The Asymptotic Safety Approach to Quantum Gravity

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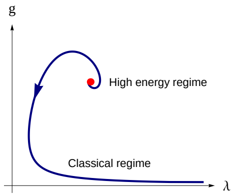

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- GR ⇒ spacetime singularities ⇒ GR makes no more predictions
- Information loss in black hole radiation (unitarity problem)
- All other interactions successfully described by QFT

Problems in Quantizing Gravity

Attempt: quantize classical Einstein-Hilbert action perturbatively

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Result: gravity is perturbatively non-renormalizable!

Approaches to Quantum Gravity

- String theory
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- Causal/euclidean dynamical triangulations
- Supergravity
- Hořava gravity

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  $\longrightarrow$ few a priori assumptions:
  supersymmetry, extra dimensions, spin foam,. . . not needed!
  $\longrightarrow$ relies on QFT concepts

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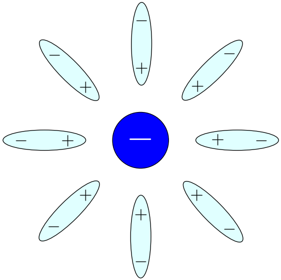
Effective description depends on scale

$\rightarrow$ parameterized by RUNNING COUPLING CONSTANTS

Why do Couplings Change?

Consider an electric charge in a vacuum



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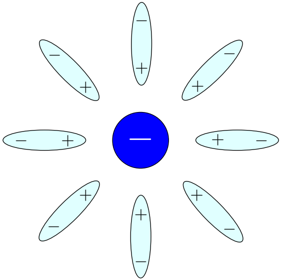

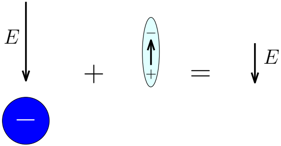

Vacuum polarization leads to screening effects
$\Rightarrow$ We see a smaller charge at large distances (low energies)

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Idea: introduce action functional established with scale dependence

$\rightarrow$ The effective average action Γ_k

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Example: scalar field theory

$$\Gamma_k[\phi] = \int \mathrm{d}^4x \left[\frac{1}{2} Z_k \, \partial^\mu\phi\partial_\mu\phi - \frac{1}{2} m_k^2 \, \phi^2 - \frac{1}{4!}\Lambda_k \, \phi^4 + \ldots\right]$$

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running couplings $\rightarrow$ explicit scale dependence

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Infinitely many effective theories, one for each energy scale k

Construction of Γ_k

Recall functional methods of QFT (scalar field, euclidean PI)

- Generating functional Z for Green's functions

$$Z[J] = \mathcal{N} \int \mathcal{D}\chi \; \exp\left(-S[\chi] + \int \mathrm{d}^d x \, J(x)\chi(x) \right)$$

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- Legendre transform of $W \Rightarrow$ effective action Γ
  Γ is the generating functional for 1PI Green's functions

$$\Gamma[\phi] = \int \mathrm{d}^d x \; J(x)\phi(x) - W[J]$$

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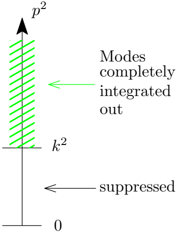

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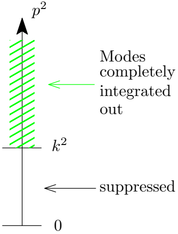
- High energy effects will be integrated out, rest is effective theory at scale k



Construction of Γ_k

$\Rightarrow$ k-dependent generating functional Z_k

$$Z_k[J] = \mathcal{N} \int \mathcal{D}\chi \; \exp\left(-S[\chi] - \Delta_k S[\chi] + \int \mathrm{d}^d x \, J(x)\chi(x) \right)$$

with $\quad \Delta_k S[\chi] = \frac{1}{2} \int \frac{\mathrm{d}^d p}{(2\pi)^d} \, \mathcal{R}_k(p^2) |\hat{\chi}(p)|^2$

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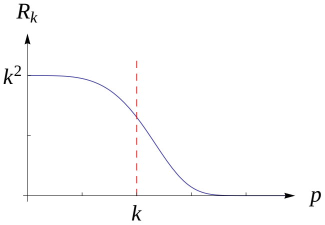

$$\mathcal{R}_k(p^2) \approx \begin{cases} k^2 & \text{for } p^2 < k^2 \\ 0 & \text{for } p^2 > k^2 \end{cases}$$

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Scale dependent field expectation value: $\phi_k[J] = \langle \chi \rangle_k^J = \frac{\delta W_k[J]}{\delta J}$

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$\Rightarrow$ Effective average action Γ_k:

$$\Gamma_k[\phi] = \widetilde{\Gamma}_k[\phi] - \frac{1}{2} \int \frac{\mathrm{d}^d p}{(2\pi)^d} \, \mathcal{R}_k(p^2) |\hat{\phi}(p)|^2$$

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- Constructed from PI $\Rightarrow$ expansion can contain *all field monomials compatible with the symmetry* ($\mathbb{Z}_2$-symmetry for scalar fields, diffeomorphism invariance for gravity), e. g.

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- Consider $k \to 0$: $\mathcal{R}_{k\to 0}(p^2) = 0 \quad \forall p^2 \quad \Rightarrow$ no cutoff

$$\boxed{\lim_{k\to 0} \Gamma_k = \Gamma}$$

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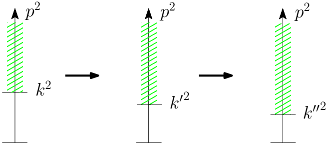
- Limit $k \to \infty$: cutoff suppresses *all* modes, except $\chi = \phi$

$$\lim_{k\to \infty} \Gamma_k = S$$

microscopic (bare) action

Meaning of the Scale Dependence

Meaning of decreasing k: successive integrating out of degrees of freedom



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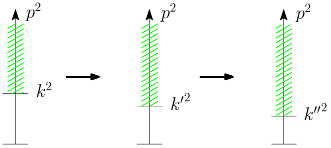

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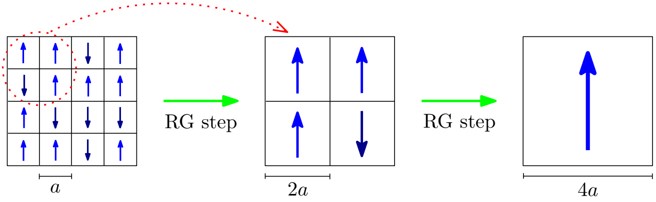
Comparable to discrete blockspin transformations



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- Take a scale derivative of Γ_k $\Rightarrow$ Wetterich equation

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\Big[\big(\Gamma_k^{(2)} + \mathcal{R}_k\big)^{-1} k\partial_k \mathcal{R}_k\Big]$$

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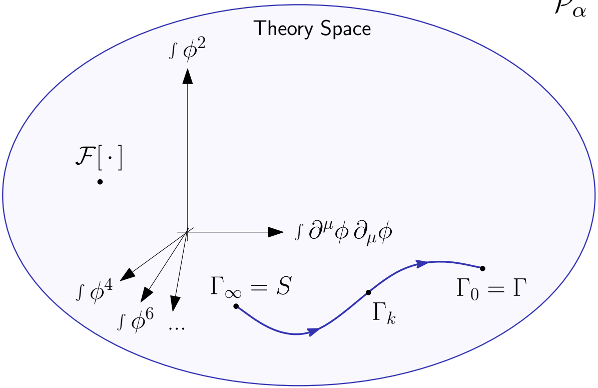
- Properties of the FRGE:

• functional integro-differential equation • non-linear
• exact ($\rightarrow$ non-perturbative) • UV finite • IR finite
• independent of PI formulation (holds for all Γ_k)

Evolution of Γ_k in Theory Space

$$\Gamma_k[\phi] = \sum_{\alpha=1}^{\infty} \bar{g}_\alpha(k)\mathcal{P}_\alpha[\phi]$$

$\bar{g}_\alpha(k)$: dimensionful running couplings
$\mathcal{P}_\alpha$: basis functionals



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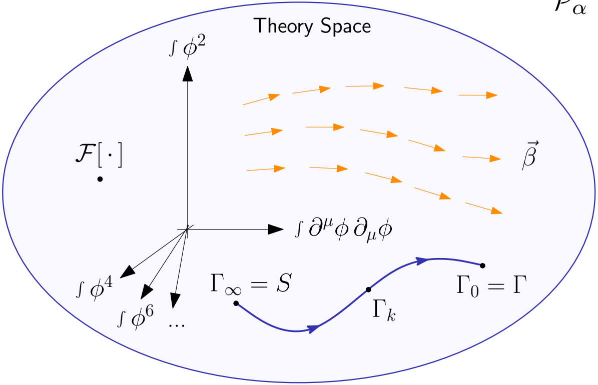

$\vec{\beta}$ given by FRGE

$$k\partial_k\,\bar{g}_\alpha(k) = \bar{\beta}_\alpha$$

$\Downarrow$

running of $\bar{g}_\alpha(k)$

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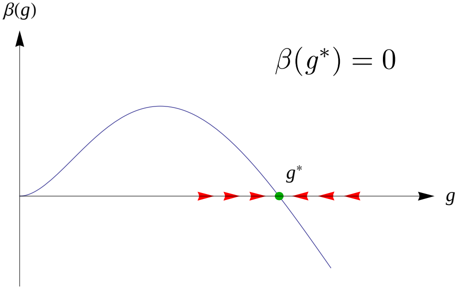
Idea (for 1 coupling) $\to$



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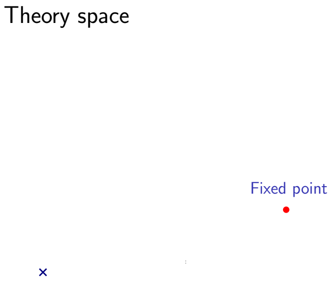

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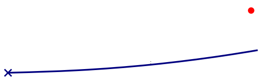

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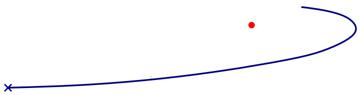

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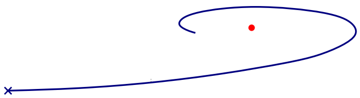

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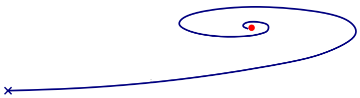

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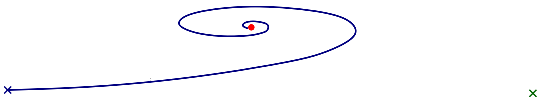

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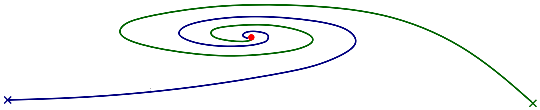

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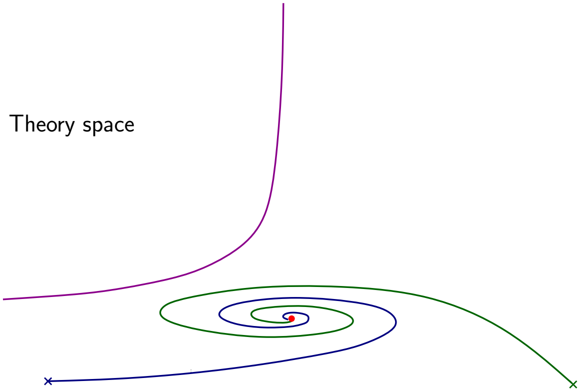

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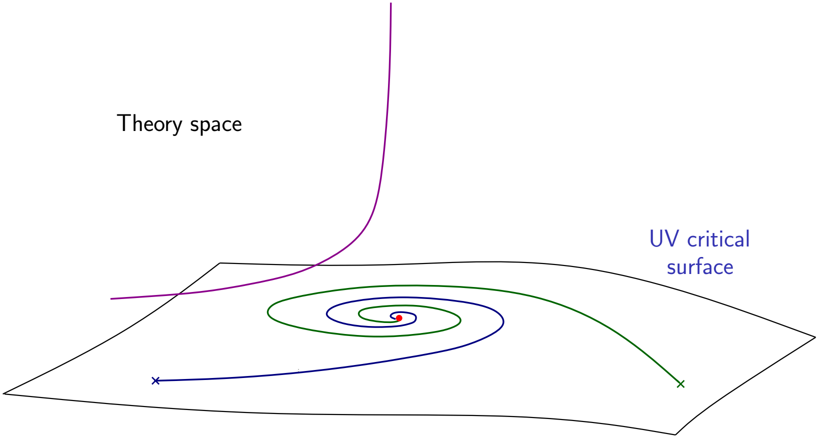

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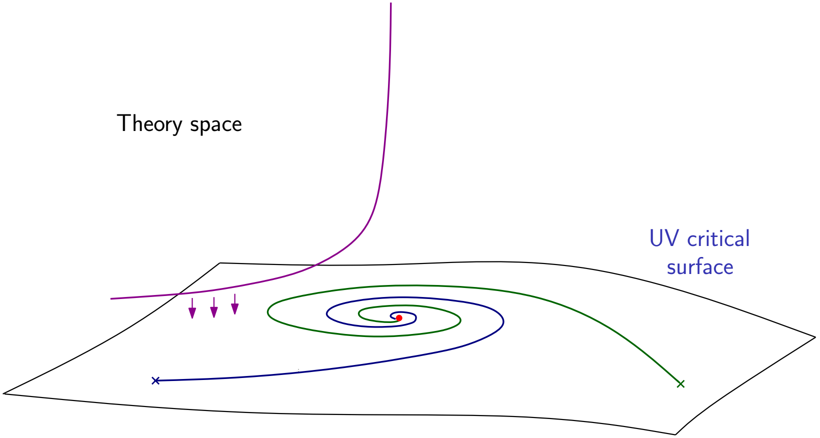

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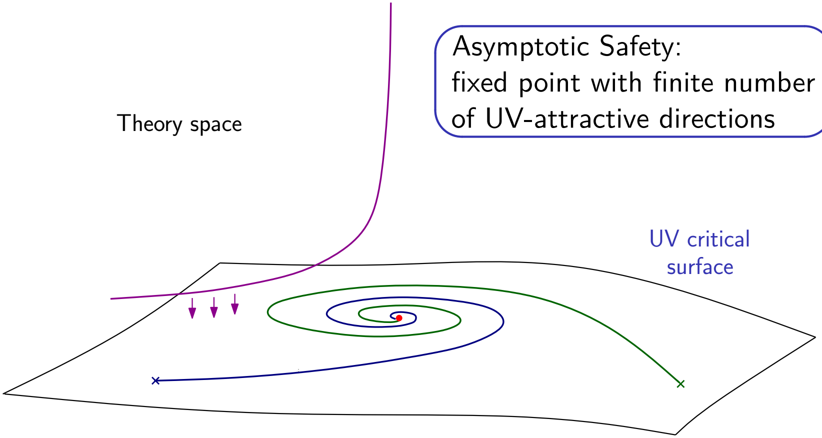

Asymptotic Safety:
fixed point with finite number of UV-attractive directions

Critical Exponents

Tool to find out whether a given direction is UV-attractive

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- If $\mathrm{Re}(\theta) > 0$ the corresponding direction is UV-attractive
  If $\mathrm{Re}(\theta) < 0$ the corresponding direction is UV-repulsive

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- Exploits non-perturbative character of the FRGE
- Gives finite number of differential equations

Outline

The Einstein-Hilbert Truncation

Ansatz for Γ_k

- Classical Einstein-Hilbert action, but with running couplings, plus gauge fixing action term

$$\Gamma_k[g] = \frac{1}{16\pi G_k} \int \mathrm{d}^d x \sqrt{g} \left\{ -R(g) + 2\bar{\lambda}_k \right\} + \Gamma_k^{\mathrm{gf}}$$

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Procedure

- Insert Γ_k into the FRGE
- Extract differential equations for G_k and $\bar{\lambda}_k$

Deriving the Flow Equations

$$k\partial_k \Gamma_k \;=\; \frac{1}{2}\,\mathrm{Tr}\Big[\,\big(\Gamma_k^{(2)} + \mathcal{R}_k\big)^{-1}\; k\partial_k \mathcal{R}_k\,\Big]$$

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$$-\frac{1}{16\pi}\,k\partial_k\left(\frac{1}{G_k}\right)\int \mathrm{d}^d x\,\sqrt{g}\,R$$

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heat-kernel techniques

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$$(\ldots)\cdot\int \mathrm{d}^d x\,\sqrt{g} + (\ldots)\cdot\int \mathrm{d}^d x\,\sqrt{g}\,R + \ldots$$

Compare coefficients of $\int \mathrm{d}^d x\sqrt{g}$ and $\int \mathrm{d}^d x\sqrt{g}R$

Deriving the Flow Equations

- $\Rightarrow$ evolution equations for G_k and $\bar{\lambda}_k$
- Dimensionless Newton's constant and cosmological constant

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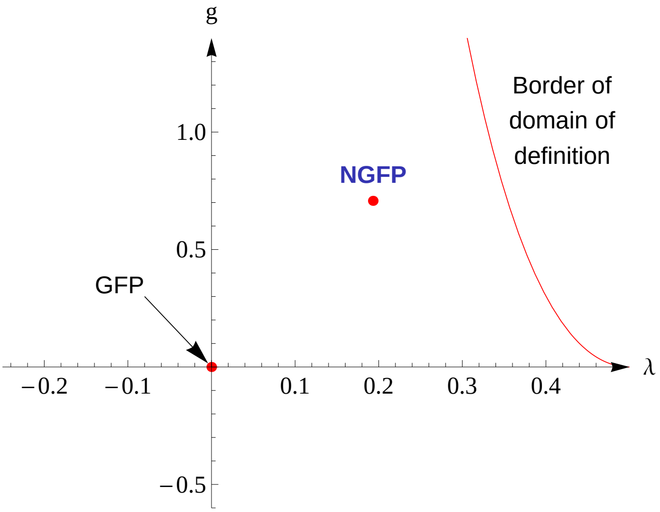
- $\Rightarrow$ evolution equations for g_k and λ_k (analytical!)

$$k\partial_k g_k = (d - 2 + \eta_N) g_k$$

$$k\partial_k \lambda_k = (\eta_N - 2)\lambda_k + 2\pi g_k (4\pi)^{-d/2} \Big[2d(d+1)\Phi^1_{\frac{d}{2}}(-2\lambda_k) - d(d+1)\eta_N \widetilde{\Phi}^1_{\frac{d}{2}}(-2\lambda_k) - 8d\Phi^1_{\frac{d}{2}}(0) \Big]$$

with threshold functions Φ, $\widetilde{\Phi}$ and anomalous dimension η_N

Numerical Solution in Four Dimensions



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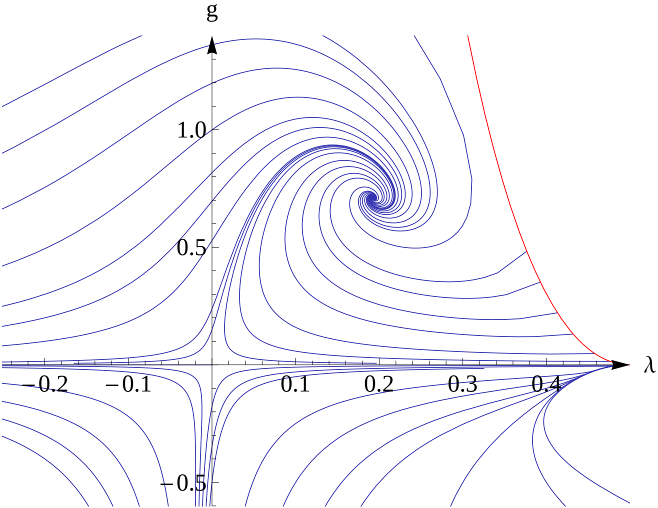

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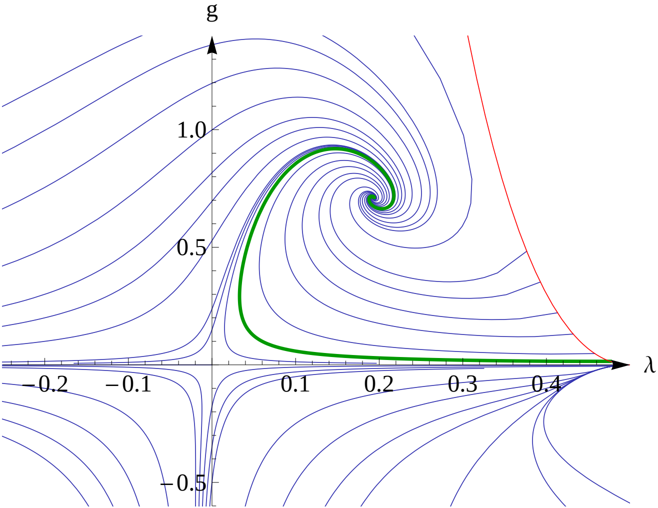

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- Existence of NGFP in $d = 4$ for all cutoff functions

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$\Rightarrow$ supports Asymptotic Safety scenario

Einstein-Hilbert Truncation: Extensions

Check validity of truncation ansatz

- Cutoff dependence, gauge dependence
- More general truncations
  - R^2, $R^2 + C^2$, $f(R)$, ... • running ghost sector
  - running gauge fixing term • inclusion of matter fields
  - bimetric truncations • inclusion of boundary terms
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Results

- Non-trivial fixed point always exists
- UV critical surface finite dimensional (?)

Outline

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$$H = -\frac{1}{2m}(\nabla - ieA)^2 - \frac{e}{2m}\sigma \cdot B$$

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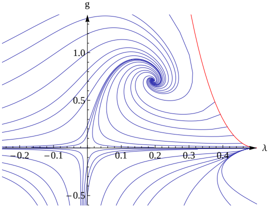
- Two different effects → Competing? Relative contribution?

Separating Minimal and non-Minimal Terms

Recall result from
Einstein-Hilbert truncation

- In particular: existence of non-trivial UV fixed point
- UV-attractive



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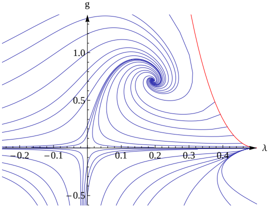

Repeat calculation, but take into account minimal and non-minimal contributions separately

- Existence of fixed point, UV-attractive directions
- Flow diagrams

Result – non-Minimal Terms Only

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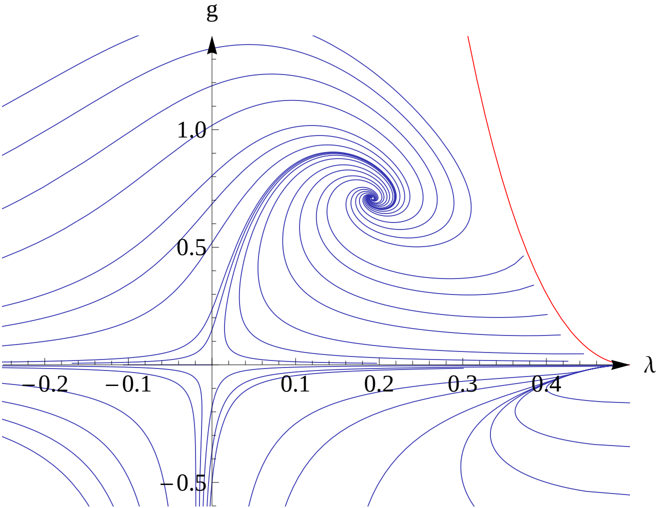

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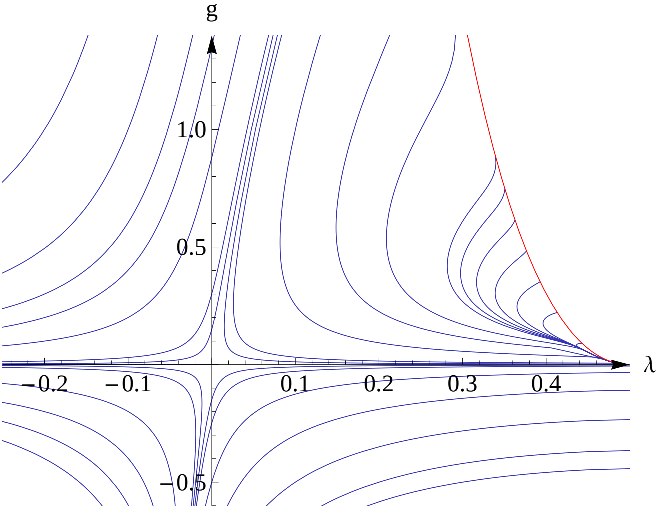

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	g^*	λ^*	$(g^*\lambda^*)$	$\text{Re}(\theta)$	$\text{Im}(\theta)$
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Existence of NGFP due to non-minimal terms only

- Holds for *all* cutoffs
- Method applicable to other theories
  ($\Rightarrow$ reason for asymptotic freedom in QCD)

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- Strong indications that gravity is asymptotically safe