On the general structure of the effective (average) action

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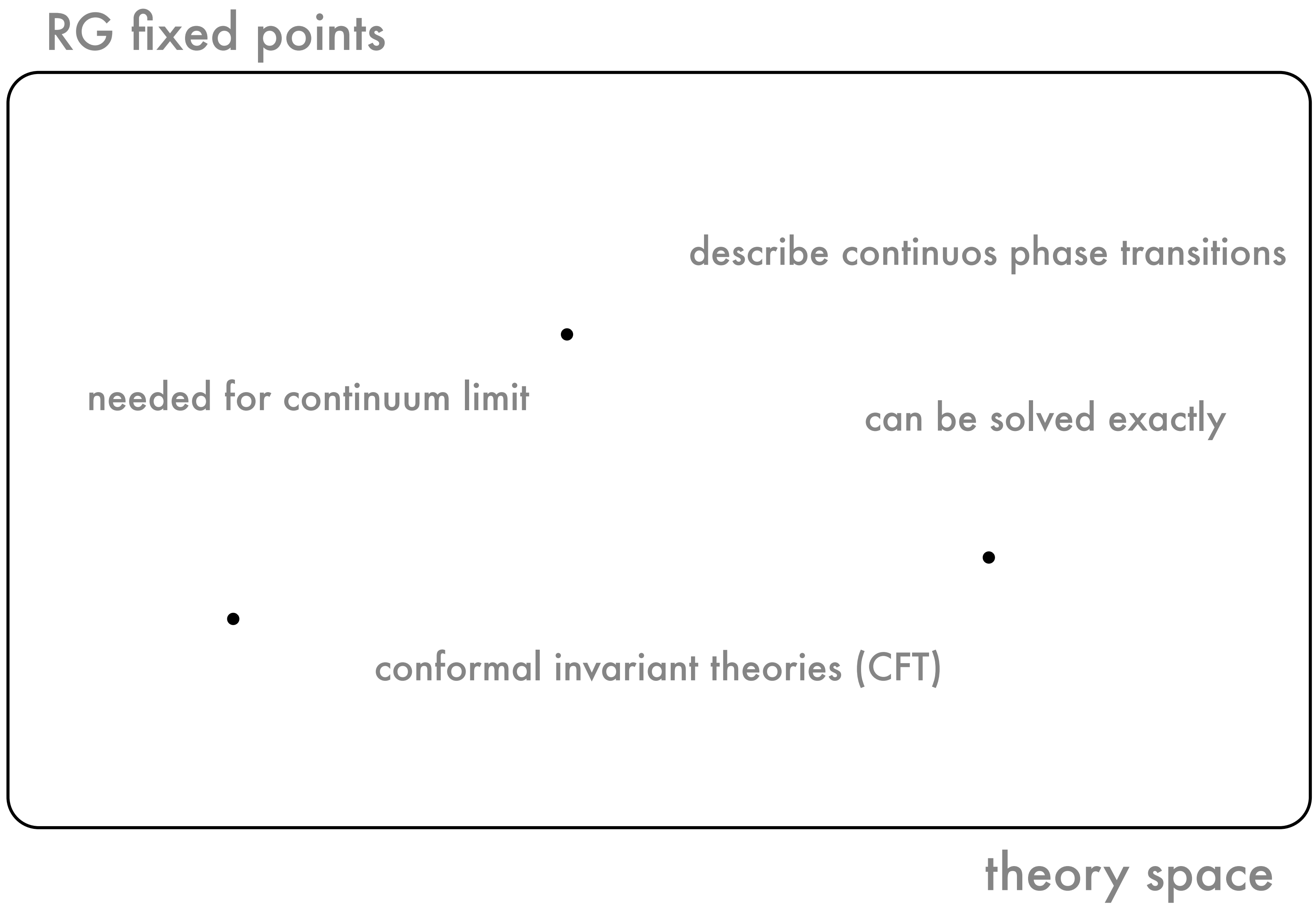
RG theory

RG flow

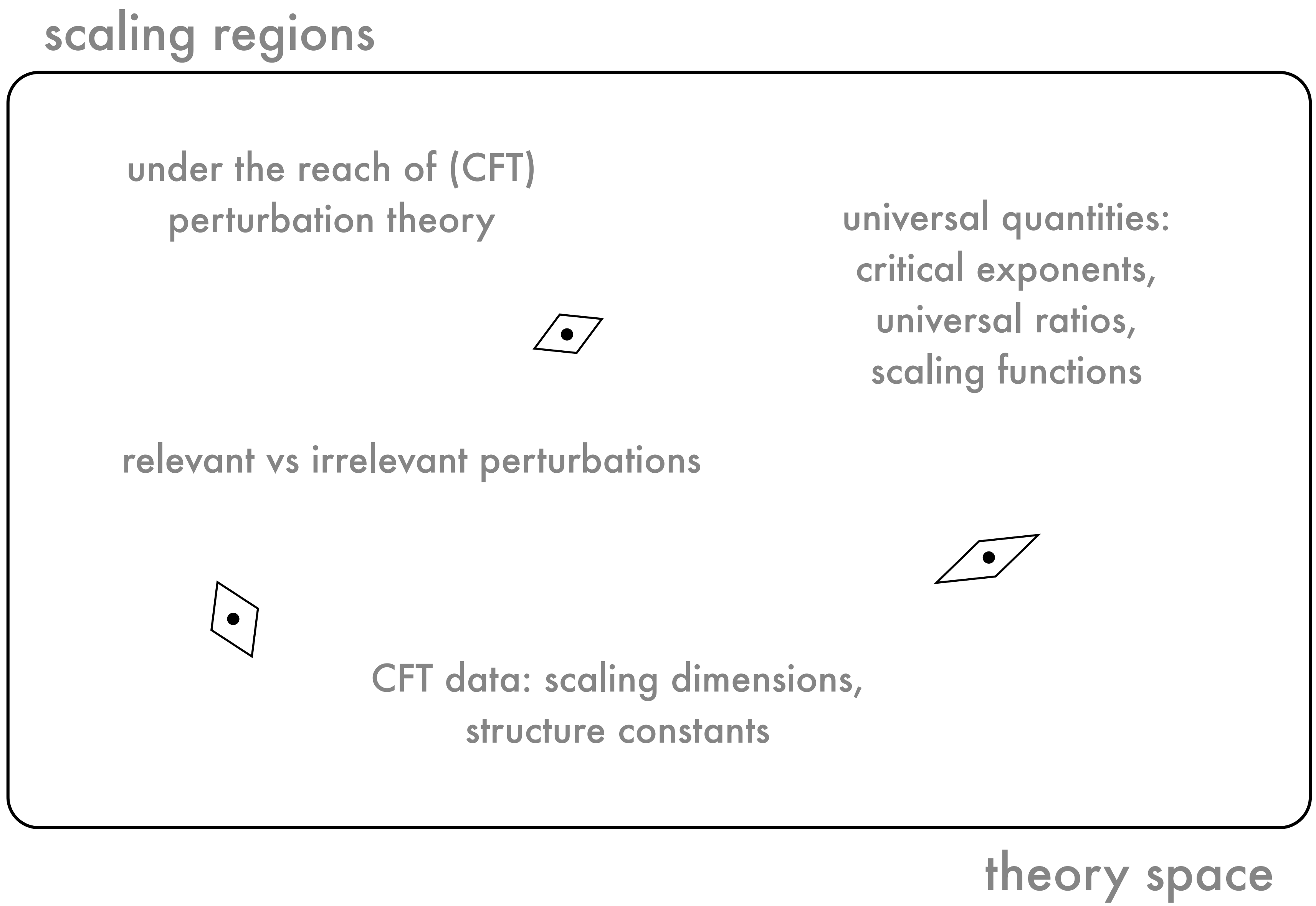
every theory consistent with the symmetries

theory space

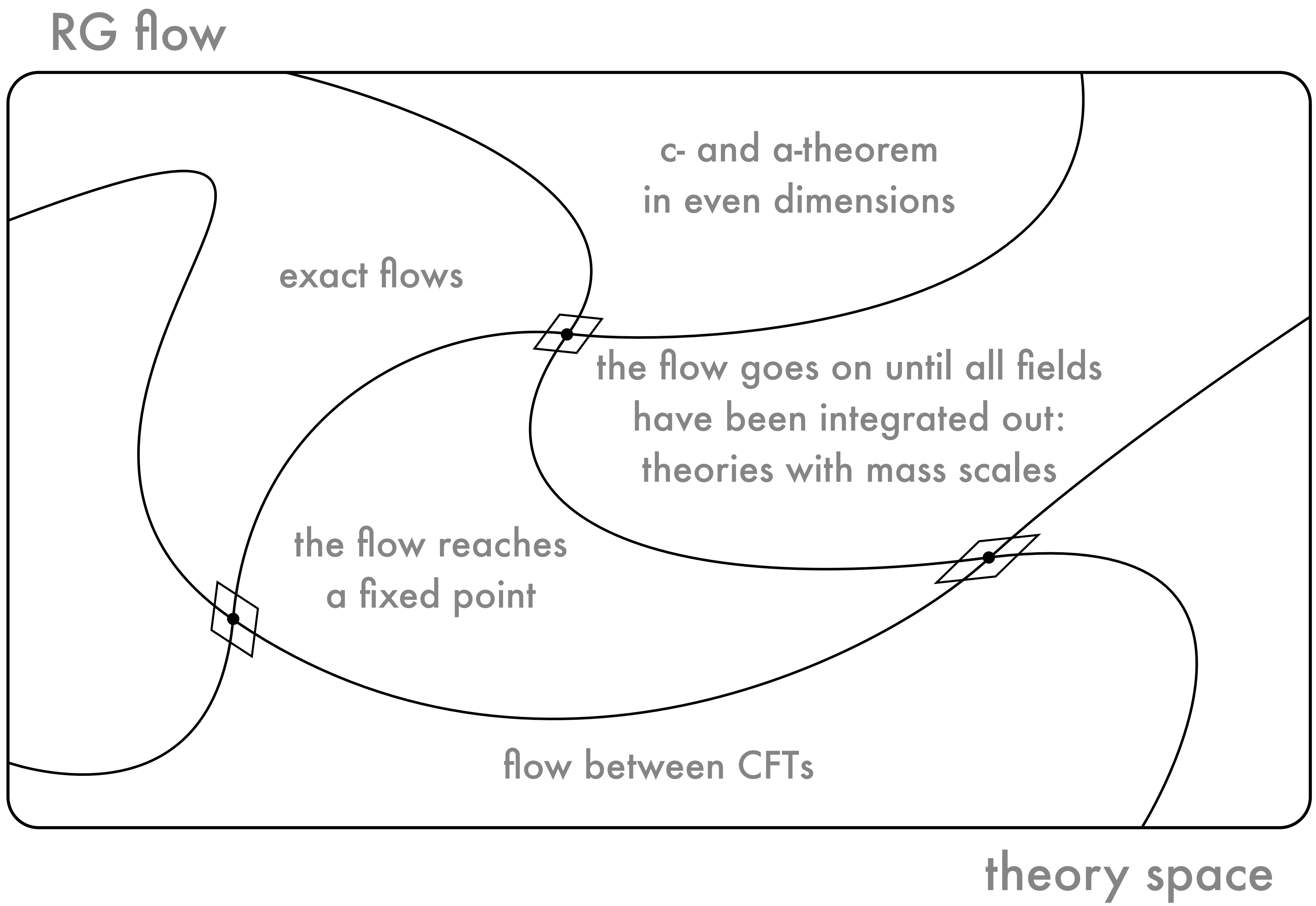
RG theory


RG fixed points
describe continuos phase transitions
needed for continuum limit
can be solved exactly
conformal invariant theories (CFT)
theory space

RG theory

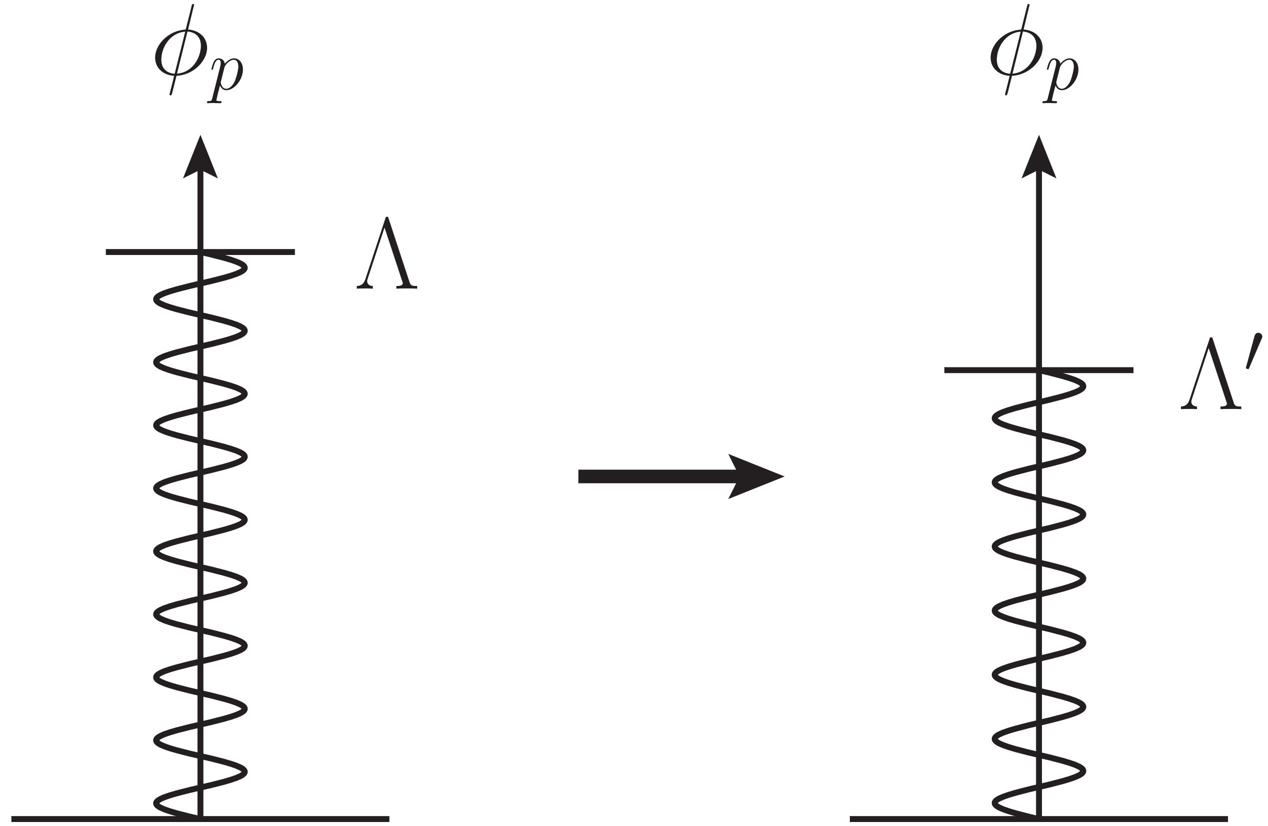

scaling regions
under the reach of (CFT)
perturbation theory
universal quantities:
critical exponents,
universal ratios,
scaling functions
relevant vs irrelevant perturbations
CFT data: scaling dimensions,
structure constants
theory space

RG theory


RG flow
c- and a-theorem
in even dimensions
exact flows
the flow goes on until all fields
have been integrated out:
theories with mass scales
the flow reaches
a fixed point
flow between CFTs
theory space

Exact RG flows

the path integral is a sum over field modes: do it step by step!

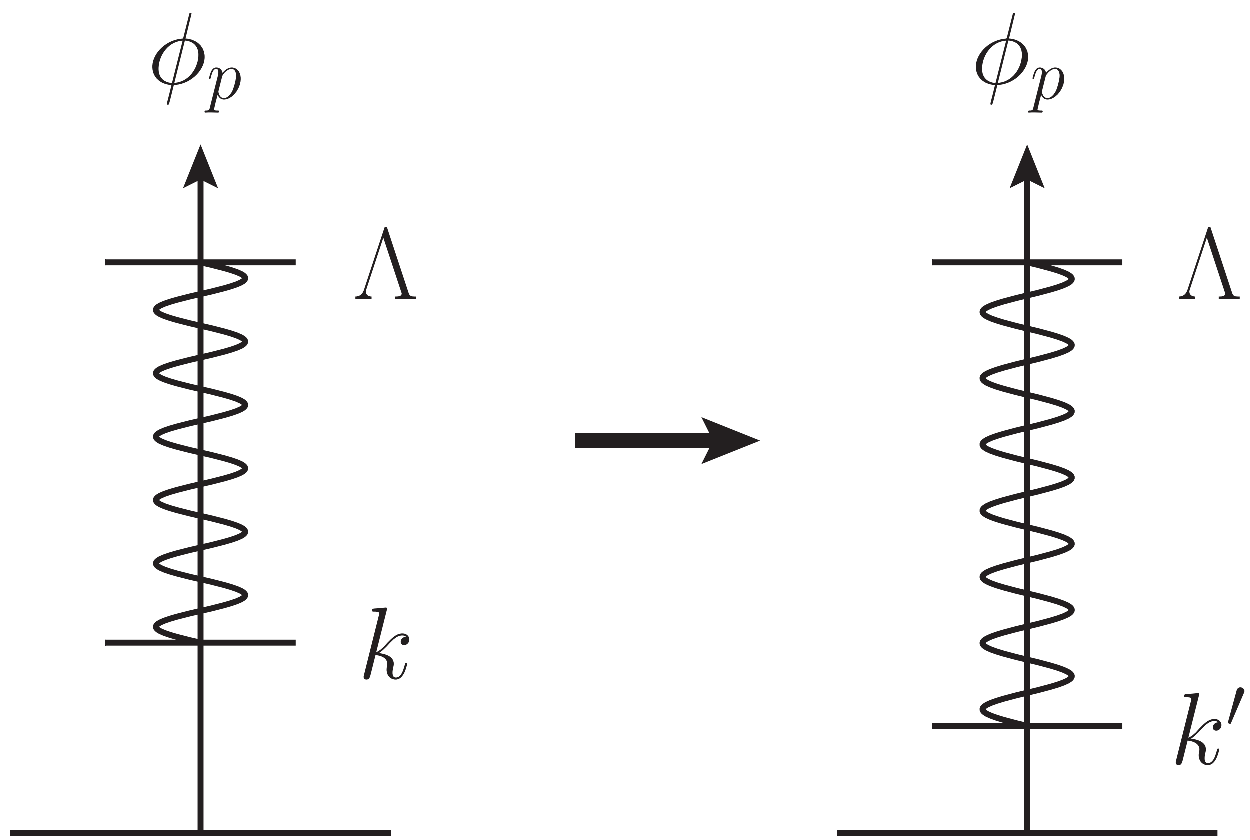


$$S_\Lambda \quad \to \quad S_{\Lambda'} \quad \to \quad S_{\Lambda''} \quad \to \quad \cdots$$

the RG flow is generated by varying the UV scale

Exact RG flows

the path integral is a sum over field modes: do it step by step!

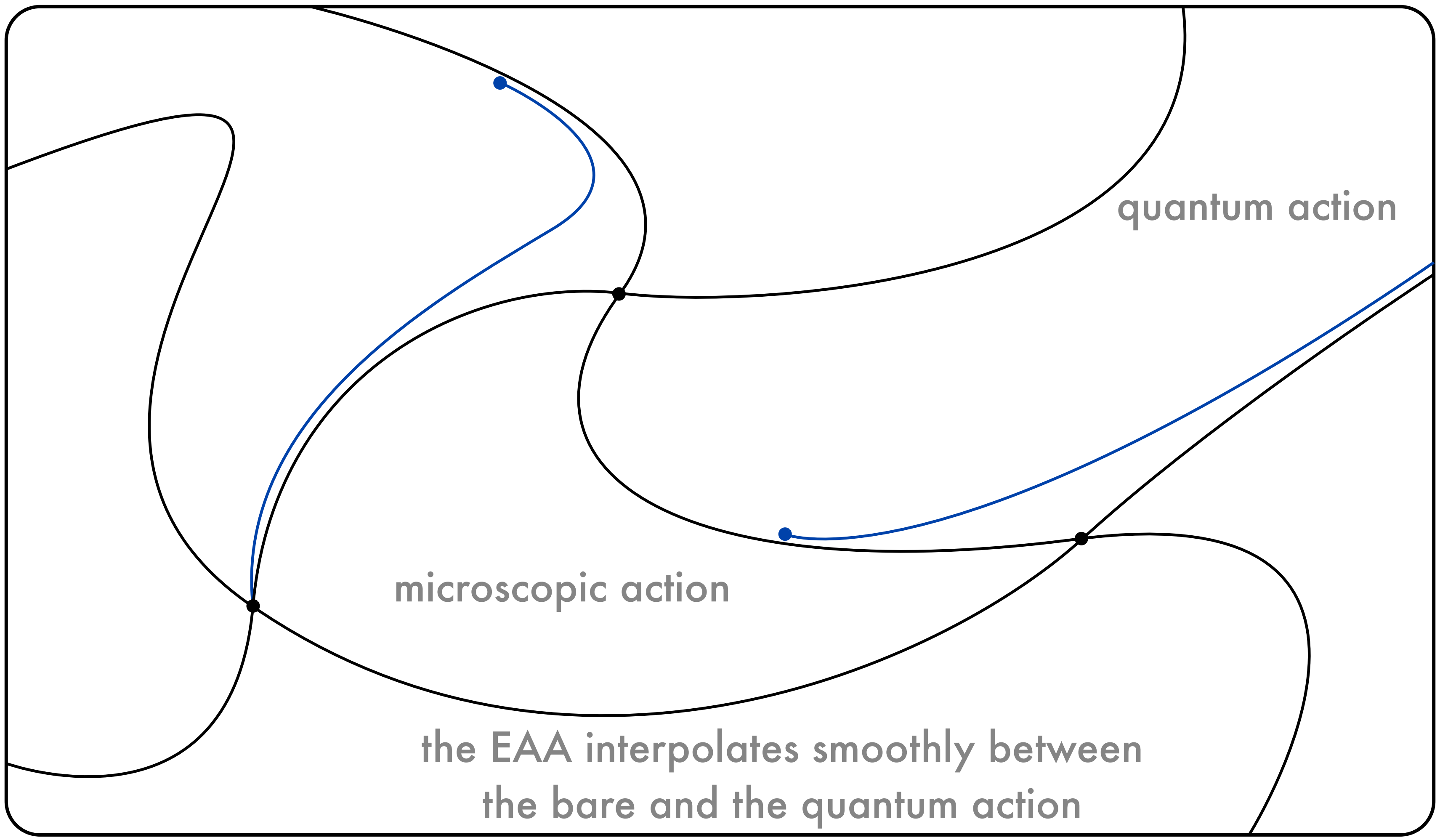


$$\Gamma_k \quad \to \quad \Gamma_{k'} \quad \to \quad \Gamma_{k''} \quad \to \quad \cdots$$

the RG flow is generated by varying the IR scale

Exact RG flows

RG flow of the effective average action



theory space

Exact RG flows

Anatomy of an equation:

exact closed equation

Hessian

non-linear

$$\partial_t \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left(\frac{\delta^2 \Gamma_k[\varphi]}{\delta\varphi\delta\varphi} + R_k \right)^{-1} \partial_t R_k$$

functional

UV finite

IR finite

integro-differential equation

Which is the general form of the effective (average) action?

Clue I: derivative expansion

Clue II: scale anomaly

Clue III: conformal anomaly

Clue IV : RG transformations can be reabsorbed by rescaling the metric

Clue 1: derivative expansion

expand the effective (average) action in an operator basis:

$$\Gamma_k[\varphi, g] = \sum_A \lambda_{A,k} \int \sqrt{g}\, \mathcal{O}^A[\varphi, g]$$

clue 1

Clue 1: derivative expansion

expand the effective (average) action in a operator basis:

$$\Gamma_k[\varphi, g] = \sum_A \lambda_{A,k} \int \sqrt{g}\, \mathcal{O}^A[\varphi, g]$$

clue 1

$$\begin{aligned}\Gamma_k[\varphi, g] &= \int \sqrt{g}\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \lambda_{2,k}\int \sqrt{g}\frac{1}{2}\varphi^2 \\ &+ \lambda_{4,k}\int \sqrt{g}\frac{1}{4!}\varphi^4 + \lambda_{6,k}\int \sqrt{g}\frac{1}{6!}\varphi^6 + \dots \\ &= \int \sqrt{g}\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \int \sqrt{g} V_k(\varphi)\end{aligned}$$

local potential approximation (LPA): all momentum dependence of the proper-vertices is dropped

Clue 1: derivative expansion

expand in powers of (momenta) derivatives and parametrize the effective average action in terms of running functions:

$$\Gamma_k[\varphi] = \int d^d x \left\{ V_k(\varphi) + \frac{1}{2} Z_k(\varphi)(\partial\varphi)^2 + \frac{1}{2} W_{1,k}(\varphi) \left(\partial^2\varphi\right)^2 \right.$$
$$\left. + \frac{1}{2} W_{2,k}(\varphi)(\partial\varphi)^2 \varphi \partial^2 \varphi + \frac{1}{4} W_{3,k}(\varphi)(\partial\varphi)^4 \right\} + O(\partial^6)$$

project the exact RG equation to obtain a set of coupled partial differential equations involving the running functions:

$$\partial_t V_k(\varphi) = \mathcal{B}_V^d [V_k(\varphi), Z_k(\varphi), ...]$$
$$\partial_t Z_k(\varphi) = \mathcal{B}_Z^d [V_k(\varphi), Z_k(\varphi), ...]$$
$$\vdots$$

the flow equations are valid in arbitrary dimension

Clue 1: derivative expansion

choose a cutoff shape function to obtain an explicit equation:

$$\partial_t V_k(\varphi) = c_d \frac{k^d}{1 + V_k''(\varphi)/k^2}$$

introduce dimensionless variables

$$\varphi = k^{d/2-1}\tilde{\varphi} \qquad\qquad V_k(\varphi) = k^d \tilde{V}_k(\tilde{\varphi})$$

to obtain the partial differential equation for the dimensionless effective potential:

$$\partial_t \tilde{V}_k(\tilde{\varphi}) = -d\,\tilde{V}_k(\tilde{\varphi}) + (d/2-1)\,\tilde{\varphi}\,\tilde{V}_k'(\tilde{\varphi}) + \frac{c_d}{1 + \tilde{V}_k''(\tilde{\varphi})}$$

Clue 1: derivative expansion

$$\tilde{\beta}_{\lambda_2} = -2\tilde{\lambda}_2 - c_d \frac{\tilde{\lambda}_4}{(1+\tilde{\lambda}_2)^2}$$

$$\tilde{\beta}_{\lambda_4} = (d-4)\tilde{\lambda}_4 + 6c_d \frac{\tilde{\lambda}_4^2}{(1+\tilde{\lambda}_2)^3} - c_d \frac{\tilde{\lambda}_6}{(1+\tilde{\lambda}_2)^2}$$

$$\tilde{\beta}_{\lambda_6} = (2d-6)\tilde{\lambda}_6 - 90c_d \frac{\tilde{\lambda}_4^3}{(1+\tilde{\lambda}_2)^4} + 30c_d \frac{\tilde{\lambda}_4\tilde{\lambda}_6}{(1+\tilde{\lambda}_2)^3} - c_d \frac{\tilde{\lambda}_8}{(1+\tilde{\lambda}_2)^2}$$

$$\tilde{\beta}_{\lambda_8} = (3d-8)\tilde{\lambda}_8 + 2520c_d \frac{\tilde{\lambda}_4^4}{(1+\tilde{\lambda}_2)^5} - 1260c_d \frac{\tilde{\lambda}_4^2\tilde{\lambda}_6}{(1+\tilde{\lambda}_2)^4} + 70c_d \frac{\tilde{\lambda}_6^2}{(1+\tilde{\lambda}_2)^3} + $$

$$+ 56c_d \frac{\tilde{\lambda}_4\tilde{\lambda}_8}{(1+\tilde{\lambda}_2)^3} - c_d \frac{\tilde{\lambda}_{10}}{(1+\tilde{\lambda}_2)^2}$$

$$\tilde{\beta}_{\lambda_{10}} = \ldots$$

the effective potential is
the generating function
for the beta functions

Clue 1: derivative expansion

at a fixed point of the RG flow the beta functions vanish:

$$\tilde{\beta}_A(\tilde{\lambda}^*) = 0$$

to obtain universal quantities we linearize the flow around the fixed point:

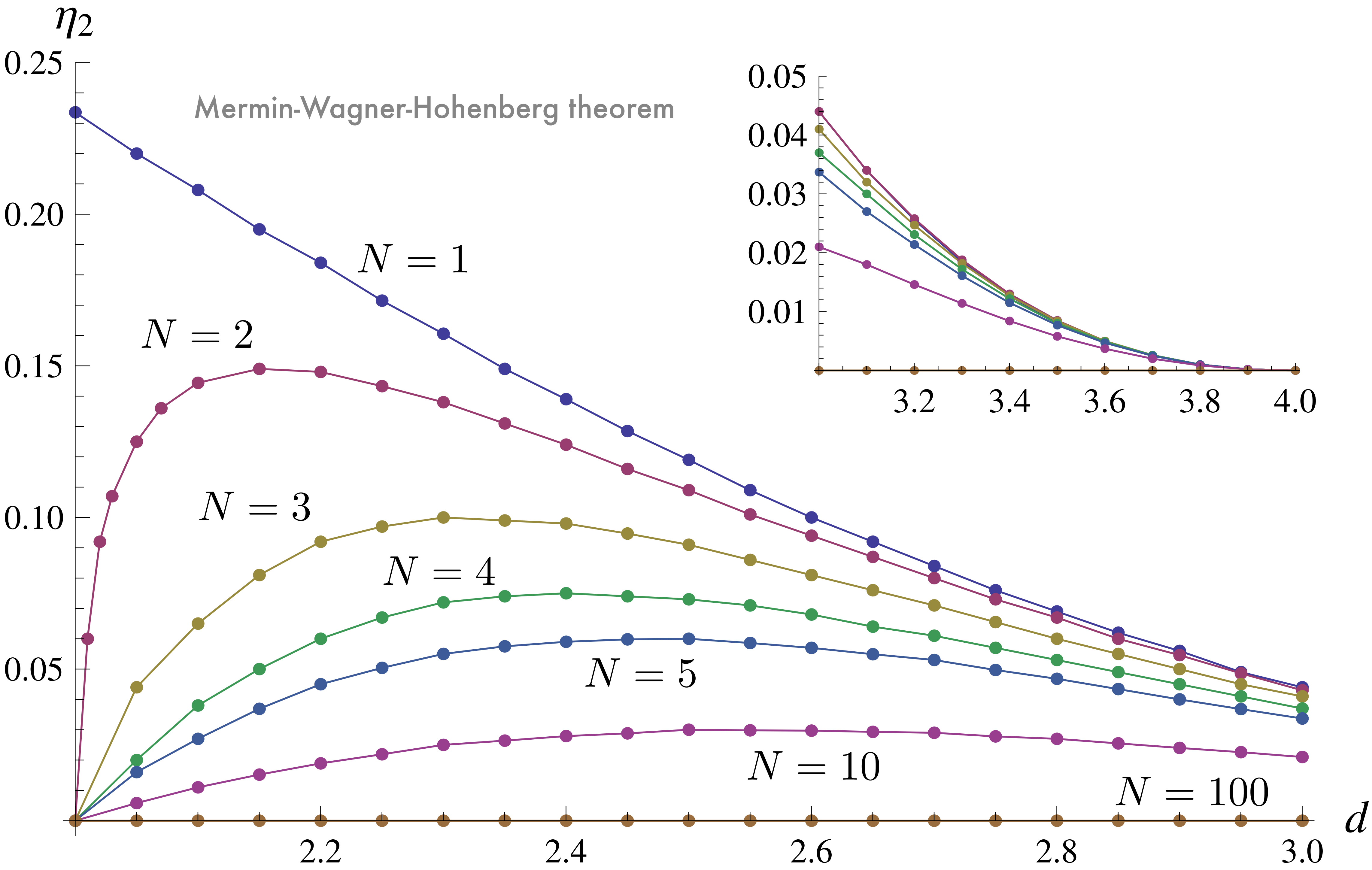
$$\tilde{\beta}_A(\delta\tilde{\lambda}) = \tilde{\beta}_A(\tilde{\lambda}^*) + \sum_B \left.\frac{\partial\tilde{\beta}_A}{\partial\tilde{\lambda}_B}\right|_* \delta\tilde{\lambda}_B + \frac{1}{2}\sum_{B,C} \left.\frac{\partial\tilde{\beta}^2_A}{\partial\tilde{\lambda}_C\partial\tilde{\lambda}_B}\right|_* \delta\tilde{\lambda}_B\delta\tilde{\lambda}_C + ...$$

the beta functions carry CFT data: scaling dimensions and structure constants:

$$\tilde{\beta}_A(\tilde{\mu}) = d_A\tilde{\mu}_A + \sum_{B,C} C_{ABC}\tilde{\mu}_B\tilde{\mu}_C + ...$$

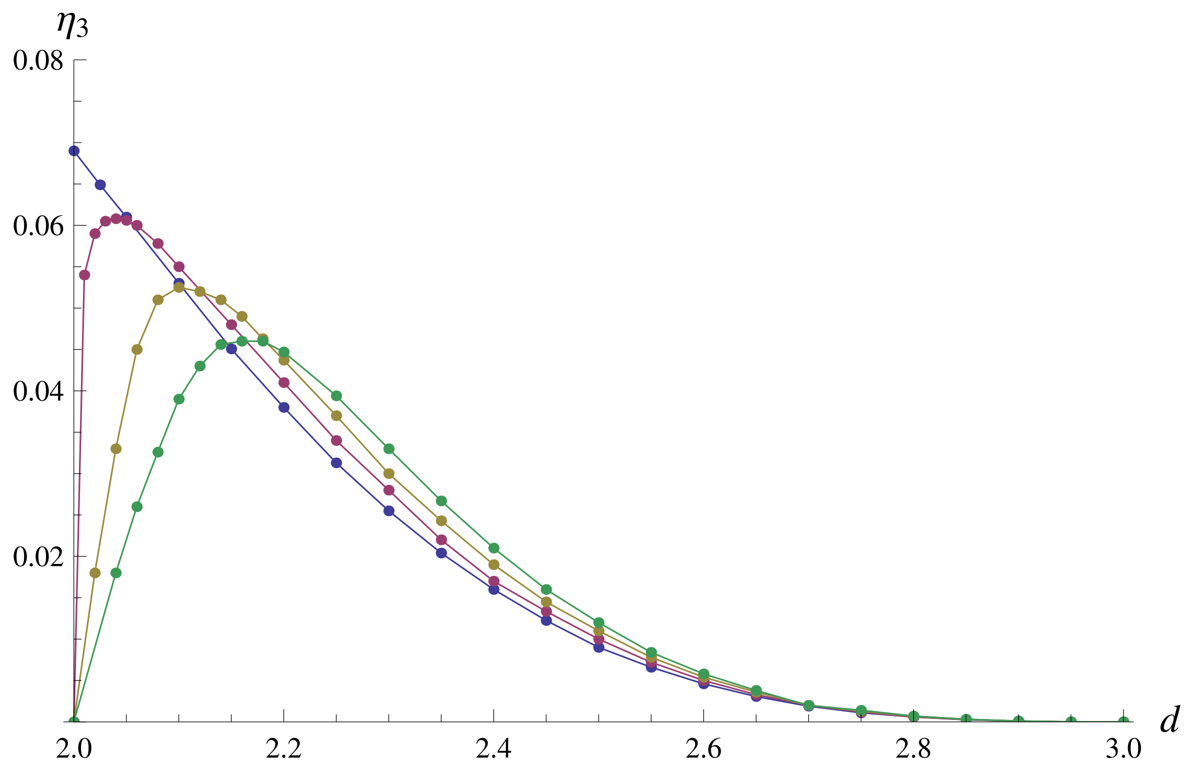
$$\tilde{\mu}_A = D_{AB}\delta\tilde{\lambda}_B$$

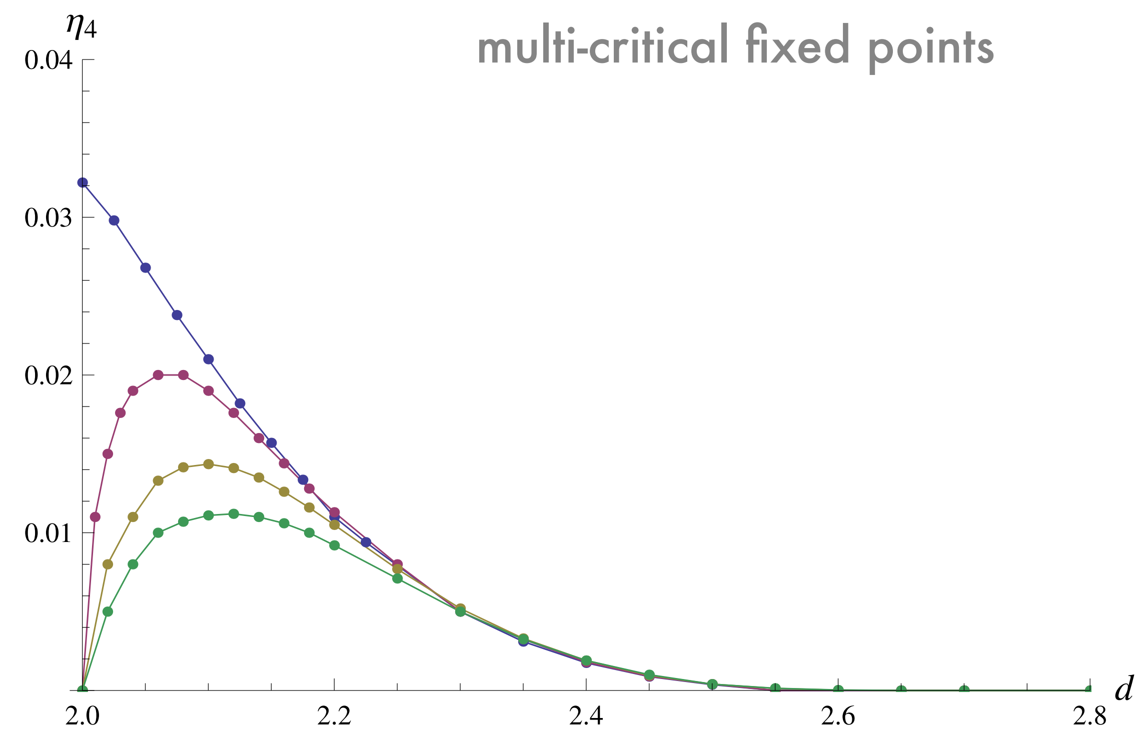
Clue 1: derivative expansion



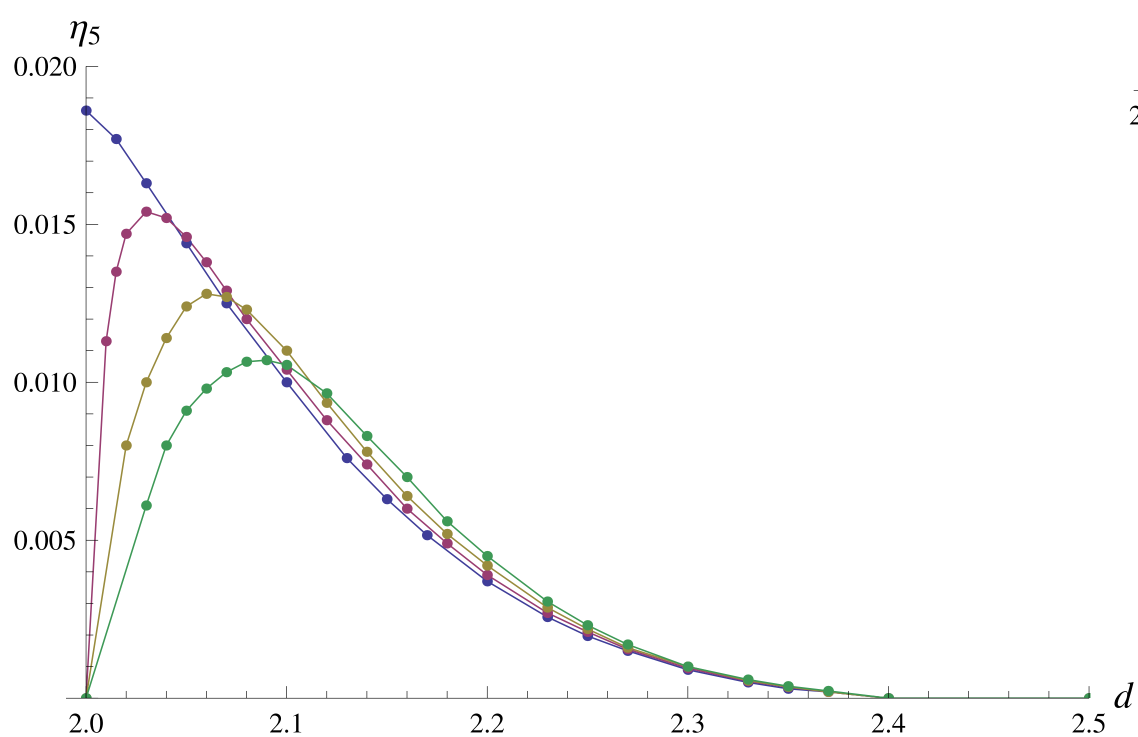
$O(N)$ models

Clue 1: derivative expansion





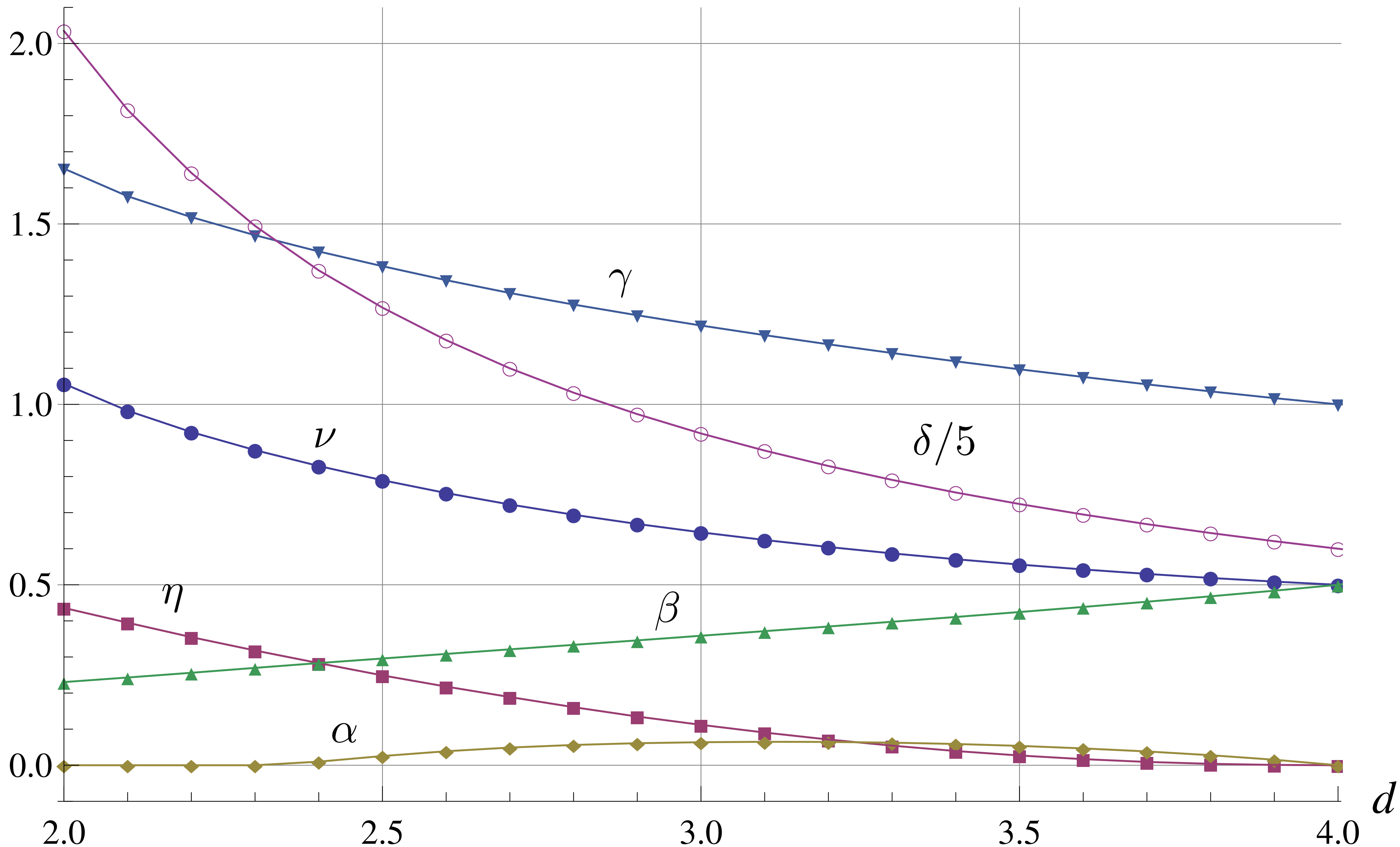
multi-critical fixed points



$$d_{c,n} = 2 + \frac{2}{n-1} = \infty, 4, 3, \frac{8}{3}, \frac{5}{2}, ...$$

minimal models of CFT

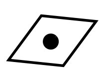
Clue 1: derivative expansion



Clue II: scale anomaly

$$S = \int \sqrt{g} \left[\frac{1}{2}\phi\Delta\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right]$$

$d = 4$



Gaussian fixed point

"classical" scale anomaly:

$$\delta_\sigma S = 2m^2 \int \sqrt{g}\frac{1}{2}\phi^2\sigma$$

Clue II: scale anomaly

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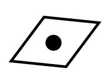

Gaussian fixed point

"classical" scale anomaly:

$$\delta_\sigma S = 2m^2 \int \sqrt{g}\frac{1}{2}\phi^2\sigma$$

quantum scale anomaly:

$$\delta_\sigma \Gamma_{1-\mathrm{loop}} = -\beta_m(m_R^2)\int \sqrt{g}\frac{1}{2}\phi^2\sigma - \beta_\lambda(\lambda_R)\int \sqrt{g}\frac{1}{4!}\phi^4\sigma$$

Clue II: scale anomaly

$$S = \int \sqrt{g} \left[\frac{1}{2}\phi\Delta\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right]$$

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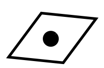

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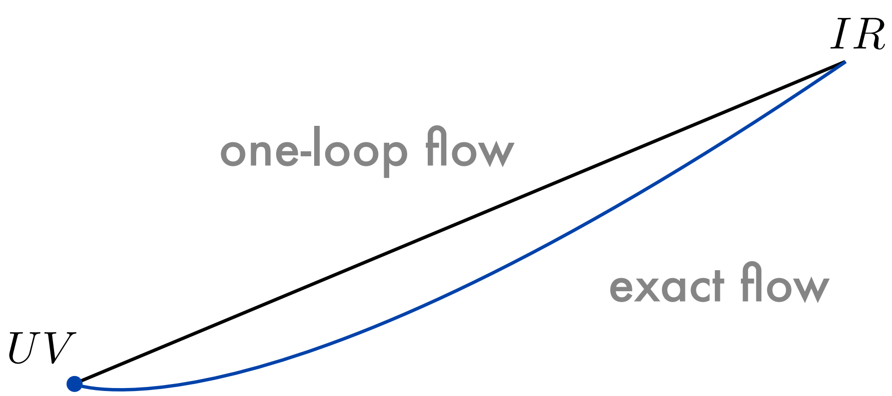
$$\beta_m = \frac{m^2\lambda}{(4\pi)^2} \qquad \beta_\lambda = \frac{3\lambda^2}{(4\pi)^2}$$

Clue II: scale anomaly

$d = 4$

$$\delta_\sigma \Gamma = \delta_\sigma S + \delta_\sigma \Gamma_{1-\text{loop}}$$

$$= -\left(\beta_m(m_R^2) - 2m_R^2\right) \int \sqrt{g}\frac{1}{2}\phi^2\sigma - \beta_\lambda(\lambda_R) \int \sqrt{g}\frac{1}{4!}\phi^4\sigma$$



quantum energy-momentum tensor:

$$\delta_\sigma \Gamma = \int \sqrt{g}\,\langle T^\mu_\mu\rangle\,\sigma$$

$$\langle T^\mu_\mu\rangle = -\left(\beta_m(m_R^2) - 2m_R^2\right) \int \sqrt{g}\frac{1}{2}\phi^2 - \beta_\lambda(\lambda_R) \int \sqrt{g}\frac{1}{4!}\phi^4$$

Clue II: scale anomaly

$d = 4$

$$\tilde{\beta}_{m^2} = -2\tilde{m}_k^2 - \frac{1}{2(4\pi)^2}\frac{\lambda_k}{(1+\tilde{m}_k^2)^2} = -2\tilde{m}_k^2 - \frac{\lambda_k}{2(4\pi)^2} + \frac{\tilde{m}_k^2\lambda_k}{(4\pi)^2} + ...$$

$$\beta_{m^2}(m_0^2, \lambda_0) - 2m_0^2 = \lim_{k\to 0} k^2 \tilde{\beta}_{m^2}(\tilde{m}_k^2, \lambda_k) = -2m_0^2 + \frac{m_0^2\lambda_0}{(4\pi)^2} + ...$$

Clue II: scale anomaly

$$d = 4$$

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$$\tilde{\beta}_\lambda = \frac{3}{(4\pi)^2}\frac{\lambda_k^2}{(1+\tilde{m}_k^2)^3} = \frac{3\lambda_k^2}{(4\pi)^2} + ...$$

$$\beta_\lambda(\lambda_0) = \lim_{k\to 0} k^2 \tilde{\beta}_\lambda(\lambda_k) = \frac{3\lambda_0^2}{(4\pi)^2}$$

Clue II: scale anomaly

scale anomaly (classical + quantum):

$$\left\langle T^{\mu}_{\ \mu} \right\rangle = - \sum_A \left(\beta_A + d_A \lambda_A \right) \mathcal{O}^A$$

the trace of the quantum energy-momentum
tensor vanishes at fixed points

$$\left\langle T^{\mu}_{\ \mu} \right\rangle = 0$$

Clue II: scale anomaly

scale anomaly (classical + quantum):

$$\langle T^\mu_\mu \rangle = -\sum_A (\beta_A + d_A \lambda_A)\, \mathcal{O}^A$$

the trace of the quantum energy-momentum
tensor vanishes at fixed points

$$\langle T^\mu_\mu \rangle = 0$$

clue II:

$$\langle T^\mu_\mu \rangle_k = -\sum_A k^{d_A} \tilde{\beta}_A \mathcal{O}^A$$

Clue III: conformal anomaly

minimally coupled scalar field on a two dimensional manifold:

$$S = \frac{1}{2}\int \sqrt{g}\phi\Delta\phi$$

$d = 2$

the bare action is Weyl invariant

the non trivial part of the EAA is purely gravitational:

$$\Gamma_k[g] = \int \sqrt{g}\,(a_k + b_k R + R\,c_k(\Delta)R) + O(R^3)$$

running structure function

the exact RG equation is a trace of a function covariant operator:

$$\partial_t \Gamma_k[g] = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k(\Delta)}{\Delta + R_k(\Delta)}$$

Clue III: conformal anomaly

$d = 2$

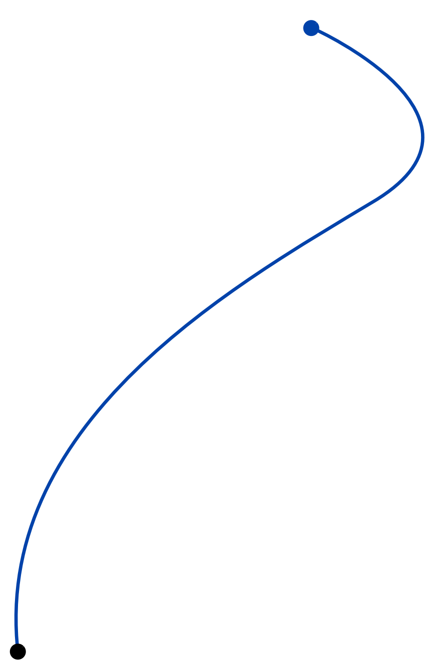
integrate the flow equation from the UV scale to the IR scale:

renormalization conditions
enter as initial conditions

$$S = \frac{1}{2}\int \sqrt{g}\phi\Delta\phi$$

$$a_k = a_\Lambda - \frac{1}{4\pi}(\Lambda^2 - k^2)$$

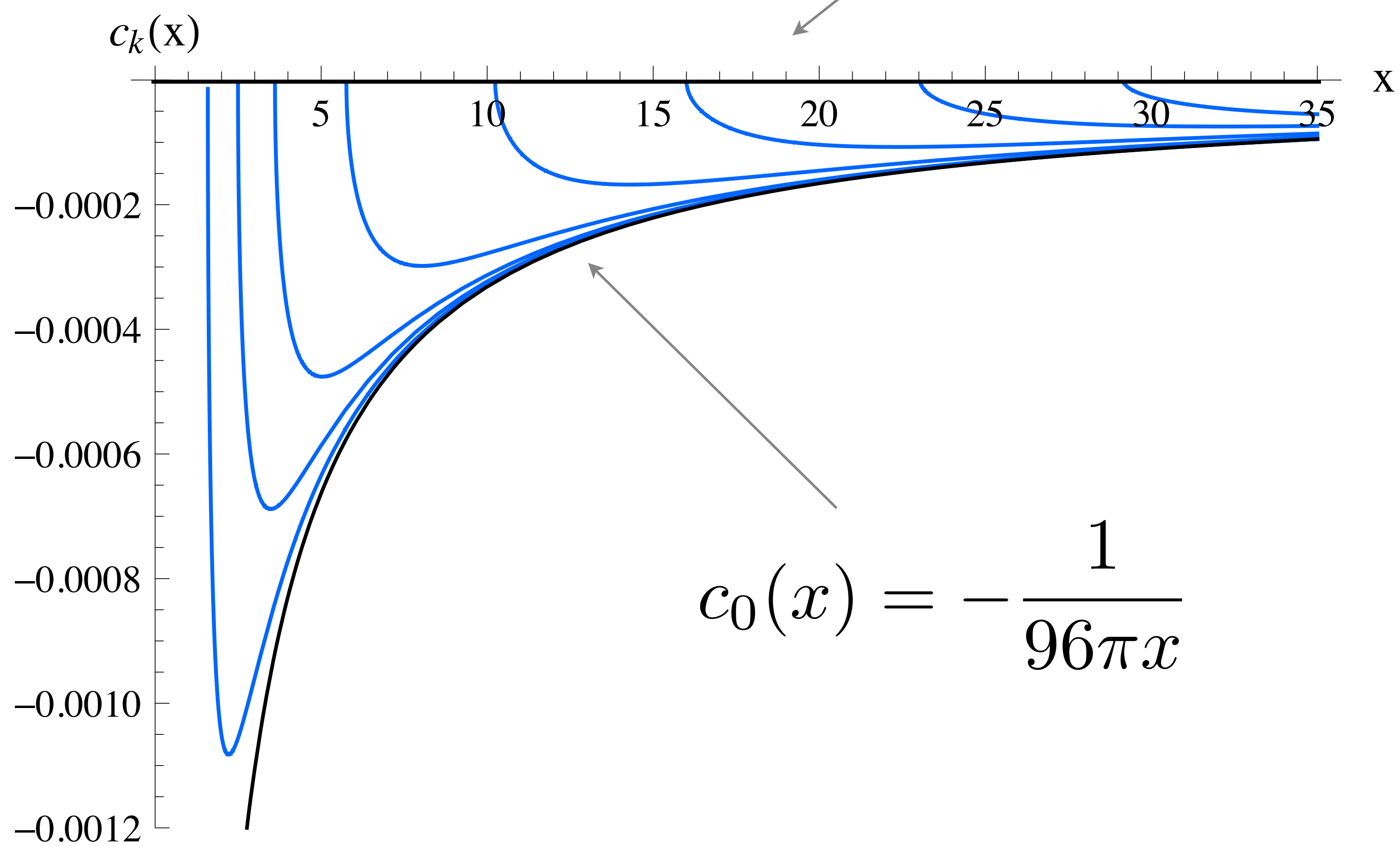
$$b_k = b_\Lambda - \frac{1}{24\pi}\log\frac{\Lambda}{k}$$



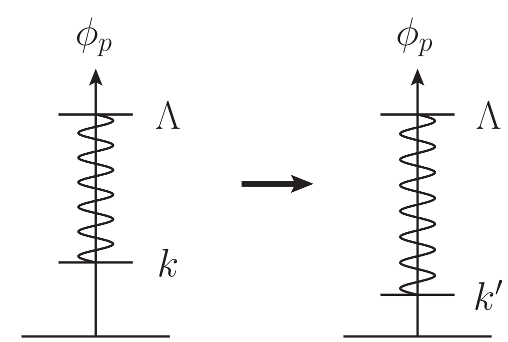
$$S^{c=1}_{CFT}[\phi, g] = \frac{1}{2}\int \sqrt{g}\phi\Delta\phi + S_P[g]$$

Gaussian in curved space

$$c_\infty(x) = 0$$



$$c_0(x) = -\frac{1}{96\pi x}$$



Clue III: conformal anomaly

$d = 2$

non-local term in the curved fixed point CFT action:

$$\Gamma_{CFT}[\phi, g] = S_{CFT}[\phi, g] + c S_P[g]$$

$$S_P[g] = -\frac{1}{96\pi} \int d^2x \sqrt{g} R \frac{1}{\Delta} R$$

Clue III: conformal anomaly

$d = 2$

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exact quantum energy-momentum tensor:

$$\langle T^{\mu\nu}\rangle = \frac{c}{48\pi}\left[-2\nabla^\mu\nabla^\nu\frac{1}{\Delta}R - \left(\nabla^\mu\frac{1}{\Delta}R\right)\left(\nabla^\nu\frac{1}{\Delta}R\right) + \right.$$
$$\left. -2g^{\mu\nu}R + \frac{1}{2}g^{\mu\nu}\left(\nabla^\alpha\frac{1}{\Delta}R\right)\left(\nabla_\alpha\frac{1}{\Delta}R\right)\right]$$

conformal anomaly:

$$\left\langle T^\mu_\mu\right\rangle = -\frac{c}{24\pi}R$$

Clue III: conformal anomaly

$d = 2$

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exact quantum energy-momentum tensor:

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$$\left. -2 g^{\mu\nu} R + \frac{1}{2} g^{\mu\nu} \left(\nabla^\alpha \frac{1}{\Delta} R \right) \left(\nabla_\alpha \frac{1}{\Delta} R \right) \right]$$

conformal anomaly:

$$\langle T^\mu_\mu \rangle = -\frac{c}{24\pi} R$$

clue III:

$$-\frac{c_k}{96\pi} \int d^2x \sqrt{g} R \frac{1}{\Delta} R \quad \in \quad \Gamma[\varphi, g]$$

Clue IV : RG transformations can be reabsorbed by rescaling the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$[ds^2] = k^{-2}$ $\quad [x^\mu] = k^{-1}$ $\quad [g_{\mu\nu}] = k^0$ $\quad$ dim-less metric

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$$\tilde{g}_{\mu\nu} = k^{-2} \tilde{g}_{\mu\nu} \qquad \tilde{\varphi} = k^{\Delta_\varphi} \tilde{\varphi}$$

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$$\tilde{g}_{\mu\nu} = k^{-2} \tilde{g}_{\mu\nu} \qquad \tilde{\varphi} = k^{\Delta_\varphi} \tilde{\varphi}$$

$$k \to e^\sigma k \qquad \tilde{g}_{\mu\nu} \to e^{2\sigma} \tilde{g}_{\mu\nu} \qquad \tilde{\varphi} \to e^{-\Delta_\varphi \sigma} \tilde{\varphi}$$

σ constant

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σ constant

$$g_{\mu\nu} \to g_{\mu\nu} \qquad \varphi \to \varphi$$

Clue IV : RG transformations can be reabsorbed by rescaling the metric

$$\Gamma_k[\varphi, g] = \Gamma_{e^\sigma k}[e^{-\Delta_\varphi \sigma}\varphi, e^{2\sigma} g]$$

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$$\Gamma_{e^\sigma k}[e^{-\Delta_\varphi \sigma}\varphi, e^{2\sigma} g] = \sum_A \lambda_A(e^\sigma k) \int \sqrt{g}\, \mathcal{O}^A[\varphi, g]$$

$$= \Gamma_k[\varphi, g] + \left[\sigma \sum_A \beta_A + \frac{1}{2}\sigma^2 \sum_{A,B} \beta_B \frac{\partial \beta_A}{\partial \lambda_B} + ...\right] \int \sqrt{g}\, \mathcal{O}^A[\varphi, g]$$

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$$\int \sqrt{g}\, \mathcal{O}^A \frac{1}{\Delta} R \to 2\sigma \int \sqrt{g}\, \mathcal{O}^A + ...$$

$$\partial_t^2 \lambda_A = \partial_t \beta_A = \sum_B \beta_B \frac{\partial \beta_A}{\partial \lambda_B}$$

Ansatz for the general form of the effective (average) action

$d = 2$

clue I

$$\Gamma_k[\varphi, g] = \sum_A \lambda_A \int \sqrt{g}\, \mathcal{O}^A[\varphi, g]$$

clue II

$$- \frac{1}{2} \sum_A \beta_A \int \sqrt{g}\, \mathcal{O}^A[\varphi, g] \frac{1}{\Delta} R$$

clue III

$$- \frac{c_k - c_\Lambda}{96\pi} \int \sqrt{g}\, R \frac{1}{\Delta} R + \ldots$$

clue IV

LPA and the c-theorem

$d = 2$

extend a given truncation:

$$\Gamma_k[\varphi] = \int \left[V_k(\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + ... \right]$$

↓

$$\Gamma_k[\varphi, g] = \int \sqrt{g} \left[V_k(\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + ... \right.$$
$$- \frac{1}{2}\partial_t V_k(\varphi)\frac{1}{\Delta}R + ...$$
$$\left. - \frac{c_k - c_\Lambda}{96\pi} R \frac{1}{\Delta} R + ... \right]$$

LPA and the c-theorem

$d=2$

non-perturbative flow for the c-function:

$$\partial_t c_k = 24\pi\ \partial_t \Gamma_k[0, e^{-2\tau}\delta]\Big|_{\int(\partial\tau)^2}$$

LPA and the *c*-theorem

$d = 2$

non-perturbative flow for the c-function:

$$\partial_t c_k = 24\pi \; \partial_t \Gamma_k[0, e^{-2\tau}\delta]\Big|_{\int(\partial\tau)^2}$$

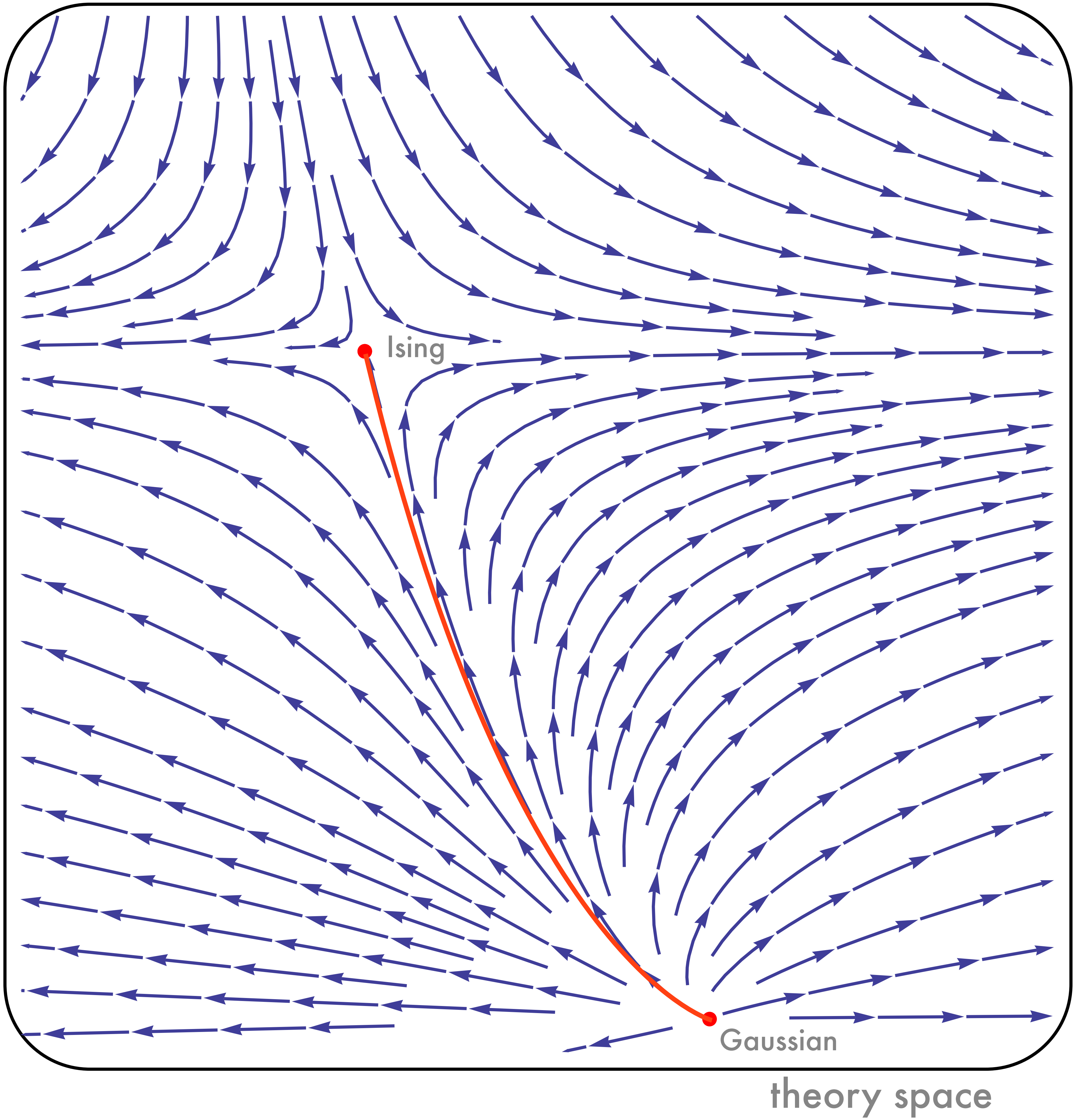
the c-function with in the LPA:

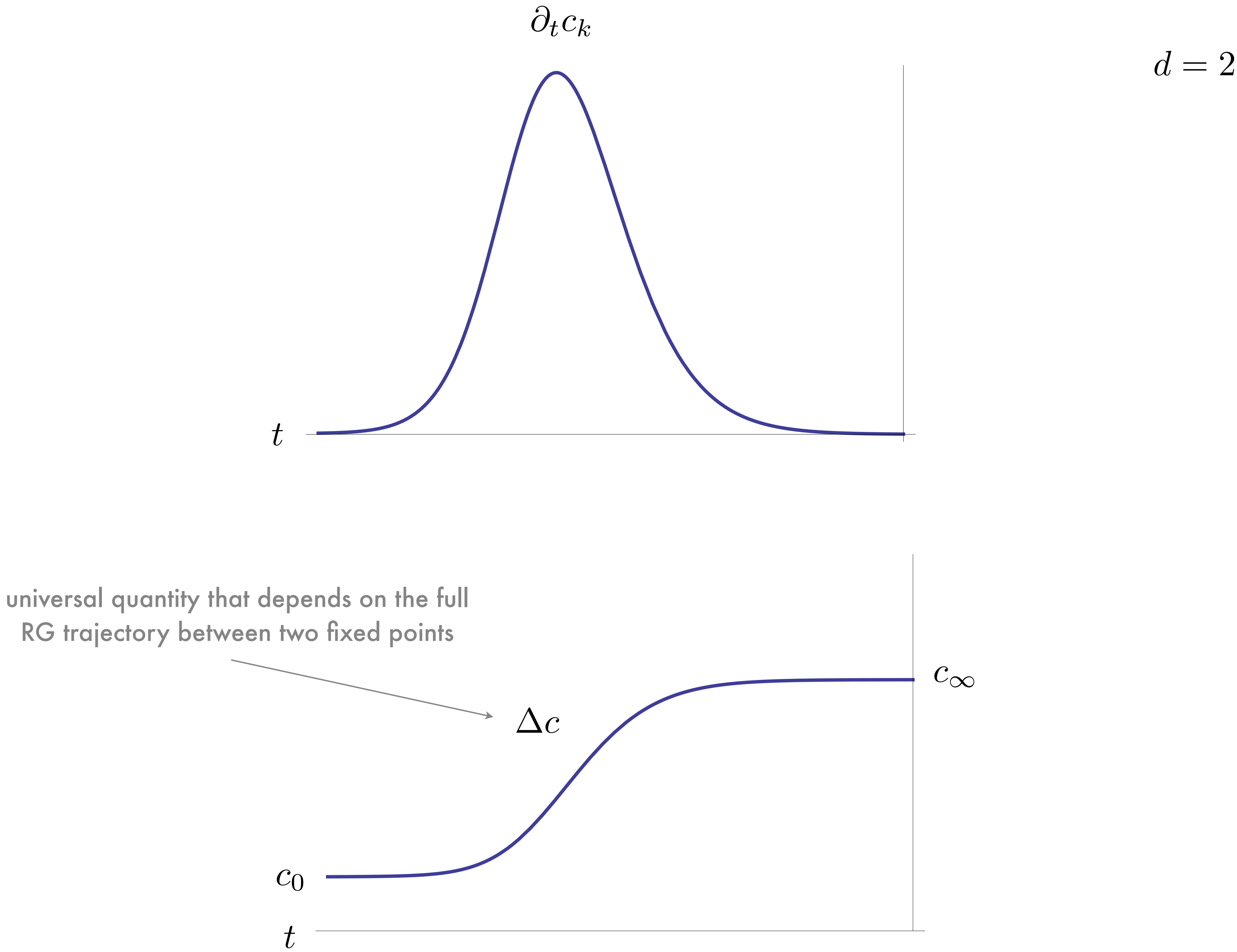
$$\begin{aligned}\partial_t c_k &= \frac{12}{(1+\tilde{m}_k^2)^4}\left(\tilde{\beta}_{m^2}\right)^2 \\ &= \frac{12}{(1+\tilde{m}_k^2)^4}\left(2\tilde{m}_k^2 + \frac{1}{4\pi}\frac{\tilde{\lambda}_k}{(1+\tilde{m}_k^2)^2}\right)^2\end{aligned}$$

the c-theorem is satisfied within our truncation!

$$\partial_t c_k \geq 0$$

$d = 2$




$\partial_t c_k$
$d=2$
t
universal quantity that depends on the full RG trajectory between two fixed points
Δc
c_∞
c_0
t

Switch on gravity!

$d = 2$

$$\Gamma_k[g] = \int \sqrt{g} \left[-\frac{1}{16\pi G_k} R + ... \right.$$
$$\left. -\frac{1}{4}\partial_t \left(-\frac{1}{16\pi G_k} \right) R \frac{1}{\Delta} R + ... \right]$$

$$= \int \sqrt{g} \left[-\frac{1}{16\pi G_k} R + ... \right.$$
$$\left. -\frac{c_k - c_\Lambda}{96\pi} R \frac{1}{\Delta} R + ... \right]$$

Switch on gravity!

$d = 2$

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$$\downarrow$$

$$\partial_t \left(-\frac{1}{16\pi G_k} \right) = \frac{c_k - c_\Lambda}{24\pi}$$

Switch on gravity!

$d = 2$

minimally coupled scalar:

$$\partial_t \left(-\frac{1}{16\pi G_k} \right) = \frac{1}{4\pi} \left(\frac{1}{6} \right) \qquad \frac{c_k - c_\Lambda}{24\pi} = \frac{1}{4\pi} \left(\frac{1}{6} \right)$$

$$c_k = c_\Lambda + 1 \qquad c_\infty = 0 \quad \Rightarrow \quad c_0 = 1$$

Switch on gravity!

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$$c_k = c_\Lambda + 1 \qquad c_\infty = 0 \quad \Rightarrow \quad c_0 = 1$$

gravity:

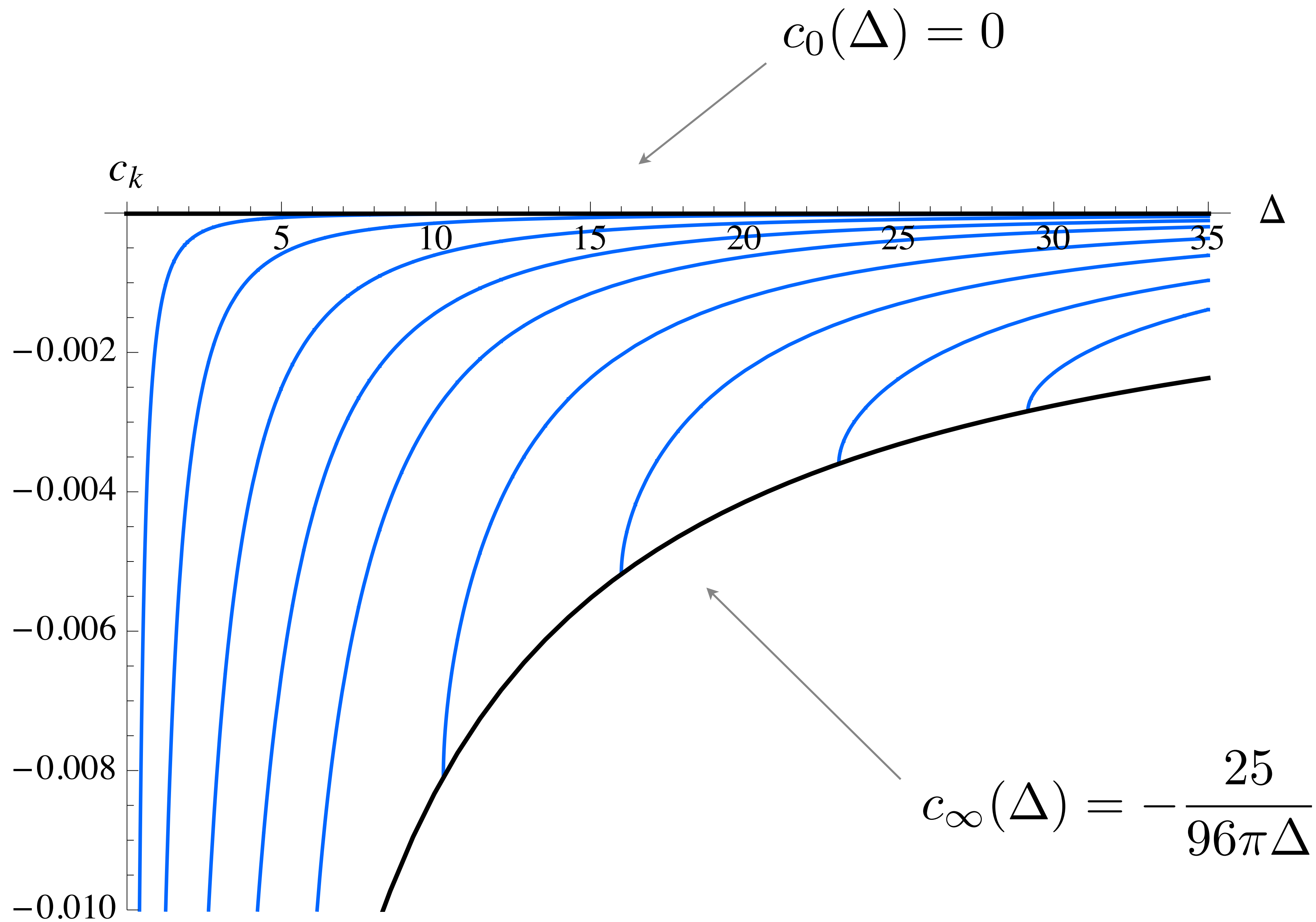
$$\partial_t \left(-\frac{1}{16\pi G_k} \right) = \frac{1}{4\pi} \left(-\frac{25}{6} \right) \qquad \partial_t G_k = -\frac{2}{3} 25\, G_k^2$$

$$\frac{1}{4\pi} \left(-\frac{25}{6} \right) = \frac{c_k - c_\Lambda}{24\pi} \quad \Rightarrow \quad c_k = c_\Lambda - 25$$

$$c_0 = 0 \quad \Rightarrow \quad c_\infty = 25$$

Switch on gravity!

$d = 2$



$$\Gamma_k[g] = \int \sqrt{g}\,(a_k + b_k R + R\,c_k(\Delta)R) + O(R^3)$$

Conclusions & Outlook

a way to extend a given ansatz to capture
all the aspects of the effective (average) action

non-perturbative definition of the c- and a-functions

framework to calculate approximated c- and a-functions

it applies to gravity: scaling relations
and connection with asymptotic safety