Event Generators: Automated Computation of Leading Order QCD Matrix Elements Part 2

Daniel Götz & Christopher Schwan

Institut für Physik
WA THEP
JGU Mainz

November 25, 2011

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

1) Reminder: Color Decomposition and Berends-Giele Recursion

2) Computation of Observables and Helicity Treatment

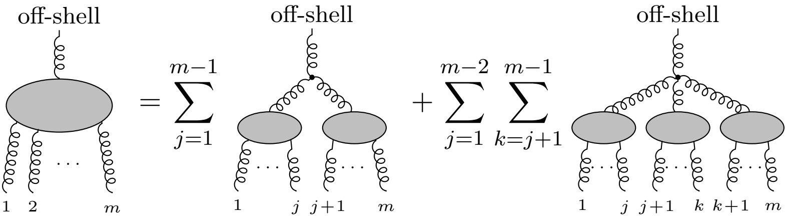
3) Data and Results (Christopher)

Reminder: Computation of Scattering Amplitudes at LO

Matrix elements can be computed using *color decomposition*:

$$\mathcal{A} = \sum_{P \in S_{n-1}} c_P A_P, \qquad |\mathcal{A}|^2 = \sum_{\substack{P \in S_{n-1} \\ P' \in S_{n-1}}} A_P^\dagger C_{PP'} A_{P'}$$

A_P are the *partial amplitudes*, which depend on momenta and helicites of the external particles, and their permutation given by P. They can be computed via the Berends-Giele recursion relations



Computation of Observables

We cannot measure matrix elements! Instead we measure observables $\mathcal{O}$:

$$\langle\mathcal{O}\rangle \propto \int \mathrm{d}\phi_{n-2}\, \mathcal{O}(\phi_n)\, \overline{|\mathcal{A}(\phi_n)|}^2$$

Integration over final state momenta (phase space) performed by Monte Carlo integration methods.

Computation of Observables

We cannot measure matrix elements! Instead we measure observables $\mathcal{O}$:

$$\langle \mathcal{O} \rangle \propto \int \mathrm{d}\phi_{n-2}\, \mathcal{O}(\phi_n)\, \overline{|\mathcal{A}(\phi_n)|}^2$$

Integration over final state momenta (phase space) performed by Monte Carlo integration methods.

Monte Carlo integration:

Evaluate integrand at N randomly chosen phase space points ϕ_{n-2} and take average as estimate for the integral. (N large!)

$$\int f(\vec{x})\,\mathrm{d}\vec{x} \;\approx\; \frac{1}{N}\sum_{i=1}^{N} f(\vec{x}_i)$$

Helicity Treatment

Most colliders measure *unpolarized* cross sections, not sensitive to particle helicities/spins!

$\curvearrowright$ Unpolarized amplitude by summation over helicities:

$$\overline{|\mathcal{A}|}^2 = \frac{1}{4} \underbrace{\sum_{\lambda_1} \sum_{\lambda_2} \cdots \sum_{\lambda_n}}_{2 \cdot 2 \cdots 2 = 2^n \text{ helicities}} |\mathcal{A}_{\lambda_1 \lambda_2 \cdots \lambda_n}|^2$$

$\Rightarrow$ Have to evaluate matrix element 2^n times, once for every helicity configuration of external particles!

Helicity Treatment

Most colliders measure *unpolarized* cross sections, not sensitive to particle helicities/spins!

$\curvearrowright$ Unpolarized amplitude by summation over helicities:

$$\overline{|\mathcal{A}|}^2 = \frac{1}{4} \underbrace{\sum_{\lambda_1} \sum_{\lambda_2} \cdots \sum_{\lambda_n}}_{2 \cdot 2 \cdots 2 = 2^n \text{ helicities}} |\mathcal{A}_{\lambda_1 \lambda_2 \cdots \lambda_n}|^2$$

$\Rightarrow$ Have to evaluate matrix element 2^n times, once for every helicity configuration of external particles!

$\Rightarrow$ very slow, but:

We can do better!

Helicity Sampling

Neccessary property for polarization vectors:

$$\sum_{\lambda=\pm} \epsilon_\mu^\lambda (\epsilon_\nu^\lambda)^* \; = \; \epsilon_\mu^+ (\epsilon_\nu^+)^* + \epsilon_\mu^- (\epsilon_\nu^-)^*$$

Idea: Use different polarization to achieve same result w/o sum!

Helicity Sampling

Neccessary property for polarization vectors:

$$\sum_{\lambda=\pm} \epsilon_\mu^\lambda (\epsilon_\nu^\lambda)^* \; = \; \epsilon_\mu^+ (\epsilon_\nu^+)^* + \epsilon_\mu^- (\epsilon_\nu^-)^*$$

Idea: Use different polarization to achieve same result w/o sum!

Naive attempt:

$$\epsilon_\mu = \epsilon_\mu^+ + \epsilon_\mu^-$$

$$\epsilon_\mu (\epsilon_\nu)^* \; = \; \epsilon_\mu^+ (\epsilon_\nu^+)^* + \epsilon_\mu^- (\epsilon_\nu^-)^* + \epsilon_\mu^+ (\epsilon_\nu^-)^* + \epsilon_\mu^- (\epsilon_\nu^+)^*$$

Helicity Sampling

Neccessary property for polarization vectors:

$$\sum_{\lambda=\pm} \epsilon_\mu^\lambda (\epsilon_\nu^\lambda)^* \;=\; \epsilon_\mu^+ (\epsilon_\nu^+)^* + \epsilon_\mu^- (\epsilon_\nu^-)^*$$

Idea: Use different polarization to achieve same result w/o sum!

Better idea:

$$\epsilon_\mu(\theta) = \mathrm{e}^{\mathrm{i}\theta} \epsilon_\mu^+ + \mathrm{e}^{-\mathrm{i}\theta} \epsilon_\mu^-, \qquad 0 \le \theta < 2\pi$$

$$\frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta \, [\epsilon_\mu(\theta) \, \epsilon_\nu^*(\theta)] \;=\; \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta \Big[\epsilon_\mu^+ (\epsilon_\nu^+)^* + \epsilon_\mu^- (\epsilon_\nu^-)^* + \mathrm{e}^{2\mathrm{i}\theta} \epsilon_\mu^+ (\epsilon_\nu^-)^* + \mathrm{e}^{-2\mathrm{i}\theta} \epsilon_\mu^- (\epsilon_\nu^+)^* \Big]$$

Helicity Sampling

Neccessary property for polarization vectors:

$$\sum_{\lambda=\pm} \epsilon_\mu^\lambda (\epsilon_\nu^\lambda)^* \;=\; \epsilon_\mu^+ (\epsilon_\nu^+)^* + \epsilon_\mu^- (\epsilon_\nu^-)^*$$

Idea: Use different polarization to achieve same result w/o sum!

Better idea:

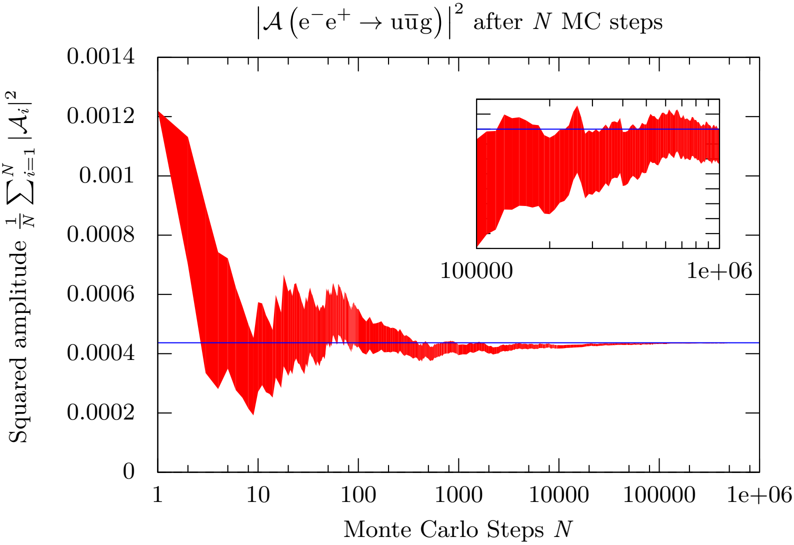
$$\epsilon_\mu(\theta) = \mathrm{e}^{\mathrm{i}\theta} \epsilon_\mu^+ + \mathrm{e}^{-\mathrm{i}\theta} \epsilon_\mu^-, \qquad 0 \le \theta < 2\pi$$

$$\frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta \, [\epsilon_\mu(\theta)\, \epsilon_\nu^*(\theta)] \;=\; \sum_{\lambda=\pm} \epsilon_\mu^\lambda (\epsilon_\nu^\lambda)^*$$

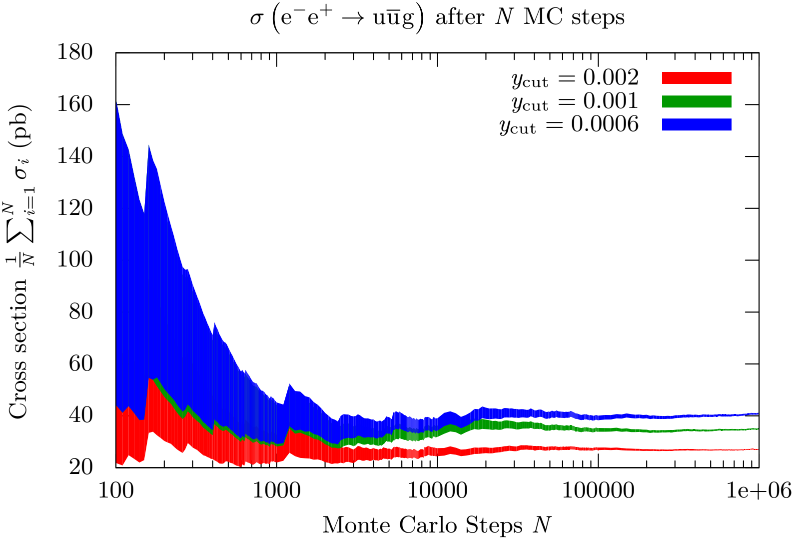
$\Rightarrow$ Integration can be combined with phase space integral!

Saves factor of 2^n evaluations of squared matrix elements!

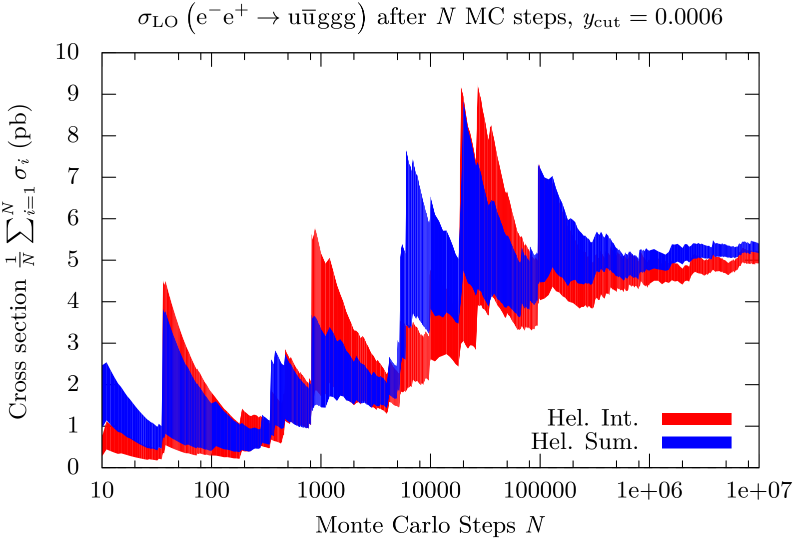
MC Integration (Helicity)



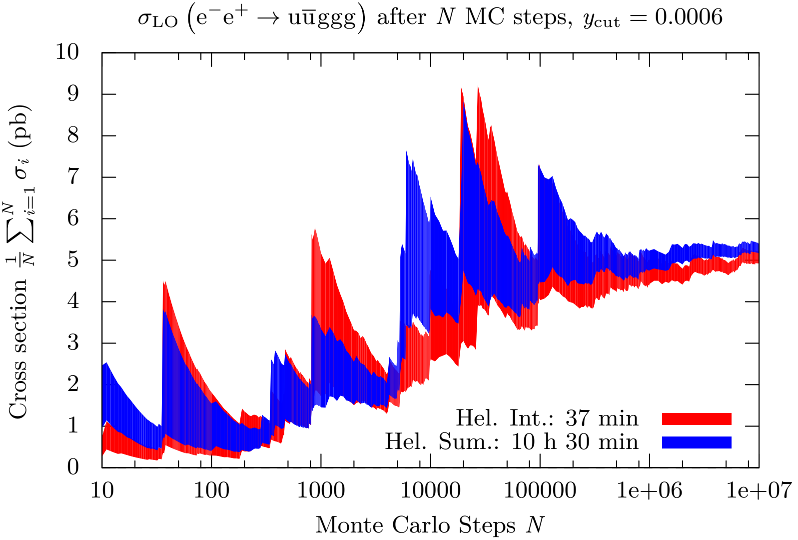
MC Integration (Helicity+Phase Space)



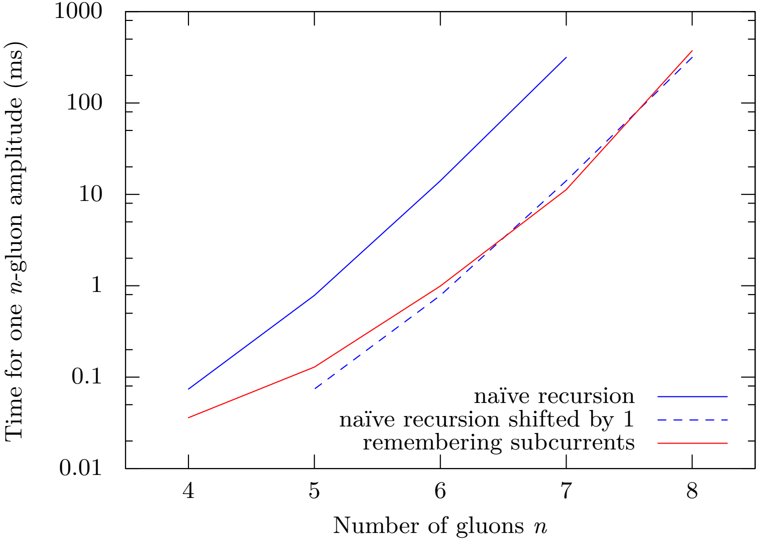
MC Integration (Helicity+Phase Space)



MC Integration (Helicity+Phase Space)



Storing Subcurrents: What's the Benefit for Gluon Processes: $gg \to (n-2)g$?



Comparison with MadGraph 5 (MG5)

- Comparison of our program with MG5 is a bit like comparing apples and oranges, but timings give "a feeling"
- MG5 is not automated: you need to ...
  - Specify process first (e.g. $gg \to ggggg$) in MG5's interface which then will generate Fortran code
  - Compile generated program (may take a lot of time and memory)
  - Execute the program (very fast).
- Our program is automated: a user specifies a process and our program outputs the result. There are no intermediate steps

Comparison with MG5: Timings

- Computing time for LEP amplitude of the type $\left|\mathcal{A}\left(e^-e^+ \to q\bar{q} + ng\right)\right|^2$ in milliseconds:

n	MG5[1]	CFD+BG[2]	
		Hel. Sum.	Hel. Int.
0	4(2)	0.7180(45)	0.05189(71)
1	4(2)	1.5623(50)	0.06110(20)
2	5(2)	3.7940(41)	0.08882(20)
3	7(2)	18.171(10)	0.2847(10)
4	70(5)	213.80(82)	1.876(21)
5	564(10)	3950.9(2.0)	45.25(13)
6	?[3]	272270(98)	1240.09(30)

[1] Execution time only

[2] CFD = Color Flow Decomposition, BG = Berends-Giele-Type Recursion

[3] Compilation aborted after 4h using more than 8 GB memory