QCD effects in precision analysis of the process $B \to K^* l^+ l^-$

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Outline

- Introduction
- *B*-physics experiment
- Theory tools:
  - $\mathcal{H}^{\rm eff}_{\Delta B=1}$ and pQCD at large scales
  - Low scale matrix elements
- Charm-quark loop in $B \to K^* \mu^+ \mu^-$
- OPE on light cone and sum rules
- Summary

LHC data agree with SM predictions well

Primary goal – search (and discovery) of Higgs boson and/or clarification of the mechanism of electroweak symmetry breaking in the standard model

Hope to detect a type of NP beyond SM as well

As for SM, the gauge sector is transparent:
observed fermionic quanta – quarks and leptons
with interaction guided by gauge invariance

Higgs sector and flavor structure
– Yukawa couplings, CKM scheme, CP violation –
no clear principle

Flavor sector is certainly a place for NP

NP search with flavor

Extensive experimental study of B-physics:
CDF, BELLE, BaBar with good results

LHCb – dedicated detector for B-decay

Radiative decays $B \to X_s \gamma$, $B \to K^* \gamma$ $(q^2 = 0)$

Rare semileptonic $B \to K \ell^+ \ell^-$ $(q^2 \neq 0)$

Rare $B \to K^*(K\pi)\ell^+\ell^-$ with four particle final state:
resonance approximation $M^2_{K\pi} = M^2_{K^*}$

Sophisticated angular analysis is possible: large statistics

Angular analysis

Differential rate for $B \to K^*(K\pi)\mu^+\mu^-$

$$\frac{1}{\Gamma}\frac{d^4\Gamma}{d\cos\theta_l d\cos\theta_K d\phi dq^2} = \frac{9}{16\pi}$$

$$\Big(F_L cos^2\theta_K + \frac{3}{4}(1-F_L)(1-cos^2\theta_K)+$$

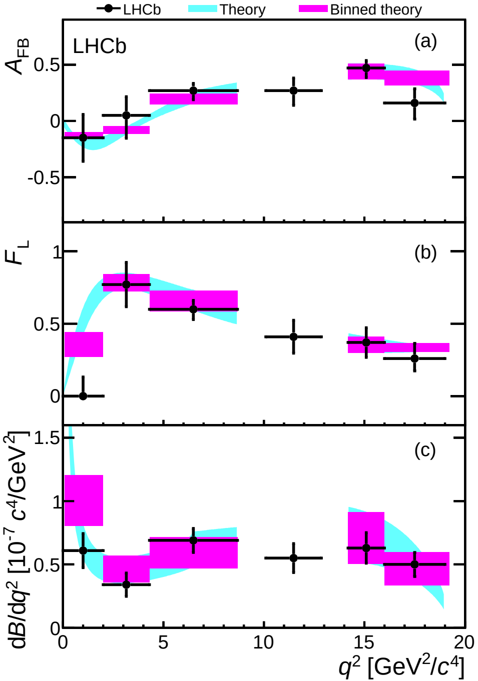
$$F_L cos^2\theta_K(2cos^2\theta_l - 1)+$$

$$\frac{1}{4}(1-F_L)(1-cos^2\theta_K)(2cos^2\theta_l - 1)+$$

$$S_3(1-cos^2\theta_K)(1-cos^2\theta_l)\cos 2\phi+$$

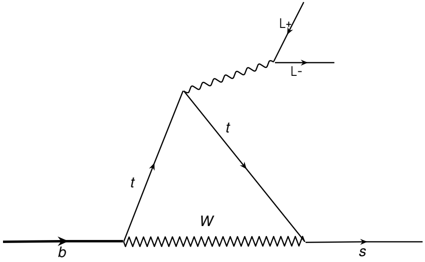
$$\frac{3}{4}A_{FB}(1-cos^2\theta_K)cos\theta_l+$$

$$A_{Im}(1-cos^2\theta_K)(1-cos^2\theta_l)\sin 2\phi\Big)$$



LHCb
arXiv:1112.3515v3
[hep-ex]

FCNC - loop mediated processes

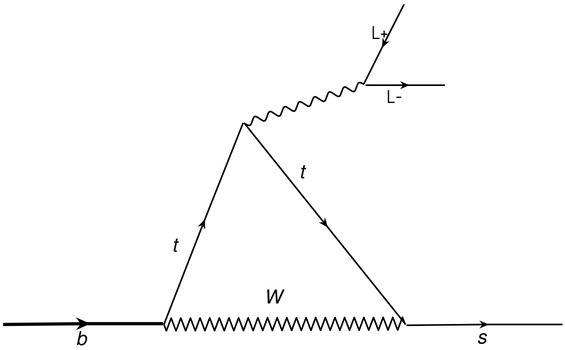


Loop diagram for
$b \to s\ell^+\ell^-$
a SM skeleton
for $B \to K^*\ell^+\ell^-$

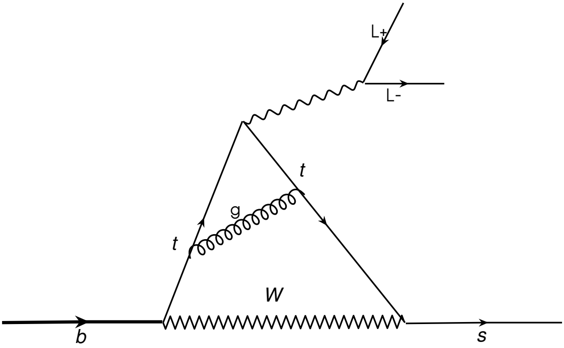
The data constrain parameters of SM and have sensitivity to new physics
Upgrades: superB, BelleII, super LHCb...
Presence of loops strongly suppresses rates in SM but provides sensitivity to NP that requires high precision

Theory description

Problem of precison check: QCD effects
The strong coupling constant is large and one expects corrections... At the parton level (quark diagram) the LO contribution is

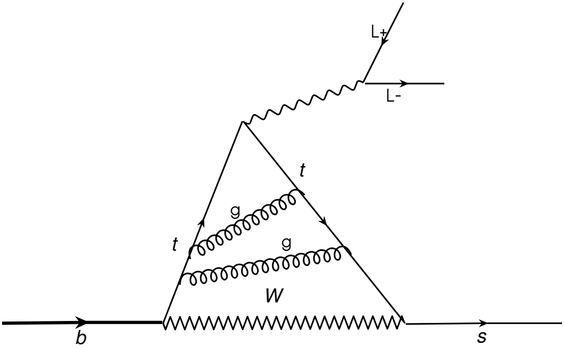


Theory description



The NLO (first iteration) contributions are not small

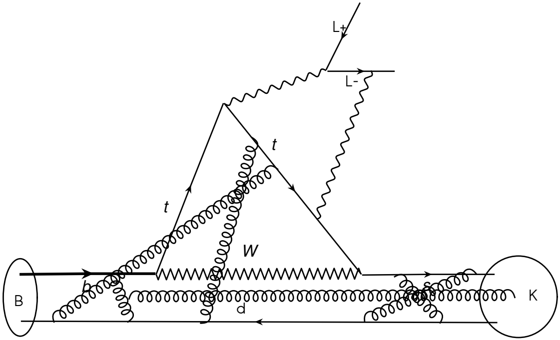
Theory description



Even NNLO (second iteration) are not negligible

And even more

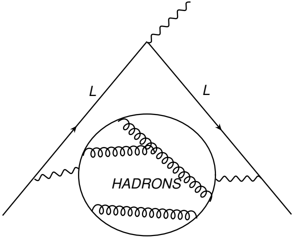
QCD variables are quarks-qluons $\{q, g\}$ while experimental modes are hadrons $B, K, K^*, \pi...$ Therefore,

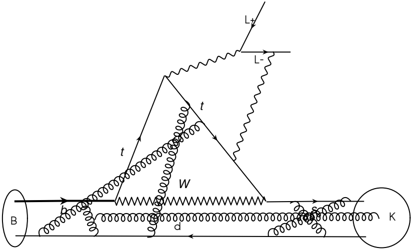


Theory has no tool to built hadrons from quarks...

Comparison with MAMM

Example of muon MM anomaly g-2.
Same logics of loop sensitivity to NP





Muon case is simpler as it has only leptonic external states.
Potentially new particles run in the loops

Theory tool - separation of scales

Scale separation and Effective Theories

EW scale of SM is $v = 250\ \mathrm{GeV}$ or in practice M_W, M_Z, m_t
For $\Delta B = 1$ processes at m_b with $\mu \sim E \sim m_b$
OPE allows for control of $\alpha_s^n ln(M_W/m_b)^k$ within pQCD
("integrating out heavy particles")

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

$C_i(\alpha_s, \mu)$ – short-distance pQCD coeffs,
$O_i(\mu)$ – composite local operators
This factorization is of pure PT origin and under control through PT series in $\alpha_s(\mu)$ for $C_i(\alpha_s(\mu))$

Operators should be at scales $m_b \sim 5\ \mathrm{GeV}$
RG runs down coefficients from scale $\mu \sim M_t, M_Z, M_W$

$$C_i(m_b) = U_{ij}(M_W, m_b, \alpha_s(m_b < \mu < M_W))C_j(M_W)$$

Known two iterations for initial values (NNLO)

$$C_i(M_W) = C_i^{LO} + \alpha_s(\mu)C_i^{NLO} + \alpha_s(\mu)^2 C_i^{NNLO},$$

give few % accuracy ($\alpha_s(M_Z) = 0.12$). Transition matrix

$$U(M_W, m_b, \alpha_s) = U^{LO} + \frac{\alpha_s(\mu)}{\pi}U^{NLO} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 U^{NNLO}$$

depends on mixing γ and running β along the trajectory $M_W > \mu > m_b$ and known with two iterations.
With $\alpha_s(m_b) = 0.20$ the accuracy of $C_i(m_b)$ is at the level of percents.

SM amplitudes as ME of effective operators

At LO in G_F the SM amplitudes reduce to

$$Amp(B \to K^* \ell^+ \ell^-) = -\langle K^* \ell^+ \ell^- \mid H_{eff} \mid B \rangle + \mathcal{O}(\text{dim } 8)$$

$$\sim C_i(\mu = m_b) \langle K^* \ell^+ \ell^- \mid O_i(\mu) \mid B \rangle + \mathcal{O}\left(\frac{m_b^2}{M_W^2, m_t^2}\right)$$

$C_i(\mu)$ are so precise that the next interation can require em corr and also terms of order $m_b^2/M_W^2 \sim (1/20)^2$. Top quark mass uncertainty becomes important.
This is a triumph of pQCD at large scales.
The rest is hadronic matrix elements of $O_i(m_b)$.
And troubles begin.

Operators giving leading contributions

$$ME_i = \langle K^* \ell^+ \ell^- \mid O_i(m_b) \mid B \rangle$$

contain no large logs but depend on IR structure of QCD
– hadrons as bound states
Operators

$$O_9 = \frac{\alpha_{em}}{4\pi} (\bar{s}_L \gamma_\rho b_L) \left(\bar{\ell} \gamma^\rho l \right), \qquad O_{10} = \frac{\alpha_{em}}{4\pi} (\bar{s}_L \gamma_\rho b_L) \left(\bar{l} \gamma^\rho \gamma_5 l \right)$$

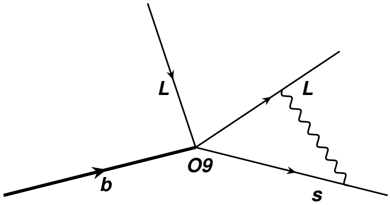
allow for tree level computation through

$$\langle K^* \ell^+ \ell^- \mid O_9(\mu) \mid B \rangle = \langle K^* \mid J_h(\mu) \mid B \rangle \langle \ell^+ \ell^- \mid J_l(\mu) \mid 0 \rangle$$

Completely factorizable

Relevant operators and corrections

Contributions due to $O_{9,10}$ are dominant
Expressible through hadronic form factors of local operators



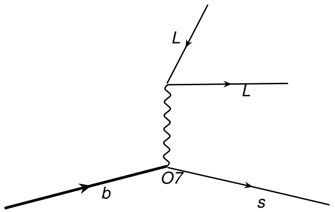
Still there are interactions at low energies that violate this approximation.
This is em correction...

Relevant operators

Operator

$$O_{7\gamma} = -\frac{e}{16\pi^2}\bar{s}\sigma_{\mu\nu}(m_s L + m_b R)bF^{\mu\nu}$$

requires low energy em vertex to allow for the process



and also reduces to hadronic form factors.

Contributions of loop operators

Contribution of O_9 is parametrized

$$\langle K^*(p)|\bar{s}_L\gamma_\rho b_L|B(p+q)\rangle = \epsilon_{\rho\alpha\beta\gamma}\epsilon^{*\alpha}q^\beta p^\gamma \frac{V(q^2)}{m_B+m_{K^*}}$$

$$-i\epsilon^*_\rho(m_B+m_{K^*})A_1(q^2) + i(2p+q)_\rho(\epsilon^* q)\frac{A_2(q^2)}{m_B+m_{K^*}}$$

Form factors $A_1(q^2)$, $A_2(q^2)$, $V(q^2)$ require nonPT methods to be used
At present the main tool is LCSR with typical accuracy of order of 10%

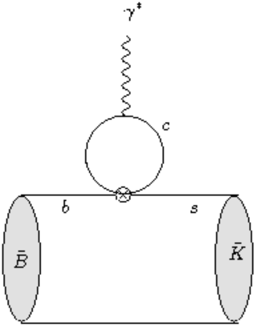
There appear still more complicated objects

Charm loops picture

Tree-level four-quark charm operators O_1 and O_2

$$O_1 = (\bar{s}_L \gamma_\rho c_L)(\bar{c}_L \gamma^\rho b_L) , \qquad O_2 = \left(\bar{s}^j_L \gamma_\rho c^i_L\right)\left(\bar{c}^i_L \gamma^\rho b^j_L\right)$$

have large Wilson coeff and important. They lead to non-local low energy contributions of the form



There are also pinquin operators that generate the same problem but their coefficients are smaller.

This is a generic QCD problem.

Two kinematical regions:

large recoil or small q^2 ($1 < q^2 < 6$, basically $q^2 < 4m_c^2$ – we neglect Cabibbo suppressed u-contributions)

QCDF is used – kaon is energetic with scaling $E_K \sim m_b$ in B-meson rest frame. Expansion in $1/m_b$ and $1/E_K$ is used (HQET+SCET techniques).

Expansions are non local, require LC DA's of kaon and B-meson, the expansion parameter $\Lambda_{\mathrm{QCD}}/E_K$ is not small, problems with higher orders in $\Lambda_{\mathrm{QCD}}/E_K$...

Advantage – form factor symmetries and reduction of the number of independent form factors at LO

Low-recoil region

Second region: Low recoil - large q^2

Applicability range $M^2_{\psi_n} < q^2 < m_b^2 = 23\ \mathrm{GeV}$
It allows for a local OPE over $1/m_b, 1/\sqrt{q^2}$ in HQET framework.

Problems: expansion is in timelike region ("on the cut") and close to resonance region, in fact expansion is over $1/(m_b^2 - 4m_c^2)$ that decreases the region of applicability, efficient for penguin with light quarks...

Seems, low recoil region is now theoretically cleaner

Charm-loops amplitude

An estimate of a new effect at low q^2 region
Contribution to the $A(B \to K^* \ell^+ \ell^-)$ reads

$$Amp^{O_{1,2}} = -(4\pi\alpha_{em} Q_c) \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\bar{\ell}\gamma^\mu \ell}{q^2} \mathcal{H}_\mu^{(B\to K^*)}(p,q)$$

with $Q_c = 2/3$ and

$$\mathcal{H}_\mu^{(B\to K^*)}(p,q) = i \int dx\ e^{iqx} \times$$

$$\langle K^*(p)| T \bar{c}\gamma_\mu c(x) \Big[C_1 O_1(0) + C_2 O_2(0) \Big] |B(p+q)\rangle$$

Charm-loops amplitude essence

Key quantity ($O_{1,2} \sim (\bar{s}c)(\bar{c}b)$)

$$T_\mu = TO_{1,2}(0)J_\mu(x) = T\bar{c}\gamma_\mu c(x)O_{1,2}(0)$$

is a nonlocal amplitude that is expanded on LC

$$TO(0)J_\mu(x) = \bar{s}\Gamma b \otimes C(x) + \bar{s}Gb \otimes C_G(x) + \ldots$$

At LO in α_s and LO operator, the coefficient C is given by the two-point product

$$C \to (c^i(x)\bar{c}^j(x))J_\mu(0)$$

It can be computed in PT at $q^2 \ll 4m_c^2$ that leads to fact approximation (LO of QCDF)

Charm-loops amplitude at LO = factorization

Thus at LO the amplitude

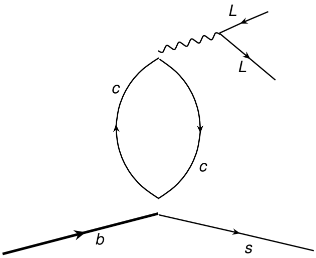
$$\mathcal{H}_\mu^{(B\to K^*)}(p,q)|_{\rm LO} = \left(\frac{C_1}{3} + C_2\right) \langle K^*(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle$$

where both $O_{1,2}$ contribute and the local operator

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho})\frac{9}{32\pi^2}\, g(m_c^2, q^2)\bar{s}_L\gamma^\rho b_L$$

reduces ME to $B \to K^*$ form factors.

Factorized charm loop

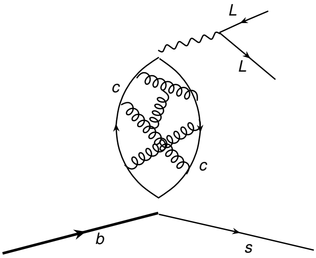


The LO charm-loop coefficient function is given by its imaginary part

$$\frac{1}{\pi}\,\mathrm{Im}_s g(m_c^2, s) = \frac{4}{9}\sqrt{1-\frac{4m_c^2}{s}}\left(1+\frac{2m_c^2}{s}\right)\Theta(s-4m_c^2)$$

It is “short-distance” charm-loop effect.

A model generalization

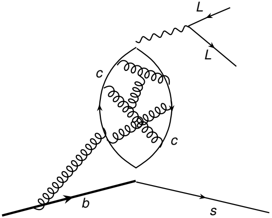


One can generalize the contribution of charm loop using physical representation through dispersion relation in the variable q^2

$$\frac{1}{\pi}\,\mathrm{Im}_s g(m_c^2, s) = \sum_i f_i^2 \delta(s - M_{\psi_i}^2) + \mathrm{cont(DD)}$$

This expression leads to the same behavior at low q^2 but can also be continued to large q^2 giving a model of analytic continuation.

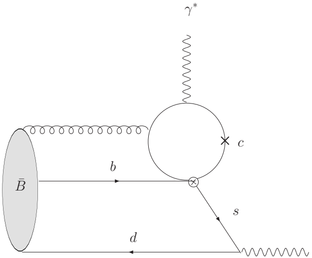
Corrections to factorization (LO) are both PT and nonPT.



Even NLO PT corrections violate factorization.

I discuss the nonPT soft gluon corrections

A.Khodjamirian,Th.Mannel,AAP,Y.-M.Wang, JHEP09(2010)089



Here $B \to K^*$ matrix element contains soft-gluon emission from the charm loop.

NonFactorized charm loop

The c-quark loop with the emitted gluon generates the nonlocal effective operator $\widetilde{O}_\mu$

One casts the soft-gluon emission part to a form

$$\mathcal{H}_\mu^{(B\to K^*)}(p,q)|_{nonfact} = 2C_1 \langle K^*(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle$$

where $\widetilde{\mathcal{O}}_\mu(q)$ is a convolution of the coefficient function with the nonlocal operator

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L$$

This matrix element resembles a nonforward distribution with different initial and final hadrons.

Spectral density of coeff function

The coeff function $I_{\mu\rho\alpha\beta}(q,\omega)$ is given by its spectral density

$$\frac{1}{\pi}\mathrm{Im}\, I_{\mu\rho\alpha\beta}(q,\omega) = \frac{m_c^2\Theta(\tilde{q}^2 - 4m_c^2)}{4\pi^2\tilde{q}^2\sqrt{\tilde{q}^2(\tilde{q}^2 - 4m_c^2)}}$$

$$\int_0^1 du \Big\{ \bar{u}\tilde{q}_\mu\tilde{q}_\alpha g_{\rho\beta} + u\tilde{q}_\rho\tilde{q}_\alpha g_{\mu\beta} - \Big[u + \frac{(\bar{u}-u)\tilde{q}^2}{4m_c^2}\Big]\tilde{q}^2 g_{\mu\alpha}g_{\rho\beta}\Big\}$$

with $\tilde{q} = q - u\omega n_-$, so that $\tilde{q}^2 \simeq q^2 - 2u\omega m_b$

Non-factorizable amplitude reads

$$\mathcal{H}_\mu^{(B\to K^*)}(p,q) = (C_1/3 + C_2)\langle K^*(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle$$

$$+2C_1\langle K^*(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle = \epsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}q^\beta p^\gamma H_1(q^2)$$
$$+i[(m_B^2 - m_{K^*}^2)\epsilon_\mu^* - (\epsilon^* q)(2p+q)_\mu]H_2(q^2)$$
$$+i(\epsilon^* q)[q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(2p+q)_\mu]H_3(q^2)$$

and $\mathcal{H}_i(q^2) = (C_1/3 + C_2)V_i(q^2) + 2C_1\tilde{V}_i(q^2)$ with $\tilde{V}(q^2)$ being soft-gluon amplitude.
Numerically, $C_1 = 1.12$, $C_2 = -0.27$ that suppresses (enhances) $V(q^2)$ ($\tilde{V}(q^2)$)

LC sum rules

We compute soft-gluon emission amplitude $\tilde{V}_i$ using LCSR with the B-meson DA's

Take correlation function

$$\mathcal{F}_{\nu\mu}(p,q) = i \int dy e^{ip\cdot y} \langle 0|T j_\nu^{K^*}(y) \widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle$$

with $j_\nu^{K^*} = \bar{d}\gamma_\nu s$ and extract a residue

$$\mathcal{F}_{\nu\mu}(p,q) = \frac{f_{K^*}\epsilon_\nu \langle K^*(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle}{m_{K^*}^2 - p^2} + \int_{s_h}^{\infty} \frac{ds\, \rho_{\nu\mu}(s,q^2)}{s-p^2}$$

where f_{K^*} is the K^* residue into the current and higher mass states are represented by the integral from s_h

B-meson DA decomposition

In theory the correlator is computed using B-meson DA with independent components Ψ_A, Ψ_V, X_A, Y_A

$$\langle 0|\bar{d}_\alpha(y)\delta[\omega - \frac{(in_+\mathcal{D})}{2}]G_{\sigma\tau}(0)b_\beta(0)|\bar{B}(v)\rangle$$

$$= \frac{f_B m_B}{2}\int d\lambda e^{-i\lambda yv}\Big[(1+\not{v})\Big\{(v_\sigma\gamma_\tau - v_\tau\gamma_\sigma)\big[\Psi_A - \Psi_V\big]$$

$$-i\sigma_{\sigma\tau}\Psi_V - \frac{y_\sigma v_\tau - y_\tau v_\sigma}{v\cdot y}X_A + \frac{y_\sigma\gamma_\tau - y_\tau\gamma_\sigma}{v\cdot y}Y_A\Big\}\gamma_5\Big]_{\beta\alpha}$$

where f_B is the B-meson decay constant

B-meson DA model

Model DA for B-meson

$$\Psi_A(\lambda,\omega) = \Psi_V(\lambda,\omega) = \frac{\lambda_E^2}{6\omega_0^4}\omega^2 e^{-(\lambda+\omega)/\omega_0}$$

$$X_A(\lambda,\omega) = \frac{\lambda_E^2}{6\omega_0^4}\omega(2\lambda-\omega)e^{-(\lambda+\omega)/\omega_0}$$

$$Y_A(\lambda,\omega) = -\frac{\lambda_E^2}{24\omega_0^4}\omega(7\omega_0-13\lambda+3\omega)e^{-(\lambda+\omega)/\omega_0}$$

Here $\omega_0 = 1/\lambda_B$ of the B-meson two-particle DA ϕ_+^B

Normalization of the three-particle DA's is $\lambda_E^2 = 3/2\lambda_B^2$

Results for C_9: $O_9 = sbll$

$$\Delta C_9^{\bar{c}c,M_i} = (C_1 + 3C_2) g(m_c, q) + 2C_1 \tilde{g}^{\bar{c}c,M_i}$$

with

$$\tilde{g}^{\bar{c}c,M_1}(q^2) = -\frac{(m_B + m_{K^*})}{q^2} \frac{\tilde{V}_1(q^2)}{V^{BK^*}(q^2)}$$

$$\tilde{g}^{\bar{c}c,M_2}(q^2) = \frac{(m_B - m_{K^*})}{q^2} \frac{\tilde{V}_2(q^2)}{A_1^{BK^*}(q^2)}$$

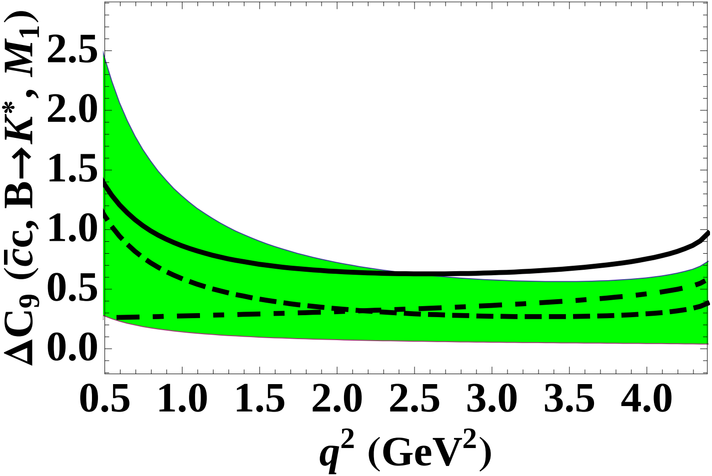
$$\tilde{g}^{\bar{c}c,M_3}(q^2) = \frac{m_B + m_{K^*}}{q^2} \frac{\tilde{V}_2(q^2)}{A_2^{BK^*}(q^2)} + \frac{1}{m_B - m_{K^*}} \frac{\tilde{V}_3(q^2)}{A_2^{BK^*}(q^2)}$$

$C_1 = 1.12$, $C_2 = -0.27$ that makes $C_1 + 3C_2 = 0.31$

$$C_9 = 4.2$$

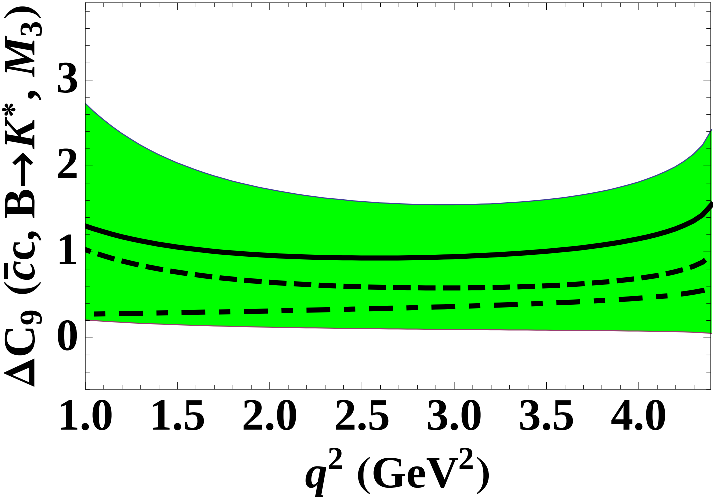
Results for K^*

Coeff ΔC_9 for $\mathcal{M}_1$ amplitude



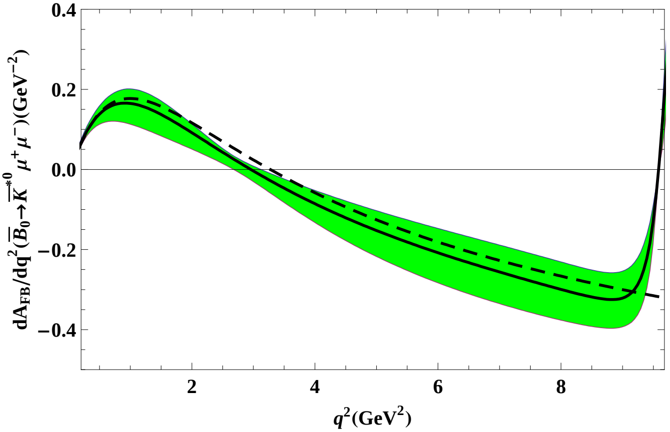
total – solid, soft-gluon – dashed
factorizable – dash-dotted

Coeff ΔC_9 for $\mathcal{M}_3$ amplitude



total – solid, soft-gluon – dashed
factorizable – dash-dotted

$q_0^2 = 2.9 \pm 0.3$ while without soft-gluon $q_0^2 = 3.2$



Forward-backward asymmetry for $\bar{B}_0 \to \bar{K}^*\mu^+\mu^-$ decay with charm-loop effect (solid), without this effect (dashed).

Summary

c-quark operators come with large Wilson coeffs. The accuracy of their ME should be high:

- new effect of soft gluons violating factorization has been considered in the OPE near LC
- LCSR with B-meson DA for quantitative analysis. Soft-gluon contribution is enhanced by Wilson coefficient for $B \to K^* \ell^+ \ell^-$ and numerically important
- the magnitude of the effect varies with observables and can reach $\sim 10\%$ in K^*-meson decays being smaller for kaons.

Accuracy of ME is low compared to one of Wilson coeff. It is still comparable with exp data precision but requires a lot of further improvement...

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C.Bobeth, P.Gambino, M.Gorbahn, U.Haisch,- ’11

c-loops:
G.Buchalla, G.Isidory, S.J.Rey,- ’98