Skyrmions in Composite Higgs Models

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CP^3 - Origins
Particle Physics & Cosmology

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December 4, 2012

based on arXiv:1012.5288, 1103.5990, 1111.2047
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Outline

The Standard Model as an effective theory

Standard Model Lagrangian

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} - \frac{1}{4} W^i_{\mu\nu} W^{\mu\nu\, i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad \text{gauge group}$$

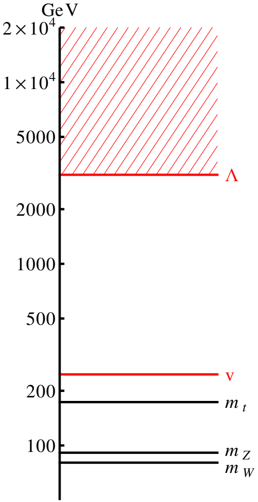
$$+ i\, \overline{q_L}\, \not{D}\, q_L + i\, \overline{u_R}\, \not{D}\, u_R + i\, \overline{d_R}\, \not{D}\, d_R + \ldots \qquad \text{fermions}$$

$$+ \frac{v^2}{4} \operatorname{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma \right) + \frac{v}{\sqrt{2}}\, \bar{q}^{(i)}_L \Sigma \begin{pmatrix} Y^{(i,j)}_u u^{(j)}_R \\ Y^{(i,j)}_d d^{(j)}_R \end{pmatrix} + \ldots \qquad \text{EWSB \& mass}$$

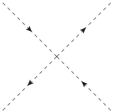
3 massless scalars $\Sigma = \exp\left[i\, \pi^i\, \sigma^i / v \right]$ become the longitudinal polarisation of the gauge bosons

$\rightarrow$ valid theory up to $\Lambda = 4\pi\, v \cong 3.1$ TeV

The Standard Model as an effective theory



Problem:
scattering of W bosons



$$\propto \frac{s+t}{v^2}$$

perturbative unitarity is lost
already at 1.7 TeV

The Standard Model as an effective theory

Standard Model Lagrangian

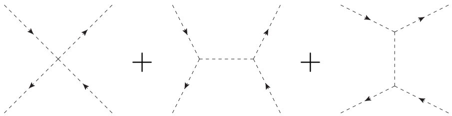
$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu\,i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$+i\,\overline{q_L}\,\not{D}\,q_L + i\,\overline{u_R}\,\not{D}\,u_R + i\,\overline{d_R}\,\not{D}\,d_R + \ldots$$

$$+\frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu\Sigma^\dagger D^\mu\Sigma\right)\left(1 + 2\,a\,\frac{h}{v} + b\,\frac{h^2}{v^2} + \ldots\right)$$

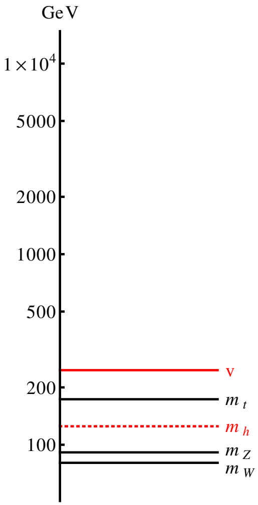
$$+\frac{v}{\sqrt{2}}\;\bar{q}_L^{(i)}\Sigma\begin{pmatrix} Y_u^{(i,j)}u_R^{(j)} \\ Y_d^{(i,j)}d_R^{(j)}\end{pmatrix}\left(1 + c_{u,d}^{(i,j)}\,\frac{h}{v} + \ldots\right) + \ldots$$

$$+\frac{1}{2}\,\partial_\mu h\,\partial^\mu h - V(h)$$



$$\propto\;(1-a^2)\,\frac{s+t}{v^2}$$

The Standard Model Higgs doublet



For $a = b = c = 1$, the fields $\left(h, \pi^i\right)$ form an electroweak doublet

$$\frac{(v+h)^2}{4}\,\mathrm{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right) \equiv \frac{1}{2} D_\mu H^\dagger D^\mu H$$

with non-vanishing vev $\langle H \rangle \neq 0$

The theory becomes renormalisable and unitary up to arbitrarily high energies

The hierarchy problem

Radiative corrections to the Higgs mass are quadratically divergent

$\frac{3}{4}\frac{m_h^2}{v^2}\frac{\Lambda^2}{32\pi^2}$ $\quad -6\,y_t^2\frac{\Lambda^2}{32\pi^2}$ $\quad \left(\frac{9}{4}\,g^2+\frac{1}{2}\,g'^2\right)\frac{\Lambda^2}{32\pi^2}$

Renormalisability $\Rightarrow$ cancellation by a counter-term

$$\begin{aligned} m_{\rm phys}^2 &= m_{\rm bare}^2 + \delta m^2 \\ (125\ {\rm GeV})^2 &\approx \left(10^{15}\ {\rm GeV}\right)^2 - \left(10^{15}\ {\rm GeV}\right)^2 \end{aligned}$$

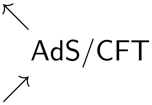
Still, the cancellation required is highly unnatural.

Solutions to the hierarchy problem

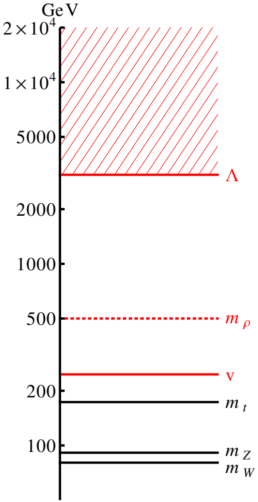
Scalar masses are not protected by any symmetry

Light scalar particles can naturally exist:

- in supersymmetric theories,
  as part of a supermultiplet
- in strongly-coupled theories,
  as (pseudo-)Goldstone bosons
  (like the π's in QCD)
- in extra-dimensional models,
  as the extra-dimensional component
  of a gauge field (zero mode)



Technicolor / Holographic higgsless models



The longitudinal components of the $W^{\pm}$ and Z gauge bosons are Goldstone bosons of the strong sector

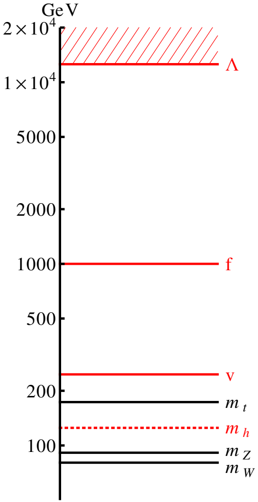
$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr} \left(D_\mu \Sigma^\dagger D^\mu \Sigma \right)$$

EWSB triggered by the strong dynamics

The 125 GeV particle could be a light resonance of the strong sector (dilaton)

However disfavoured by electroweak precision data (it's not a Higgs!)

(Holographic) Composite Higgs models



EWSB is triggered by a Higgs doublet, i.e. four pseudo-Goldstone bosons with a potential generated radiatively

$$\mathcal{L} = \frac{f^2}{4} \operatorname{Tr} \left(D_\mu \Sigma^\dagger D^\mu \Sigma \right)$$

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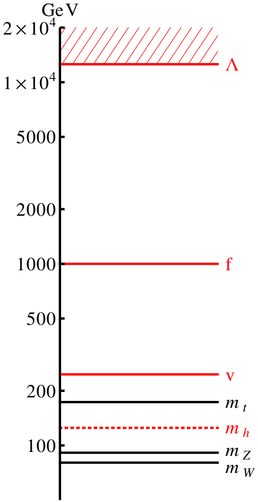

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Why holography?

Such a model may require scalar fields in the 4D strongly-coupled description
$\rightarrow$ provided by a warped 5D

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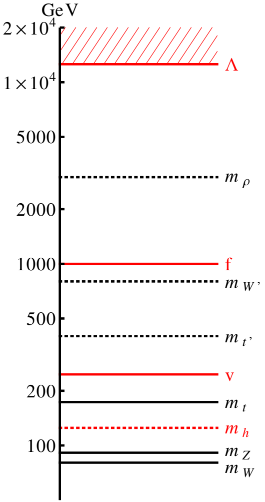

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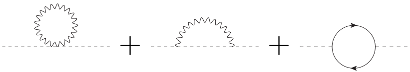
Realistic models contains additional particles below the cutoff

Little Higgs models

EWSB is generated radiatively by gauge boson and fermion loops



$$\approx \frac{\Lambda^2}{(4\pi)^2} \approx f^2$$

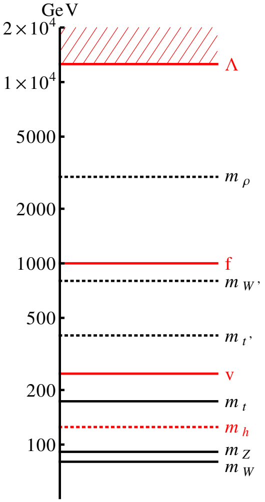
The Little hierarchy problem

v is naturally generated around the scale f

- $f \lesssim 1$ TeV requires a moderate amount of fine-tuning
  $\rightarrow$ holographic composite Higgs models
- $f \gtrsim 1$ TeV requires additional symmetry to cancel the loops
  $\rightarrow$ little Higgs models

cancellation of quadratic divergent contributions to the Higgs potential through collective symmetry breaking

Phenomenology of composite Higgs / little Higgs models



Weak regime phenomenology:

- non-standard Higgs couplings
- top partners
- vector resonances
- additional gauge fields W', Z'
- flavour physics
- ...

Strong regime phenomenology:

- new heavy stable particles ?
  $\rightarrow$ AdS/CFT techniques ?
  $\rightarrow$ topological soliton models

The chiral low-energy effective theory of QCD

In the absence of quark masses, the QCD Lagrangian has a global chiral symmetry

$$U(N_f)_L \times U(N_f)_R$$
$$= SU(N_f)_V \times SU(N_f)_A \times U(1)_V \ [\times U(1)_A]$$

Quarks condensate and break the chiral symmetry

$$\langle \overline{q_L^a} q_R^b \rangle \propto \delta^{ab}$$

$\rightarrow$ only the $SU(N_f)_V \times U(1)_V$ symmetry survives

$\Rightarrow$ $N_f^2 - 1$ massless Goldstone bosons

$$\Sigma = \exp\left[i\,\pi^a\,T^a/f_\pi\right] \qquad\qquad \langle \Sigma \rangle = \mathbb{1}$$

σ-model description for the Goldstone bosons

Non-linear sigma-model

$$\mathcal{L} = \frac{f_\pi^2}{4} \operatorname{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right) + \mathcal{O}\left[(\partial\Sigma)^4\right]$$

No dependence on the gauge group, only on the flavour symmetry

Describes the pion's electromagnetic and self-interactions:
e.g. in the two-flavour case $N_f = 2$, $\Sigma = \exp\left(i\,\boldsymbol{\pi}\cdot\boldsymbol{\sigma}/f_\pi\right)$

$$\mathcal{L} = \frac{1}{2}\left|D_\mu \boldsymbol{\pi}\right|^2 + \frac{1}{6 f_\pi^2}\left[\left|\boldsymbol{\pi}\cdot D_\mu \boldsymbol{\pi}\right|^2 - \boldsymbol{\pi}^2\left|D_\mu \boldsymbol{\pi}\right|^2\right] + \ldots$$

But this Lagrangian also contains skyrmions

What are skyrmions?

Skyrmions are topological solitons

- extended field configurations, with finite size and finite energy
- stable at the classical level since they cannot be deformed into the vacuum by infinitesimal transformations

The topological index of a field configuration can be expressed as an integral

$$\mathcal{B} = \frac{1}{24\pi^2}\epsilon_{ijk}\int \mathrm{d}^3x\,\mathrm{Tr}\left(\Sigma^\dagger\partial_i\Sigma\,\partial_j\Sigma^\dagger\partial_k\Sigma\right) \in \mathbb{Z}$$

and corresponds to the baryon number of the theory E. Witten (1983)

$\Rightarrow$ the skyrmions in QCD are baryons

Topology of the vacuum manifold

The presence of topological defects is characterised by the homotopy groups of the vacuum manifold G/H

			example: $SU(N)$
$\pi_0(G/H)$	$\leftrightarrow$	domain walls	0
$\pi_1(G/H)$	$\leftrightarrow$	cosmic strings	0
$\pi_2(G/H)$	$\leftrightarrow$	monopoles	0
$\pi_3(G/H)$	$\leftrightarrow$	skyrmions	$\mathbb{Z}$

The Skyrme model

In general $\pi_3(G/H) \neq 0$ is not sufficient to guarantee the presence of skyrmions, higher order terms are needed to stabilise the skyrmion size and mass

Derrick (1964)

The Skyrme Lagrangian

Add a single higher-order term, antisymmetric in Lorentz indices

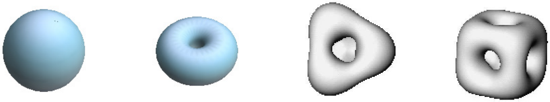
$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{Tr}\left(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left(\left[\Sigma^\dagger\partial_\mu\Sigma, \Sigma^\dagger\partial_\nu\Sigma\right]\right)^2$$

T. H. R. Skyrme (1961)

- Stabilises the size and mass of the skyrmion
- Only quadratic in time derivatives
  $\Rightarrow$ canonical quantisation procedure

Skyrmions in QCD

Bogomolny bound: $E \geq 6\pi^2 \frac{f_\pi}{e} |\mathcal{B}|$



…

$\mathcal{B} =$	± 1	± 2	± 3	± 4
$E =$	$72.9 \frac{f_\pi}{e}$	$139.6 \frac{f_\pi}{e}$		

Baryon properties with $f_\pi \cong 93$ MeV (exp. data) and $e \cong 4.25$:

	Skyrme model	Experiment
M_N	946 MeV	939 MeV
$\mu_{I=1}$	2.24	2.35
$r^2_{E,I=0}$	0.51 fm^2	0.62 fm^2
$r^2_{M,I=1}$	0.64 fm^2	0.73 fm^2
g_A	0.66	1.26

F. Meier, H. Walliser (1997)

Limitations of the Skyrme model

Good agreement with the experimental data already with a single free parameter, but

- the skyrmion lives above the cutoff of the theory
- all terms in the derivative expansion can equally contribute to the skyrmion mass
- in general, no formal proof that the skyrmion corresponds to a physical state in the quantum theory, although works in some non-QCD-like theories R. Auzzi, S. Bolognesi, M. Shifman (2007-2009)

Solution from AdS/CFT?

- stable skyrmion are present in extra-dimensional models
- their size is larger than the cutoff length

A. Pomarol, A. Wulzer (2007)

Skyrmions in composite Higgs & little Higgs models

Are there stable skyrmions in composite Higgs & little Higgs models ?

Differences with respect to QCD:

1. Possibly different symmetry breaking pattern
   $\rightarrow$ different vacuum topology
   $\rightarrow$ not all models have skyrmions

2. f is much larger (of order 10^4 times)
   $\rightarrow$ much larger skyrmion mass, expected 1–100 TeV
   $\rightarrow$ much smaller radius / annihilation cross-section

3. Different interplay with gauge fields
   $\rightarrow$ skyrmion decay?
   $\rightarrow$ electric charge

Relic density

Skyrmions are thermally produced in the early universe, independently of the production mechanism

Skyrmion decay can occur through instanton effects, depending on the gauge group
$\rightarrow$ suppression factor $\exp\left(-8\pi^2/g^2\right)$ make them long-lived

- If neutral, could provide the observed dark matter relic density, for skyrmion masses in the range 1–10 TeV
  "topological dark matter" Murayama, Shu (2009)
- If charged, affect dramatically the early universe cosmology

Models

A (non-exhaustive) list of composite / little Higgs models

Model		*Coset*	$\mathcal{B}$
Min. Composite Higgs	Agashe et al. (2004)	$SO(5)/SO(4)$	0
Beyond the MCHM	Gripaios et al. (2009)	$SO(6)/SO(5)$	
"Minimal Moose"	Arkani-Hamed et al. (2002)	$(SU(3)\times SU(3))/SU(3)$	$\mathbb{Z}$
with custodial sym.	Chang, Wacker (2004)	$(SO(5)\times SO(5))/SO(5)$	
"Bestest LH"	Schmaltz et al. (2010)	$(SO(6)\times SO(6))/SO(6)$	
Littlest Higgs	Arkani-Hamed et al. (2002)	$SU(5)/SO(5)$	$\mathbb{Z}_2$
"Antisym. condensate"	Low et al. (2002)	$SU(6)/Sp(6)$	0
LH from simple group	Kaplan et al. (2003)	$SU(4)/SU(3)$	
Simple two LH	Skiba, Terning (2003)	$SU(9)/SU(8)$	
Custodial littlest higgs	Chang (2003)	$SO(9)/(SO(5)\times SO(4))$	

Example: the minimal moose

$$\mathcal{L} = \frac{f^2}{4} \mathrm{Tr}\left(D_\mu \Sigma \, D^\mu \Sigma^\dagger\right) + \text{higher order terms}$$

$$\Sigma(x) \in SU(3) \qquad\qquad \langle \Sigma(x) \rangle = \mathbb{1}$$

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Global symmetry breaking pattern $SU(3)_L \times SU(3)_R \to SU(3)_V$

$\to$ 8 Goldstone bosons

Gauge an $SU(2) \times U(1)$ subgroup, $\mathbf{8} = \mathbf{3}_0 \oplus \mathbf{2}_{\pm 1/2} \oplus \mathbf{1}_0$

$$D_\mu \Sigma = \partial_\mu \Sigma - i g W_\mu^a \left[Q^a, \Sigma\right] - i g' B_\mu \left[Y, \Sigma\right] + \ldots$$

$$Q_a = \frac{1}{2} \begin{pmatrix} \sigma_a & \\ & \end{pmatrix} \qquad Y = \frac{1}{6} \begin{pmatrix} \mathbb{1} & \\ & -2 \end{pmatrix}$$

Skyrme Lagrangian

As in QCD, add the (gauged) Skyrme term

$$\mathcal{L} = \frac{f^2}{4}\,\mathrm{Tr}\left(D_\mu\Sigma\, D^\mu\Sigma^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left(\left[\Sigma^\dagger D_\mu\Sigma, \Sigma^\dagger D_\nu\Sigma\right)\right|^2$$

In the absence of gauge fields, the skyrmion configuration minimising the energy is obtained with a hedgehog ansatz

$$\Sigma = \exp\left[2i\,F(r)\,\hat{x}_i T_i\right] \qquad T_i \in SU(3)_V \qquad [T_i, T_j] = i\,\epsilon_{ijk}\,T_k$$

Spherical symmetry:

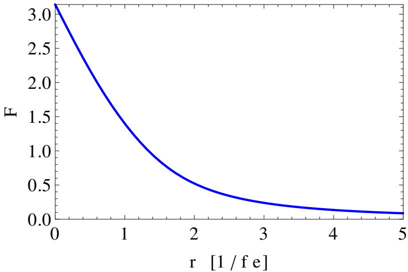
$SO(3)$ rotation in space $\equiv$ global transformation $\subset SU(3)_V$

Classical skyrmion solution

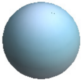
Find the function $F(r)$ which minimises the energy

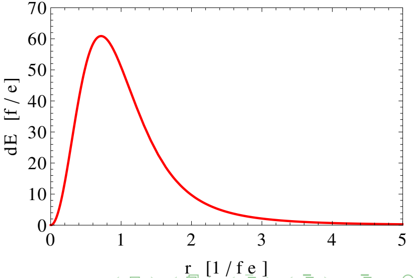
$$E_0 = 72.9\ \frac{f}{e}$$

$$\langle r^2 \rangle = \left(\frac{1.06}{fe}\right)^2$$



Spherical energy distribution





The effects of gauge fields

$SU(2) \times U(1)$ gauge fields

- break the global $SU(3)$ symmetry of the model
- can possibly reduce the classical mass of the skyrmion

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- can possibly reduce the classical mass of the skyrmion

The minimal energy solution is obtained with the
Skyrme-Wu-Yang ansatz Brihaye, Tchrakian (1998) Brihaye, Hill, Zachos (2004)

$$\Sigma = \exp\left[2i\, F(r)\, \hat{x}_i Q_i\right]$$

$$W_i^a = \frac{a(r)}{2g\, r}\, \epsilon_{iak}\, \hat{x}_k \qquad W_0^a = 0 \qquad B_\mu = 0$$

- Spherically symmetric
- Makes use of the $SU(2)_W$ generators only

Skyrme-Wu-Yang ansatz and EWSB

Energy functional

$$E[F,a] = 2\pi \frac{f}{e} \int_0^\infty \mathrm{d}r \left[\frac{e^2}{g^2} \left(2\left(a'\right)^2 + \frac{a^2(a+2)^2}{r^2} \right) + \left(r^2 + 2(1+a)^2 \sin^2 F \right) \left(F'\right)^2 + (1+a)^2 \sin^2 F \left(2 + (1+a)^2 \frac{\sin^2 F}{r^2} \right) \right]$$

Skyrme-Wu-Yang ansatz and EWSB

Energy functional

$$\begin{aligned} E[F,a] &= 2\pi \frac{f}{e}\int_0^\infty \mathrm{d}r \left[\frac{e^2}{g^2}\left(2\,(a')^2 + \frac{a^2(a+2)^2}{r^2}\right)\right. \\ &\quad + \left(r^2 + 2(1+a)^2 \sin^2 F\right)\left(F'\right)^2 \\ &\quad \left. +(1+a)^2 \sin^2 F \left(2 + (1+a)^2 \frac{\sin^2 F}{r^2}\right)\right] \end{aligned}$$

Electroweak symmetry breaking

$$\langle h \rangle = v \quad \Leftrightarrow \quad \langle \Sigma \rangle = \begin{pmatrix} 1 & & \\ & \cos(v/f) & i\sin(v/f) \\ & i\sin(v/f) & \cos(v/f) \end{pmatrix}$$

Skyrme-Wu-Yang ansatz and EWSB

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$$E[F,a] = 2\pi \frac{f}{e} \int_0^\infty \mathrm{d}r \left[\frac{e^2}{g^2} \left(2\,(a')^2 + \frac{a^2(a+2)^2}{r^2} \right) + \frac{\epsilon}{2} a^2 + \mathcal{O}\left(\epsilon^2\right) + \left(r^2 + 2(1+a)^2 \sin^2 F\right) \left(F'\right)^2 + (1+a)^2 \sin^2 F \left(2 + (1+a)^2 \frac{\sin^2 F}{r^2} \right) \right]$$

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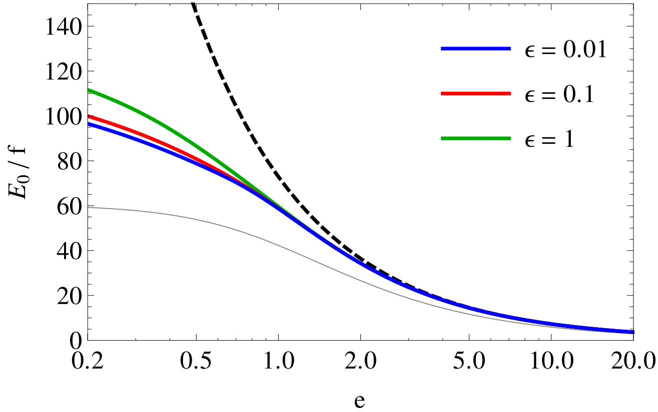
$\rightarrow$ preserves spherical symmetry at LO

Classical energy of the skyrmion



- E_0 tends to the ungauged solution in the limit $e \to \infty$ (vanishing Skyrme term)
- Still always the lightest solution at the classical level

Quantisation

The lowest physical states are obtained by quantisation of zero-modes

- translations and boosts
- rotations
- global $SU(2)_W$ transformations
- global $U(1)_Y$ transformation

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- ~~translations and boosts~~
- rotations $\rightarrow$ spin
- global $SU(2)_W$ transformations $\rightarrow$ isospin
- ~~global $U(1)_Y$ transformations~~

Since both are equivalent, for the lightest states

spin = isospin

The lightest skyrmion state is either

- an electroweak singlet boson
- an electroweak doublet fermion

Spin statistics of the skyrmion

The low-energy theory is a purely bosonic theory:

How could the skyrmion not be a boson?

Straightforward to compute: Does the action picks up a phase when performing a 2π rotation of the skyrmion?

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If the space of all possible field configurations is disconnected, then this is possible

Finkelstein, Rubinstein (1968)

$$\pi_4\big(SU(N \geq 3)\big) = 0 \qquad \Rightarrow \qquad \text{not for QCD}$$

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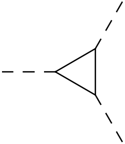
Answer: the σ-model Lagrangian written so far is incomplete

Anomalies

Anomalies in the UV theory must be matched in the low-energy description by the introduction of a non-local term



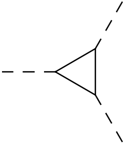
Wess, Zumino (1971) Witten (1983)

$$\Gamma = -\frac{i\,N_c}{240\pi^2}\int_{\mathcal{M}_5} \mathrm{d}^5x\,\epsilon^{\mu\nu\rho\sigma\tau}\,\mathrm{Tr}\left(\Sigma^\dagger\partial_\mu\Sigma\,\partial_\nu\Sigma^\dagger\partial_\rho\Sigma\,\partial_\sigma\Sigma^\dagger\partial_\tau\Sigma\right)$$

$(N_c \in \mathbb{Z})$ + 4D terms required by gauge invariance

Anomalies

Anomalies in the UV theory must be matched in the low-energy description by the introduction of a non-local term



Wess, Zumino (1971) Witten (1983)

$$\Gamma = -\frac{i\, N_c}{240\pi^2}\int_{\mathcal{M}_5} \mathrm{d}^5x\, \epsilon^{\mu\nu\rho\sigma\tau}\, \mathrm{Tr}\left(\Sigma^\dagger\partial_\mu\Sigma\,\partial_\nu\Sigma^\dagger\partial_\rho\Sigma\,\partial_\sigma\Sigma^\dagger\partial_\tau\Sigma\right)$$

$(N_c \in \mathbb{Z})$ $+$ 4D terms required by gauge invariance

- Induces anomalous pion decay in QCD
- (In)famous in LH for breaking T-parity C. .T. Hill, R. J. Hill (2007)

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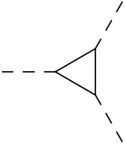

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- Induces anomalous pion decay in QCD
- (In)famous in LH for breaking T-parity — C. .T. Hill, R. J. Hill (2007)
- Fixes the spin statistics of the skyrmion

  N_c even $\Leftrightarrow$ boson $\qquad$ N_c odd $\Leftrightarrow$ fermion

  ($N_c = 3$ in QCD, hence the nucleons are fermions)

Summary: the skyrmion in composite Higgs models

In any composite Higgs model, the electroweak gauge group must be preserved by the global symmetry breaking at scale f

Since at the energy scale of the skyrmion $M \gg f$, the $SU(2)_W$ symmetry is approximately preserved, the lightest skyrmion is always obtained from a Skyrme-Wu-Yang ansatz along the $SU(2)_W$ group.

Independently of the model, if skyrmions are present,
the lightest state is either

- an electroweak singlet boson
- an electroweak doublet fermion

depending on the coefficient N_c of the WZW term

WZW term and hypercharge

The Skyrme-Wu-Yang skyrmion configuration is built out of $SU(2)_W$ generators

$\Rightarrow$ no charge under $U(1)_Y$ from local terms

$$\Sigma_0 = \begin{pmatrix} \exp\left[iF(r)\hat{x}_i\sigma_i\right] & \\ & 1 \end{pmatrix} \qquad Y = \frac{1}{6}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\Rightarrow \qquad \delta_Y \Sigma \propto [Y, \Sigma_0] = 0$$

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But there is a contribution from the non-local WZW term

$$Y(\Sigma_0) = \frac{1}{6} N_c$$

Electric charge of the skyrmion

In our toy model:

- if N_c is even

$$q = \frac{N_c}{6}$$

- if N_c is odd

$$q = \frac{N_c \pm 3}{6}$$

Two possibilities for having $q = 0$

- $N_c = 0 \quad \Longleftrightarrow \quad$ anomaly free UV completion (if any?)
- $N_c = 3 \quad$ as in QCD

In any other case the lightest skyrmion state is electrically charged!

A way out? Extending the symmetry

Generalise the toy model to $SU(N)$, $N > 3$

$$Q_a = \frac{1}{2}\begin{pmatrix} \sigma_a & \\ & \end{pmatrix} \qquad Y = \begin{pmatrix} y_0\,\mathbb{1} & & \\ & y_0 - \frac{1}{2} & \\ & & \ddots \end{pmatrix}$$

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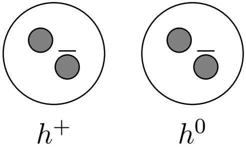
Neutral skyrmion requires

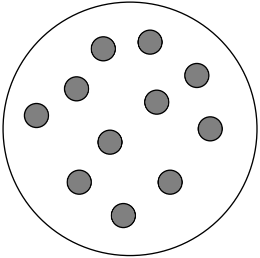
- $y_0 = 0$ for even N_c $\quad \Longrightarrow \quad Q_{em} = \mathrm{diag}\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\right)$
- $y_0 = \pm 1/\left(2N_c\right)$ for odd N_c

In terms of fermions

The Higgs fields are fermion- antifermion bound states



Skyrmions are bound states of N_c fermions



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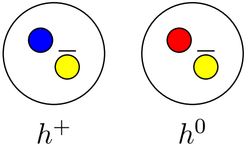

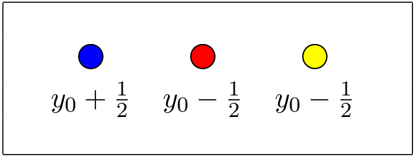

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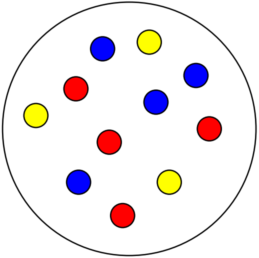

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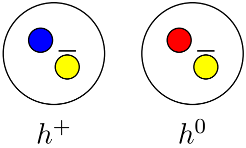

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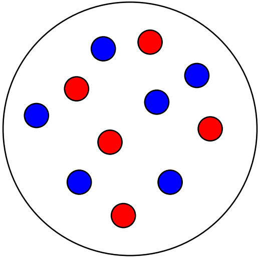

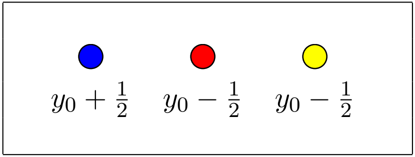

$q = y_0\, N_c$ for N_c odd

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SO(N) × SO(N) / SO(N)

The lightest skyrmion is also obtained with the Skyrme-Wu-Yang ansatz (using a real representation for $SU(2)_W$)

But there is no WZW term and the underlying theory must be free of chiral anomalies

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$\Longrightarrow$ electrically neutral

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Open questions: What is the 4D UV completion? What is the microscopic nature of the skyrmion? Can it be dark matter?

SU(N) / SO(N)

The Goldstone field $\Sigma(x)$ is taken in the two-index symmetric representation of $SU(N)$
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There is a unique implementation of the hypercharge that provides a Higgs doublet

SU(N) / SO(N) : classical skyrmion configuration

The skyrmion is constructed using the Cartan embedding of a $SU(N)$ hedgehog, written in terms of broken generators

$\Longrightarrow$ the skyrmion can not live along the $SU(2)_W$ gauged subgroup

No Skyrme-Wu-Yang ansatz, but the lightest skyrmion still makes use of the $SU(2)$ gauge fields to lower its mass (while remaining spherically symmetric)

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Also: due to the group structure, the coefficient N_c of the Wess-Zumino-Witten term can take half-integer values

SU(N) / SO(N) : electric charge of the skyrmion

There are two skyrmion implementations Σ_+ and Σ_-, identical in mass, size and charge under $SU(2)_W$

Only differ by the hypercharge

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All purely fermionic UV completions have $N_c \geq 2$ and cannot have neutral skyrmions!

Weakly-coupled UV completions exist Csaki, Heinonen, Perelstein, Spethmann (2008)
but the nature of the skyrmion is not clear in that case

The "little Higgs crisis"

Model	*Coset*	*Status*
Min. Composite Higgs	$SO(5)/SO(4)$	
Beyond the MCHM	$SO(6)/SO(5)$	
"Minimal Moose"	$(SU(3) \times SU(3))/SU(3)$	
with custodial sym.	$(SO(5) \times SO(5))/SO(5)$	
"Bestest LH"	$(SO(6) \times SO(6))/SO(6)$	
Littlest Higgs	$SU(5)/SO(5)$	
"Antisym. condensate"	$SU(6)/Sp(6)$	
LH from simple group	$SU(4)/SU(3)$	
Simple two LH	$SU(9)/SU(8)$	
Custodial littlest higgs	$SO(9)/(SO(5) \times SO(4))$	

- Electroweak precision tests require custodial symmetry
- No collective Higgs quartic in models with gauge singlets

Schmaltz, Thaler (2009)

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Conclusions

- Skyrmions appear in many models of strongly-coupled electroweak physics, not only in composite Higgs / little Higgs models B. Campbell, J. Ellis, K. Olive (2012) J. Ellis, M. Karliner, M. Praszalowicz (2012)

- Large mass but small cross-section could make them naturally abundant in the universe

- The lightest skyrmion states prefer to be charged under the electroweak gauge group
  For composite Higgs models, it can be complicated (or impossible) to have the correct charge assignments both for the Goldstone bosons (Higgs doublet) and for the skyrmions

- Need for a better understanding of the strong dynamics beyond the effective approach