QCD resummation for Drell-Yan-like processes Beyond the Standard Model.

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Outline.

1. Motivation for precision calculations.
2. Resummation and parton showering.
3. Numerical results, including uncertainties, for Z' and supersymmetry at the LHC.
4. Summary - conclusions.

To start: a simple question.

- **One of the LHC purposes: which model of new physics is the correct one?**
  - We need **data**.
  - We need **theoretical predictions** for all models.
  - We need **reliable** predictions. [that is the aim of this talk].

Confront data and theory.

- **How to make reliable predictions? - toy case.**
  - Process: **Drell-Yan lepton pair production at the Tevatron**.
  - Considered observables:
    - ◇ the lepton-pair **invariant-mass distribution** $\frac{\mathrm{d}\sigma}{\mathrm{d}M}$.
    - ◇ the lepton-pair **transverse-momentum distribution** $\frac{\mathrm{d}\sigma}{\mathrm{d}p_T}$.
  - No new physics [for the moment...].

QCD factorization theorem.

$$\frac{d\sigma}{d\omega} = \sum_{ab} \int dx_a \, dx_b \, \mathbf{f_{a/p_1}}(x_a; \mu_F) \, \mathbf{f_{b/p_2}}(x_b; \mu_F) \, \frac{d\sigma_{\mathbf{ab}}}{d\omega}(\ldots, \mu_F) \ ,$$

where ω is any kinematical variable.

- **Long-distance and short-distance physics factorize.**
  $\Rightarrow$ **Convolution** of parton densities and the partonic distribution.
- **Long-distance physics: parton densities $\mathbf{f_{a/p_1}}$, $\mathbf{f_{b/p_2}}$.**
  * Fitted from experimental data.
  * Depend on the momentum fractions x_i of parton i in the proton p_i.
- **Short-distance physics: differential partonic cross section $d\sigma_{\mathbf{ab}}$.**
  * Use of **QCD perturbation theory**.

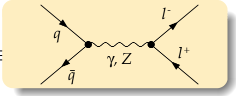
$$d\sigma = d\sigma^{(0)} + \alpha_s \, d\sigma^{(1)} + \ldots .$$

  * Calculation of the matrix elements order by order.
- **Introduction of the unphysical factorization scale μ_F.**
  * Separate long distances from short distances.

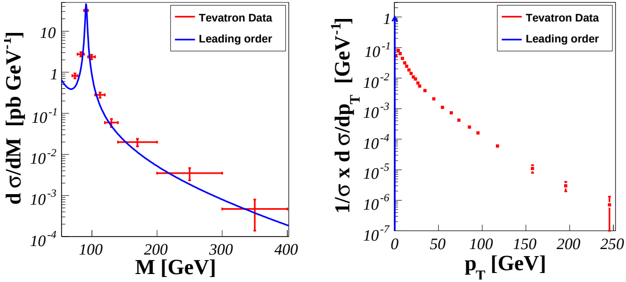
First guess: leading order predictions.

- **First easy naive approach: matrix element calculation at leading order:**

$$\mathrm{d}\sigma \approx \mathrm{d}\sigma^{(0)} \quad \text{with} \quad \mathrm{d}\sigma^{(0)} \equiv$$



- **Confrontation between theory and Tevatron data** [DØ collaboration (2005, 2008)].



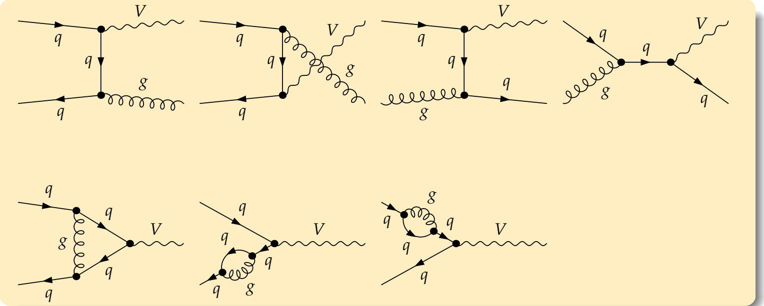
Disagreement between theory and experiment.

Improvement: next-to-leading order predictions (1).

- **Improvement of the predictions: next-to-leading order calculation**.

$$d\sigma \approx d\sigma^{(0)} + \alpha_s\, d\sigma^{(1)}.$$

$\sigma^{(1)} \equiv$



- **Partonic invariant-mass and transverse-momentum distributions at** $\mathcal{O}(\alpha_s)$,

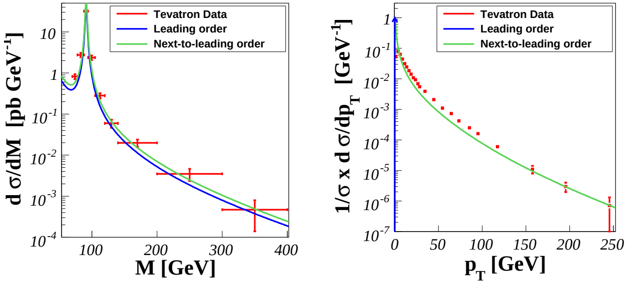
$$\frac{d\sigma}{dM} = d\sigma^{(0)}(M)\,\delta(1-z) + \alpha_s\, d\sigma^{(1)}(M,z) + \mathcal{O}(\alpha_s^2),$$

$$\frac{d^2\sigma}{dM\, dp_T} = d\sigma^{(0)}(M)\,\delta(p_T)\delta(1-z) + \alpha_s\, d\sigma^{(1)}(M,z,p_T) + \mathcal{O}(\alpha_s^2)\ ,$$

where $z = M^2/s$.

Second try: next-to-leading order predictions (2).

- **Confrontation between theory and Tevatron data** [DØ collaboration (2005, 2008)].



> * Invariant-mass distribution: **good agreement**.
>
> * p_T-distribution:
> - ◇ **Very good agreement in the large-p_T region**.
> - ◇ **Underestimation in the intermediate-p_T region**.
> - ◇ **Divergence in the small-p_T region**.

- **How to improve NLO predictions** [in particular for the small-p_T region]?

Investigation of the next-to-leading order contributions (1).

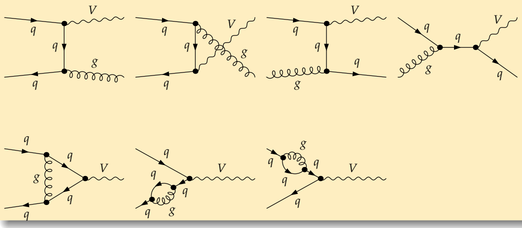
- **Partonic invariant-mass and transverse-momentum distributions at $\mathcal{O}(\alpha_s)$**,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M} = \mathrm{d}\sigma^{(0)}(M)\,\delta(1-z) + \alpha_s\,\mathrm{d}\sigma^{(1)}(M,z) + \mathcal{O}(\alpha_s^2),$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M\,\mathrm{d}p_T} = \mathrm{d}\sigma^{(0)}(M)\,\delta(p_T)\delta(1-z) + \alpha_s\,\mathrm{d}\sigma^{(1)}(M,z,p_T) + \mathcal{O}(\alpha_s^2)\ ,$$

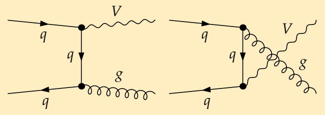
where $z = M^2/s$.

- $\mathrm{d}\sigma^{(1)}$ **contains three different pieces**.
  * **Real gluon emission** diagrams.
  * **Quark-gluon** channels.
  * **Virtual loop** contributions.



Investigation of the next-to-leading order contributions (2).

- **Amplitude for soft real gluon emission.**



$$iM = g_s T^a \ \bar{v}(k_2) \left[\frac{\not\epsilon^*(k_g) \left(\not k_g + \not k_2 \right) \Gamma^\mu_{qqV}}{2 k_2 \cdot k_g} - \frac{\Gamma^\mu_{qqV} \left(\not k_g + \not k_1 \right) \not\epsilon^*(k_g)}{2 k_1 \cdot k_g} \right] u(k_1)$$

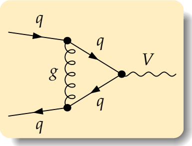
$$\approx g_s T^a \left[\frac{\epsilon^* \cdot k_2}{k_2 \cdot k_g} - \frac{k_1 \cdot \epsilon^*}{k_1 \cdot k_g} \right] \bar{v}(k_2)\ \Gamma^\mu_{qqV} u(k_1)$$

$$= g_s T^a \left[\frac{\epsilon^* \cdot k_2}{k_2^0 \mathbf{k_g^0}(\mathbf{1} + \cos\theta)} - \frac{k_1 \cdot \epsilon^*}{k_1^0 \mathbf{k_g^0}(\mathbf{1} - \cos\theta)} \right] \mathbf{iM}^{\rm Born} \ .$$

Soft and collinear radiation diverges and factorizes.

Investigation of the next-to-leading order contributions (3).

- **Amplitude for the virtual contribution (soft gluons in the loop).**



$$iM = (i\,g_s^2)\bar{v}(k_2)\int \mathrm{d}k_g \,\frac{\gamma_\nu\left(\not{k}_2+\not{k}_g\right)\Gamma^\mu_{qqV}\left(\not{k}_1-\not{k}_g\right)\gamma^\nu}{k_g^2\,(2k_1\cdot k_g)(2k_2\cdot k_g)}\,u(k_1)$$

$$\approx (i\,g_s^2)\int \mathrm{d}k_g\,\frac{k_1\cdot k_2}{k_g^2\,(k_1\cdot k_g)(k_2\cdot k_g)}\,iM^{\mathrm{Born}}$$

$$= (i\,g_s^2)\int \mathrm{d}k_g\,\frac{k_1\cdot k_2}{k_g^2\,(k_1^0\mathbf{k_g^0}(\mathbf{1}-\cos\theta))(k_2^0\mathbf{k_g^0}(\mathbf{1}+\cos\theta))}\,\mathbf{iM}^{\mathrm{Born}}\ .$$

The virtual contributions diverge and factorize.

The problem of the soft and collinear radiation (1).

- **Sum of the two contributions.**

$$d\sigma^{(1)} = d\sigma^{(1,\text{loop})} + d\sigma^{(1,\text{real})} \ .$$

  * **Cancellation** of the poles.
  * **Infrared behaviour:** logarithmic terms in the distributions,

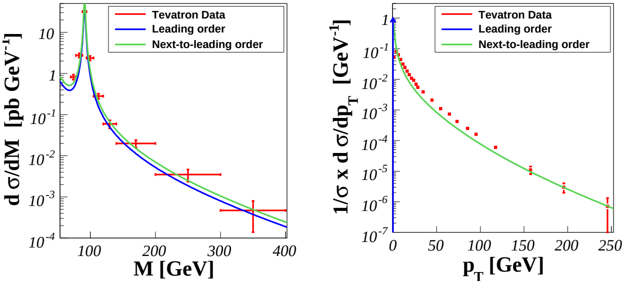
$$\alpha_s \left(\frac{\ln(1-z)}{1-z}\right)_+ \qquad \text{and} \qquad \frac{\alpha_s}{p_T^2} \ln \frac{M^2}{p_T^2} \ .$$

  * **Problems at $z \lesssim 1$ or small p_T.**

The fixed-order theory is unreliable in these kinematical regions.

The problem of the soft and collinear radiation (2).

- **Confrontation between theory and Tevatron data** [DØ collaboration (2005, 2008)].



* Invariant-mass distribution:

  ◇ Convolution with the steeply falling parton densities at large z.

  ◇ **Next-to-leading order calculation reliable**.

* p_T-distribution:

  ◇ **Next-to-leading order calculation reliable for the large p_T**.

  ◇ Behaviour in the small-p_T region: $\propto \frac{\alpha_s}{p_T^2} \ln \frac{M^2}{p_T^2}$.

Improvements.

Improvements of the next-to-leading order calculation.

- Matching with a resummation calculation.
  - * **Correct treatment** of the soft and collinear radiation.
  - * **Perturbative method**.
  - * Soft and collinear radiation taken into account to all orders.
  - * **Parton-level calculation.**
- Matching with a parton shower algorithm.
  - * **Approximation of the resummation calculation**.
  - * **Suitable for a proper description of the collision**.
  - * **Beyond the parton level**.

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Perturbative QCD in Mellin space (1).

- **Doubly-differential cross section - factorization theorem.**

$$M^2 \frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}M^2\mathrm{d}p_T^2}\left(\frac{M^2}{S_h}\right) = \sum_{ab}\int \mathrm{d}x_a\,\mathrm{d}x_b\,\mathrm{d}z \left[x_a\ f_{a/A}(x_a;\mu^2)\right]\left[x_b\ f_{b/B}(x_b;\mu^2)\right] \times \left[z\ \hat\sigma_{ab}\left(z, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\right)\ \delta\left(\frac{M^2}{S_h} - x_a x_b z\right)\right] .$$

  - We set $\mu_F = \mu_R = \mu$ to simplify.
  - S_h is the hadronic center of mass energy.
  - $\hat\sigma_{ab}$ is the **hard-scattering function**.
  - Beware: slight change of notations.

- **Mellin transformation of a function F with respect to a variable X.**

$$F(N) = \int_0^1 \mathrm{d}X\ X^{N-1} F(X) \ .$$

  - ***N is the variable conjugate to X.***

Perturbative QCD in Mellin space (2).

- **Mellin transform of the hadronic cross section with respect to M^2/S_h.**

$$M^2 \frac{\mathrm{d}^2\sigma_{AB}}{\mathrm{d}M^2 \mathrm{d}p_T^2}(N-1) = \sum_{ab} \Big[f_{a/A}(N;\mu^2)\Big] \Big[f_{b/B}(N;\mu^2)\Big] \Big[\hat{\sigma}_{ab}\Big(N, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\Big)\Big] .$$

  * The convolution is now a standard **product**.

- **Mass factorization and the partonic cross section.**

$$M^2 \frac{\mathrm{d}^2\sigma_{ab}}{\mathrm{d}M^2 \mathrm{d}p_T^2}(N-1) = \sum_{cd} \Big[\phi_{c/a}(N;\mu^2)\Big] \Big[\phi_{d/b}(N;\mu^2)\Big] \Big[\hat{\sigma}_{cd}\Big(N, M^2, \frac{M^2}{p_T^2}, \frac{M^2}{\mu^2}\Big)\Big] .$$

  * We introduce the **parton-in-parton distributions** $\phi(x,\mu)$.
    ◇ They are defined at **fixed longitudinal momentum fraction** x.
    ◇ They contain the **collinear singularities** of $\mathrm{d}^2\sigma_{ab}$ for $p_T \neq 0$.
  * The hard-scattering function $\hat{\sigma}_{cd}$ is then **infrared-safe**.
  * Factorization scheme: finite pieces of $\hat{\sigma}_{cd}$ in the densities ϕ.
  * **$\overline{\text{MS}}$-scheme: the densities are pure divergences** (up to $\ln(4\pi) - \gamma_E$).

Perturbative QCD in Mellin space (3).

- **Evolution equations** [Altarelli, Parisi (1977)].
  * The evolution of the parton-in-parton densities are governed by:

$$\frac{\partial \phi_{c/a}}{\partial \ln \mu^2}(N,\mu^2) = \sum_b P_{cb}\big(N, \alpha_s(\mu^2)\big)\ \phi_{b/a}(N,\mu^2) \ ,$$

  * We introduce the **splitting functions** P_{cb}.
  * The splitting functions can be calculated **perturbatively**.

$$P_{cb}\big(N,\alpha_s(\mu^2)\big) = \sum_{n=1}^{\infty} \alpha_s^n(\mu^2) P_{cb}^{(n)}(N) \ .$$

- **The QCD evolution operator** $E_{ab}(N,\mu^2,\mu_0^2)$, solution of

$$\frac{\partial E_{ab}}{\partial \ln \mu^2}(N,\mu^2,\mu_0^2) = \sum_c P_{ac}\big(N,\alpha_s(\mu^2)\big)\ E_{cb}(N,\mu^2,\mu_0^2) \ .$$

  * Allow to define ϕ in a **compact form**.

$$\phi_{c/a}(N,\mu^2) = \sum_b E_{cb}\ (N,\mu^2,\mu_0^2)\phi_{b/a}(N,\mu_0^2) \ .$$

  * Leading order: E_{ab} **written in an exponential form** [Furmanski, Petronzio (1982)].

Resummation philosophy (1).

- **Inputs to compute (differential) cross sections**.
  - * **Parton densities**.
    - ◇ **Fitted** from experiment.
    - ◇ **Universal**.
    - ◇ Obey to **Altarelli-Parisi equations**.
  - * **Hard-scattering function**.
    - ◇ **Perturbatively** computable.
    - ◇ **Ultraviolet divergences**: renormalization.
    - ◇ **Infrared divergences**: Bloch-Nordsieck [Bloch, Nordsieck (1937)].
- **The next-to-leading order quantity $\hat{\sigma}^{(1)}_{ab}$**.
  - * Infrared safe.
  - * **Definite logarithmic structure**: issues at phase space boundaries.
    - ◇ Each term is thus **either** soft or collinear.
    - ◇ Remark: $\mathrm{d}^2\sigma_{AB}/\mathrm{d}M^2\mathrm{d}p_T^2$ is then infrared sensitive.
    - ◇ **Can be resummed to all orders**.

Resummation philosophy (2).

- **We consider an infrared sensitive quantity R.**
  - Depends on a **hard scale M**.
  - Depends on a **scale m measuring the distance from the critical region**.
  - Contains **large ratios of scales**.
- **Resummation to all orders.**

$$R(M^2, m^2) = H\big(M^2/\mu^2\big)\ S\big(m^2/\mu^2\big)$$

  - **Separation** of the two scales $\equiv$ **refactorization**.
  - Remark: refactorization holds in conjugate spaces (*e.g.*, Mellin space).
  - S and H obey to

$$\frac{\partial H}{\partial \ln \mu^2} = -\frac{\partial S}{\partial \ln \mu^2} = \gamma_S(\mu^2) \ .$$

  - **Choice of $\mu = M$, introduction of the Sudakov form factor.**

$$\mathbf{R(M^2, m^2) = H(1) S(1)} \exp\left[- \int_{\mathbf{m^2}}^{\mathbf{M^2}} \frac{\mathrm{d}\mathbf{q^2}}{\mathbf{q^2}} \gamma_{\mathbf{S}}(\mathbf{q^2}) \right] \ .$$

  - **No large ratios of scale anymore.**

The threshold resummation formalism (1).

- **Reorganization of the $\alpha_s^n(\ln^m(1-z)/1-z)_+$ terms (with $m \leq 2n-1$).**
  * In Mellin space: $\ln^{m+1} N$.
  * **Important in the $N \to \infty$ limit.**
- **Off-diagonal splitting functions negligible.**
  * **Subdominant** in the large N limit.
  * The **p_T-integrated partonic cross section** can then be written as

$$M^2 \frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}M^2}(N-1) = \Big[\phi_{a/a}(N;\mu^2)\Big]\Big[\phi_{b/b}(N;\mu^2)\Big]\Big[\hat\sigma_{ab}\Big(N, M^2, \frac{M^2}{\mu^2}\Big)\Big] + \mathcal{O}\Big(\frac{1}{N}\Big) .$$

- **Refactorization** [Sterman (1987)].

$$M^2 \frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}M^2}(N-1) = \Big[\psi_{a/a}(N;M^2)\Big]\Big[\psi_{b/b}(N;M^2)\Big] S_{ab}\Big(N, \frac{M^2}{\mu^2}\Big) \times \Big[H_{ab}\Big(M^2, \frac{M^2}{\mu^2}\Big)\Big] + \mathcal{O}\Big(\frac{1}{N}\Big) .$$

  * Factorization of the N-dependence.

The threshold resummation formalism (2).

- **Refactorization** [Sterman (1987)].

$$M^2 \frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}M^2}(N-1) = \Big[\psi_{a/a}(N;M^2)\Big]\Big[\psi_{b/b}(N;M^2)\Big]\, S_{ab}\Big(N,\frac{M^2}{\mu^2}\Big) \times \Big[H_{ab}\Big(M^2,\frac{M^2}{\mu^2}\Big)\Big] + \mathcal{O}\Big(\frac{1}{N}\Big)\ .$$

  * H_{ab} organizes infrared-safe coefficients (independent of N).
    - **Perturbatively** computable.
  * The parton-in-parton distributions $\psi_{i/i}$.
    - Dependence on the **energy** (not on the momentum) fraction.
    - **Satisfy**

$$\frac{\partial \psi_{i/i}}{\partial \ln \mu^2}(N,\mu^2) = \gamma_i\big(\alpha_s(\mu^2)\big)\ \psi_{i/i}(N,\mu^2)\ ,$$

    - $\gamma_i \equiv N$-independent parts of the splitting functions.
    - **Perturbatively** computable.
  * Large-angle soft gluon emission is included in S_{ab}.
    - Computable in the **eikonal approximation**.

The threshold resummation formalism (3).

- **Mass factorization.**

$$\hat{\sigma}_{ab}\Big(N, M^2, \frac{M^2}{\mu^2}\Big) = H_{ab}\Big(M^2, \frac{M^2}{\mu^2}\Big) \times \frac{\psi_{a/a}(N; M^2)\psi_{b/b}(N; M^2)}{\phi_{a/a}(N; \mu^2)\phi_{b/b}(N; \mu^2)} S_{ab}(N, \frac{M^2}{\mu^2}) + \mathcal{O}\Big(\frac{1}{N}\Big) \ .$$

  * Evolution of the parton densities: **can be solved in the large N limit**.
  * The eikonal function S_{ab} **exponentiates** [Gatheral (1983)].

- **Exponentiation.**

$$\hat{\sigma}_{ab}\Big(N, M^2, \frac{M^2}{\mu^2}\Big) = H_{ab}\Big(M^2, \frac{M^2}{\mu^2}\Big) \exp\Big[G_{ab}(N, M^2, \frac{M^2}{\mu^2})\Big] + \mathcal{O}\Big(\frac{1}{N}\Big) \ .$$

  * The G_{ab}-function can be rewritten in terms of **radiation factors**.

$$G_{ab} = \ln \Delta_a + \ln \Delta_b + \ln \Delta_{ab} \ .$$

The threshold resummation formalism (4).

- **The radiation factors are integrals of the running coupling constant.**

$$\ln \Delta_i\Big(N, M^2, \frac{M^2}{\mu^2}\Big) = \int_0^1 \mathrm{d}z \frac{z^{N-1}-1}{1-z} \int_{\mu^2}^{(1-z)^2 M^2} \frac{\mathrm{d}q^2}{q^2} A_i(\alpha_s(q^2)) \ ,$$

$$\ln \Delta_{ab}\Big(N, M^2, \frac{M^2}{\mu^2}\Big) = \int_0^1 \mathrm{d}z \frac{z^{N-1}-1}{1-z} D_{ab}\Big(\alpha_s((1-z)^2 M^2)\Big) \ .$$

  * The function A_i collects **soft and collinear radiation**.
  * The function D_{ab} collects **large-angle soft-radiation**.
  * All functions can be computed **perturbatively**.

- **Final resummation formula.**

$$\hat\sigma_{ab}\Big(N, M^2, \frac{M^2}{\mu^2}\Big) = \mathcal{H}_{ab}\Big(M^2, \frac{M^2}{\mu^2}\Big) \exp\Big[\mathcal{G}_{ab}\Big(N, M^2, \frac{M^2}{\mu^2}\Big)\Big] + \mathcal{O}\Big(\frac{1}{N}\Big) \ .$$

  * **All non-logarithmic pieces have been absorbed in the hard function $\mathcal{H}$.**
  * **Improvement 1**: inclusion (and exponentiation) of terms in $1/N$ in the diagonal splitting functions [Krämer, Laenen, Spira (1998)].
  * **Improvement 2**: inclusion (and exponentiation) of terms in $1/N$ in the non-diagonal splitting functions [Kulesza, Sterman, Vogelsang (2002)].

Three resummation formalisms.

- **Based on similar factorization properties**.
- ***p_T*-resummation** [Catani, de Florian, Grazzini (2001); Bozzi, Catani, de Florian, Grazzini (2006)]
  - * Universal formalism ≡ process-independent Sudakov form factor.
  - * Resums $\frac{\alpha_s}{p_T^2} \ln \frac{M^2}{p_T^2}$.
- **Threshold resummation** [Sterman (1987); Catani, Trentadue (1989,1991)]
  - * Resums $\left(\frac{\ln(1-z)}{1-z}\right)_+$.
- **Joint resummation** [Bozzi, BenjF, Klasen (2008)]
  - * Universal formalism ≡ process-independent Sudakov form factor.
  - * Resums both types of logarithms.

The resummed component (1).

- **Based on factorization properties**.
  - **Holds in non-physical conjugate spaces**.
  - Mellin N-space (N conjugate to M^2/S_h).
  - Impact parameter b (conjugate to p_T) $\leftrightarrow$ joint/p_T resummation.

$$\mathrm{d}\sigma_{AB}^{(\mathrm{res})}(N-1,b) = \sum_{a,b} f_{a/A}(N)\, f_{b/B}(N)\, \mathcal{W}_{ab}(N,b),$$

$$\mathcal{W}_{ab}(N,b) = \mathcal{H}_{ab} \exp\Big\{\mathcal{G}_{ab}(N,b)\Big\}.$$

- The $\mathcal{H}$-coefficient:
  - Contains **real and virtual collinear radiation, hard contributions**.
- The Sudakov form factor $\mathcal{G}$:
  - Contains **the soft-collinear radiation**.

The resummed component (2).

$$\mathcal{W}_{ab}(N,b) = \mathcal{H}_{ab} \exp\Big\{\mathcal{G}_{ab}(N,b)\Big\}.$$

- The $\mathcal{H}$-coefficient:
  - * Contains **real and virtual collinear radiation, hard contributions**.
  - * **Can be computed perturbatively as series in α_s**, from fixed-order results.
  - * Is process-dependent.
- The Sudakov form factor $\mathcal{G}_{ab}$:
  - * Contains **the soft-collinear radiation**.
  - * **Can be computed perturbatively as series in α_s log.**
  - * Is process-independent (universal).
  - * Contains the full color and spin structure.

Matching to the fixed order (1).

- **Fixed-order calculations**.
  - * **Reliable far from the critical kinematical regions**.
  - * **Spoiled in the critical regions**.
- **Resummation**.
  - * **Needed in the critical regions.**
  - * **Not justified far from the critical regions.**
- **Intermediate kinematical regions:**
  - * Both fixed order and resummation contribute.

Information from both fixed order and resummation is required.
$\Rightarrow$ consistent matching procedure.

Matching to the fixed order (2).

- **Matching procedure:**
  - Addition of both resummation and fixed-order results.
  - Subtracting the **expansion** in α_s of the resummed result.
  - No double-counting of the logarithms.

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(\mathrm{F.O.})} + \mathrm{d}\sigma^{(\mathrm{res})} - \mathrm{d}\sigma^{(\mathrm{exp})}.$$

- **Effects of the matching procedure**:
  - Far from the critical regions, $\mathrm{d}\sigma^{(\mathrm{res})} \approx \mathrm{d}\sigma^{(\mathrm{exp})}$ $\equiv$ **perturbative theory.**
  - In the critical regions, $\mathrm{d}\sigma^{(\mathrm{F.O.})} \approx \mathrm{d}\sigma^{(\mathrm{exp})}$ $\equiv$ **pure resummation.**
  - In the intermediate regions: **both contribute.**

Parton showers (1).

- **The parton splitting factorizes $\Rightarrow$ iterative splitting.**

$$
\begin{aligned}
a(t) &\to b(z) + c \\
&\qquad b(t') \to d(z') + e \\
&\qquad\qquad d(t'') \to \ldots
\end{aligned}
$$

where t is the ordering variable and z the momentum fraction.

- **Ingredient 1: Altarelli-Parisi splitting kernels $P_{ba}(z)$.**
- **Ingredient 2: No emission probability - the Sudakov form factor.**
  * **Conservation of probability** for the branching of a parton:

$$
\begin{aligned}
1 &= P_{\rm no\ emis}(t+{\rm d}t, t) + P_{\rm emis}(t+{\rm d}t, t) \\
&= P_{\rm no\ emis}(t+{\rm d}t, t) + \frac{{\rm d}t}{t} \sum_b \int {\rm d}z \frac{\alpha_s(t)}{4\pi} P_{ab}(z) \ .
\end{aligned}
$$

  * Solving the equation defines the **Sudakov form factor**,

$$
\boldsymbol{\Delta}(\mathbf{t_1}, \mathbf{t_2}) = P_{\rm no\ emis}(t_1, t_2) = \exp\left[- \int_{t_1}^{t_2} \frac{{\rm d}t}{t} \sum_b \int {\rm d}z \frac{\alpha_s}{4\pi} P_{ba}(z) \right] \ .
$$

Parton showers (2).

Evolution equation for the parton a to the cut-off scale t_0.

$$\phi_a(t,E) = \Delta_a(t,t_0) + \sum_b \int_{t_0}^{t} \frac{\alpha_s(t')}{4\pi}\frac{\mathrm{d}t'}{t'}\mathrm{d}z\,\Delta_a(t,t')\,P_{ab}(z)\,\phi_b\big(t',zE\big)\,\phi_c\big(t',(1-z)E\big)$$

- **Derivation of a parton shower algorithm.**
  - **Ordered Markov chain** (t-variable).

$$Q_0^2 \ll t_1 \ll t_2 \ll \ldots \ll t_N \ll Q^2$$

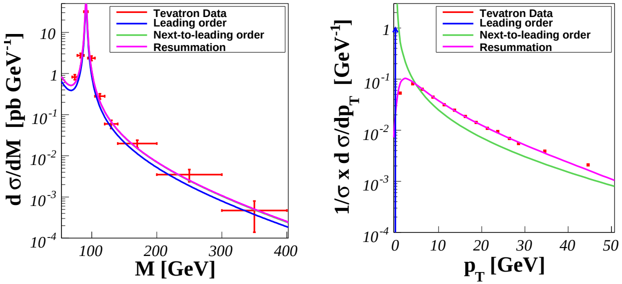
  - Choice of t: different shower algorithms.
- **Limitations.**
  - **Leading logarithms,**
  - **Large number of colors,**
  - **Collinear and/or soft-collinear radiation.**
- **Improvements require matrix exponentiation $\Rightarrow$ soft-gluon resummation.**

Outline.

1. Motivation for precision calculations.
2. Resummation and parton showering.
3. Numerical results, including uncertainties, for Z' and supersymmetry at the LHC.
4. Summary - conclusions.

Resummation vs. Tevatron data.

- **Confrontation between theory and Tevatron data** [D∅ collaboration (2005, 2008)].



* Invariant-mass distribution: **good agreement**.
(no change with respect to next-to-leading order).

* p_T-distribution: **good agreement**.
(big improvement with respect to next-to-leading order).

Grand Unified Theories and Z' bosons.

- **Grand Unified Theories: generalities.**
  - * **Unification** of the Standard Model gauge groups:

$$\mathbf{G} \supset \mathbf{SU(3)_C} \times \mathbf{SU(2)_L} \times \mathbf{U(1)_Y}.$$

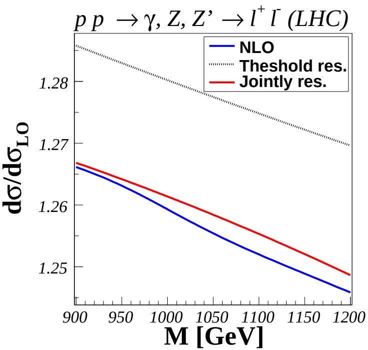
  - * **Breaking** to the SM at high energy scale:
    - ◇ Possible appearance of **additional $U(1)$ symmetries**.
    - ◇ **Extra neutral gauge bosons Z'**.

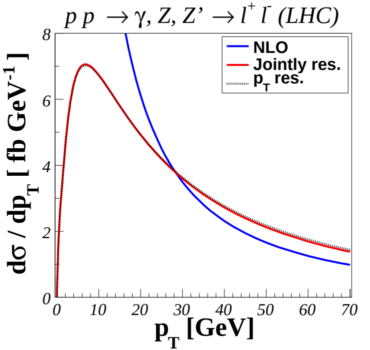
- **Considered theoretical model** [Green, Schwarz (1984); Hewett, Rizzo (1989)].
  - * Ten-dimensional string theories $E_8 \times E_8$:
    - ◇ **Anomaly-free and contains chiral fermions**.
    - ◇ Compactified to E_6.
  - * **Breaking to the SM gauge groups**

$$\begin{aligned} E_6 &\to SO(10) \times \mathbf{U(1)}_\psi \\ &\to SU(5) \times \mathbf{U(1)}_\chi \times \mathbf{U(1)}_\psi \\ &\to \mathbf{SU(3)_C} \times \mathbf{SU(2)_L} \times \mathbf{U(1)_Y} \times \mathbf{U(1)}_\chi \times \mathbf{U(1)}_\psi . \end{aligned}$$

Additional bosons Z_ψ and $Z' \equiv Z_\chi$.

Resummation for Z' production at the LHC.

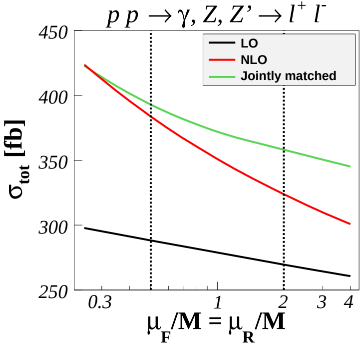




[BenjF, Klasen, Ledroit, Li, Morel (2008)]

- **Scenario**: production of a Z' of 1 TeV at the LHC, at 14 TeV.
- **Mass-spectrum normalized to leading order.**
  - * Resummation/NLO: **additional increase of the K-factor** (few percents).
  - * Resummation **effects reduced due to parton densities**.
  - * Resummation formalism choice: small uncertainties (few percents).
- **Transverse-momentum distribution:**
  - * Resummation/NLO: **finite results at small p_T**; peak around 10 GeV.
  - * Good agreement between the two resummation formalisms.

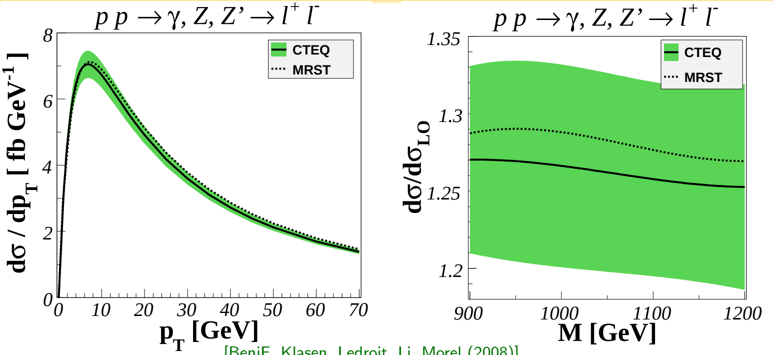
Uncertainties: scale variations.



[BenjF, Klasen, Ledroit, Li, Morel (2008)]

- **Scenario**: production of a Z' of 1 TeV at the LHC, at 14 TeV.
- **Total cross section** (900 GeV $\leq M \leq$ 1200 GeV).
  - * Leading order: full dependence related to μ_F ($\sim$ 7%).
  - * Next-to-leading order: introduction of μ_R and the qg channel ($\sim$ 17%).
  - * **Resummation: reduction of scale dependence ($\sim$ 9%).**

Uncertainties: parton densities.



[BenjF, Klasen, Ledroit, Li, Morel (2008)]

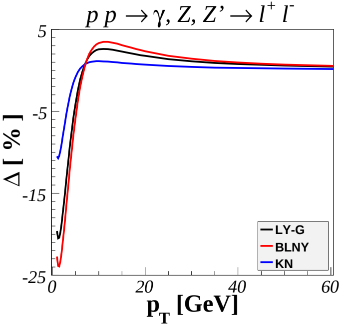
- **Scenario**: production of a Z' of 1 TeV at the LHC, at 14 TeV.
- **CTEQ vs. MRST.**
  - p_T-spectrum: similar shapes but a bit **harder for MRST**.
  - Mass-spectrum: different shapes.
- **Variations along 20 directions for the CTEQ densities**.
  - Variations along the PDF fits: **modest uncertainties** ($\sim$ 10%).
  - Similar to scale dependence.

Non-perturbative effects.

- **Important non-perturbative effects in the p_T-distributions.**
  * Intrinsic p_T of the partons inside the hadrons.
  * **Modification of the Sudakov form factor**,

$$\mathcal{G}(N,b) \to \mathcal{G}(N,b) + F^{\rm NP}_{ab}.$$

- **Form factors** [Ladinsky, Yuan (94); Landry, Brock, Nadolsky, Yuan (03); Konyshev, Nadolsky (06)].
  * **Obtained from experimental data (fits) and assumed universal.**



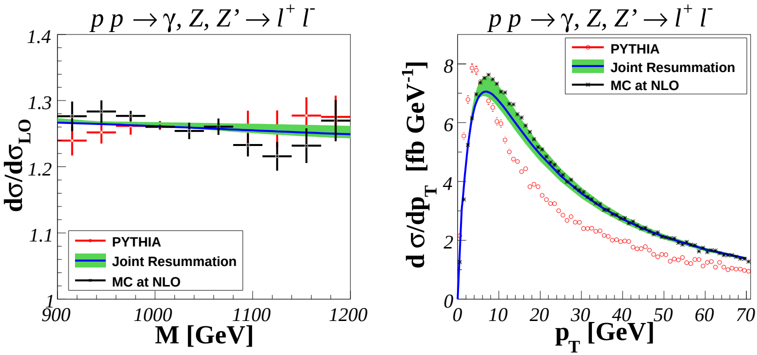
[BenjF, Klasen, Ledroit, Li, Morel (2008)]

- **Non-perturbative effects under good control for $p_T > 5$ GeV.**

Monte Carlo and resummation for BSM processes.

- **Soft and collinear radiation ≡ Sudakov form factor**.
  - * Parton showers in general: leading logarithms, color,...
  - * Momentum conservation at each branching: **(leading logs)**$_+$, e.g. PYTHIA.
  - * **Resummation: next-to-leading logarithms.**
- **Matched with matrix elements**.
  - * Monte Carlo codes in general: leading order.
  - * **Sometimes next-to-leading order**: e.g. MC@NLO and POWHEG.
  - * **Resummation: next-to-leading order.**
- **Comparison: resummation vs. PYTHIA vs. MC@NLO.**
  - * **PYTHIA**: virtuality-ordered showers; nice process library.
  - * **MC@NLO**: angular-ordered showers; precision MC generator.
  - * **Resummation**: best precision.

Comparison: PYTHIA, MC@NLO and joint resummation.



[BenjF, Klasen, Ledroit, Li, Morel (2008)]

- 1 TeV Z'; PYTHIA (LO/LL$_+$), MC@NLO (NLO/LL), resummation (NLO/NLL).
- **Mass-spectrum normalized to leading order:**
  - PYTHIA (*power shower*): mass-spectrum multiplied by a K-factor of 1.26.
  - **Good agreement between MC@NLO and resummation**.
- **Transverse-momentum distribution:**
  - **PYTHIA spectrum much too soft, peak not well predicted**.
  - **Good agreement between MC@NLO and resummation**.

The Minimal Supersymmetric Standard Model (MSSM).

- High energy extension to Standard Model.
- Symmetry between fermions and bosons.

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle$$
$$Q|\text{Fermion}\rangle = |\text{Boson}\rangle \quad \text{where } Q \text{ is a SUSY generator.}$$

- The MSSM: **one** single supersymmetric (SUSY) generator Q.

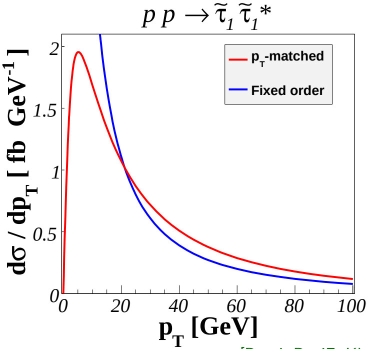
The MSSM: one SUSY partner for each SM particle.

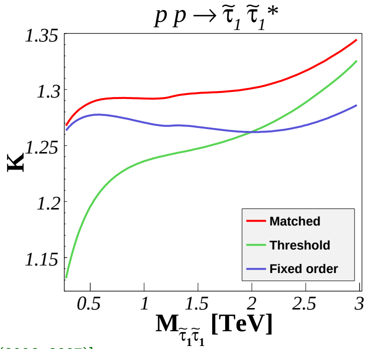
* Quarks ⇔ squarks.
* Leptons ⇔ sleptons.
* Gauge/Higgs bosons ⇔ gauginos/higgsinos ⇔ charginos/neutralinos.
* Gluon ⇔ gluino.

Some features of the MSSM.

- **Introduction of the SUSY particles in the theory**.
  - * **Solution to the hierarchy problem** (stabilization of the Higgs mass).
  - * **Gauge coupling unification** at high energy.
  - * **Dark matter candidate** $\Leftrightarrow$ lightest SUSY particle stable and neutral.
- **No SUSY discovery until now!**
  - * **SUSY must be broken.**
  - * SUSY masses at a higher scale than Standard Model (SM) masses.
  - * More than **100 new free parameters**.
  - * Simplified benchmark scenarios:
    - ◇ Minimal supergravity (mSUGRA).
    - ◇ Gauge-mediated SUSY-breaking (GMSB).
    - ◇ ...

Resummation for slepton pair production at the LHC.

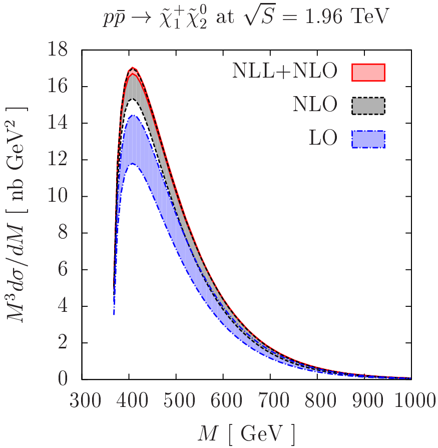




[Bozzi, BenjF, Klasen (2006, 2007)]

- **SUSY scenario**: slepton masses ≈ 100-200 GeV.
- **Resummation effects**:
  - * **Finite results at small p_T.**
  - * Matching: **important effects at intermediate p_T.**
  - * Small M: $\mathrm{d}\sigma^{(\mathrm{res})} \approx \mathrm{d}\sigma^{(\mathrm{exp})} \equiv$ **perturbative theory**.
  - * Large M: $\mathrm{d}\sigma^{(\mathrm{F.O.})} \approx \mathrm{d}\sigma^{(\mathrm{exp})} \equiv$ **pure resummation**.

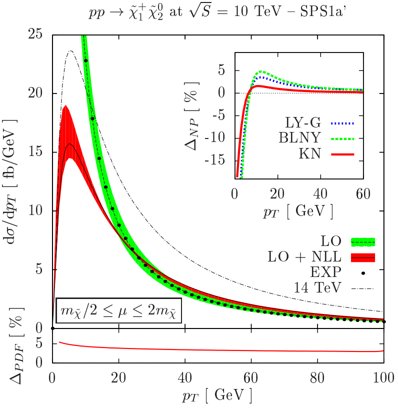
Uncertainties: chargino-neutralino associated production.



[Debove, BenjF, Klasen (2011)]

- **Scenario**.
  - * SPS1a'.
  - * $\approx$ 180 GeV gauginos.
  - * Tevatron collider (1.96 TeV).
- **Invariant-mass spectrum**
  - * **NLO**: 20-25% increase.
  - * **Resummation**: moderate increase.
- **Scale dependence** ($M/2 \le \mu_R = \mu_F \le 2M$).
  - * **NLO: Large dependence**.
  - * **Resummation: reduced dependence**.

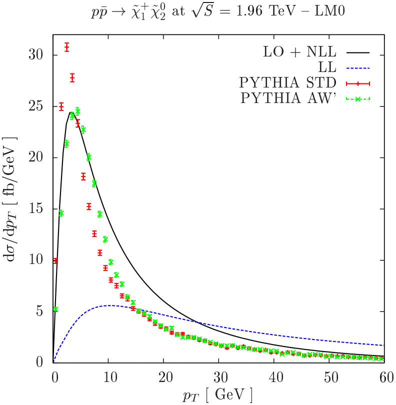
Uncertainties: chargino-neutralino associated production.



[Debove, BenjF, Klasen (2010)]

- **Scenario**.
  - * $\approx$ 180 GeV gauginos.
  - * LHC collider (10 TeV & 14 TeV).
- p_T**-spectrum**
  - * **Next-to-leading** logarithms.
  - * $\mathcal{O}(\alpha_s)$ fixed-order.
  - * **Small** p_T: expansion $\approx$ fixed-order.
  - * **Large** p_T: expansion $\approx$ resummation.
  - * **Intermediate** p_T: enhancement.
- **Scale dependence** ($M/2 \leq \mu_R = \mu_F \leq 2M$).
  - * **Reduction** of the uncertainties.
  - * Less than **5%** for $p_T > 5$ GeV.
- **Parton densities dependence** (44 CTEQ sets).
  - * **4-5%** uncertainties for all p_T.
  - * Similar to **weak boson** production.
- **Non perturbative effects at low** p_T**.**
  - * **Under control** for $p_T > 5$ GeV.
- **Uncertainties under control for** $p_T > 5$ **GeV.**

Comparison: PYTHIA and p_T-resummation.



[Debove, BenjF, Klasen (2010)]

- **Scenario**.
  - * ≈ 110 GeV gauginos.
  - * Tevatron collider.
- PYTHIA **predictions**.
  - * Used for SUSY **experimental analyses**.
  - * **Leading log** Sudakov form factor.
  - * **Two tunes**.
    - ◇ CDF-AW.
    - ◇ Our tune AW'.
- **Two set of resummed predictions.**
  - * **Leading logaritmic** approximation.
  - * **Next-to-leading logaritmic** results.
- PYTHIA **results**.
  - * **Improves** the LL picture.
  - * **Intrinsic p_T** helps to reproduce NLL.
  - * **Underestimation** for intermediate p_T.
  - * **Direct impact for experimental analyses**.

Outline.

Summary - conclusions.

- **Soft and collinear radiation:**
  - * Large logarithmic corrections in p_T- and invariant-mass spectra.
  - * **Need for resummation (or parton showers)**.
- **p_T, threshold and joint resummations have been implemented.**
  - * Reliable perturbative results.
  - * Correct quantification of the soft-collinear radiation.
  - * **Important effects**, even far from the critical regions.
  - * **Uncertainties from scales and parton densities under good control**.
  - * **Reduced dependence on non-perturbative effects.**
- **Comparison with Monte Carlo generators**
  - * **Significant shortcomings in normalization and shapes for PYTHIA**.
  - * **MC@NLO reaches (almost) the same precision level as resummation.**
    BUT: easier implentation in the analysis chains of any experiment.