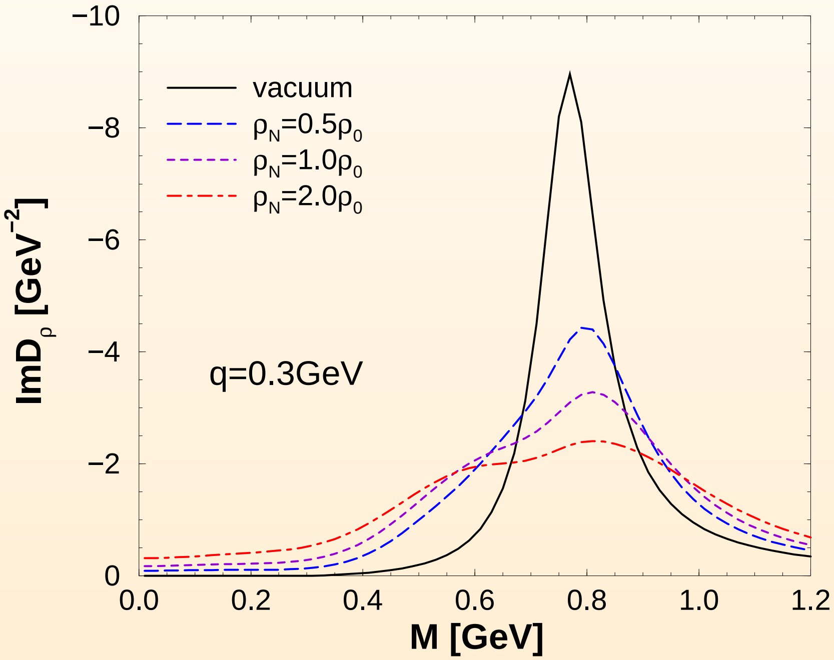
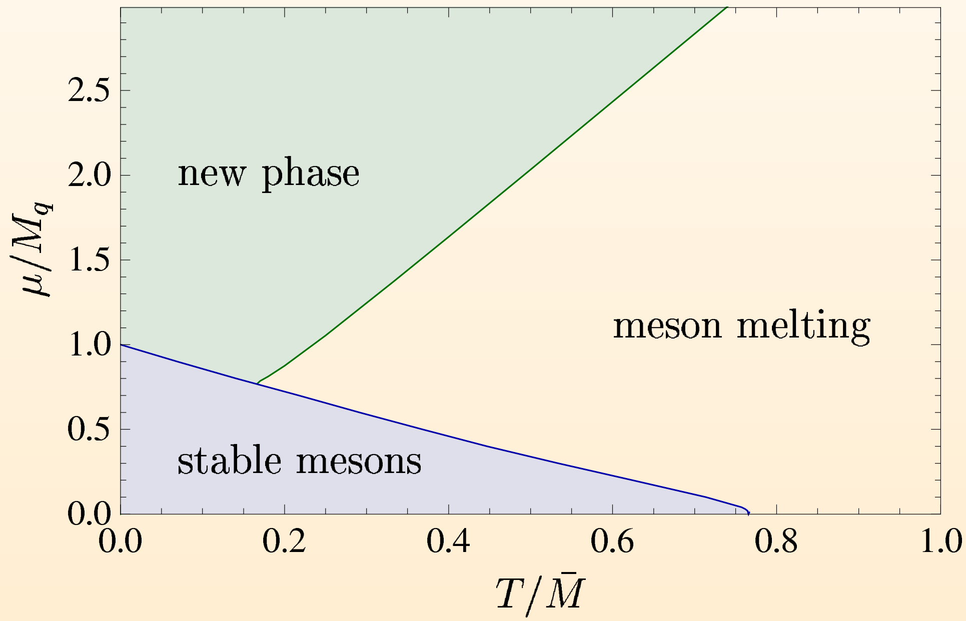


Field theory (Rapp, Wambach 2000)

Chemical Potential and Finite Density for Isospin ($SU(2)$)

u- and d-Quarks

Phase diagram



Instability!

New solution to the equations of motion with lower free energy

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The new solution contains a condensate $\langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \; bosons \, \rangle$

3. Superfluidity and Superconductivity

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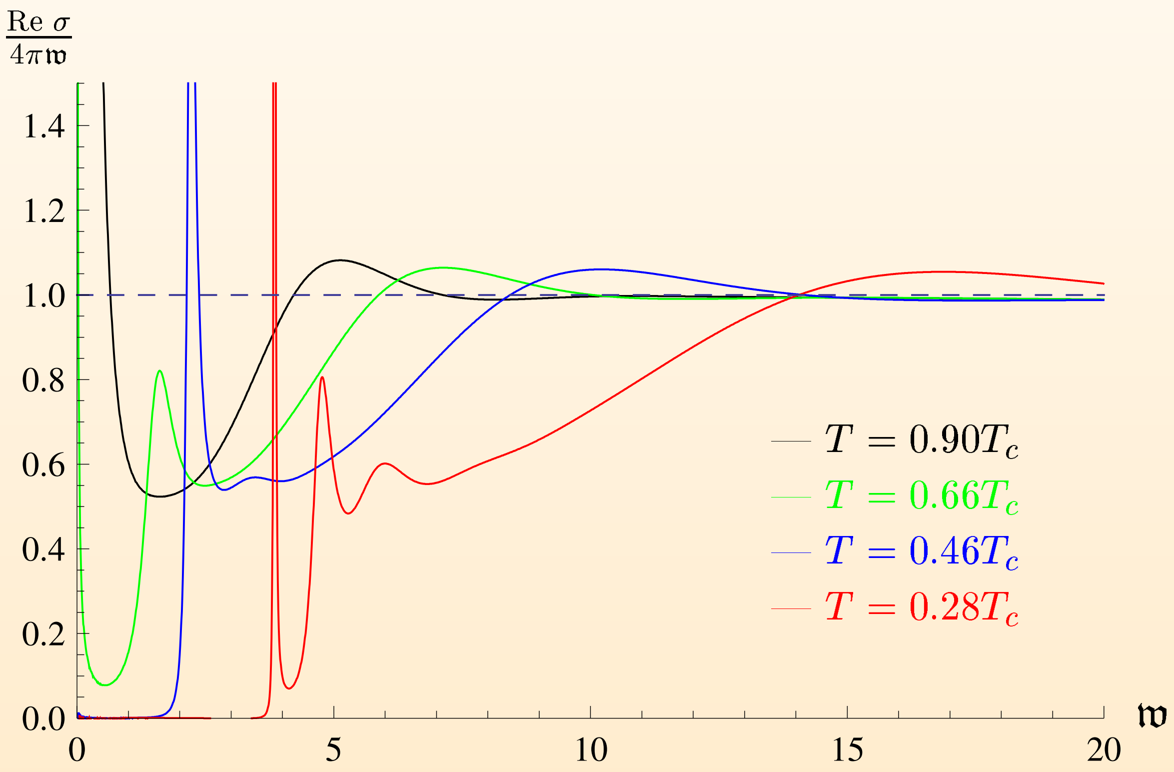
The new solution contains a condensate $\langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \, bosons \, \rangle$

ρ meson condensate (p-wave, triplet pairing)

Ammon, J.E., Kaminski, Kerner 2008

The new ground state is a superfluid.

Frequency-dependent conductivity from spectral function



$$\mathfrak{w} = \omega/(2\pi T)$$

Prediction: Frictionless motion of mesons through the plasma

Quantum Phase Transition

Two chemical potentials: Isospin and Baryon Chemical Potential

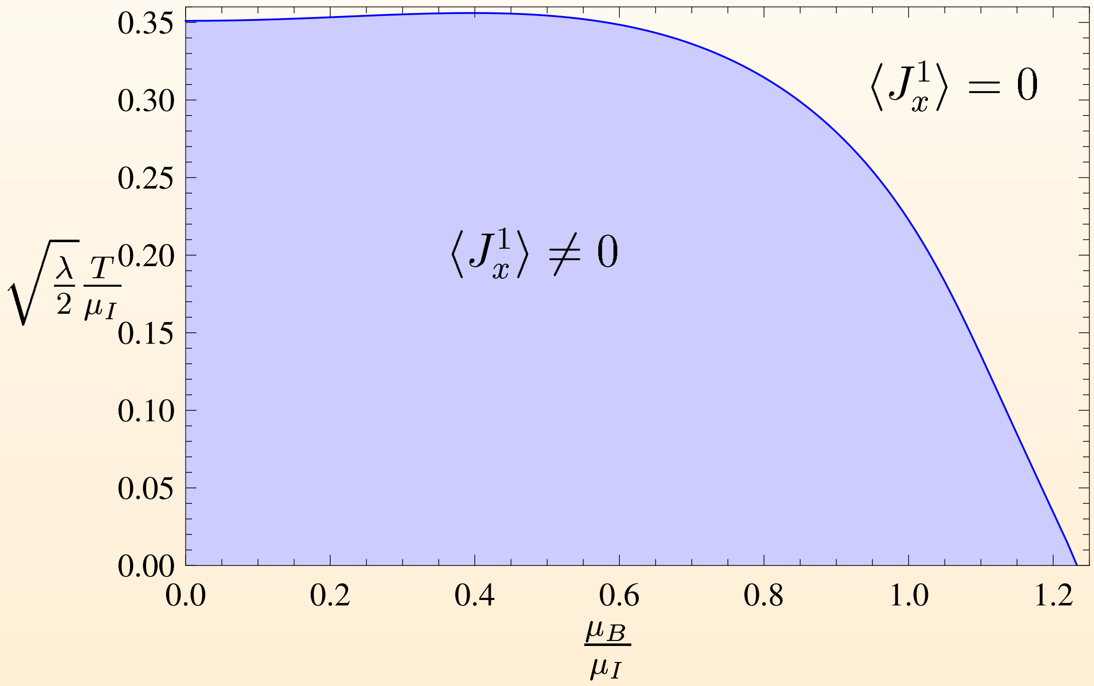
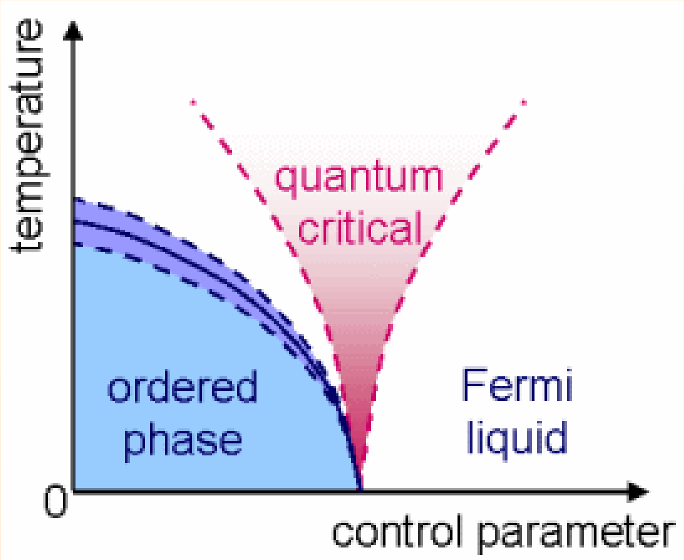
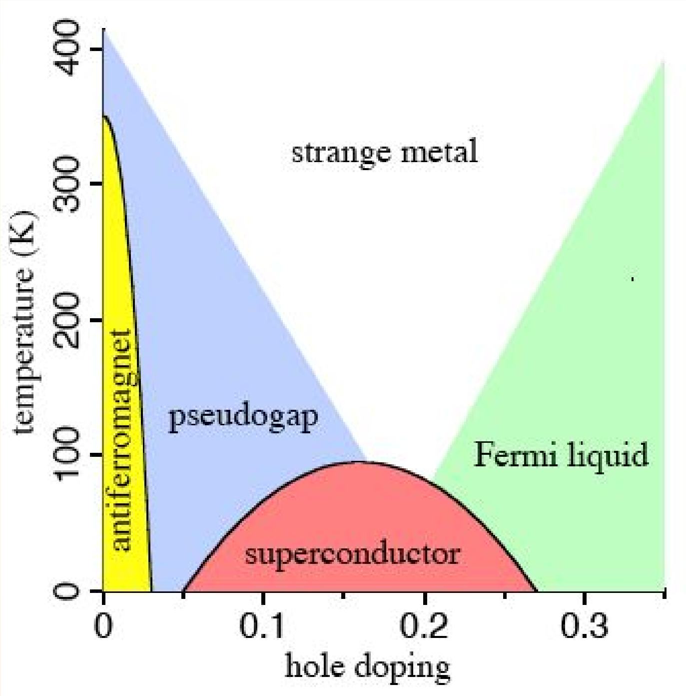


Figure by Patrick Kerner

Example for Quantum Phase Transition

Phase diagrams



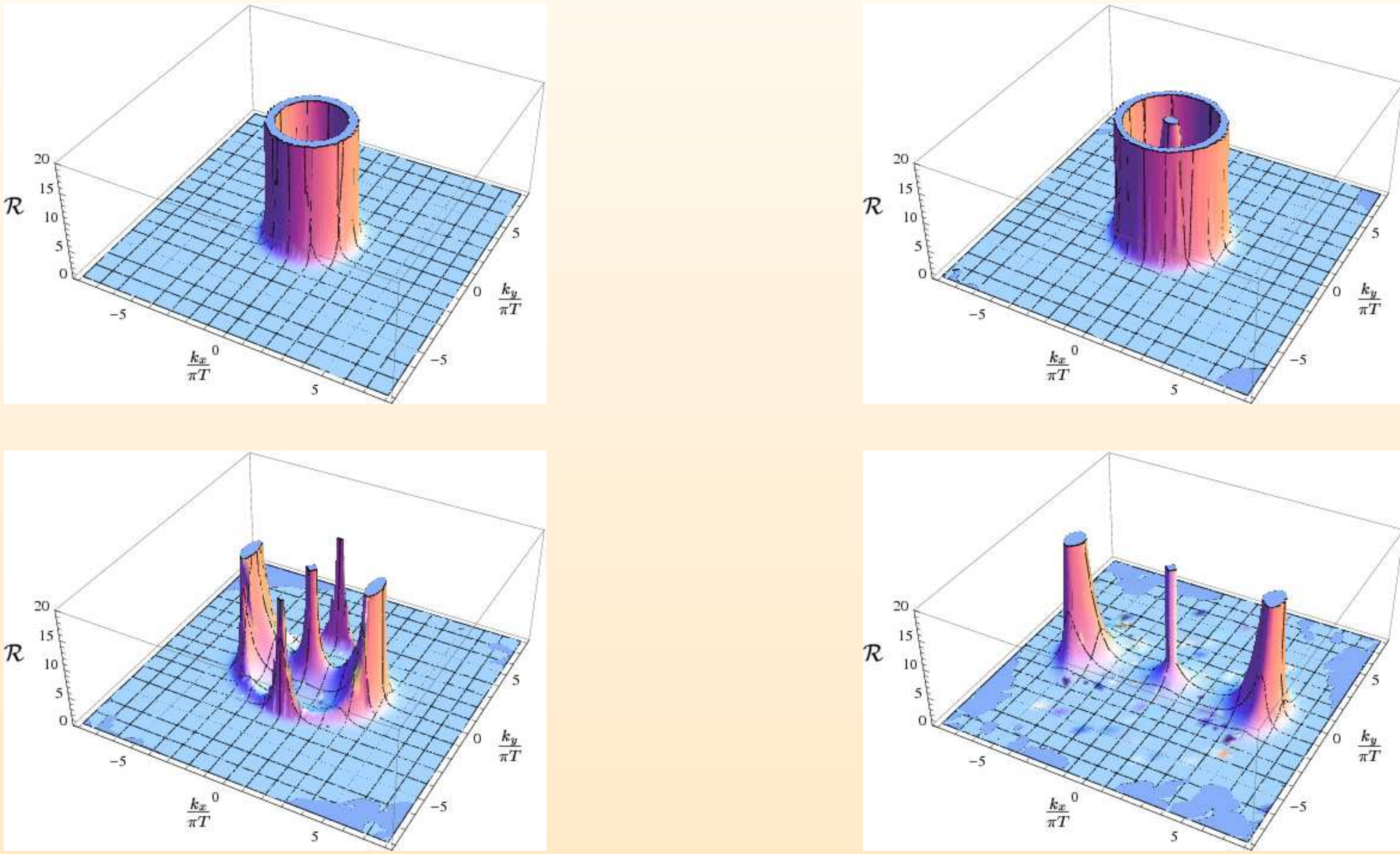


Quantum phase transition

Superconductor

Fermionic Excitations in p-wave Superconductors

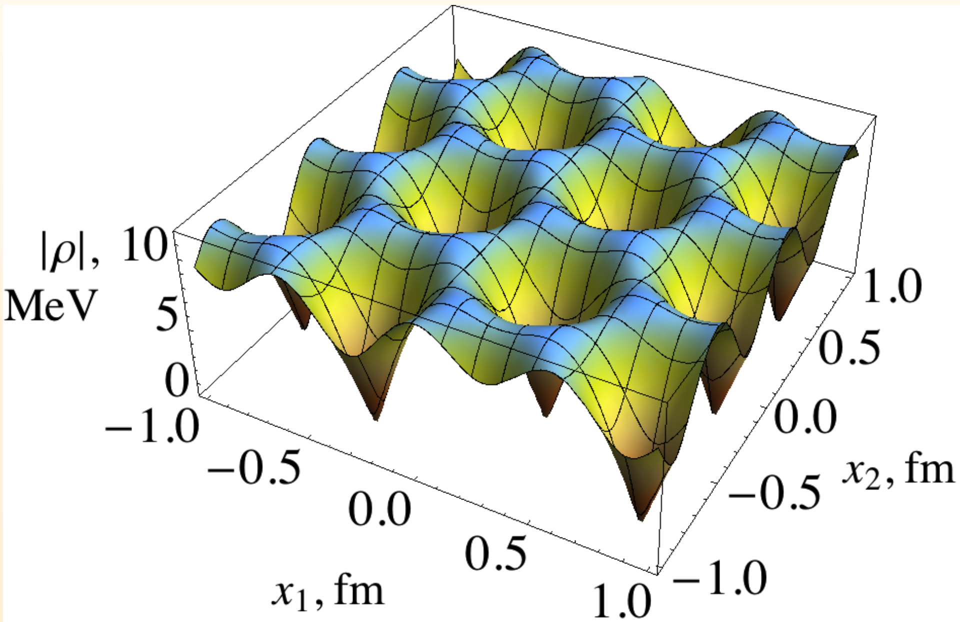
Ammon, J.E., Kaminski, O'Bannon 2010



B-field induced rho meson condensation

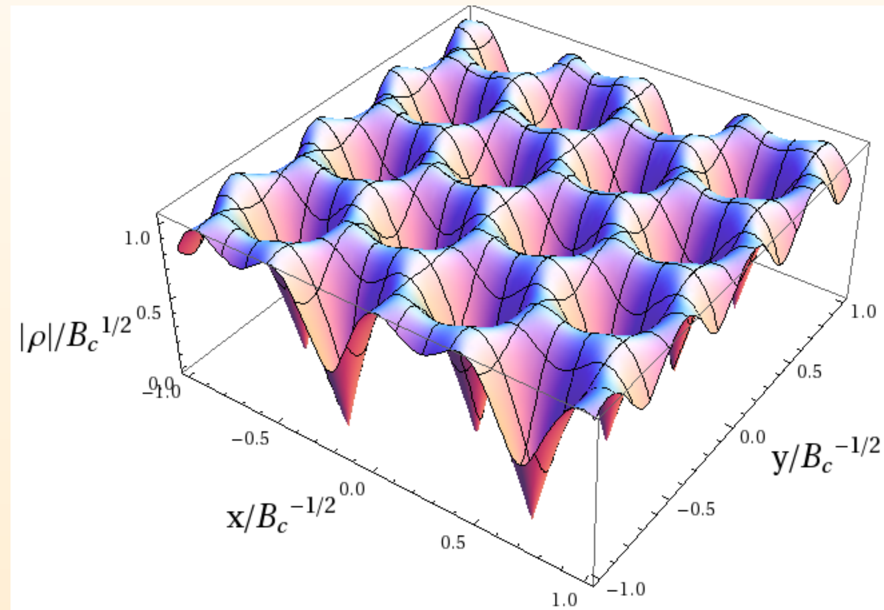
Field theory calculation

(Chernodub)



Gauge/gravity calculation

(Bu, J.E., Shock, Strydom)



Outlook: Universality and Homes’ Law

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Universal result from gauge/gravity duality for hydrodynamics:

$$\frac{\eta}{s} = \frac{\hbar}{k_B}\frac{1}{4\pi}$$

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Candidate:

Homes' Law for superconductors:

$$n_s(T=0) = C\sigma(T_c)T_c$$

n_s: Superfluid density; σ: Conductivity

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Found experimentally to great accuracy

Outlook: Homes' Law and Universality

Relation between η/s and Homes' Law:

Outlook: Homes' Law and Universality

Relation between η/s and Homes' Law:

Shortest possible dissipation time in strongly coupled systems:

Planckian time: $\tau = \frac{\hbar}{k_B T_c}$

Outlook: Homes' Law and Universality

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Work in progress: Obtain this relation within gauge/gravity duality

Conclusion and Outlook

Generalized AdS/CFT correspondence: Gauge/Gravity Duality

- New relation between fundamental and applied aspects of theoretical physics

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- Identification of Universal Behaviour

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- Quantum phase transitions

Conclusion and Outlook

Generalized AdS/CFT correspondence: Gauge/Gravity Duality

- New relation between fundamental and applied aspects of theoretical physics
- Identification of Universal Behaviour
- Predictions for mesons in the quark-gluon plasma
- Quantum phase transitions
- Comparison and combination with other approaches
- New relations between different areas of physics