Construction of explicit de Sitter vacua in type IIB flux compactifications

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arXiv:1107.2115, 1208.3208, 1212.4530, 1306.1237
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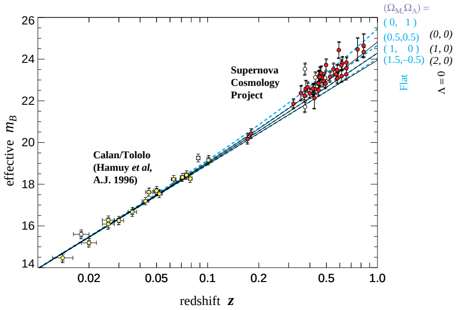
Theorie-Palaver, Universität Mainz, 11 June 2013

Outline:

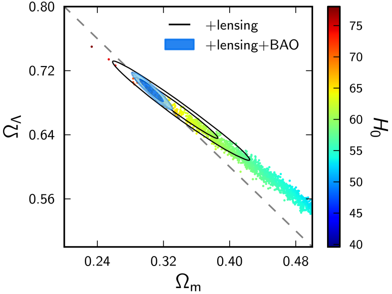
1. Introduction: De Sitter vacua in string theory
2. Complex structure moduli stabilization
3. Kähler uplifted de Sitter vacua
4. A consistent global model
5. Conclusions

1. Introduction: De Sitter vacua in type IIB

Motivation: The cosmological constant problem (I)



[Perlmutter et al. 1998], [Riess et al. 1998]



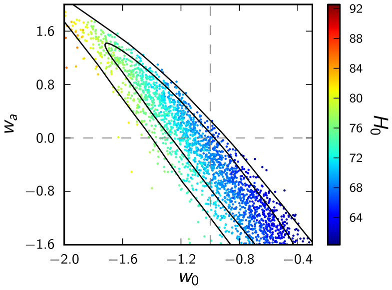
[Planck 2013]

$$\Rightarrow \Omega_\Lambda \simeq 0.68 \quad \Leftrightarrow \quad \rho_\Lambda \sim 10^{-120} M_{\mathrm{P}}^4 \quad \text{Tiny!}$$

Motivation: The cosmological constant problem (II)

What is dark energy?

- Equation of state $w = \dfrac{p}{\rho}$ is consistent with -1
- Simplest explanation:
  $G_{\mu\nu} = T^{\Lambda}_{\mu\nu} = -\Lambda\eta_{\mu\nu}$
  (Cosmological constant Λ)
  $\Rightarrow$ de Sitter space
- For scalar field ϕ at local minimum ϕ_0: $\Lambda = V_{\text{eff}}(\phi_0)$



[Planck '13]

Parametrize $w = w_0 + (1-a)w_a$

Motivation: String theory and dark energy (I)

- 10D $\rightarrow$ 4D: Typically $\mathcal{O}(100)$ scalar moduli fields ϕ_i with $V_{\text{eff}}(\phi_i)$
- 5th forces and cosmological constraints: $m_{\phi_i} \gtrsim 30$ TeV
  $\Rightarrow$ **Moduli stabilization required!**
- ϕ_i: Kähler $T_i = t_i + i\,\tau_i$, Complex structure $U_i = \nu_i + i\,u_i$,
  Axio-dilaton $\tau = \sigma + i\,s$, $s = g_s^{-1}$
- String theory seems to contain an enormous number of vacua:
  Flux quanta $-5 \leq n_i \leq 5$ for 100 moduli $\Rightarrow \sim 10^{100}$ vacua
  [Bousso, Polchinski '00] $\Rightarrow$ Lots of tuning available in principle!
- **Does string theory provide such an extremely tunable example = our universe?** $\Rightarrow$ Highly non-trivial constraint on the theory!

$\Rightarrow$ **Try to find explicit examples of dS vacua in string theory!**

Motivation: String theory and dark energy (II)

Questions we do not address:

- *Why* is Λ so small?
  - Anthropic explanation? [Weinberg '89], [Susskind '05]
  - Statistical preference? [Sumitomo,Tye '11 & '12]
- Standard model [Blumenhagen,Cvetic,Ibanez,Lüst,Uranga,..],[Gmeiner,Honecker '08]
- Inflation [Kachru,Kallosh,Linde,Maldacena,McAllister,Trivedi '03, ...]

4D effective field theory description:

- String scale
- Kaluza-Klein scale
- Moduli stabilization scale
- SUSY breaking scale
- ...
- Cosmological constant scale

Effective supergravity description of type IIB string theory

$\mathcal{N} = 1$, 4D effective potential completely determined by Kähler potential K and superpotential W:

$$V = e^K \left(K^{\alpha\bar{\beta}} D_\alpha W \overline{D_\beta W} - 3|W|^2 \right)$$

For $\hat{\xi} \ll \hat{\mathcal{V}}$ this is to 0-th order the positive semi-definite potential

[Balasubramanian,Berglund,Conlon,Quevedo'05],[Westphal,MR'11]

$$V = e^K \left(K^{\tau\bar{\tau}} \left|D_\tau W_0\right|^2 + K^{a\bar{b}} D_a W_0 \overline{D_b W_0} \right) + \mathcal{O}(\frac{\hat{\xi}}{\hat{\mathcal{V}}})$$

due to no-scale structure [Cremmer,Ferrara,Kounnas,Nanopoulos'83]

- **Every SUSY extremum for the τ, U_a is a minimum for $\hat{\mathcal{V}} \gg \hat{\xi}$**
- **Separation of scales $\Rightarrow$ Stabilize U_a first**

Ingredients for de Sitter vacua in type IIB/F-theory

What is the structure of K and W?

- Quantized RR and NS-NS fluxes $\int F_3, H_3 \in \mathbb{Z}$

  $\Rightarrow$ Flux superpotential $W_0 \propto \int F_3 - \tau \int H_3$

  $\Rightarrow$ SUSY vacua $(D_a W = 0)$ for the U_a and τ

  [Giddings,Kachru,Polchinski '01]

- Non-perturbative effects $W = W_0 + A_i\, e^{-a_i\, T_i}$

  [Kachru,Kallosh,Linde,Trivedi '03]

- Leading α'-corr. to the Kähler pot. $K = -2\ln\left[\hat{\mathcal{V}}(T_i) + \alpha'^3 \hat{\xi}(\tau)\right]$

  [Becker,Becker,Haack,Louis '02]

Uplifting to de Sitter

$W_0 \ll 1$, α'-correction negligible [KKLT '03]

$W_0 \neq 0$, α'-correction [Balasubramanian,Berglund '05]

$\hat{\mathcal{V}} >>> \hat{\xi}$ W_0 arbitrary → **LVS** [Balasubramanian, Berglund,Conlon,Quevedo '05]

$\hat{\mathcal{V}} >> \hat{\xi}$ $W_0 \sim \mathcal{O}(1-100)$ → **Kähler uplifting** [Westphal '06]

KKLT

- $\bar{D}3$ branes
- F-terms from matter fields [Lebedev,Nilles,Ratz'06]
- F-terms from metastable vacua in gauge theories [Intriligator,Seiberg,Shih'07]

LVS

- $\bar{D}3$ branes
- D-terms [Burgess,Kallosh,Quevedo'03, Haack,Krefl,Lüst,Van Proyen,Zagermann'06] [Cicoli,Krippendorf,Mayrhofer,Quevedo,Valandro'12]

Kähler uplifting

- F-terms from Kähler moduli + α'-correction sufficient for dS

2. Complex structure moduli stabilization of $\mathbb{CP}_{11169}[18]$

$$\mathbb{CP}_{11169}: \ (x_1, x_2, x_3, x_4, x_5) \sim (\lambda x_1, \lambda x_2, \lambda x_3, \lambda^6 x_4, \lambda^9 x_5)$$

e.g. $x_1^{18} + x_2^{18} + x_3^{18} + x_4^3 + x_5^2 = 0 \qquad (h^{1,1} = 2 \text{ and } h^{2,1} = 272)$

Complex structure moduli stabilization of $\mathbb{CP}_{11169}[18]$ (I)

Need to find $f = \int F_3 \in \mathbb{Z}$ and $h = \int H_3 \in \mathbb{Z}$ with

$$D_a W = 0\,, a = 1, .., h^{2,1} = 272$$ [Giddings,Kachru,Polchinski'01]

Effective theory completely determined by $\mathcal{N} = 2$ prepotential:

$$\mathcal{G}(U_1, \ldots, U_{h^{2,1}}) = \sum_{i+j\leq 3} c_{ij} U_a^i U_b^j + \xi + \mathcal{G}_{\text{instanton}}(e^{-2\pi U_1}, \ldots, e^{-2\pi U_{h^{2,1}}})$$

- $W_0 = \int (F_3 - \tau H_3) \wedge \Omega(U_a) = (f - \tau h) \cdot (\mathcal{G}_{U_a}, U_a)$ [Gukov,Vafa,Witten'00]
- $K_{\text{cs}} = -\ln\left[i(\bar{U}_a \mathcal{G}_{U_a} - U_a \bar{\mathcal{G}}_{U_a})\right]$
- **Large complex structure limit:** $\left|\frac{\mathcal{G}_{\text{instanton}}}{\mathcal{G}_{\text{cubic}}}\right| \leq \epsilon_{LCS}$

Complex structure moduli stabilization of $\mathbb{CP}_{11169}[18]$ (II)

- Discrete symmetry $\mathbb{Z}_6 \times \mathbb{Z}_{18}$: $U_a\,, a = 1, .., h^{2,1}_{\text{inv.}} = 2$ and $\tilde{U}_a\,, a = 3, .., 272$ [Greene,Plesser'89], [Candelas,Font,Katz,Morrison'94]
- Switch on flux only on $h^{2,1}_{\text{inv.}} = 2$ [Giryavets,Kachru,Tripathy,Trivedi '03]
- Symmetry $\Rightarrow D_{\tilde{U}_a} W_0 = 0$ at $\tilde{U}_a = 0$ for $a = 3, \ldots, 272$
- $\Rightarrow$ **Only need to solve $\mathbf{D_\phi W|_{\tilde{U}_a=0} = 0}$ for $\boldsymbol{\phi = \tau, U_1, U_2}$**
- $\Rightarrow V \sim |D_a W|^2$ ensures stable minimum for all 272 U_a in the large volume limit!

Scanning all vacua with paramotopy

see e.g. [Li '03], [Sommese,Wampler '05]

Have to solve polynomial system $P(x) = (p_1(x), \ldots, p_m(x))^T = 0$ with $x = (x_1, \ldots, x_m)^T$

- Maximal number of isolated solutions in $\mathbb{C}^m$: $\prod_{i=1}^{m} d_i$, with d_i the degree of the ith polynomial. (*Classical Bézout bound*)

- Construct homotopy $H(x,t) = \gamma(1-t)Q(x) + t\,P(x)$, with e.g. $Q(x) = \left(x_1^{d_1} - 1, \ldots, x_m^{d_m} - 1\right) \Rightarrow$ **Easy to solve.**

- Follow paths $H(x,t) = 0$ for $0 \leq t \leq 1$ for all solutions to $Q(x) = 0$

$\Rightarrow$ **Will find all solutions to P(x) = 0**

The scan

Use to solve $\mathbf{D_\phi W(f) = 0}$ for all fluxes f with $\mathbf{L \sim f^2 \leq L_{max}}$

[Martinez-Pedrera,Mehta,Westphal,MR '12]

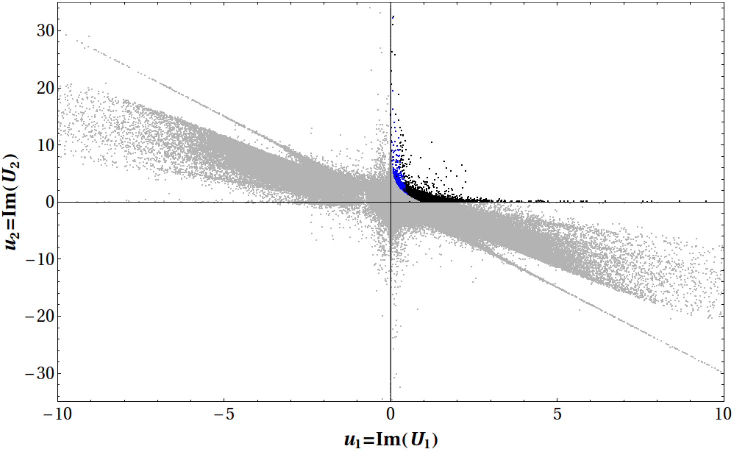
- The 10D IIB action is $SL(2,\mathbb{Z})$ invariant: $\tau \to \dfrac{a\tau + b}{c\tau + d}$,

$$G_3 = F_3 - \tau H_3 \to \frac{G_3}{c\tau + d} \quad \text{with} \quad a, b, c, d \in \mathbb{Z}\,, \quad ad - bc = 1$$

- Make sure to only consider physically inequivalent configurations!

- Our scan: ~ 50.000 parameter points ($L \leq 35$).

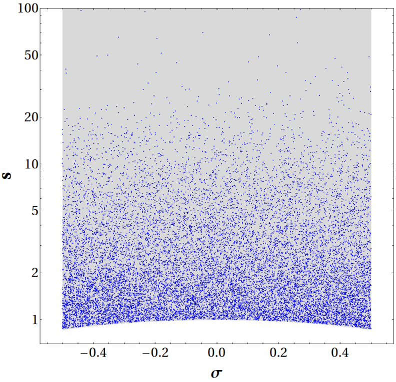
Scan results: The limit of large complex structure

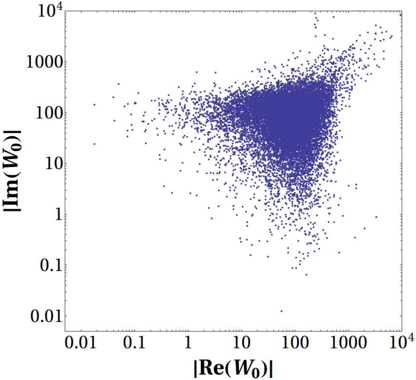


$$\left|\frac{\mathcal{G}_{\text{instanton}}}{\mathcal{G}_{\text{cubic}}}\right| \le \epsilon_{LCS} = \begin{cases} 10^{-1} & \text{(blue)} \Rightarrow 25.000 \text{ of } 500.000 \\ 10^{-2} & \text{(black)} \Rightarrow 15.000 \text{ of } \quad " \end{cases}$$

Scan results: $\tau = \sigma + i\,s$ and W_0

- $SL(2,\mathbb{Z})$ fundamental domain: $-\frac{1}{2} \leq \text{Re}(\tau) \leq \frac{1}{2}$ and $|\tau| > 1$
- Transformations: $\tau \to \tau + b\,,\ G_3 \to G_3$ and $\tau \to -1/\tau\,,\ G_3 \to G_3/\tau$



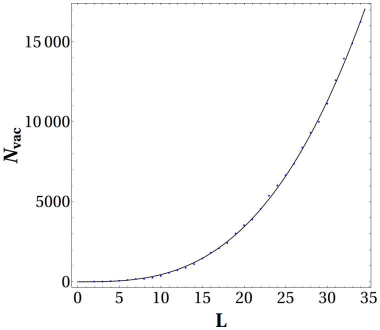


Scan results: Number of vacua N_{vac}

- $$N_{vac}^{\text{stat}} = \frac{(2\pi L)^3}{3!} \int \det(-\mathcal{R} - \mathbf{1} \cdot \omega) \simeq 0.03\, L^3$$

  [Ashok,Douglas'04],[Denef,Douglas,Florea'04]

- $N_{vac} \simeq (0.50 \pm 0.04)\, L^{2.94 \pm 0.03}$



Scan results: Tuning of the cosmological constant

- Untuned cosmological constant (cc): $\Lambda \sim \frac{m_{3/2}^2}{\hat{\mathcal{V}}}$
- Spacing in cc: $\frac{\Delta\Lambda}{\Lambda} \sim 2\,\frac{\Delta m_{3/2}}{m_{3/2}} \sim \frac{C}{L^{a\,(h_{\text{eff}}^{2,1}+1)}}$
- Fit, **Extrapolate** $\Rightarrow \frac{\Delta\Lambda}{\Lambda} \simeq (6.0 \pm 0.3)\, L^{-(0.95\pm 0.005)\,(h_{\text{eff}}^{2,1}+1)}$

$h_{\text{eff}}^{2,1}$	L	$\Delta\Lambda/\Lambda$
2	34	$7\cdot 10^{-3} \pm 5\cdot 10^{-4}$
2	500	$5\cdot 10^{-5} \pm 4\cdot 10^{-6}$
40	34	$3\cdot 10^{-58} \pm 2\cdot 10^{-58}$
40	500	$10^{-102} \pm 10^{-102}$

3. Kähler uplifted de Sitter vacua

Bottom up SUGRA analysis: Kähler uplifting

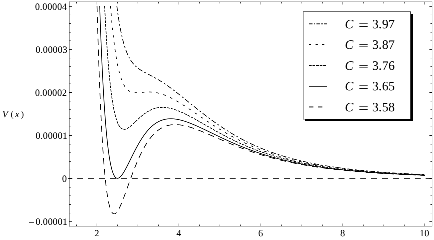
If a 3-fold is 'swiss-cheese', i.e. $\hat{\mathcal{V}}(T_i) = \gamma_1 \mathrm{Re}(T_1)^{3/2} - \sum_{i=2}^{h^{1,1}} \gamma_i \mathrm{Re}(T_i)^{3/2}$

one can perform a complete perturbative moduli stabilization in the limit

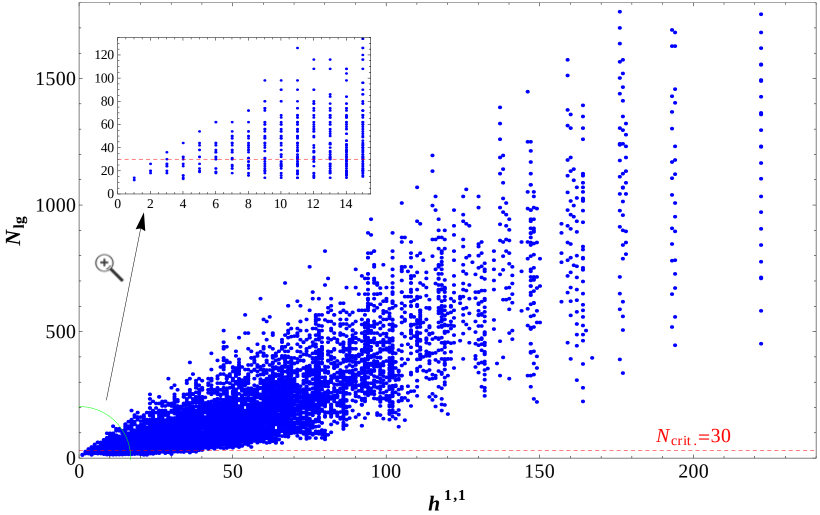
- $\hat{\mathcal{V}} \gg \hat{\xi}$ $\Rightarrow$ Large but not LARGE Volume $\hat{\mathcal{V}}$.
- $|W_0| \gg A_i e^{-a_i t_i}$, $a_i = 2\pi/N_i$ $\Rightarrow$ Non-perturbative effects are small.
- $D_i W(\tau, U_j) \simeq 0$ $\Rightarrow$ Supersymmetric stabilization to 0-th order.

Sufficient for a de Sitter vacuum: [MR, Westphal '11], [de Alwis, Givens '11]

- $3.65 < \dfrac{27|W_0|\hat{\xi}a^{3/2}}{64\sqrt{2}\gamma A} < 3.89.$
- $\hat{\mathcal{V}} \simeq \gamma N_1^{3/2} \Rightarrow$ Need $N_1 \gtrsim 30$.



Maximal gauge group ranks



⇒ $N_{lg} > 30$ quiet natural for larger $h^{1,1}$!

Constraints on a consistent global model

- Contribution of gaugino condensation to the superpotential, $A \neq 0$:
  - Rigid divisor? [Witten'96]
  - Can it be 'rigidified' by gauge flux $\mathcal{F}$? [Martucci'06, Bianchi,Collinucci,Martucci'11]
- Swiss-cheese?
- $N_1 \gg 1$ enforces factorization of D7 brane equation in coordinates $u_i \neq u_1$ [Cicoli,Mayrhofer,Valandro'11]?
- Flux: Freed-Witten anomalies?[Minasian,Moore'96, Freed,Witten'97]
- Chiral matter at brane intersections that might destroy $A \neq 0$ [Blumenhagen,Moster,Plauschinn'08]?
- Stabilization inside the Kähler cone?
- D3 tadpole: $Q^{D7-\text{stacks}} + Q^{O7} = Q^{\mathcal{F}} + Q^{RR,NS-NS} + Q^{D3-\text{branes}}$?
- $A \neq$ constant but $A(\phi_i)$

4. A consistent global model on $\mathbb{CP}_{11169}[18]$

A consistent global model on $\mathbb{CP}_{11169}[18]$

[Louis,Valandro,Westphal,MR '12]

- $\hat{\mathcal{V}} = \sqrt{\frac{2}{3}}\left(\hat{\mathcal{V}}_1 + \frac{1}{3}\hat{\mathcal{V}}_5\right)^{3/2}$ ⇒ 'Approx. swiss-cheese'
- Brane config. ($N_{lg} = 27$): $Sp(24)$ on D_1 forces $SO(24)$ on D_5.
- D_5 rigid, D_1 can be 'rigidified' by gauge flux ⇒ $Sp(24) \to SU(24)$.
- Brane intersections: Switch on gauge flux $F_{1/5} + c_1(D_{1/5})/2$ to cancel Freed-Witten anomalies and tune F_1, F_5 and B such that $\mathcal{F}_{1/5} = F_{1/5} - B$ is 'trivial' ⇒ No chiral matter or D-terms.
- D3 tadpole: $Q^{RR,NS-NS} + Q^{D3-\text{branes}} = 114$.

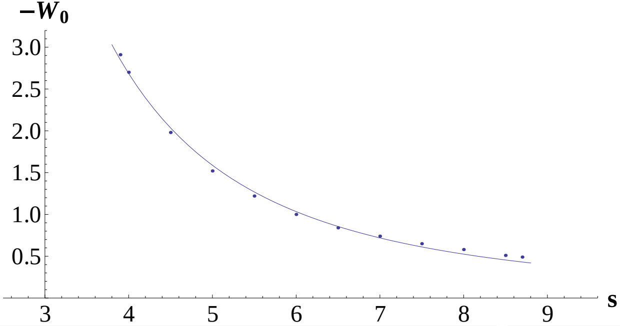
$$\Rightarrow W = W_0 + A_1\, e^{-2\pi/24\; T_1} + A_2\, e^{-2\pi/22\; T_2},\; A_1, A_2 \neq 0.$$

Kähler uplifted de Sitter vacua (I)

Global model for Kähler moduli stabilization in a de Sitter vacuum on $\mathbb{CP}_{11169}[18]$ via non-perturbative effects: [Louis,Valandro,Westphal,MR '12]

$$V = e^K \left(K^{T_i \bar{T}_j} \left[W_{T_i} \overline{W_{T_j}} + W_{T_i} \cdot \overline{W K_{T_j}} \right] + 3\hat{\xi} \frac{\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2}{(\hat{\mathcal{V}} - \hat{\xi})(\hat{\xi} + 2\hat{\mathcal{V}})^2} |W|^2 \right)$$

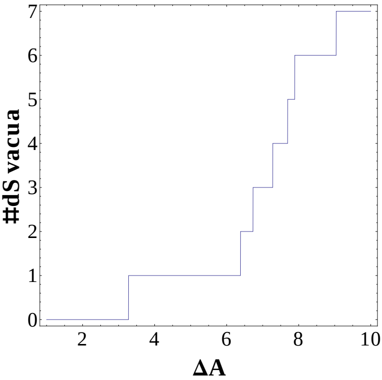
Numerically: Stable de Sitter solutions for $A_1, A_2 \sim 1$

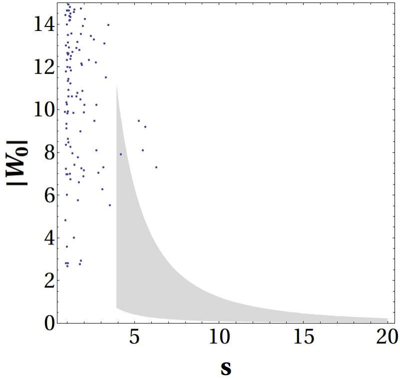


Kähler uplifted de Sitter vacua (II)

Parametrize missing knowledge of A_1, A_2 by ΔA with scaling relations:

$$W_0 \to W_0 \cdot \Delta A\,,\ \ A_1 \to A_1 \cdot \Delta A\,,\ \ A_2 \to A_2 \cdot \Delta A \quad \Rightarrow V \to V \cdot \Delta A^2$$





(Uplifting can be applied in the shaded region for $\Delta A = 4$)

Higher order α' corrections

Recently computed α'^2 correction: [Grimm,Savelli,Weissenbacher'13]

- $K = -2\ln\left[\hat{\mathcal{V}}(T_i) - \alpha'^2\,\hat{\mathcal{V}}_{D7\cap O7}(T_i) + \alpha'^3\hat{\xi}(\tau)\right]$
- Extended no-scale structure [Hebecker,von Gersdorff '05]

  $$\Rightarrow V_{\text{eff}} \sim \frac{\alpha'^3}{\hat{\mathcal{V}}^3} - \frac{\alpha'^4}{\hat{\mathcal{V}}^{10/3}} + \dots$$

- α'^2 corrections suppressed by $\hat{\mathcal{V}}^{1/3}$

Bounds on various moduli stabilization scenarios: [Pedro,Westphal,MR '13]

- KKLT: $\hat{\mathcal{V}} \gtrsim 10^2 \Rightarrow W_0 \lesssim 10^{-3}$
- LVS: $\hat{\mathcal{V}} \gtrsim 10^{10}$
- Kähler uplifting: $\hat{\mathcal{V}} \gtrsim 10^3$

5. Conclusions

Conclusions

- *All* flux vacua of reduced moduli space have been constructed for given D3-tadpole ($L = 35$) in the large complex structure limit
- $g_s \lesssim 1$ and $W_0 \sim \mathcal{O}(10^1 - 10^3)$ are preferred in our solutions
- N_{vac} and $\Delta\Lambda/\Lambda \sim 10^{-100}$ for $h^{2,1}_{\text{eff}} = 40$ and $L = 500$ consistent with semi-analytical predictions [Ashok,Douglas'04],[Denef,Douglas,Florea'04]
- Sufficient condition for de Sitter with all moduli stabilized
- Large gauge groups are quiet natural for many Kähler moduli
- Consistent global model of Kähler uplifting on $\mathbb{CP}_{11169}[18]$ $\Rightarrow \sim 10^{-4}$ flux vacua can be uplifted
- Only the dependence of the 1-loop determinant A remains implicit but negligible due to mass scale separation