Lamb shift in muonic hydrogen and the proton charge radius puzzle

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Proton charge radius puzzle

- global fit to H and D spectrum: $r_p = 0.8758(77)$ fm (CODATA 2010)
- $e - p$ scattering: $r_p = 0.8791(79)$ (Bernauer, 2010)
- from muonic hydrogen: $r_p = 0.84089(39)$ fm (PSI, 2010, 2012)

If all these measurements and Lamb shift calculations are correct, this discrepancy does not find explanation within the known description of electroweak and strong interactions.

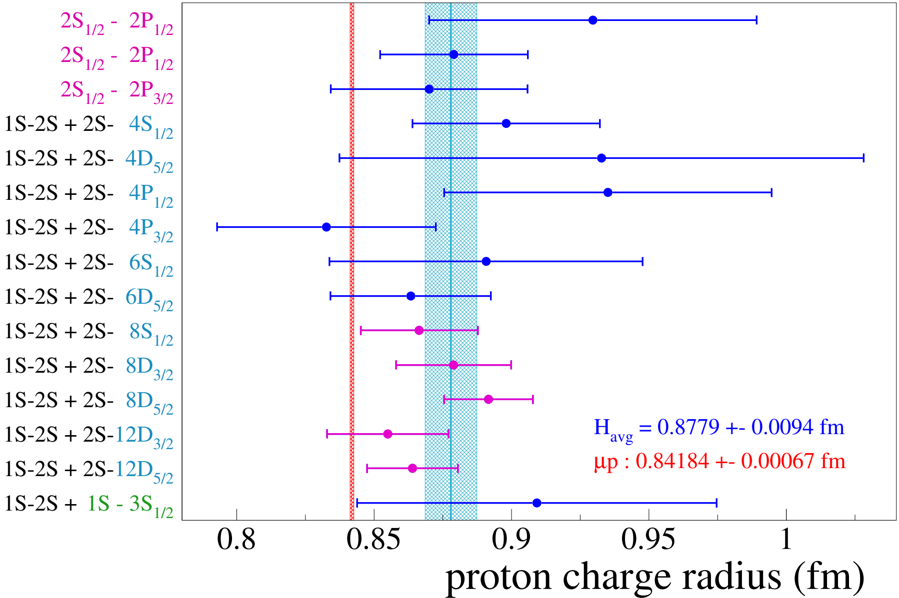
Proton charge radius and the Rydberg

- Hydrogenic energy levels depend on R_∞, r_p and other constants which uncertainties are irrelevant.

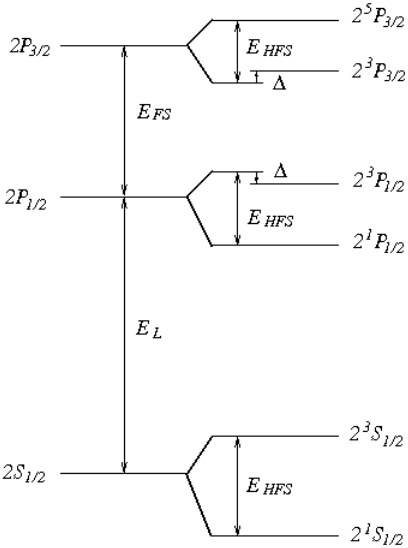
$$\begin{aligned} E &= R_\infty \, f(\alpha, m_e/m_p) \\ \delta E &= \frac{2\,\pi\,\alpha}{3}\,\phi^2(0)\,\langle r_p^2\rangle \end{aligned}$$

- Energy shift due to finite nuclear size depends mainly on r^2, the mean square nuclear charge radius.
- The remainder $\sim r^3$ is negligible for light (electronic) atoms, but not for muonic atoms !
- One fits two constants R_∞, and r_p to match the well known hydrogen $1S - 2S$ with the other transition

Experimental results for hydrogen and r_p



energy levels of μH



$E_L = 202.1\ meV$

$E_{FS} = 8.4\ meV$

$E_{HFS}(2S_{1/2}) = 22.7\ meV$

$E_{HFS}(2P_{1/2}) = 8.0\ meV$

$E_{HFS}(2P_{3/2}) = 3.4\ meV$

$\Delta = 0.1\ meV$

μH energy levels

- μH is essentially a nonrelativistic atomic system
- muon and proton are treated on the same footing
- $m_\mu / m_e = 206.768 \Rightarrow \beta = m_e/(\mu\,\alpha) = 0.737$
  the ratio of the Bohr radius to the electron Compton wavelength
- the electron vacuum polarization dominates the Lamb shift in muonic hydrogen

Theory of μH energy levels

- nonrelativistic Hamiltonian $H_0 = \frac{p^2}{2\,m_\mu} + \frac{p^2}{2\,m_p} - \frac{\alpha}{r}$

- and the nonrelativistic energy $E_0 = -\frac{m_r\,\alpha^2}{2\,n^2}$

- the evp dominates the Lamb shift

$$
\begin{aligned}
E_L = \int d^3r\, V_{vp}(r)\,(\rho_{2P} - \rho_{2S}) &= 205.007\,3\,\mathrm{meV} \\
\text{without finite size} &= 206.033\,6(5)\,\mathrm{meV}
\end{aligned}
$$

- important corrections: second order, two-loop vacuum polarization, and the muon self-energy

- other corrections are much smaller than the discrepancy of 0.3 meV.

Leading relativistic correction

Breit-Pauli Hamiltonian

$$
\begin{aligned}
H_{BP} &= H_0 + \delta H_{BP} \\
\delta H_{BP} &= -\frac{p^4}{8\,m_\mu^3} - \frac{p^4}{8\,m_p^3} - \frac{\alpha}{2\,m_\mu\,m_p}\,p^i\left(\frac{\delta^{ij}}{r} + \frac{r^i\,r^j}{r^3}\right)p^j \\
&+ \frac{2\,\pi\,\alpha}{3}\left(\langle r_p^2\rangle + \frac{3}{4\,m_\mu^2} + \frac{3}{4\,m_p^2}\right)\delta^3(r) \\
&+ \frac{2\,\pi\,\alpha}{3\,m_\mu\,m_p}\,g_\mu\,g_p\,\vec{s}_\mu\cdot\vec{s}_p\,\delta^3(r) - \frac{\alpha}{4\,m_\mu\,m_p}\,g_\mu\,g_p\,\frac{s_\mu^i\,s_p^j}{r^3}\left(\delta^{ij} - 3\,\frac{r^i\,r^j}{r^2}\right) \\
&+ \frac{\alpha}{2\,r^3}\,\vec{r}\times\vec{p}\left[\vec{s}_\mu\left(\frac{g_\mu}{m_\mu\,m_p} + \frac{(g_\mu-1)}{m_\mu^2}\right) + \vec{s}_p\left(\frac{g_p}{m_\mu\,m_p} + \frac{(g_p-1)}{m_p^2}\right)\right.
\end{aligned}
$$

Leading relativistic correction

$$
\begin{aligned}
\delta_{\rm rel} E_L &= \langle 2P_{1/2}|\delta H_{BP}|2P_{1/2}\rangle - \langle 2S_{1/2}|\delta H_{BP}|2S_{1/2}\rangle \\
&= \frac{\alpha^4\, m_r^3}{48\, m_p^2} = 0.05747\,\mathrm{meV}
\end{aligned}
$$

- valid for an arbitrary mass ratio
- quite small and highr order relativistic corrections are negligible

Leading vacuum polarization

$$
\begin{aligned}
V_{\rm vp}(r) &= -\frac{Z\,\alpha}{r}\,\frac{\alpha}{\pi}\int_4^\infty \frac{d(q^2)}{q^2}\, e^{-m_e\,q\,r}\, u(q^2) \\
u(q^2) &= \frac{1}{3}\sqrt{1-\frac{4}{q^2}}\left(1+\frac{2}{q^2}\right) \\
\delta_{\rm vp}E_L &= \langle 2P_{1/2}|V_{\rm vp}|2P_{1/2}\rangle - \langle 2S_{1/2}|V_{\rm vp}|2S_{1/2}\rangle = 205.0073\,\mathrm{meV}
\end{aligned}
$$

- the dominating part of the muonic hydrogen Lamb shift
- the expectation value is taken with nonrelativistic wave function
- the muon-proton mass ratio η is included exactly

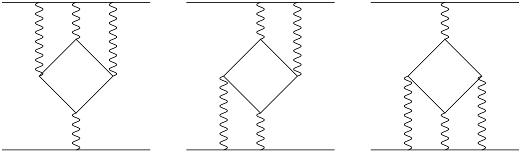
Higher order vacuum polarization

- second order V_{vp}: $\delta E_L = 0.1509$ meV
- two-loop vp: $\delta E_L = 1.5081$ meV
- three-loop vp: $\delta E_L = 0.0053$ meV
- hadronic vp: $\delta E_L = 0.0112(4)$ meV

Muonic vp is included later togethr with self-energy

Is there any further corrrection related to vp ?

Light by light diagrams



- $\delta E_L = -0.0009$ meV
- significant cancellation between diagrams
- S.G. Karshenboim *et al.*, arXiv:1005.4880

Small corrections

- relativistic correction to vp

$$\delta_{\mathrm{vp,rel}} E_L = \langle \delta_{\mathrm{vp}} H_{BP} \rangle + 2 \left\langle V_{\mathrm{vp}} \frac{1}{(E-H)'} H_{\mathrm{BP}} \right\rangle$$
$$= 0.01876\,\mathrm{meV}.$$

If one used the Dirac equation in the infinite nuclear mass limit, the obtained result would be 0.021 meV

- muon self-energy and muon vp: $\delta E_L = -0.6677$ meV
- muon self-energy combined with evp: $\delta E_L = -0.0025$ meV

Pure recoil corrections

We pass now, to remaining corrections which have an overlap with the proton elastic structure and polarizability effects, and not always are treated consistently in the literature

Pure recoil corrections of order α^5: their derivation requires full QED treatment and the obtained result

$$E(n,l) = \frac{m_r^3}{m_\mu\, m_p} \frac{(Z\,\alpha)^5}{\pi\, n^3} \quad \left\{ \frac{2}{3}\,\delta_{l0}\,\ln\left(\frac{1}{Z\,\alpha}\right) - \frac{8}{3}\,\ln k_0(n,l) - \frac{1}{9}\,\delta_{l0} - \frac{7}{3}\,a_n \right.$$
$$\left. - \frac{2}{m_p^2 - m_\mu^2}\,\delta_{l0}\left[m_p^2\,\ln\left(\frac{m_\mu}{m_r}\right) - m_\mu^2\,\ln\left(\frac{m_p}{m_r}\right)\right]\right\}$$

where

$$a_n = -2\,\left(\ln\left(\frac{2}{n}\right) + \left(1 + \frac{1}{2} + \ldots + \frac{1}{n}\right) + 1 - \frac{1}{2\,n}\right)\,\delta_{l0} + \frac{1 - \delta_{l0}}{l\,(l+1)\,(2\,l+1)}$$

is valid for an arbitrary mass of particles: $\delta E_{LS} = -0.0450\,\mathrm{meV}$

Proton self-energy

- The proton self energy leads to the modification of elastic form factors in such a way that they depend on a fictitious photon mass
- one takes the simplest possible point of view and use the formula for the low energy part of the proton self-energy

$$\begin{aligned}\delta E &= \frac{4\, m_r^3\, (Z^2\,\alpha)\,(Z\,\alpha)^4}{3\,\pi\, n^3\, m_p^2}\left(\delta_{l0}\, \ln\left(\frac{m_p}{m_r\,(Z\,\alpha)^2}\right) - \ln k_0(n,l)\right)\,. \\ &= -0.0099\,\mathrm{meV}\,.\end{aligned}$$

  the high energy part of the Lamb shift is by definition included in the charge radius and the magnetic moment anomaly
- how this definition corresponds to r_p from the electron scattering ?

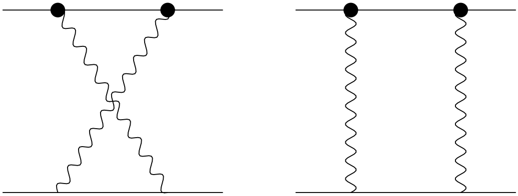
Summary of theoretical predictions

$$
\begin{aligned}
\Delta E_{\mathrm{LS}} &= 206.0336(15) - 5.2275(10)\, r_p^2 + \Delta E_{\mathrm{TPE}} \\
\Delta E_{\mathrm{FS}} &= 8.3521\,\mathrm{meV} \\
\Delta E_{\mathrm{HFS}}^{2S_{1/2}} &= 22.8089(51)\,\mathrm{meV}, \quad (\mathrm{exp.\ value}) \\
\Delta E_{\mathrm{HFS}}^{2P_{1/2}} &= 7.9644\,\mathrm{meV} \\
\Delta E_{\mathrm{HFS}}^{2P_{3/2}} &= 3.3926\,\mathrm{meV} \\
\Delta &= 0.1446\,\mathrm{meV}
\end{aligned}
$$

where $\Delta E_{\mathrm{TPE}} = 0.0351(20)$ meV is a proton structure dependent two-photon exchange contribution, on the next slide…

Nuclear structure effects

- if nuclear excitation energy is much larger than the atomic energy, the two-photon exchange scattering amplitude gives the dominating correction
- the total proton structure contribution $\delta E_L = 0.035\,1(20)$ meV is much too small to explain the discrepancy, but its calculation is uncertain [Carlson, Vanderhaeghen, 2011]



Possible sources of the proton radius discrepancy: theory

- mistake in $e - H$ calculations: all corrections calculated independently by at least two groups, uncertainty in the two-loop correction enters at 1 kHz level for 1S state, but this discrepancy corresponds to 100 kHz
- mistake in $\mu - H$: QED theory is quite simple, dominated by nonrelativistic vacuum polarization, everything checked and verified
- missing QED corrections
- significant underestimation of the proton polarizability and of the related subtraction term in dispersion relations (not known from e-p inelastic scattering, (G. Paz and R.J. Hill, J.A. McGovern)
- new interactions between the muon and the proton: a scalar with 1 MeV mass is not completely ruled out, but requires fine tuning

Possible sources of the proton radius discrepancy: experiment

- μH measurement is not verified by independent experiment
- the determination of r_p from $e - p$ scattering data requires extrapolation to $q^2 = 0$, subject of systematic uncertainties and model dependence, main issue discussed during the conference
- $2S - nS, D$ measurements (mostly from one laboratory, LKB Paris), not confirmed by independent and equally accurate measurements. Highly excited states of H are affected by various systematics. As a result the Rydberg constant might be not as accurate as claimed

New interactions

If discrepancy in r_p is to be explained by a new type of interaction between the proton (neutron) and leptons, than we have two options

- long range $\sim \lambda\!\!\!^{-}_e$, not consistent with precise measurements of the Lamb shift in H- and Li-like heavy ions at GSI
- short range $\sim$ 1fm (or shorter), can be seen in μp scatt.

Comparison of nuclear charge radii for H,D,^{3}He and ^{4}He will give hints on the range of new interactions

If it is local, than discrepancy for all these elements can be parametrized by

$$\delta E = (Z\,\delta r_p^2 + (A-Z)\,\delta r_n^2)\,\frac{2\,\delta_{l0}}{3\,n^3}\,Z^3\,\alpha^4\,\mu^3$$

Determination of r_N from muonic atoms spectra requires an accurate calculation of the nuclear polarizability correction, not necessarily easy task

Ways to go

- determine *Ry* by another accurate measurement in
  - 2S-4P in H (Garching)
  - 2S-nS,D in H (J. Flowers, NPL)
  - 1S-3S (Garching, . . .)
  - transitions between Rydberg states of heavy H-like ions (NIST)
  - 1S-2S and 1S hfs in $e\,\mu$ (A. Antonini, PSI)
- determine r_p from $2S - 2P$ transition in H: (E. Hessels)
- $\mu - p$ elastic scattering (Arrington *et al.*)
- compare charge radii from electronic and muonic spectra of other atomic systems
  - μD data are coming, r_D from very accurate H-D isotope shift (Garching)
  - $r_{\rm He}$ charge radius from 1S-2S (two-photon) transition in He^+, or $2^3S - 2^3P$ in He